



**Time dependent
Long's equation**

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Time dependent Long's equation

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Abstract

Long's equation describes steady-state two-dimensional stratified flow over terrain. Its numerical solutions under various approximations were investigated by many authors. Special attention was paid to the properties of the gravity waves that are predicted to be generated as a result. In this paper we derive a time-dependent generalization of this equation and investigate analytically its solutions under some simplifications. These results might be useful in the experimental analysis of gravity waves over topography and their impact on atmospheric modeling.

1 Introduction

Long's equation (Long, 1953, 1954, 1955, 1959) model the flow of inviscid stratified incompressible fluid in two dimensions over terrain. When the base state of the flow (that is the unperturbed flow field far upstream) is without shear the solutions of this equation are in the form of steady lee waves. Solutions of this equation in various settings and approximations were studied by many authors (Drazin, 1961; Drazin and Moore, 1967; Durran, 1992; Lily and Klemp, 1979; Peltier and Clark, 1983; Smith, 1980, 1989; Yih, 1967). The most common approximation in these studies was to set Brunt–Väisälä frequency to a constant or a step function over the computational domain. Moreover the values of the parameters β and μ which appear in this equation were set to zero. In this (singular) limit of the equation the nonlinear terms and one of the leading second order derivatives in the equation drop out and the equation reduces to that of a linear harmonic oscillator over two dimensional domain. Careful studies (Lily and Klemp, 1979) showed that these approximations are justified unless wave breaking is present in the solution (Peltier and Clark, 1983; Miglietta and Rotunno, 2014).

Long's equation provides also the theoretical framework for the analysis of experimental data (Fritts and Alexander, 2003; Shutts et al., 1988; Vernin et al., 2007; Jumper et al., 2004) under the assumption of shearless base flow. (An assumption which, in

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general, is not supported by the data.) An extensive list of references appears in Fritts and Alexander (2003), Baines (1995), Nappo (2012) and Yhi (1980).

An analytic approach to the study of this equation and its solutions was initiated recently by the current author (Humi, 2004). We showed that for a base flow without shear and under rather mild restrictions the nonlinear terms in the equation can be simplified. We also identified the “slow variable” that controls the nonlinear oscillations in this equation and using phase averaging approximation derived a formula for the attenuation of the stream function perturbation with height. This result is generically related to the presence of the nonlinear terms in Long’s equation. We explored also different formulations of this equation (Humi, 2007, 2009) and the effect of shear on the solutions of this equation (Humi, 2006, 2010).

One of the major obstacles to the application of Long’s equation in realistic applications is due to the fact that it is restricted to the description of steady states of the flow. It is therefore our objective in this paper to derive a time-dependent generalization of this equation and study the properties of its solutions. The resulting system contains two equations for the time evolution of the density and the stream function. While the equation for the stream function is rather complicated it can be simplified in two instances. The first corresponds to the classical (steady state) Long’s equation while the second is time dependent and new (as far as we know). In this paper we explore the properties of the flow in this second case which might find some applications in the analysis of experimental data about gravity waves (Vernin, 2007; Jumper, 2004; Nappo, 2012) and its application to atmospheric modeling (Jadwiga et al., 2010; Geller et al., 2013).

The plan of the paper is as follows: in Sect. 2 we derive the time dependent of Long’s equation. In Sect. 3 we consider the time evolution and proper boundary conditions on shearless flow over topography.

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2 Derivation of the time dependent Long's equation

In two dimensions (x, z) the flow of inviscid and incompressible stratified fluid is modeled by the following equations:

$$u_x + w_z = 0 \quad (1)$$

$$5 \quad \rho_t + u\rho_x + w\rho_z = 0 \quad (2)$$

$$\rho(u_t + uu_x + ww_z) = -p_x \quad (3)$$

$$\rho(w_t + uw_x + ww_z) = -p_z - \rho g \quad (4)$$

10 where subscripts indicate differentiation with respect to the indicated variable, $u = (u, w)$ is the fluid velocity, ρ is its density, p is the pressure and g is the acceleration of gravity.

