Timelapse ERT inversion approaches and their applications

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Introduction

Aim of timelapse ERT is to image resistivity changes in the subsurface in such that plausible timedependent models fit the data. Since plausibility involves temporal changes, specific inversion and regularization schemes are to be found, particularly if quantitatively reliable measures as absolute water content or ground water velocity are of interest. ERT Inversion is usually ill-posed and non-unique and needs further restrictions. The question is how we can do it best?

There is a number of factors that influence the imaging and thus determine the applicability and performance of the existing approaches, i.e. (1) contrast and heterogeneity of background model, (2) shape and contrast of the changes, (3) reproducibility of electrode positions and arrays, (4) error structure overall and correlation between frames, (5) velocity of ongoing processes, and (6) target values, whether absolute or relative changes of ρ or secondary parameters (e.g. moisture content θ) are regarded.

Generally, five main types of minimization can be distinguished:

- individual inversion of single time frames: $\boldsymbol{d^n} = \{log\ \rho_a(t = t_n)\} \rightarrow \boldsymbol{m}^n = \rho^n$
- inversion of data ratios (Schütze et al., 2002): $\boldsymbol{d}^n = \rho_a^n / \rho_a^0 \rightarrow \boldsymbol{m}^n = \{\rho^n / \rho^0\}$
- inversion with $\mathbf{m}^0$ as reference constraining $\mathbf{m}^n$-$\mathbf{m}^0$ (or alternatively $\mathbf{m}^n - \mathbf{m}^{n-1}$)
- the so-called difference inversion after LABRECQUE and YANG (2001), which additionally corrects the misfit at t_0 so that $\boldsymbol{d}^n = \{\rho_a^n / \rho_a^0\} \boldsymbol{f}(\boldsymbol{m}^0)$ (or n-1 instead of 0).
- fully discretized (4D ERT) with constraints in space and time.

Furthermore there exist different regularization schemes for the absolute models or the model differences (in the usual logarithmic domain the ratio). Mostly smoothness constrains approximating a first or second order derivative are used, sometimes with direction-dependent penalties. Alternatively, minimum length, i.e. the total deviation independent on model cell neighbourhood can be used, or any combination of them.

Synthetic 1D experiments

In order to systematically investigate the role of different inversion and regularization techniques, we simulate synthetic time-lapse experiments with each two time-steps (frames). Therefore a discretisation in time does not have to be treated separately. To keep it simple, a 1d resistivity structure is considered, but it is inverted using a fixed discretisation as in 2d or 3d. A Schlumberger depth sounding is assumed, data are contaminated with 1% correlated and 1% uncorrelated noise. Regularization strength is varied iteratively such that the data are fitted within noise ($\chi^2 = 1$). Unless stated otherwise, smoothness of 1st order is used for both inversion of background and time-lapse data.

First scenario is a shallow infiltration, i.e. a decrease of resistivity by factor 2 in the very first layer of a three-layer model describing a profile of soil, vadose zone and aquifer.

The sounding curve (Fig. 2) of the second frame is lower but the maximum is down-shifted, which leads to an increase of apparent resistivity for deeper penetrations. The latter is known leading to artifacts of increasing resistivity at depth as reported by DESCLOITRES et al. (2003).
The absolute resistivities (Fig. 2 left) are all similar and show mainly the three-layer case in a smooth representation for both frames. Consequently, the ratio images are almost identical in showing a slight increase at medium depths, except the ratio inversion, which shows a strong increase at large depths but also bad values for intermediate depths (Fig. 2 right). Reason is the disregarded deviation in the sensitivity function, which is affected by the background resistivity. In the next scenario, the described infiltration front is moving down, i.e. only changing its layer thickness, which is represented by a thin decrease of factor 3 at 0.5 – 1 m depth. Again, the absolute resistivities (Fig. 3 left) describe the smooth background model. However the ratio curves all show the expected decrease. For ratio inversion it is too shallow and smoothed, similar to the difference inversion. Both independent and reference inversion exhibit a sharper image of the change, but produce severe artifacts at greater depths. In contrast to the last examples the ratio inversion shows least artifacts.

