Nonlin. Processes Geophys. Discuss., 1, 999–1021, 2014
www.nonlin-processes-geophys-discuss.net/1/999/2014/
doi:10.5194/npgd-1-999-2014

Nonlinear Processes in Geophysics
Discussions
Open Access

Estimation of flow velocity for a debris flow via the two-phase fluid model

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Received: 10 April 2014 – Accepted: 28 May 2014 – Published: 19 June 2014

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Published by Copernicus Publications on behalf of the European Geosciences Union & the American Geophysical Union.

Abstract

The two-phase fluid model is applied in this study to calculate the steady velocity of a debris flow along a channel bed. By using the momentum equations of the solid and liquid phases in the debris flow together with an empirical formula to describe the interaction between two phases, the steady velocities of the solid and liquid phases are obtained theoretically. The comparison of those velocities obtained by the proposed method with the observed velocities of two real-world debris flows shows that the proposed method can estimate accurately the velocity for a debris flow.

1 Introduction

A debris flow is the gravity flow of soil, rock and/or water mixtures, which is frequently initiated by a landslide, and is one of the common potential hazards throughout the world. For examples, the debris flow in Zhouqu (China) on 8 August 2010 killed about 1700 people, and the debris flow that occurred in Afghanistan on 30 April 2014 killed more than 2000 people. Debris flows can often occur following brush and forest fires. They pose a significant hazard in lots of mountainous, steep areas, and have received particular attention in China, Japan, the USA, Canada, New Zealand, the Philippines, the European Alps, Kazakhstan and Russia.

The typical characteristics of the multi-phase fluid exhibited by a debris flow have been demonstrated by many field observations (O'Brien et al., 1993; Hutter et al., 1996; Hutter and Schneider, 2010a, b). It has been found that a low-viscous debris flow with a density higher than 1400 kg m^{-3} would contain a non-sediment fluid in which the diameter of granules is smaller than 0.05 mm, whereas the high-viscous debris flow with a density higher than 1900 kg m^{-3} would contain a non-sediment fluid in which the diameter of granules is smaller than 2 mm. We refer to these values as the critical diameters for a debris flow. The non-sediment fluid, composed of water and viscous, fine, non-sediment particles, is in the liquid phase in a debris flow, which behaves as non-Newtonian fluid. The solid phase is composed of particles whose diameter is larger than the critical diameter. This characteristic of a debris flow is aptly described by the two-phase fluid model (Anderson and Jackson, 1967; Iverson, 1997; Iverson and Delinger, 2001; Pitman and Le, 2005; Pudasaini, 2005, 2012). However, the two-phase fluid model describing a debris flow is still very difficult to explain via theoretical methods and to simulate accurately via numerical methods.

To understand the dynamics of the debris flow, including its initiation, runout and deposition, finding out the velocity of the debris flow is important, which would be helpful to analyze and forecast the dynamics of the debris flow and then prevent its hazards. The reason for this is that soils or rocks involved in a debris flow cause the dynamics of the debris flow to become more complicated, especially the existence of interactions between the solid particles and the fluid. As observed in natural debris flow, the velocities of the solid and liquid phases may deviate substantially from each other, essentially affecting flow mechanics (Prochaska et al., 2008; Pudasaini and Domnik, 2009; Pudasaini, 2011, 2012; Revellino et al., 2004; Rickenmann et al., 2006; Teufelsbauer et al., 2009; Uddin et al., 2001; Yang et al., 2011; Zhu, 1992). Several models have been introduced to estimate the velocity of the debris flow, such as the Fleishman formula (Fleishman, 1970) and the mean velocity formula (Takahashi, 1991; Hashimoto and Hirano, 1997; Julin and Paris, 2010; Hu et al., 2013). These models provide some rough estimations of the flow velocity and are applied to predict the risk of the debris flow. But the assumption of one-phase flow for these models leads to large modelling errors. Few theoretical results have been obtained to estimate the solid- and liquid-phase velocities for a two-phase debris flow. Although some empirical formulas are introduced to calculate the velocity of a debris flow at special location, such as the K631 debris flow that occurred at the G217 highway (Tianshan highway) in Xinjiang province and the Pingchuan debris flow that occurred at the trunk highway from Xichang to Muli in Liangshan Yi Autonomous Prefecture, Sichuan province (Chen et al., 2004, 2006). There is no general formula to calculate the velocity of a debris flow.

In this study, the two-phase flow model is applied to analyze the velocity of a debris flow. To focus on the velocity of the debris flow along the channel, a simplified, one-dimensional, two-phase model is consider here, and the motion equations governing the solid and liquid phases are deduced. Following the discussions of Bagnold (1954), the interaction between the solid and liquid phases is obtained and the velocities of the solid and liquid phases in a debris flow are obtained theoretically. This result provides a new theoretical method for estimating the velocities of the solid and liquid phases for a debris flow, which would be useful for evaluating the damage of a debris flow, estimating its arrival time, simulating its deposition area, predicting its risk, and so on. By comparing the theoretical results for the velocity and the empirical formulas for two natural debris processes, the numerical results show that the proposed method could accurately provide velocities of solid and liquid phases for a debris flow.