We can non-dimensionalize these equations by introducing

$$\bar{x} = \frac{x}{L}, \quad \bar{z} = \frac{N_0}{U_0} z, \quad \bar{u} = \frac{u}{U_0}, \quad \bar{w} = \frac{LN_0}{U_0^2} w$$

$$15 \quad \bar{\rho} = \frac{\rho}{\rho_0}, \quad \bar{p} = \frac{N_0}{gU_0\rho_0} p \quad (5)$$

where L , U_0 , and ρ_0 represent respectively characteristic length, velocity and density. N_0 is the characteristic Brunt–Väisälä frequency

$$N_0^2 = -\frac{g}{\rho_0} \frac{d\rho_0}{dz}. \quad (6)$$

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In these new variables Eqs. (1)–(4) take the following form (for brevity we drop the bars)

$$u_x + w_z = 0 \quad (7)$$

$$\rho_t + u\rho_x + w\rho_z = 0 \quad (8)$$

$$\beta\rho(u_t + uu_x + ww_z) = -\rho_z \quad (9)$$

$$\beta\rho(w_t + uw_x + ww_z) = -\mu^{-2}(\rho_z + \rho) \quad (10)$$

where

$$\beta = \frac{N_0 U_0}{g} \quad (11)$$

$$\mu = \frac{U_0}{N_0 L} . \quad (12)$$

β is the Boussinesq parameter (Shutts et al., 1988; Baines, 1995) which controls stratification effects (assuming $U_0 \neq 0$) and μ is the long wave parameter which controls dispersive effects (or the deviation from the hydrostatic approximation). In the limit $\mu = 0$ the hydrostatic approximation is fully satisfied (Baines, 1995; Nappo, 2012).

In view of Eq. (7) we can introduce a stream function ψ so that

$$u = \psi_z, \quad w = -\psi_x . \quad (13)$$

Using this stream function we can rewrite Eq. (8) as

$$\rho_t + J\{\rho, \psi\} = 0 \quad (14)$$

where for any two (smooth) functions f, g

$$J\{f, g\} = \frac{\partial f}{\partial x} \frac{\partial g}{\partial z} - \frac{\partial f}{\partial z} \frac{\partial g}{\partial x} . \quad (15)$$

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Using $\bar{\psi}$ the momentum Eqs. (9) and (10) become

$$\beta\rho(\psi_{zt} + \psi_z\psi_{zx} - \psi_x\psi_{zz}) = -\rho_x \quad (16)$$

$$\beta\mu^2\rho(-\psi_{xt} - \psi_z\psi_{xx} + \psi_x\psi_{xz}) = -\rho_z - \rho. \quad (17)$$

5 We can suppress μ from the system (Eqs. 14, 16 and 17) if we introduce the following normalized independent variables

$$\bar{t} = \frac{t}{\mu}, \quad \bar{x} = \frac{x}{\mu}, \quad \bar{z} = z, \quad \mu \neq 0. \quad (18)$$

Equations (14) and (16) remain unchanged and Eq. (17) becomes

$$\beta\rho(-\psi_{xt} - \psi_z\psi_{xx} + \psi_x\psi_{xz}) = -\rho_z - \rho \quad (19)$$

10 where we dropped the bars on t, x, z . However we observe that in these coordinates $\psi_z = u$ and $\psi_x = -\mu w$.

Thus after all these transformations the system of equations governing the flow is Eqs. (14), (16) and (19).

15 To eliminate ρ from Eqs. (16) and (18) we differentiate these equations with respect to z, x respectively and subtract. This leads to

$$\begin{aligned} &\beta\rho_z(\psi_{zt} + \psi_z\psi_{zx} - \psi_x\psi_{zz}) + \\ &\beta\rho(\psi_{zzt} + \psi_z\psi_{zzx} - \psi_x\psi_{zzz}) - \\ &\beta\rho_x(-\psi_{xt} - \psi_z\psi_{xx} + \psi_x\psi_{xz}) - \\ 20 &\beta\rho(-\psi_{xxt} - \psi_z\psi_{xxx} + \psi_x\psi_{xxz}) = \rho_x. \end{aligned} \quad (20)$$

The sum of the second and fourth terms in this equation can be rewritten as

$$\beta\rho[\nabla^2\psi]_t + J\{\nabla^2\psi, \psi\}. \quad (21)$$

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(However observe that when $\mu \neq 1$, $\nabla^2 \psi$ does not represent the flow vorticity due to the transformation Eq. (18) and therefore the sum of the two terms in Eq. (21) is not zero in general.)