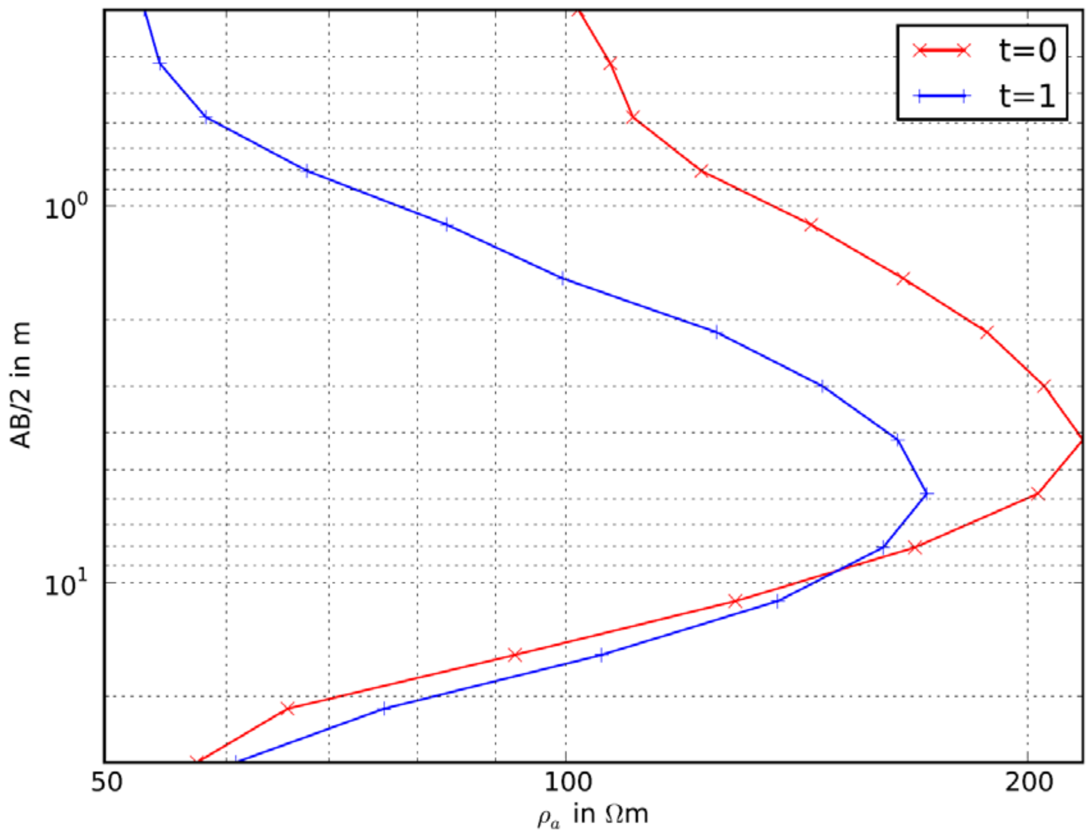

Fig. 1: Apparent resistivity curves of the two time steps for a shallow infiltration front.