This study is arranged as follows: in Sect. 2, the formulas to calculate the velocities of a debris flow are deduced, and in Sect. 3, the numerical validation of the theoretical results is made by means of two real-world debris flows. The conclusions are presented in Sect. 4 and a complete summary of mathematical notation is provided in Table A1.

2 Velocity estimation of a debris flow

Two difficulties arise in the calculation of the velocity of a debris flow: one is that the diameters of the solid particles are in a wide range, and the other is that the interaction between the solid and liquid phases is difficult to describe exactly. In order to deal with the solid particles with different diameters, the diameter-equivalent method (Brunelli, 1987), which treats all particles with different diameters as the particles with the same diameter, is applied in this study.

In order to build a simple model for a debris flow to estimate the velocities of its solid and liquid phases, the following assumptions are made:

1. In this study, the downstream direction is set as the x direction, while the vertical direction to the channel bed is the y direction (Fig. 1). We assume that the velocity along the y direction is uniform and then the debris model is simplified as a one-dimensional model.

2. There are no external materials involved in the debris flow, and there is no transformation between the solid and liquid particles. Three inner forces are involved in the model: the interactions among particles, the interactions in slurry and the interactions between particles and slurry.

3. A debris flow is assumed to be a homogeneous flow (Major and Iverson, 1999; Chen et al., 2004; Kaitna et al., 2007) and a steady flow. This means that $\frac{\partial v}{\partial t} = 0$.

Under the above assumptions and following the two-phase flow theory, the governing equations for a debris flow are obtained, which are written separately for the solid and liquid phases, denoted by subscripts s and f, respectively. The mass conservation equations for the two phases are written as

$$\frac{\partial}{\partial t}(\rho_s \varphi) + \nabla \cdot (\rho_s \varphi v_s) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}[\rho_f (1-\varphi)] + \nabla [\rho_f (1-\varphi) v_f] = 0. \tag{2}$$

The momentum equations for the two phases take the forms (with the buoyancy effect considered)

$$\varphi \rho_s \left[\frac{\partial v_s}{\partial t} + (v_s \cdot \nabla) v_s \right] = b_s + f_s - \varphi \nabla P_s, \tag{3}$$

$$(1-\varphi)\rho_f \left[\frac{\partial v_f}{\partial t} + (v_f \cdot \nabla) v_f \right] = b_f + f_f - (1-\varphi) \nabla P_f. \tag{4}$$

In this study, we are mainly concerned with the steady velocity of a debris flow. Thus the motion equation can be re-written into the following one-dimensional steady flow

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system:

$$\varphi \rho_s v_{sx} \frac{dv_{sx}}{dx} = b_{sx} + f_{sx} - \varphi \frac{dP_s}{dx}, \tag{5}$$

$$(1-\varphi)\rho_f v_{fx} \frac{dv_{fx}}{dx} = b_{fx} + f_{fx} - (1-\varphi)\frac{dP_f}{dx}. \tag{6}$$

In order to estimate the steady velocities of a debris flow using Eqs. (5) and (6), the volume forces (b_{sx} and b_{fx}), surface forces (f_{sx} and f_{fx}) and pressures (P_s and P_f) firstly need to be given. The pressure for a debris flow slurry can be calculated by

$$P = k\rho v^2, \tag{7}$$

where the density ρ takes the form

$$\rho = \varphi \rho_s + (1-\varphi)\rho_f, \tag{8}$$

and the non-uniform coefficient k is about 2.4–3.0 for a viscous debris flow, whereas k is about 3.5–4.0 for a thin debris flow. According to Eq. (7), the pressures of the solid and liquid phases in the x direction can be rewritten as

$$P_s = k\rho_s v_{sx}^2, \tag{9}$$

$$P_f = k\rho_f v_{fx}^2. \tag{10}$$

The velocity of the debris flow in x direction takes the form

$$\overline{v} = \frac{\rho_s \varphi v_{sx} + (1-\varphi)\rho_f v_{fx}}{\varphi \rho_s + (1-\varphi)\rho_f}. \tag{11}$$

In a debris flow, the solid particles move parallel to the liquid slurry. By considering the gravity and the buoyancy of solid particles, the volume force of the solid phase is written as

$$b_s = \varphi(\rho_s - \rho_f) g \sin\theta. \tag{12}$$

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The volume force of the liquid phase is written as

$$b_\mathrm{f} = (1-\varphi)\rho_\mathrm{f} g \sin\theta. \tag{13}$$

In this study, the surface forces of the solid phase $f_{\mathrm{s}x}$ is divided into two parts: the traction of slurry outside control volume, $f_{\mathrm{s}x1}$, and the force from solid particles outside control volume, $f_{\mathrm{s}x2}$. The surface forces of the liquid phase $f_{\mathrm{f}x}$ is divided into two parts: the resistance from particles outside control volume, denoted by $f_{\mathrm{f}x1}$, and the resistance from slurry outside control volume, denoted by $f_{\mathrm{f}x2}$. The particle number N in a unit volume is given by

