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Spatial analysis of oil reservoirs using DFA of geophysical data

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Abstract

We employ Detrended Fluctuation Analysis (DFA) technique to investigate spatial properties of an oil reservoir. This reservoir is situated at Bacia de Namorados, RJ, Brazil. The data corresponds to well logs of the following geophysical quantities: sonic, gamma ray, density, porosity and electrical resistivity, measured in 56 wells. We tested the hypothesis of constructing spatial models using data from fluctuation analysis over well logs. To verify this hypothesis we compare the matrix of distances among well logs with the differences among DFA-exponents of geophysical quantities using spatial correlation function and Mantel test. Our data analysis suggests that sonic profile is a good candidate to represent spatial structures. Then, we apply the clustering analysis technique to the sonic profile to identify these spatial patterns. In addition we use the Mantel test to search for correlation among DFA-exponents of geophysical quantities.

1 Introduction

To a great extend the information about petroleum reservoirs is obtained from well logs that measure geophysical quantities along drilled wells, see Asquith and Krygowski (2004). As a rule data is spatially sparse and presents strong fluctuation, therefore we have to rely on statistical methods for evaluating indices that describe the characteristics of the reservoirs, see for instance Hardy and Beir (1994) and Hewitt (1998). The question about what methods are more appropriate to fulfil this task is still open. In this work we investigate the use of fluctuation analysis to tackle this problem.

The well log data is the most valuable information that can be obtained from geological volumes and from oil reservoirs. However, the cost of drilling imposes severe limitation in the number of wells. In this situation we are faced with the problem of uncover geophysical properties over long field extensions from data collected along few drilled wells. To perform this task we have to rely on data statistics that guarantees similarities among geological structures. One goal is to draw contour lines expressing

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the variation of proprieties in the subsurface by evaluating interpolation from well logs data. This will be justified if correlations show consistent spatial patterns. The question of this article is: can we use DFA-exponent to discover spatial patterns. In other words, is DFA-exponent spatially correlated in such way we can employ it as a spatial parameter.

The Detrended Fluctuation Analysis DFA is a powerful fluctuation analysis technique introduced by Peng et al. (1995) that was developed to deal with non-stationary time series. This tool is an elegant generalization of the Hurst method, see for instance Mandelbrot (1977), that is used to compare an aleatory time series with a similar Brownian series, as well as, to evaluate correlation and anti-correlation in a series. DFA technique have been used in many areas of geophysical literature, in Padhy (2004) it is used to obtain information from seismic signals. In references Andrade et al. (2009), Chun-Feng and Liner (2005), Gholamy et al. (2008) and Tavares et al. (2005) DFA is employed to interpret and filter images of seismograms. In reference Ribeiro et al. (2011), Lozada-Zumeta et al. (2012), Marinho et al. (2013) and Dashtian et al. (2011) this technique is used, as in this manuscript, in the analysis of well logs.

When we treat with complex systems that have a huge amount of data the DFA method is attractive because it allows to summarize data into a suitable parameter. The DFA parameter summarizes the global behaviour of the full data set, it is a synthetic index of its complexity. This simple procedure allows a fast comparison among large samples. Furthermore, the first step in oil research is a geographycal analysis of the surface. To have characteristics of the geological structure of the subsurface projected into a single measurement on the ground level is an useful information. In addition, the spatial correlation between theses quantities allow us to have a better understanding of the lithology which is crucial in oil prospection.

The case study employed in this work is an oil reservoir and we apply the DFA technique over data logs of drilled wells. The oil reservoir is situated at Bacia de Namorados, an offshore field in the Rio de Janeiro State, Brazil. The five geophysical measurements available in the well logs are: sonic (DT, sonic transient time), gamma ray

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(GR, gamma emission), density (RHOB, bulk density), porosity (NPHI, neutron porosity) and electrical resistivity (ILD, deep induction resistivity). The manuscript can be summarized as follows. In Sect. 2 we perform three tasks: show the geologic data in some detail, introduce briefly the mathematics of the DFA and present the statistical methods we use in this work: spatial correlation, Mantel test and *k*-means clustering analysis technique. In Sect. 3 we show the results of the spatial correlation function and the Mantel test; we estimate that the sonic profile is the best candidate to model spatial patterns. In addition we apply clustering analysis to this geophysical quantity to create a spatial model. Finally in Sect. 4 we conclude the work and give our final remarks.

