



**Finding recurrence networks' threshold adaptively for a specific time series**

D. Eroglu et al.

# Finding recurrence networks' threshold adaptively for a specific time series

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Title Page	
Abstract	Introduction
Conclusions	References
Tables	Figures
⏪	⏩
◀	▶
Back	Close
Full Screen / Esc	
Printer-friendly Version	
Interactive Discussion	





## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract Introduction

Conclusions References

Tables Figures

⏪ ⏩

◀ ▶

Back Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

powerful to study complex, real-world systems (Trulla et al., 1996; Marwan et al., 2002; Donges et al., 2011). Examples of their successful application in real-world systems can be found in life science (Riley et al., 1999; Marwan et al., 2002; Neuman et al., 2009; Carrubba et al., 2012), Earth science (Marwan et al., 2003; Matcharashvili et al., 2008; Donges et al., 2011), astrophysics (Asghari et al., 2004; Zolotova et al., 2009), and others (Marwan, 2008).

The measures defined by the RP framework, called *recurrence quantification analysis* (RQA), are based on point density and on the length of diagonal and vertical line structures visible in the RP, being regarded as alternative measures to quantify the complexity of physical systems. In order to uncover their time-dependent behaviour, RQA measures are often computed by applying a sliding window on the time series, which then can be used to identify dynamical transitions, such as period-chaos transitions (Trulla et al., 1996) or chaos-chaos transitions (Marwan et al., 2002).

Another popular method to analyse complex systems is the complex network approach (Watts and Strogatz, 1998; Boccaletti et al., 2006). Complex network measurements are useful to investigate and understand the complex behaviour of real world systems such as social, computer (Newman, 2002), or brain networks (Singer, 1999). The adjacency matrix of a complex network explains the structure of the system, thus, determines the links between the nodes of a network. For unweighted and undirected networks, the adjacency matrix is binary and symmetric, hence very similar to a RP. In our previous work, we have shown that time series can be analysed by complex networks by identifying the RP by the adjacency matrix of a network (Marwan et al., 2009; Donner et al., 2011), forming so-called recurrence networks (RNs). Complex network measures applied on RNs have been used to investigate real-world systems such as the climate system (Donges et al., 2011) or the cardio-respiratory system (Ramírez Ávila et al., 2013). RNs have been shown to be more sensitive for the detection of periodic-chaos or chaos-periodic regime transitions than some of the standard RQA measures (Zou et al., 2010; Marwan, 2011).





## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



where  $N$  is the trajectory length,  $\Theta(\cdot)$  is the Heaviside function, and  $|\cdot|$  is the norm of the adopted phase space (Marwan et al., 2007). Thus,  $R_{i,j} = 1$  if states at times  $i$  and  $j$  are recurrent, and  $R_{i,j} = 0$  otherwise. The trajectory in the phase space can be reconstructed via time delay embedding from a time series  $\{u_i\}_{i=1}^N$  (Packard et al., 1980)

$$\mathbf{x}_i = (u_i, u_{i+\tau}, \dots, u_{i+\tau(m-1)}) \tag{2}$$

where  $m$  is the embedding dimension and  $\tau$  is the embedding delay. The embedding dimension  $m$  can be found by false nearest neighbours and the delay  $\tau$  by mutual information or auto-correlation (Kantz and Schreiber, 1997).

The main diagonal of the RP,  $R_{i,i} = 1$ , represents the line of identity (LOI). As we have mentioned the RP is a symmetric, binary matrix. The structures formed by line segments, which are parallel to the LOI in a RP, characterize typical dynamical properties. We observe homogeneously distributed recurrence points if the dynamics is white noise. If the system is deterministic, diagonal line segments which are parallel to the LOI will dominate. The dynamics is related to the length of the diagonal line segments: chaotic dynamics causes mainly short line segments, but contrary, regular (periodic) dynamics causes long line segments. The RQA quantifies this relation and can be used to detect transitions in the system's dynamics (Trulla et al., 1996; Marwan et al., 2007).