$$N = \frac{6\varphi}{\pi d_\mathrm{e}^3}. \tag{14}$$

The cross-section A_0 of solid phase taken as

$$A_0 = \frac{\pi d_\mathrm{e}^2}{4} N = \frac{3\varphi}{2d_\mathrm{e}}, \tag{15}$$

on which the pressure difference between the solid and liquid phases generates, thus using Eqs. (9) and (10), $f_{\mathrm{s}x1}$ is written as

$$f_{\mathrm{s}x1} = (P_\mathrm{f} - P_\mathrm{s})A_0 = \frac{3k\varphi}{2d_\mathrm{e}}(\rho_\mathrm{f} v_{\mathrm{f}x}^2 - \rho_\mathrm{s} v_{\mathrm{s}x}^2). \tag{16}$$

Further, the traction from slurry outside control volume $f_{\mathrm{s}x1}$ and the resistance from particles outside control volume $f_{\mathrm{f}x1}$ are equal and opposite, i.e.,

$$f_{\mathrm{s}x1} = -f_{\mathrm{f}x1}. \tag{17}$$

The force from the solid particles outside control volume mainly appears in the form of collisions among all solid particles. The mechanical effects of collision appear as the

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discrete stress among all solid particles, P_0, and the shear stress among particles, T_0. Following Bagnold (Bagnold, 1954), P_0 and T_0 can be written as

$$P_0 = 0.042\cos\alpha_i \rho_s (\lambda d_e)^2 \left(\frac{dv_{sy}}{dy}\right)^2,$$

$$T_0 = P_0 \tan\alpha_i,$$

where α_i is the collision angle among solid particles in a debris flow and λ is the linear fraction for the solid particles in a debris flow. The discrete stress P_0 and the shear stress T_0 along the downstream direction in a control volume also take the forms

$$P_0 = 0.013\rho_s (\lambda d_e)^2 \left(\frac{dv_{sy}}{dy}\right)^2,$$

$$T_0 = 0.028\rho_s (\lambda d_e)^2 \left(\frac{dv_{sy}}{dy}\right)^2,$$

and thus f_{sx2} takes the form

$$f_{sx2} = \int_0^{d_0} (P_0 + T_0) dy = \int_0^{d_0} 0.041\rho_s (\lambda d_e)^2 \left(\frac{dv_{sy}}{dy}\right)^2 dy. \quad (18)$$

As a debris flow can be regarded as a generalized Bingham flow (Takahashi, 2007; Chen et al., 2006), the rheological equation of the Bingham flow can reflect the internal viscous resistance of slurry, i.e.,

$$\tau = \tau_B + \mu \frac{dv_{fy}}{dy} - \rho_f l^2 \left(\frac{dv_{fy}}{dy}\right)^2,$$

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where l is the moving distance of framboids in the liquid slurry under the fluctuation effect, which can be written as $l = \eta y$, where η is the turbulence constant obtained by experiments and y is the internal depth of the debris flow body. Then the resistance of slurry in a control volume f_{fx2} can be written as

$$f_{fx2} = \int_0^{d_0} \tau \mathrm{d}y = \int_0^{d_0} \left[\tau_B + \mu \frac{\mathrm{d}v_{fy}}{\mathrm{d}y} - \rho_f l^2 \left(\frac{\mathrm{d}v_{fy}}{\mathrm{d}y} \right)^2 \right] \mathrm{d}y. \quad (19)$$

We assume that the velocity of slurry with respect to y satisfies a quadratic function, i.e.,

$$v_{fy} = ay^2 + by + c, \quad (20)$$

where the coefficients a, b and c are obtained by experiments. Then, using Eqs. (19) and (20), we can further obtain

$$f_{fx2} = \frac{-4\rho_f a^2 \eta^2 d_0^5}{5} - ab\rho_f \eta^2 d_0^4 - \frac{\rho_f b^2 \eta^2 d_0^3}{3} + a\mu d_0^2 + (\tau_B + \mu b) d_0. \quad (21)$$

Combining Eqs. (16) and (18) yields

$$f_{sx} = f_{sx1} + f_{sx2} = \frac{3k\varphi}{2d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2) + \int_0^{d_0} 0.041 \rho_s (\lambda d_e)^2 \left(\frac{\mathrm{d}v_{sy}}{\mathrm{d}y} \right)^2 \mathrm{d}y. \quad (22)$$

Combining Eqs. (17) and (21) yields

$$\begin{aligned} f_{fx} = f_{fx1} + f_{fx2} = &- \frac{3k\varphi}{2d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2) \\ &- \frac{4\rho_f a^2 \eta^2 d_0^5}{5} - ab\rho_f \eta^2 d_0^4 - \frac{\rho_f b^2 \eta^2 d_0^3}{3} + a\mu d_0^2 + (\tau_B + \mu b) d_0. \end{aligned} \quad (23)$$