2 Model background

2.1 The geologic data

The geologic data used in this work are from well logs located at the oil field of Bacia de Namorados, Rio de Janeiro State, Brazil. The wells are situated in an area of approximately 100 km^2 and distant 150 km from the coast. The spatial arrangement of the well logs is illustrated in Fig. 3 and the matrix of distance among pairs of well i and j is done by $d_{i,j}$. The number of records for each well is not constant, the sonic register was recorded in ($N = 17$) well logs, gamma ray ($N = 53$), density ($N = 51$), porosity ($N = 48$), and, finally, resistivity ($N = 54$). The time series of the geophysical quantities of each well log has around $N_S \approx 1000$, the exact value depends on the measurement, this data series length guarantees a good statistics for the use of DFA method (Kantelhardt et al., 2001). An example of a segment of the time series corresponding to each of the five geophysical variables is visualized in Fig. 1.

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2.2 The Detrended Fluctuation Analysis DFA

The DFA is an improved extension of the most traditional fluctuation analysis tool: the Hurst algorithm, see for instance Peng et al. (1994) and Kantelhardt et al. (2001). Here we present a brief description of the DFA algorithm, a more detailed presentation of the method can be view in Peng et al. (1995) and Ihlen (2012). Consider a time series $x_t = (x_1, x_2, \ldots, x_{N_S})$. To compute the DFA algorithm we initially integrate the series $x(t)$ creating a new variable $y(t)$:

$$y(t) = \sum_{i=1}^{t} |x_i|. \tag{1}$$

The second step of the algorithm consists in equally partition the time series into boxes of length n. Inside each box a data fitting is performed using the least square method, this auxiliary curve is the local trend $y_n(t)$ of the data. In the third step we detrend the integrated series, $y(t)$, that means, we subtract $y(t)$ from the local trend $y_n(t)$. The root mean square fluctuation, in a similar way as performed in the Hurst algorithm, is found using the relation:

$$F(n) = \sqrt{\frac{1}{N_S} \sum_{i=1}^{N_S} (y(t) - y_n(t))^2}. \tag{2}$$

The fourth step consists in estimating Eq. (2) over all blocks of size n. Usually $F(n)$ increases with n, a linear increasing of $F(n)$ with n in a log-log scale is a signature of a fractal behaviour. The exponent α of the relation:

$$F(n) = n^{\alpha} \tag{3}$$

is known as the DFA-exponent. The most important equation of this theoretical development is Eq. (3) that provides a relationship between the average root mean square

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fluctuation, $F(n)$, as a function of the box size n. In this work we have computed α with help of the algorithm available in Matlab. In Fig. 2 we show, as an illustration, the curve of $F(n)$ vs. n for two distinct well for gamma-ray and sonic data.

We performed similar analysis for the available well logs of all geophysical quantities. For 98 % of cases the correlation coefficient of the adjusted line in the log-log plot fulfil the relation $R^2 \leq 0.95$, for R the Pearson correlation. The cases that do not follow this condition were discarded from the statistics.

2.3 Statistical analysis

In the paragraphs that follows we show the statistical methods explored in the paper. All statistical analysis were performed using R language, see the reference R Development Core Team (2008).

2.3.1 Spatial correlation

To test the spatial correlation among variables, the most simple statistics is the correlation function, $\mathrm{Corr}(\tau)$, for τ the correlation length. To test spatial correlation between DFA-exponent and distance we start ranking all $d_{i,j}$ of the distance matrix. For each g^t geophysical quantity the difference $\Delta\alpha^t$ is ordered according to the distances. $\mathrm{Corr}^t(\tau)$ is estimated as follows:

$$\mathrm{Corr}^t(\tau) = \frac{\sum_{l=1}^{\mathrm{Num}} \Delta\alpha^t(d)\Delta\alpha^t(d+\tau)}{\mathrm{Num}\ \mathrm{sd}(\Delta\alpha^t)} \tag{4}$$

where $0 \leq \tau \leq \mathrm{Num}$. To compute $\mathrm{Corr}(\tau)$ the quantity $\Delta\alpha$ is transformed to $\Delta\alpha \to \Delta\alpha - \mu$ for μ the average of $\Delta\alpha$, the correlation function is evaluated over zero means series. The standard deviation, $\mathrm{sd}(\Delta\alpha)$, in the denominator normalizes adequately the function such that $\mathrm{Corr}(0) = 1$.