Recurrence networks are based on the recurrence matrix, Eq. (1) which is a  $N \times N$  matrix where  $N$  is the length of the phase space trajectory (the number of time steps). We now consider these time steps as nodes of a network; if the nodes are sufficiently close to each other, in other words, if the space vectors are neighbours, there is a link between them. In network theory, connections between network nodes can be described with the adjacency matrix  $\mathbf{A}$ , with  $A_{i,j} = 1$  if there is a link between nodes  $i$  and  $j$ , otherwise  $A_{i,j} = 0$ . To obtain the adjacency matrix from the recurrence matrix, we discard self-loops in the recurrence matrix, i.e.

$$A_{i,j} = R_{i,j} - \delta_{i,j} \tag{3}$$



## 3 Applications

### 3.1 Logistic map

As a first application we compare some RN measures for using first the adaptive and then the constant threshold approach by analysing the logistic map,

$$x_{i+1} = ax_i(1 - x_i). \quad (5)$$

It is one of the most popular iterated maps which has different regimes for different control parameter  $a$ . The detection of the transitions of the logistic map between these different regimes was studied with RP and RN previously (Trulla et al., 1996; Marwan et al., 2009). The logistic map shows interesting dynamics in the range of the control parameter  $a \in [3.5, 4.0]$ , which is studied here with a step size of  $\Delta a = 0.0005$ , there occur e.g. periodic and chaotic regimes, bifurcations, inner and outer crises. We compute a time series of length  $N = 5000$  for each value of  $a$ . In order to discard transients, we delete the first 2000 values, resulting in time series consisting of 3000 values that have been used for all analysis of the logistic map in this paper.

As the regular threshold selection method, we use the recurrence rate method to choose a threshold value: a threshold is selected in such a way that the recurrence rate RR is constant even for different time series with different dynamics (e.g. different values of  $a$ ) (Marwan et al., 2007). In this paper, we use  $RR = 5\%$  for further analysis.

Now we compute the RNs by using the given threshold selection techniques  $\epsilon$  and  $\epsilon_c$  for each control parameter  $a$ . We then calculate transitivity  $T$  and betweenness centrality BC as the complex networks measures in order to detect the transitions from periodic to chaotic, chaotic to periodic states, bifurcations and inner(outer)-crisis. The network transitivity is given by,

$$T = \frac{\sum_{i,j,k} A_{i,j} A_{j,k} A_{k,i}}{\sum_{i,j,k} A_{k,i} A_{k,j}}. \quad (6)$$

## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



The average betweenness centrality of network,

$$BC = \frac{1}{N} \sum_v \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}, \quad (7)$$

where  $\sigma_{st}$  is the total number of shortest paths from node  $s$  to node  $t$  and  $\sigma_{st}(v)$  is number of those paths that pass through  $v$ . As mentioned in the previous chapter, not all complex network measures can be applied to a disconnected network. However, it would cause problems for computing the measures on RNs calculated by using the constant threshold technique, since the network could be disconnected. For instance, to compute the average shortest path length or assortativity for an entire network, the network must be connected. Disconnected nodes of the network could be discarded from the calculation, but in this case, we would lose information. In the adaptive threshold case, we could calculate all these measurements on the entire network since the selection of the adaptive threshold ensures that the recurrence network is connected.

Both threshold selection methods could detect transitions between dynamical regimes (periodic-chaos or chaos-periodic) very well. Transitivity gives large values for the chaotic regime and small values for periodic. In the betweenness centrality case, it is contrary to transitivity, large values for periodic and small values for the chaotic regimes. The transitivity and betweenness centrality for the constant threshold selection case,  $T_{\text{constant}}$  and  $BC_{\text{constant}}$ , cannot distinguish between different periodic dynamics, i.e. cannot detect certain bifurcation points such as for period doublings, e.g. at  $a \approx 3.544, 3.564, 3.84$ . Contrary, in the adaptively chosen threshold case,  $T_{\text{adaptive}}$  and  $BC_{\text{adaptive}}$  are sensitive to these bifurcations (Figs. 2 and 3). Thus, the adaptive threshold case allows also the detection of period-period transitions.

### 3.2 Application to palaeoclimate record

The study of palaeoclimate variation helps in understanding and evaluating possible future climate change. Lake sediments provide valuable archives of past climate variations.

## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion





## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



In the following we will focus on a well dated high resolution climate archive from palaeolake Lisan located beneath the archaeological site of Massada in the near East (Prasad et al., 2004, 2009). The sediments from the Upper Member were deposited (26–18 cal ka BP) when the lake reached its highest stands (Bartov et al., 2003; Torfstein et al., 2013). The sedimentary sequence contains varves comprising seasonally deposited primary (evaporitic) aragonite and silty detritus (Prasad et al., 2004). The pure aragonite sublaminae were precipitated from the upper layer of the lake during summer evaporation. Their formation requires inflow of  $\text{HCO}_3^-$  ions into the lake from the catchment area during winter floods (Stein et al., 2003) that also bring in silty detrital material. One detrital and overlying aragonite sublaminae constitute a varve. Previous studies (Prasad et al., 2004; Torfstein et al., 2013) indicate that small ice-rafting events (denoted as *a*, *b*, *c*, and *d*), as well as prominent Heinrich events in the North Atlantic, are associated with the Eastern Mediterranean arid intervals. The study of seasonal sublaminae yields evidence of decadal to century scale arid events that correlate with cooler temperatures at higher latitudes. Analyses in the frequency domain indicate the presence of periodicities centered at 1500, 500, 192, 139, 90, and 50–60 yr, suggesting a solar forcing on climate (Prasad et al., 2004).

We use the yearly sampled pure aragonite proxy ( $\text{CaCO}_3$ ) from the palaeolake Lisan for our RN analysis (Fig. 4a). We use a time delay embedding with dimension  $m = 3$  and delay  $\tau = 2$  (these parameters have been computed by standard procedure using false nearest neighbours and mutual information (Packard et al., 1980; Kantz and Schreiber, 1997)) for reconstructing the phase space. To detect dynamical transitions in the palaeoclimate data, we adopt a sliding window of  $W$  data points with a step size of  $\Delta W$ . RNs are computed for each window of the time series one by one. We have chosen a sampling window size of  $\Delta T = 100$  yr with 90 % overlap corresponding to a time window size of  $W \approx 100$  data points (since there are some gaps in the data, it is not exactly 100). The time series' length is  $N = 7665$  and the total number of the windows analysed is

$$\frac{N - W}{\Delta W} \approx 755.$$

Transitivity and betweenness centrality is then calculated within these windows (Fig. 4b and c). As we have shown for the logistic map, transitivity and betweenness centrality are both sensitive to detect transitions. Larger values of transitivity  $T$  refer to regular behaviour, whereas smaller values to more irregular dynamics in the considered window of the time series.

The grey shaded horizontal band in Fig. 4b and c is the confidence interval of the network measures. We apply a simple statistical test in order to see whether the characteristics of the dynamics at a certain time differs from the general characteristics of the dynamics. In order to apply this statistical test, we use the following approach. We create surrogate data segments of length  $W$  by drawing data points randomly from the entire time series and we compute the RN and the network measures from such a surrogate segment. We repeat this 10 000 times and have an empirical test distribution of transitivity  $T$  and betweenness centrality BC. A confidence interval is then estimated from these distributions by their 0.05 and 0.95 quantiles.

Previous studies (Prasad et al., 2004) had identified multiple climate fluctuations in the varved Lisan record and correlated them with the Greenland oxygen isotope data (indicative of temperature changes, Stuiver and Grootes, 2000) and ice rafting events in the north Atlantic (Bond et al., 1997). The blue and orange vertical bars in Fig. 4 delineate periods of cooling and warming respectively in the higher latitudes that resulted in drier and wetter episodes in the eastern Mediterranean.

The network measures  $T$  and BC both indicate well abrupt transitions (Fig. 4b and c). In particular for  $T$ , the values jump between high and low values.  $T$  reveals epochs of significantly low values at around 25.8–25.6, 25.2–25.1, 24.3–24.2, 24.0–23.9, 22.8–22.6, 22.3–22.1, 21.5–21.1, 21.7, 20.6–20.5, 20.1–19.9, 19.8–19.6, and 19.3–18.9 cal ka BP. The periods 25.8–25.6, 22.3–22.1, 21.5–21.1, and 19.3–18.9 cal ka BP correspond to the known Bond events  $d$ ,  $c$ ,  $b$ , and  $a$ , and the epoch between 24.3 and

## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

23.9 cal ka BP coincides with the Heinrich H2 event. During the interstadial peaks IS2 at 23.8–23.7 and 23.3–23.2 cal ka BP,  $T$  shows significant high values, almost reaching the value one. BC exhibits a rather similar behavior of abrupt transitions like  $T$ , but with opposite sign. A general observation is that low values in  $T$  can be found during dry but high values during wet regimes, and that such regimes change abruptly.