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To simplify the calculation, the velocity of the solid phase in the y direction and the effect of turbulence in slurry are ignored. As d_0 is usually small enough, the high-order terms of d_0 can be ignored and the surface forces f_{sx} and f_{fx} can be written as

$$f_{sx} = \frac{3k\varphi}{2d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2), \quad (24)$$

$$f_{fx} = -\frac{3k\varphi}{2d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2) + (\tau_B + \mu b)d_0. \quad (25)$$

Substituting Eqs. (9), (12) and (24) into Eq. (5) yields

$$\rho_s v_{sx}\frac{dv_{sx}}{dx} = (\rho_s - \rho_f)g\sin\theta + \frac{3k}{2d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2) - k\rho_s\frac{dv_{sx}^2}{dx}. \quad (26)$$

Substituting Eqs. (10), (13) and (25) into Eq. (6) yields

$$\rho_f v_{fx}\frac{dv_{fx}}{dx} = \rho_f g\sin\theta - \frac{3k\varphi}{2(1-\varphi)d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2) + \frac{d_0}{1-\varphi}(\tau_B + \mu b) - k\rho_f\frac{dv_{fx}^2}{dx}. \quad (27)$$

Furthermore, Eqs. (26) and (27) can be rewritten as

$$(2k+1)\frac{1}{2}\rho_s\frac{dv_{sx}^2}{dx} = (\rho_s - \rho_f)g\sin\theta + \frac{3k}{2d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2), \quad (28)$$

$$(2k+1)\frac{1}{2}\rho_f\frac{dv_{fx}^2}{dx} = \rho_f g\sin\theta - \frac{3k\varphi}{2(1-\varphi)d_e}(\rho_f v_{fx}^2 - \rho_s v_{sx}^2) + \frac{(\tau_B + \mu b)d_0}{1-\varphi}. \quad (29)$$

Adding Eqs. (28) and (29) together, we obtain

$$\frac{2k+1}{2}\left[\varphi\rho_s\frac{dv_{sx}^2}{dx} + (1-\varphi)\rho_f\frac{dv_{fx}^2}{dx}\right] = \varphi(\rho_s - \rho_f)g\sin\theta + (1-\varphi)\rho_f g\sin\theta + (\tau_B + \mu b)d_0. \quad (30)$$

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Integrating from 0 to x for the two sides of Eq. (30) leads to

$$\frac{1}{2}\left[\varphi\rho_\mathrm{s}v_{\mathrm{s}x}^2 + (1-\varphi)\rho_\mathrm{f}v_{\mathrm{f}x}^2\right] = \frac{x}{2k+1}\{[\varphi\rho_\mathrm{s} + (1-2\varphi)\rho_\mathrm{f}]g\sin\theta + (\tau_\mathrm{B}+\mu b)d_0\}. \quad (31)$$

Subtracting Eq. (29) from Eq. (28) leads to

$$\frac{1}{2}\left(\rho_\mathrm{s}\frac{\mathrm{d}v_{\mathrm{s}x}^2}{\mathrm{d}x} - \rho_\mathrm{f}\frac{\mathrm{d}v_{\mathrm{f}x}^2}{\mathrm{d}x}\right) = -\frac{3k}{(2k+1)(1-\varphi)d_\mathrm{e}}\frac{1}{2}(\rho_\mathrm{s}v_{\mathrm{s}x}^2 - \rho_\mathrm{f}v_{\mathrm{f}x}^2) - \frac{1}{2k+1}[(2\rho_\mathrm{f}-\rho_\mathrm{s})g\sin\theta + (\tau_\mathrm{B}+\mu b)d_0]. \quad (32)$$

Solving this above equation yields

$$\frac{1}{2}\left(\rho_\mathrm{s}v_{\mathrm{s}x}^2 - \rho_\mathrm{f}v_{\mathrm{f}x}^2\right) = \frac{d_\mathrm{e}(1-\varphi)}{3k}\left[(2\rho_\mathrm{f}-\varphi\rho_\mathrm{s})g\sin\theta + (\tau_\mathrm{B}+\mu b)d_0\right]\left[\exp\left(\frac{-3k}{(2k+1)(1-\varphi)d_\mathrm{e}}x\right)-1\right]. \quad (33)$$

The velocities of the solid and liquid phases for a debris flow are then obtained via Eqs. (31) and (33).

$$\frac{1}{2}\rho_\mathrm{s}v_{\mathrm{s}x}^2 = \{[\varphi\rho_\mathrm{s} + (1-2\varphi)\rho_\mathrm{f}]g\sin\theta + (\tau_\mathrm{B}+\mu b)d_0\}\frac{x}{2k+1} - \frac{d_\mathrm{e}(1-\varphi)^2}{3k}[(2\rho_\mathrm{f}-\rho_\mathrm{s})g\sin\theta + (\tau_\mathrm{B}+\mu b)d_0]\left[1-\exp\left(\frac{-3k}{(2k+1)(1-\varphi)d_\mathrm{e}}x\right)\right], \quad (34)$$