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2.3.2 Mantel test

Mantel test is a statistical tool to test correlation between two symmetrical matrices of the same rank. The rationale of this test is to employ matrix elements in the same way as vectors of objects, in this way Mantel test is quite similar to Pearson test that search for correlation between two vectors. In the Mantel test we transform each matrix into a vector to evaluate the correlation, see Sokal and Rohlf (1995).

We compute two distinct set of tests: in the first we check for correlation between the matrix of distances of well logs $d_{i,j}$ and the differences matrix of DFA-exponent: $\Delta^t \alpha_{i,j} = |\alpha_i^t - \alpha_j^t|$ of any geophysical variable g^t. In a second moment we compare the DFA among geophysical quantities applying Mantel test between matrices of $\Delta^t \alpha_{i,j}$ and $\Delta^s \alpha_{i,j}$ of geophysical quantities g^t and g^s. Of course we evaluate this test only over pairs i and j of well logs that have available data for both g^t and g^s.

2.3.3 Clustering analysis

For the geophysical quantities that show spatial correlation we search for spatial patterns. In this article we use k-means – a standard tool of clustering analysis to perform this task. The k-means methodology works by creating groups using a metric criterion. The user of the method choose a fixed number k of subsets, or clusters, and a optimization algorithm select elements according to the distance to k centroids.

In our study we find that only one geophysical quantity present significant spatial correlation, the sonic variable. To use k-means methodology it is necessary to have at least three input variables. For obtaining the two additional parameters we employ the following strategy: we use the upper and lower values of the error interval of the fitting of the curve defined by Eq. (3).

We use a Monte Carlo test to check if k-clusters method create groups that are closer, in a metric sense, than groups generated by an aleatory way. We define an index Ω of neighbourhood in the following way. We create balls of radius b attached to each well logs. For all pairs of well logs we perform the computation: if the balls of two well logs overlap and belong to the same group we count $\Omega \rightarrow \Omega + 1$ otherwise we do nothing. The index Ω is normalized by the number of groups and the maximal number of elements in each k-group. After that we shuffle the well logs over the k-groups and compute Ω_{shuffled} over the shuffled data. We estimate a p value as the probability of Ω being larger than the Ω_{shuffled} distribution.

3 Results

3.1 Spatial correlation

We initially compute the function $\text{Corr}(\tau)$ for $0 \leq \tau \leq 80$ for all geophysical variables. We expect that in case α variables of any geophysical quantity g^t shows spatial correlation the function $\text{Corr}(\tau)$ should decreases with τ. To analyse the decay of $\text{Corr}(\tau)$ of the geophysical variables we fit a linear curve and test how significant is its decay. The result of the fitting of the geophysical quantities is shown in Table 1, this result indicates that the only quantity that reveals a significant decay is the sonic data, all the other quantities show $p > 0.05$ for the linear fitting test.

3.2 Mantel test

In Table 1 it is also shown the results of the analysis of Mantel test for all geophysical variables. Here it is computed the correlation between two matrices: $d_{i,j}$, the matrix of distance between two wells, and $\Delta\alpha_{i,j} = \alpha_i - \alpha_j$, the matrix of difference between DFA-exponent α for the same wells. The correlation parameter of the test is indicated by r while p is the p value of the significance test. In agreement with the output of the correlation function analysis the smallest p value is attributed to the sonic variable. This result justify the use of sonic data for constructing spatial patterns, the subject of the next section.

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We use Mantel test not only to analyse the correlation between distances and DFA-exponent, but also to perform a comparison between distinct geophysical quantities. That means we compare matrices $\Delta\alpha^t$ and $\Delta\alpha^s$ of geophysical quantities g^t and g^s. The result of this analysis is shown in Table 2. We plot only the p value of the test in the table, the major agreement observed was between variables: resistivity and porosity, which is followed by density and sonic.

3.3 Clustering analysis

The sonic variable has revealed a good candidate to generate spatial patterns. In Fig. 3 we plot the oil reservoir area with well logs, the axis x and y represent the spatial coordinates, we use metric arbitrary units. The points in the figure represent the coordinates of the well logs. In Fig. 3a we use the fixed number of clusters $k = 3$ while in Fig. 3b we use $k = 4$. Elements in the same cluster are indicated by a common symbol, these two pictures suggests that sonic variable is indeed a good geophysical quantity to model spatial formations.