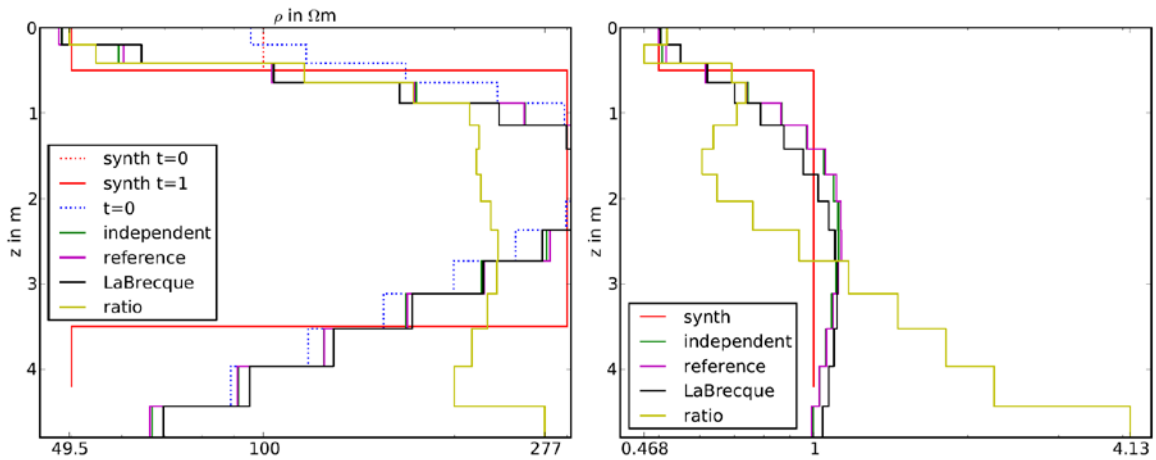

Fig. 2: Absolute resistivity (left) and relative changes (right) of different time-lapse strategies for a shallow infiltration front.

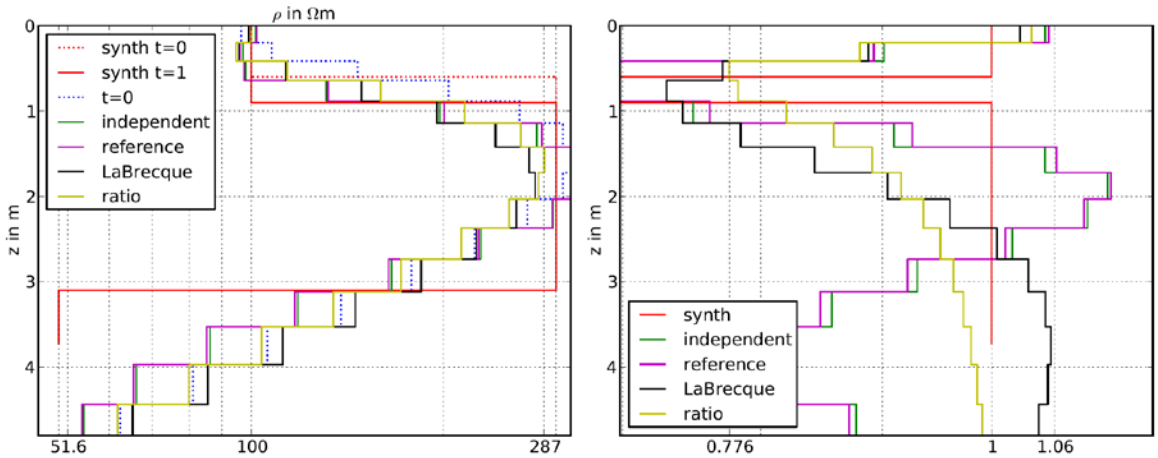

Fig. 3: Absolute resistivity (left) and relative changes (right) for the infiltration front moving down.

In the next example we consider a conductive tracer injection at a certain depth for the same three-layer case. The synthetic ratio is similar but inside the second layer and with a stronger contrast of 20. Therefore absolute and relative resistivity are similar. All methods are able to see the decrease but smoothed and too deep. At least the ratio inversion sees a sharper image.