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$$
\begin{aligned}
&\frac{1}{2}\rho_\text{f} v_{\text{f}x}^2 \\
&= \{[\varphi\rho_\text{s} + (1-2\varphi)\rho_\text{f}]g\sin\theta + (\tau_\text{B} + \mu b)d_0\}\frac{x}{2k+1} \\
&\quad + \frac{d_\text{e}(1-\varphi)\varphi}{3k}[(2\rho_\text{f} - \rho_\text{s})g\sin\theta + (\tau_\text{B} + \mu b)d_0]\left[1 - \exp\left(\frac{-3k}{(2k+1)(1-\varphi)d_\text{e}}x\right)\right],
\end{aligned} \tag{35}
$$

where x denotes the distance from the calculation point to the initial point in flow area.

3 Results and discussion

In this study, we developed a new formula to estimate the solid- and liquid-phase velocities in a debris flow, which is useful for understanding the dynamics of the debris flow. By the discussion in Sect. 2, Eq. (31) provides the total kinetic energy of a debris flow element, which is

$$
\{[\varphi\rho_\text{s} + (1-2\varphi)\rho_\text{f}]g\sin\theta + (\tau_\text{B} + \mu b)d_0\}\frac{x}{2k+1}.
$$

The total kinetic energy is combined from two parts: the kinetic energy derived by gravity M_1 and the kinetic energy derived by the yielding stress M_2, which are given by

$$
M_1 = [\varphi\rho_\text{s} + (1-2\varphi)\rho_\text{f}]g\sin\theta\frac{x}{2k+1}, \tag{36}
$$

$$
M_2 = (\tau_\text{B} + \mu b)d_0\frac{x}{2k+1}. \tag{37}
$$

However, Eq. (33) provides the kinetic energy difference between two phases – the solid and liquid phases– and it describes the interaction between two phases. The parameter d_c is referred to as the characteristic scale of a debris flow, which is defined

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by

$$d_c = \frac{d_e(1-\varphi)}{3k}.$$

Following from this, the kinetic energy change due to the interaction between two phases is divided into two parts: the kinetic energy derived by gravity G_1 and the kinetic energy derived by the yielding stress G_2, which are given by

$$G_1 = (2\rho_f - \rho_s) g \sin\theta d_c \left[1 - \exp\left(\frac{-x}{d_c(2k+1)}\right)\right], \tag{38}$$

$$G_2 = (\tau_B + \mu b) d_0 d_c \left[1 - \exp\left(\frac{-x}{d_c(2k+1)}\right)\right]. \tag{39}$$

Then the velocities of the solid and liquid phases in a debris flow are given by

$$v_s^2 = \frac{2}{\rho_s}[M_1 + M_2 - (1-\varphi)G_1 - (1-\varphi)G_2],$$

$$v_f^2 = \frac{2}{\rho_f}(M_1 + M_2 + \varphi G_1 + \varphi G_2).$$

In this section, we will give some numerical examples to show the dynamics of a debris flow along the channel. Figure 2 shows some numerical results for the solid- and liquid-phase velocities for an example debris flow. The figure indicates that the liquid phase is faster than the solid phase, and the ratio of the velocities for two phases is about 0.790. The solid- and liquid-phase velocities at a point 300 m along the channel are shown in Fig. 3 for the different solid volume fractions; it can be seen that the velocity of a debris flow decreases as the solid volume fraction increases. The solid- and liquid-phase velocities at 300 m along the channel are shown in Fig. 4 for the different equivalent diameters of solid particles, and here it can be seen that, as the equivalent diameter of solid particles increases, the solid-phase velocity of a debris flow decreases very slowly whereas the liquid-phase velocity increases very slowly.

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In order to validate the estimation of velocities, in this section, two real-world debris flow – the K631 debris flow that occurred at the G217 highway (Tianshan highway) in Xinjiang province and the Pingchuan debris flow that occurred at the trunk highway from Xichang to Muli in Liangshan Yi Autonomous Prefecture, Sichuan province – are considered. The velocities obtained by observations for the two debris flows, one a viscous debris flow and the other a dilute debris flow, are $11.59\,\mathrm{m\,s^{-1}}$ and $9.70\,\mathrm{m\,s^{-1}}$, respectively. Particles more than 2 cm in diameter are regarded as solid particles, and the others are classified as slurry (Chen et al., 2006). The related parameters were obtained through analyzing samples at the locale. The comparison of the theoretical results and the experiential results shows that the estimation method for the velocities of a debris flow can be widely used for a real-world debris flow (see Table 1).

4 Conclusions

A one-dimensional model for a debris flow is introduced to estimate the velocities of the solid and liquid phases. By applying the specific form of the volume force and the surface forces for the solid and liquid phases, theoretical results are used to estimate the velocities of the solid and liquid phases. These results are found to be valid by comparing the theoretical results with the experiential formula for two real-world debris flows. Furthermore, the theoretical methods can estimate the velocities of a debris flow with different solid volume fractions and different equivalent diameters, which makes the theoretical results more useful for tracing a debris flow, simulating the deposition area and predicting the risk for a debris flow.