To test how good is the spatial formation of the clustering analyse we employ a Monte Carlo test. We estimated the proper Ω value and found $p = 0.005$ for $k = 3$ and $p = 0.16$ for $k = 4$ using an optimal ball size b. We checked the k-means clustering technique for the other quantities: sonic, resistivity, porosity and gamma-ray. We use $3 \leq k \leq 6$ for all these geophysical data set and we found no $p > 0.05$, that means, no evidence of significant spatial cluster formation. This result is an indirect evidence that only sonic variable is a good choice to formation of spatial patterns.

4 Final remarks

The issue of this manuscript is to test the hypothesis that we can use DFA-exponent α from log wells as integrated indices projected over the earth surface to reveal spatial structures. Each α is an index that summarizes the structure of fluctuation of a

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geophysical quantity over geologic layers of thousand meters deep. The challenge is to use the information of the fluctuation from a set of distinct well logs distributed over several kilometres to construct spatial patterns.

The results of Mantel test and spatial correlation function indicate that the only geophysical parameter we can rely on this global approach to model spatial patterns is the sonic. We use partitioning by k-means, a standard technique of cluster analyse appropriate to represent spatial models. A visual inspection of the spatial patterns, as well as a Monte Carlo test, verify that sonic data forms good spatial models for $k = 3$ and 4. In opposition, other geophysical quantities do not show significant results in Monte Carlo test.

In addition to spatial analysis, we also used Mantel test to search for correlation among geophysical quantities. In a previous work (Ribeiro et al., 2011), using the same data set, but applying a different methodology, it was found that the only pair of geophysical variables that shows significant correlation was density and sonic ($p = 0.01$). In this work the pairs of quantities that show greater significance were porosity and resistivity ($p = 0.088$) is closely followed by density and sonic ($p = 0.13$). The paper Ferreira et al. (2009) has also found a major correlation between sonic and density using standard correlation matrix. For both methodologies the pair density and sonic seems to be correlated, this property is probably related to the trivial fact that sound speed increases with density, see for instance Feynman and Leighton (1964). A result that is close to our result. As the methodologies of these works are not the identical we do not expect the same result, indeed, small discrepancies are acceptable in statistical treatments. This last result is in agreement with Dashtian et al. (2011) that have used cross-correlation analysis between well logs and found that sonic, porosity and density are more correlated among them than with gamma-ray.

To conclude the work we go back to the initial question of the manuscript: is it possible to create spatial models using fluctuation analysis? The answer to this question is yes, but a yes without enthusiasm. The sonic variable has shown enough spatial correlation to perform this task, but the density, which is the quantity the most correlated to sonic does not share the same property. In a future work we intend to test the combination of distinct geophysical quantities in the formation of spatial patterns.

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Table 1. The results of spatial correlation: the decaying of the spatial correlation and Mantel test. The linear fitting of the correlation function is indicated in table as well as the output of the Mantel test. The result indicate that only sonic data is appropriate for constructing spatial analysis. The geophysical quantities are indicated in the table: sonic (SO), density (DE), gamma ray (GR), electrical resistivity (RE), and porosity (PO).

	Spatial correlation			Mantel test	
	F	ρ	p	r	p
PO	0.002	0.00003	0.96	−0.021	0.64
RE	0.11	0.002	0.74	0.016	0.51
GR	1.05	0.015	0.31	−0.028	0.73
SO	9.03	0.12	0.004	0.181	0.06
DE	0. 64	0.01	0.43	0.023	0.34

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Table 2. This symmetric table shows the p value of the Mantel test of hypothesis for correlation among de DFA-exponent of geophysical quantities. The test is performed between each pair of five geophysical variables: porosity (PO), resistivity (RE), gamma ray (GR), density (DE), and sonic (SO).