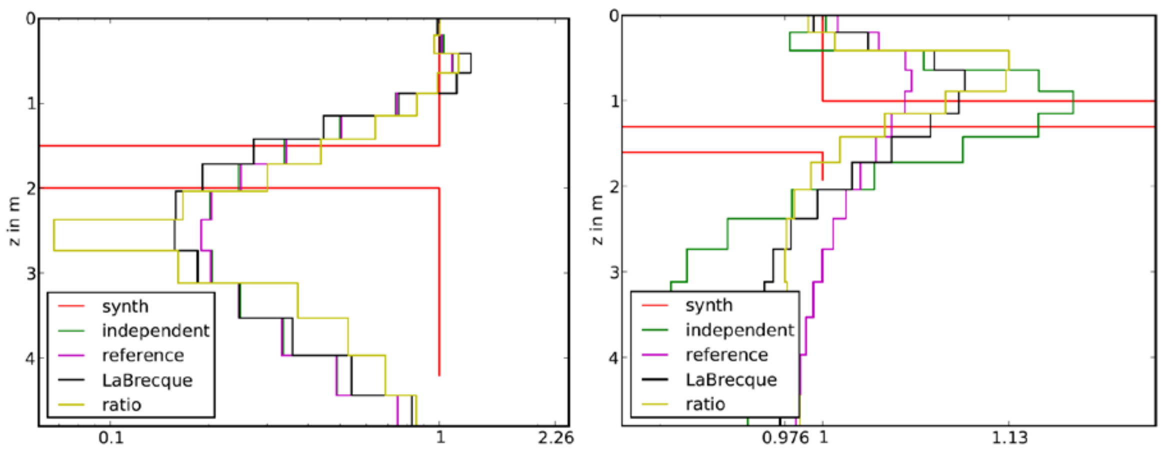

Fig. 4: Relative changes of a synthetic tracer injection (left) and movement (right).

Further, an already injected tracer is moving down, leaving a combined increase/decrease pattern. All methods can see the increase, but are shallow. The following decrease is observed and much too deep. Amazingly, the independent inversion yields the most realistic curve.
The difference inversion seems to be the most stable approach concerning imaging properties and artifacts, particularly if the amount of correlated noise is increased. However, if we use uncorrelated noise only, the method becomes very similar to reference inversion, but with higher smoothness due to the superposition of noise from both time steps.
Additionally to inversion techniques, we want to investigate how different regularization techniques affect the results of difference/reference inversion. The first time step is processed with smoothness of first order, whereas for the changes zero, first, second and combinations of zeroth with first and second order are considered (Fig. 5).