Acknowledgements. This work was supported by the National Natural Science Foundation of China (grant no. 11071238) and the National Center for Mathematics and Interdisciplinary Sciences, CAS and the Key Lab of Random Complex Structures and Data Science, CAS.

References

Anderson, T. B. and Jackson, R.: Fluid mechanical description of fluidized beds: equations of motion, Ind. Eng. Chem. Fundam., 6, 527–539, 1967.

Bagnold, R. A.: Experiments on a gravity-free dispersion of large solid spheres in a Newtonian fluid under shear, Proc. R. Soc. Lond. Ser. A, 225, 49–63, 1954.

Brunelli, S.: Trasporto solido annuo dei corsi d'acqua in funzione delle loro caratteristiche idrologiche e morfologiche, Tesi di Laurea, Università degli Studi di Padova, Padova, 1987 (in Italian).

Chen, H., Tang, H., Ma, Y., and Wu, S.: Research and Control of Debris Flow Along Highway, China Communications Press, Beijing, 86–116, 2004 (in Chinese).

Chen, H., Tang, H., and Chen, Y.: Research on method to calculate velocities of solid phase and liquid phase in debris flow, Appl. Math. Mech., 27, 357–364, 2006 (in Chinese).

Fleishman, S. M.: Seli, Gidrometeoizdat, Leningrad, 1970 (in Russian).

Hashimoto, H. and Hirano, M.: A flow model of hyperconcentrated sand-water mixtures, in: Proc. 1st Int. Conf. on Debris-Flow Hazards Mitigation: Mechanics, Prediction, and Assessment, San Francisco, California, 7-9 August, ASCE, New York, 464–473, 1997

Hu, K., Tian, M., and Li, Y.: Influence of flow width on mean velocity of debris flows in wide open channel, J. Hydraul. Eng., 139, 65–69, 2013.

Hutter, K. and Schneider, L.: Important aspects in the formulation of solid-fluid debris-flow models, Part I. Thermodynamic implications, Continuum Mech. Therm., 22, 363–390, 2010a.

Hutter, K. and Schneider, L.: Important aspects in the formulation of solid-fluid debris-flow models, Part II. Constitutive modelling, Continuum Mech. Therm., 22, 391–411, 2010b.

Hutter, K., Svendsen, B., and Rickenmann, D.: Debris flow modelling: a review, Continuum Mech. Therm., 8, 1–35, 1996.

Iverson, R. M.: The physics of debris flows, Rev. Geophys., 35, 245–296, 1997.

Iverson, R. M. and Denlinger, R. P.: Flow of variably fluidized granular masses across three-dimensional terrain, 1. Coulomb mixture theory, J. Geophys. Res., 106, 537–552, 2001.

Julien, P. Y. and Paris, M. A.: Mean velocity of mudflows and debris flows, J. Hydraul. Eng., 136, 676–679, 2010.

Kaitna, R., Rickenmann, D., and Schatzmann, M.: Experimental study on rheologic behaviour of debris flow material, Acta Geotech., 2, 71–85, 2007.

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Major, J. J. and Iverson, R. M.: Debris-flow deposition: effects of pore-fluid pressure and friction concentrated at flow margins, Geol. Soc. Am. Bull., 111, 1424–1434, 1999.
O'Brien, J. S., Julien, P. J., and Fullerton, W. T.: Two-dimensional water flood and mudflow simulation, J. Hydraul. Eng., 119, 244–261, 1993.
Pitman, E. B. and Le, L.: A two-fluid model for avalanche and debris flows, Phil. Trans. R. Soc. A, 363, 1573–1601, 2005.
Prochaska, A. B., Santi, P. M., Higgins, J. D., and Cannon, S. H.: A study of methods to estimate debris flow velocity, Landslides, 5, 431–444, 2008.
Pudasaini, S. P.: Some exact solutions for debris and avalanche flows, Phys. Fluids, 23, 043301, doi:10.1063/1.3570532, 2011.
Pudasaini, S. P.: A general two-phase debris flow model, J. Geophys. Res., 117, F03010, doi:10.1029/2011JF002186, 2012.
Pudasaini, S. P. and Domnik, B.: Energy considerations in accelerating rapid shear granular flows, Nonlinear Proc. Geoph., 16, 399–407, 2009.
Pudasaini, S. P., Wang, Y., and Hutter, K.: Modelling debris flows down general channels, Nat. Hazards Earth Syst. Sci., 5, 799–819, doi:10.5194/nhess-5-799-2005, 2005.
Revellino, P., Hungr, O., Guadagno, F. M., and Evans, S. G.: Velocity and runout simulation of destructive debris flows and debris avalanches in pyroclastic deposits, Campania region, Italy, Environ. Geol., 45, 295–311, 2004.
Rickenmann, D., Laigle, D., McArdell, B., and Hübl, J.: Comparison of 2D debris-flow simulation models with field events, Computat. Geosci., 10, 241–264, 2006.
Takahashi, T.: Debris flow, IAHR monograph series, A. A. Balkema, Rotterdam, 1991.
Takahashi, T.: Debris flow: Mechanics, Prediction and Countermeasures, Taylor & Francis, London, 2007.
Teufelsbauer, H., Wang, Y., Chiou, M. C., and Wu, W.: Flow-obstacle interaction in rapid granular avalanches: DEM simulation and comparison with experiment, Granul. Matter, 11, 209–220, 2009.
Uddin, M. S., Inaba, H., Yoshida, Y., Itakura, Y., and Kasahara, M.: Large motion estimation by gradient technique-application to debris flow velocity field, Phys. Chem. Earth C, 26, 633–638, 2001.
Yang, H., Wei, F., Hu, K., Chernomorets, S., Hong, Y., Li, X., and Xie, T.: Measuring the internal velocity of debris flows using impact pressure detecting in the flume experiment, J. Mt. Sci., 8, 109–116, 2011.