A high transitivity value indicates a more regular deposition of aragonite, and, thus, a more regular, or even periodic climate variability. This could be an indication for a dominant role of the (more or less periodic) solar forcing via its influence on the temperature in the higher latitudes. During phases of a colder North Atlantic, the solar forcing become less important but regional climate effects more important and dominating, causing a more complex, irregular climate variability, finally indicated by low values of  $T$ .

Combining the maxima of  $T$  and minima of BC, we can identify the above mentioned periods of non-regular climate dynamics. Most of these periods correspond to cold events, e.g. the Bond events and Heinrich event, and the found Lisan lake events L3 till L13 (Prasad et al., 2004). Several regular periods can be identified, some of them coinciding with the warm period during the interstadial IS2. Few remaining periods of high or low regularity have not yet been identified in the literature so far and call for further investigation.

The abrupt changes in  $T$  are available due to the adaptive threshold. By using a constant threshold,  $T$  varies only slowly and more gradual. Defining the time points of the climate regime shifts becomes more difficult in this case.

## 4 Conclusions

We have represented a novel method to chose a recurrence threshold adaptively and compared with the constant threshold selection technique. The selection of recurrence thresholds for recurrence plots and recurrence networks is a crucial step for these techniques. So far, the threshold had to be chosen arbitrarily, taking into account different

## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

criteria and application cases as well as requiring some expertise. Here we have proposed a novel technique to determine such a threshold value automatically depending on the time series. Such adaptive threshold is directly derived from the topology of the recurrence network. It is selected in such a way that the recurrence network does not have unconnected components. We have discussed transitivity and betweenness centrality measures of the complex network approach. Both measures are related to the regularity of the dynamics.

We have compared the novel adaptive threshold selection with the arbitrarily selected threshold by applying them to the logistic map. Although both methods distinguish the dynamical regimes clearly, the adaptively chosen threshold approach detects much more bifurcations, in particular such as period doubling. Such bifurcations are important characteristics of the dynamical systems, since these bifurcations route to chaos from periodicity.

Moreover, we have used our approach to investigate a palaeoclimate proxy record from the palaeolake Lisan representing the climate variability in the near East between 27 and 18 cal ka BP. Both transitivity and betweenness centrality measures clearly identified transitions between wet and dry (and vice versa) periods by an abrupt decrease of dynamical regularity, perhaps due to a reduced solar influence. Our method identified some transitions which have not been known so far from the literature and require further investigation, e.g. by analyzing other proxy records from this region. By choosing the adaptive threshold, we have been able to identify the transitions more clearly than by using the arbitrary selected threshold approach.

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30

### Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

- Donner, R. V., Small, M., Donges, J. F., Marwan, N., Zou, Y., Xiang, R., and Kurths, J.: Recurrence-based time series analysis by means of complex network methods, *Int. J. Bifurcat. Chaos*, 21, 1019–1046, doi:10.1142/S0218127411029021, 2011. 805
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## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

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## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

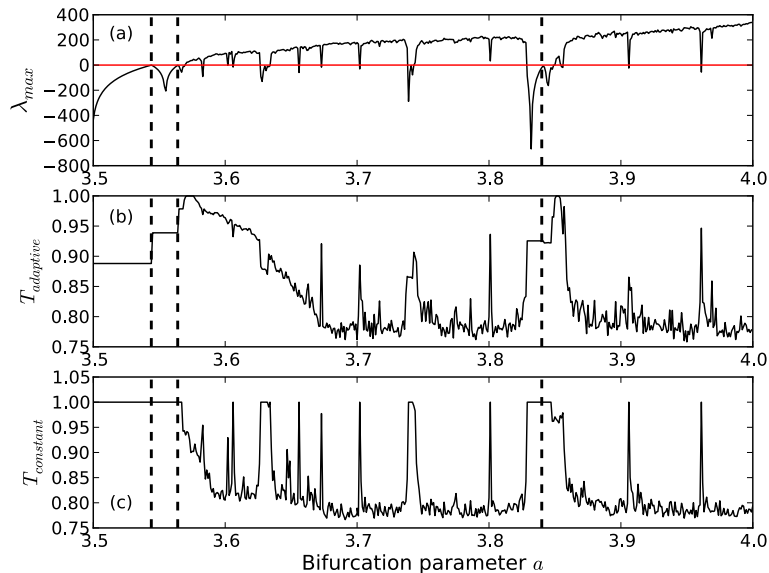






## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.