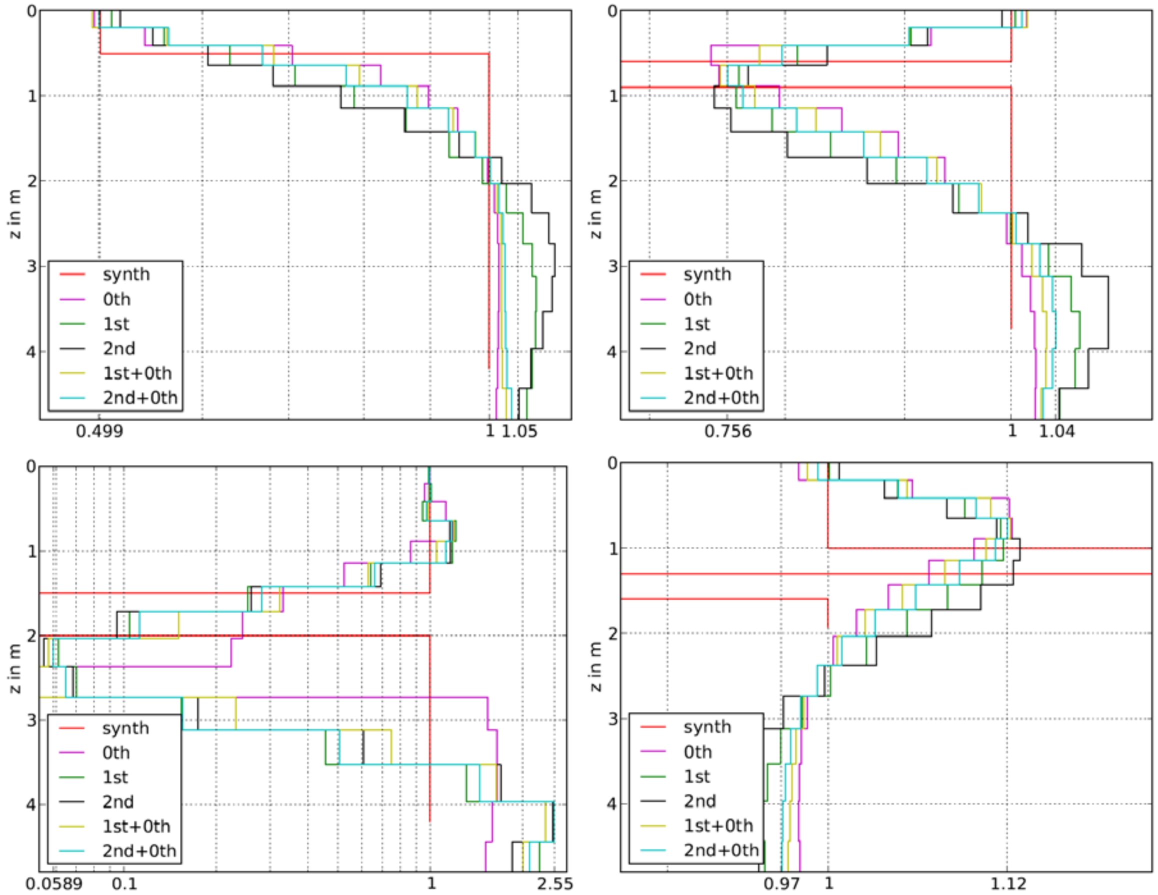

Fig. 5: Resistivity ratios of different time-lapse regularizations for shallow water (top) and tracer (bottom) infiltration (left) and movement (right).

For the shallow infiltration example we see that 0th order alone or in combination does not lead to artificial increase due to smoothness. This holds also for the second example (moving infiltration), where 0th order is showing the change too shallow and 2nd order too deep. A combination of 0th and 1st (or 2nd) order performs best.
In example three (tracer injection) 0th order is too deep, although with least artifacts. The others are similar, but 0th + 1st order performs best. When the tracer moves down, 0th order is too shallow again and 2nd order shows the best agreement, although overly smoothing at depth.

Real data

We tested the methods for a tracer injection experiment presented by KURAS et al. (2009), a cross-hole ERT data set using 9 boreholes with 10 cm spaced electrodes. The BERT algorithm after GÜNTHER et al. (2006) with a regular discretisation of rectangular cells (5 cm x 5 cm) was used. Figure 6 shows the resistivity ratios for two time steps (4 hours and 12 hours) using different inversion and regularization techniques. Generally, the conducting tracer can be seen by all methods but with different imaging properties.

Individual inversion (first row) yields strong artifacts at the borehole electrodes, probably due to systematic error sources such as positioning inaccuracies. This effect was similarly observed in other reference inversions and therefore the misfit removal after LABRECQUE and YANG (2001) was used for the following inversions. Constraining the models to the predecessor (second row) leads to decreases followed by increases due to combination of two ratios.

From the inversions with regularization orders 0, 1 and 2 (lines 3-5) the classical smoothness constraints (1st/2nd with slightly decreased vertical weights) exhibit the largest effects, but also the largest artifacts above and below the tracer. Minimum-length regularization of the model difference does not show such artifacts but increases at the electrodes interrupting the tracer shape. If (isotropic) smoothness and pure deviation are combined (last line), the least artifacts are observed, but the shape of the tracer remains interrupted. Further tuning may lead to even nicer images, however it is not clear beforehand which method is best and how reality looks like.

Conclusions

There is a huge number of different timelapse approaches and options concerning minimization methods and regularization types. All schemes are generally similar, especially for small contrasts, but can produce significant artifacts, particularly when resistivity ratios are of interests. The best method is not clear beforehand and depends on the background model, shape and contrast of the changes, but also on noise conditions.

Difference inversion (LaBrecque's method) turns out to be a safe choice for all considered models but could decrease resolution in case of negligible systematic errors. The reference model inversion is most general and works with arbitrary measuring sequences and even electrode positions for the different time-steps. Ratio inversion achieved most contrasted models but can yield wrong depths and resistivities due to wrong sensitivity. Therefore it has to be treated with care and should only be used for quasi-homogeneous $\mathbf{m}^0$.

Different regularization schemes applying to the model differences can have significant impact on the results. After our experience, a mix of first order smoothness constraints and simple damping produces least artifacts. Movements of small units should not be constrained to each other (as in 4D approaches) but to a background model. In all cases, only with a good m0 model successfull time-lapse ERT can be performed. Synthetic studies should be carried out before the measurement and help interpreting the solutions by understanding the nature of imaging.