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Zhu, P.: A discussion on the velocity of debris flow, Erosion, Debris Flows and Environment in Mountain Regions (Proceedings of the Chengdu Symposium, July 1992), IAHS Publ., 209, 369–374, 1992.

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Table 1. The results of velocity calculation for the K631 (G217 highway) and Pingchuan debris flows.

Name	φ	ρ_s ($kg\,m^{-3}$)	ρ_f ($kg\,m^{-3}$)	d_e (m)	v_s ($m\,s^{-1}$)	v_f ($m\,s^{-1}$)	$\overline{v}_1$ ($m\,s^{-1}$)	$\overline{v}_2$ ($m\,s^{-1}$)	$\overline{v}_3$ ($m\,s^{-1}$)
K631	0.0902	2500	1660	0.1033	8.43	11.97	11.59	11.72	11.51
Pingchuan	0.0497	2400	1500	0.0816	8.97	10.41	9.70	11.14	10.30

$\overline{v}_1$ is the velocity of debris flow obtained from field observations, $\overline{v}_2$ is the velocity of debris flow calculated by Chen et al. (2006), and $\overline{v}_3$ is the velocity of debris flow calculated from Eq. (11).

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Table A1. Notation.

ρ	the density of debris flow
ρ_s	the density of solid particles
ρ_f	the density of liquid slurry
v_s	the velocity of solid constituent
v_f	the velocity of liquid constituent
d_e	the equivalent diameter of solid particles
d_0	the equivalent radius of control volume for debris flow
g	the gravity acceleration
P	the pressure of debris flow
θ	the gradient of debris flow groove
φ	the solid volume fraction
b_s	the volume force of solid phases
b_f	the volume force of liquid phases
P_s	the pressure of solid phases
P_f	the pressure of liquid phases
f_s	the surface forces of solid phases
f_f	the surface forces of liquid phases
v	the velocity of debris flow body
k	the nonuniform coefficient of debris flow body
τ_B	the yielding stress of debris flow slurry
μ	the viscous coefficient of debris flow slurry

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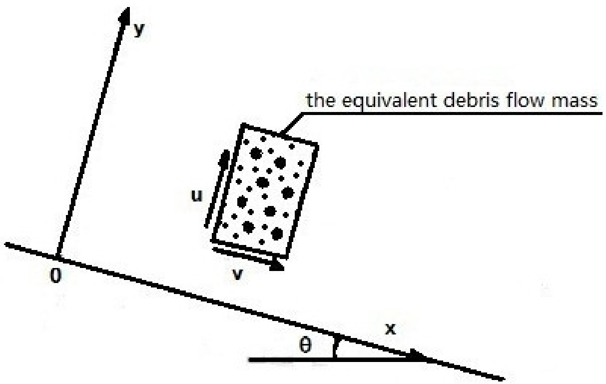

Figure 1. Velocity analysis of the debris flow.

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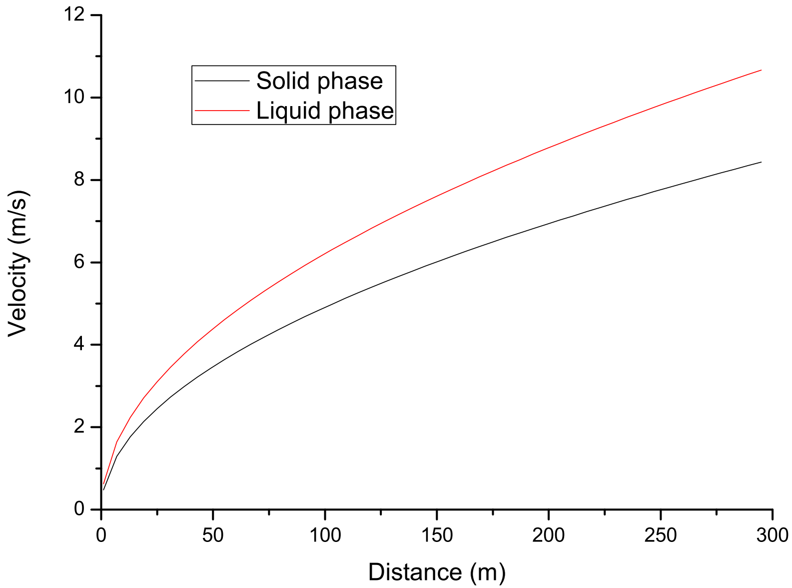