**Fig. 2.** (a) Lyapunov exponent and transitivity using (b) adaptive threshold and (c) constant threshold for the logistic map. Dashed lines show certain bifurcation points before the chaotic regime.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

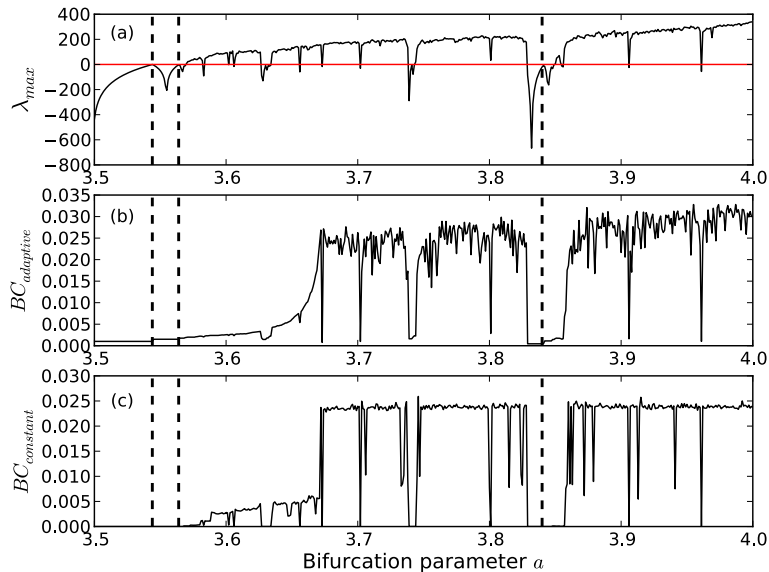
Full Screen / Esc

Printer-friendly Version

Interactive Discussion

## Finding recurrence networks' threshold adaptively for a specific time series

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**Fig. 3.** (a) Lyapunov exponent and betweenness centrality using (b) adaptive threshold and (c) constant threshold for the logistic map. Dashed lines show certain bifurcation points before the chaotic regime.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

## Finding recurrence networks' threshold adaptively for a specific time series

D. Eroglu et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

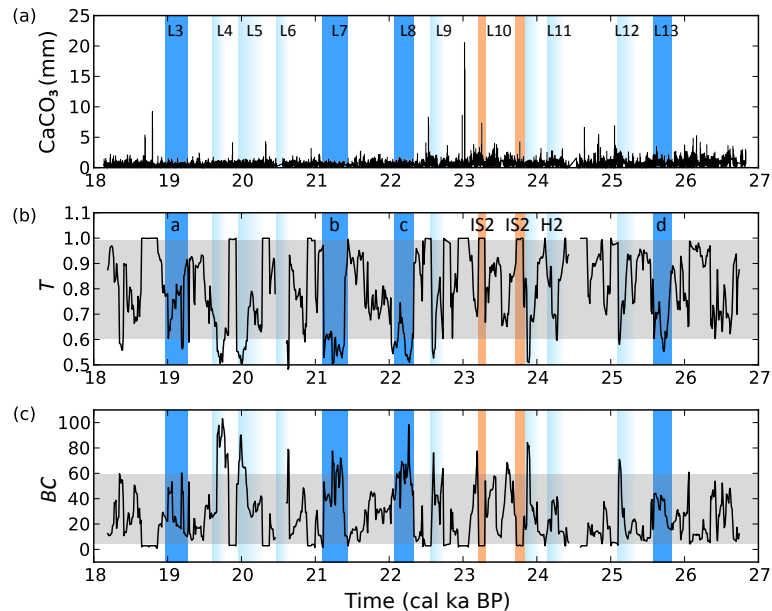
Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



**Fig. 4.** (a) Aragonite ( $\text{CaCO}_3$ ) record from palaeolake Lisan, (b) transitivity, and (c) betweenness centrality results of RN using the adaptive threshold. Abrupt changes in  $T$  and  $BC$  indicate transitions between different climate regimes. Dry events in Lake Lisan (cooling of the higher latitudes) are marked by blue bars and two interstadial peaks (warming) by orange bars. The gray shaded band is the 90% confidence interval for the networks measures.