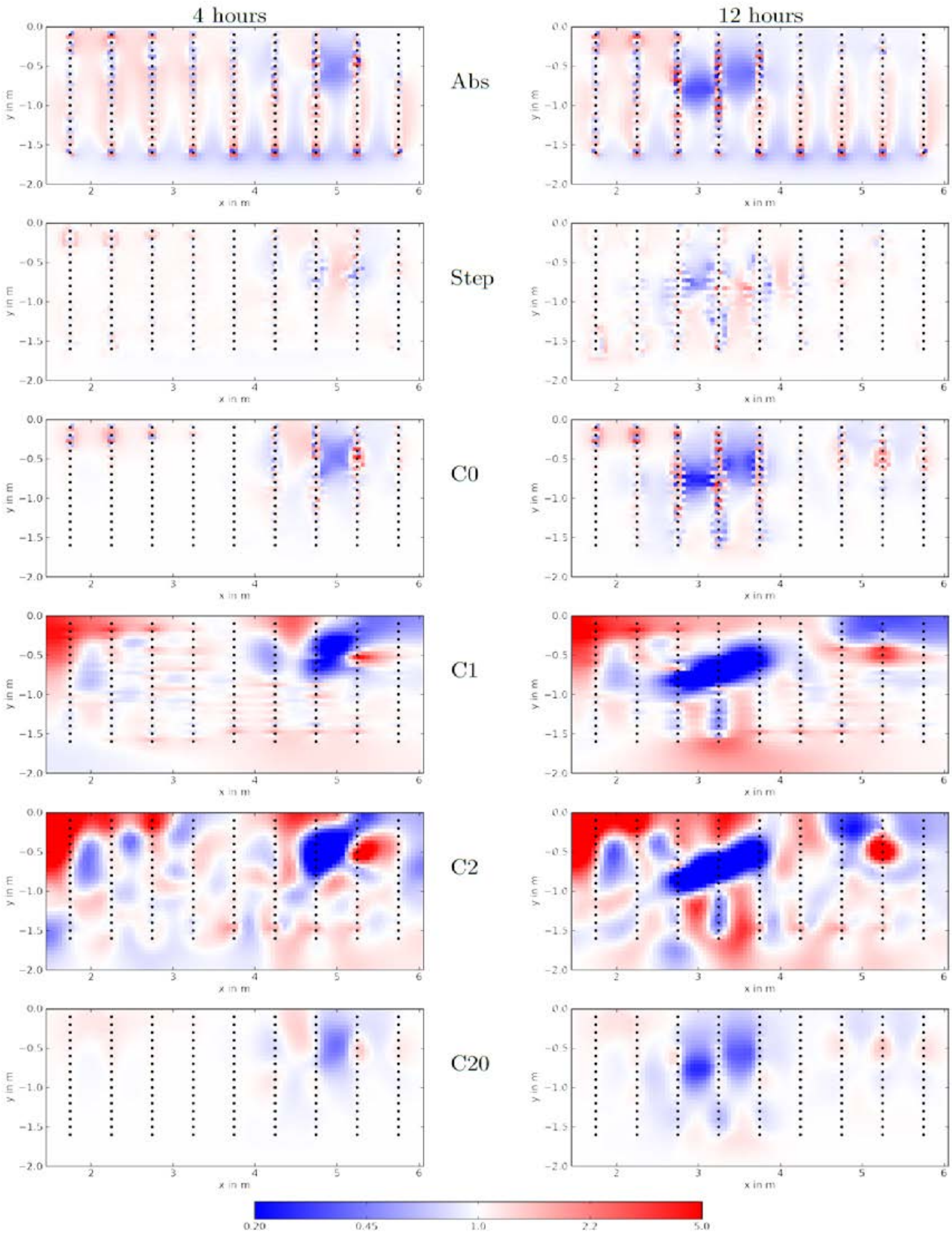

Fig. 6: Resistivity ratio for timesteps 4 hours (left) and 12 hours (right) and inversion schemes: individual inversion, step-wise constrained and difference inversion using constraint orders 0, 1, 2 and 0+2.

References

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