Figure 2. Solid- and liquid-phase velocities of a debris flow along the channel: $\rho_s = 2400\,\mathrm{kg\,m^{-3}}$, $\rho_f = 1500\,\mathrm{kg\,m^{-3}}$, $d_e = 0.10\,\mathrm{m}$, $\varphi = 0.10$, $\theta = 30^\circ$, $(\tau_B + \mu b)d_0 = 100$, $k = 3.72$, $g = 9.8$, $x \in (0, 300)$.

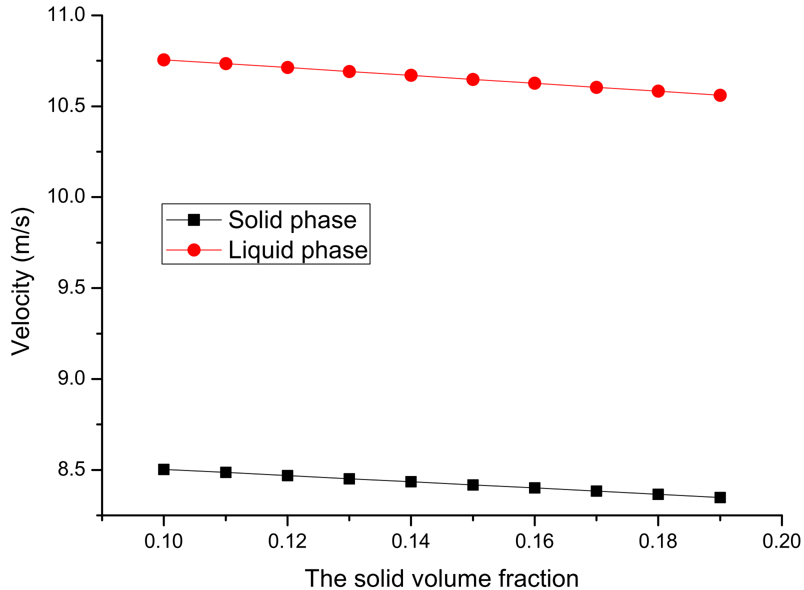

Figure 3. Solid- and liquid-phase velocities of a debris flow along the channel: $\rho_s = 2400\,\mathrm{kg\,m^{-3}}$, $\rho_f = 1500\,\mathrm{kg\,m^{-3}}$, $d_e = 0.10\,\mathrm{m}$, $\varphi = 0.10 \sim 0.19$, $\theta = 30^\circ$, $(\tau_B + \mu b)d_0 = 100$, $k = 3.72$, $g = 9.8$, $x = 300$.

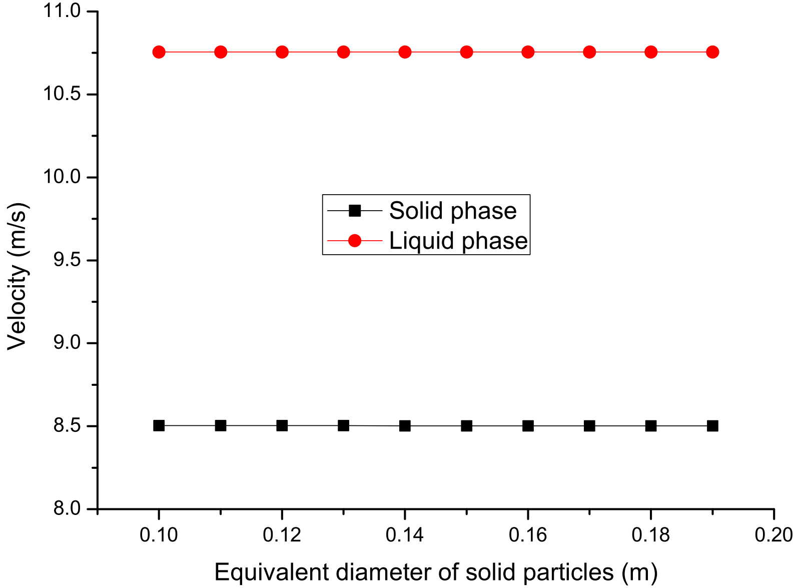

Figure 4. Solid- and liquid-phase velocities of a debris flow along the channel: $\rho_s = 2400\,\mathrm{kg\,m^{-3}}$, $\rho_f = 1500\,\mathrm{kg\,m^{-3}}$, $d_e = 0.10\,\mathrm{m} \sim 0.19\,\mathrm{m}$, $\varphi = 0.10$, $\theta = 30^\circ$, $(\tau_B + \mu b)d_0 = 100$, $k = 3.72$, $g = 9.8$, $x = 300$.

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