	RE	GR	DE	SO
PO	$p = 0.088$	$p = 0.74$	$p = 0.95$	$p = 0.21$
RE	–	$p = 0.73$	$p = 0.62$	$p = 0.44$
GR	–	–	$p = 0.62$	$p = 0.61$
DE	–	–	–	$p = 0.13$

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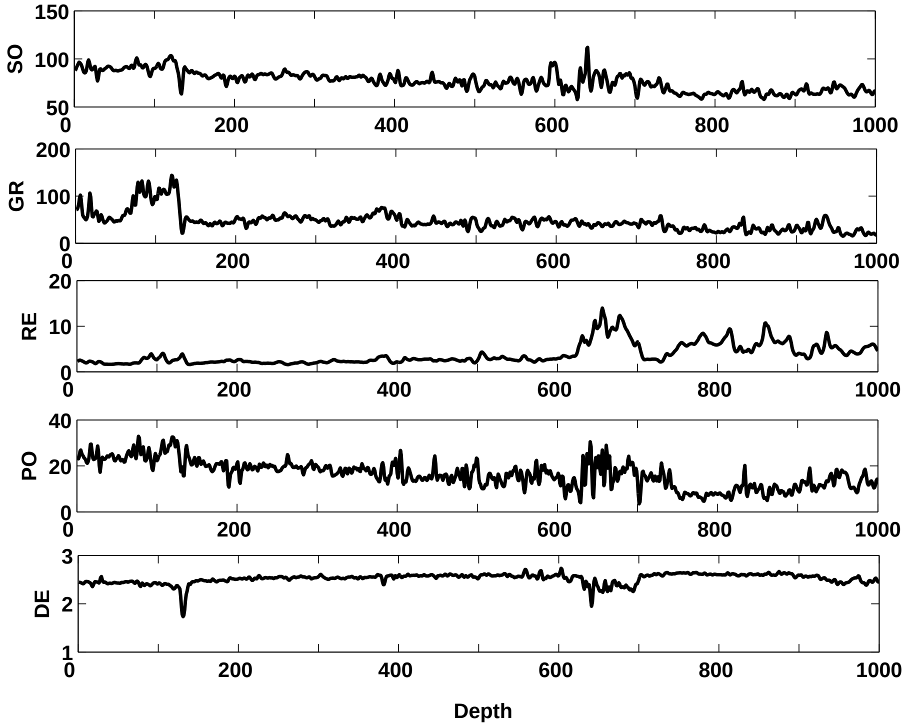

Fig. 1. A segment of a typical measurement, for an arbitrary well, of the geophysical properties vs. depth (in meters): sonic (SO), gamma ray (GR), density (DE), porosity (PO), and resistivity (RE).

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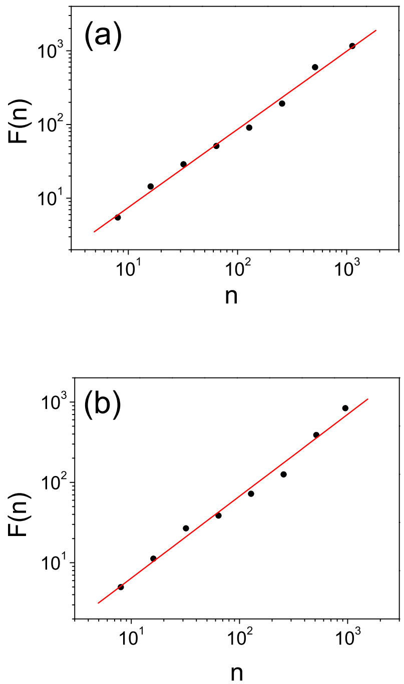

Fig. 2. A typical plot illustrating DFA scaling property: $F(n)$ vs. n, the curve of Eq. (3). The good fitting of most curves in a log-log scale reveals the fractal characteristic of geophysical data. In **(a)** the well 2 of gamma ray data and in **(b)** the well 17 of sonic data.

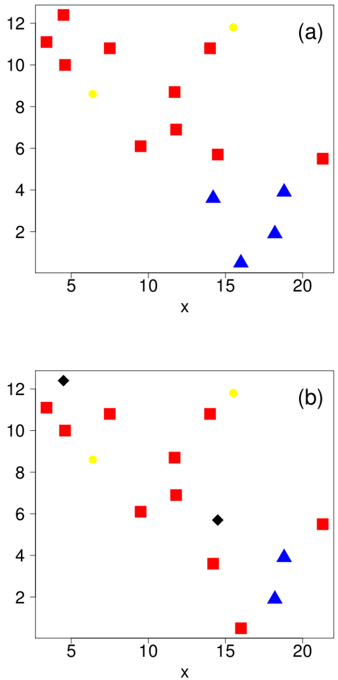

Fig. 3. Clustering analysis patterns for sonic data **(a)** $k = 3$, and **(b)** $k = 4$. Both figures show a satisfactory cluster formation in this data as confirmed by Monte Carlo test.

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