To reduce the first and third terms in Eq. (20) we use Eq. (14). We obtain

$$\begin{aligned}
 & \beta[\rho_z(\psi_{zt} + \psi_z\psi_{zx} - \psi_x\psi_{zz})] \\
 & \quad - \beta[\rho_x(-\psi_{xt} - \psi_z\psi_{xx} + \psi_x\psi_{xz})] \\
 & = \beta[\rho_z(\psi_{zt} + \rho_z\psi_z\psi_{zx} - (\rho_t + \rho_x\psi_z)\psi_{zz} \\
 & \quad + \rho_x\psi_{xt} + (\psi_x\rho_z - \rho_t)\psi_{xx} - \rho_x\psi_x\psi_{xz}] \\
 & = \beta \left\{ \rho_z\psi_{zt} + \rho_x\psi_{xt} - \rho_t\nabla^2\psi + \frac{1}{2}J\{(\psi_x)^2 + (\psi_z)^2, \rho\} \right\}.
 \end{aligned}
 \tag{22}$$

Combining the results of Eqs. (21) and (22) Eq. (20) becomes

$$\begin{aligned}
 & \rho \left[(\nabla^2\psi)_t + J\{\nabla^2\psi, \psi\} \right] \rho_z\psi_{zt} + \rho_x\psi_{xt} \\
 & \quad + \left[-\rho_t\nabla^2\psi + \frac{1}{2}J\{(\psi_x)^2 + (\psi_z)^2, \rho\} \right] = \frac{J\{\rho, z\}}{\beta}.
 \end{aligned}
 \tag{23}$$

Thus we have reduced the original four Eqs. (1)–(4) to two Eqs. (14) and (23). This system of equations can be considered as the generalization of Long's equation to time dependent flows. While Eq. (23) is rather complicated in general it can be simplified further in two special cases. The first is when one considers the steady state of the flow. This restriction leads to Long's equation (Long, 1953; Smith, 1989; Baines, 1995).

The second case happens when $\nabla^2\psi = 0$ i.e ψ is harmonic. (Observe however that this does not imply that the vorticity is zero due to the transformation (Eq. 18) unless $\mu = 1$.) Equation (23) becomes

$$\rho_z\psi_{zt} + \rho_x\psi_{xt} + \frac{1}{2}J\{(\psi_x)^2 + (\psi_z)^2, \rho\} = \frac{J\{\rho, z\}}{\beta}.
 \tag{24}$$

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However if $\nabla^2 \psi = 0$ we can define $v_1 = \psi_z$ and $v_2 = -\psi_x$. (These definitions use the stretched coordinates of Eq. 18). This implies that,

$$(v_1)_z - (v_2)_x = 0.$$

This implies that there exists a function η so that

$$5 \quad \eta_x = v_1, \quad \eta_z = v_2.$$

That is

$$\eta_x = \psi_z, \quad \eta_z = -\psi_x.$$

Physically these relations imply that $\eta_x = u$ and $\eta_z = \mu w$.

Replacing ψ by η in Eq. (24) yields

$$10 \quad J \left\{ \eta_t + \frac{1}{2} [(\eta_x)^2 + (\eta_z)^2] + \frac{z}{\beta}, \rho \right\} = 0. \quad (25)$$

Hence

$$\eta_t + \frac{1}{2} [(\eta_x)^2 + (\eta_z)^2] + \frac{z}{\beta} = R(\rho) \quad (26)$$

where $R(\rho)$ is a parameter function that can be determined from the asymptotic conditions on the flow. To summarize: the equations of the flow in this case are

$$15 \quad \rho_t + \eta_x \rho_x + \eta_z \rho_z = 0 \quad (27)$$

(which replaces Eq. 14), and Eq. (26).

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Other reductions of Eq. (23)

The reduction of Eq. (23) was carried out above under the assumption $\nabla^2\psi = 0$. However it can be generalized to case $\nabla^2\psi = a$ where a is a constant. To this end we define

$$v_1 = \psi_z, \quad v_2 = -\psi_x + ax.$$

5 Therefore

$$(v_1)_z - (v_2)_x = 0,$$

which implies that there exists a function η so that

$$\eta_x = v_1, \quad \eta_z = v_2.$$

Hence

$$10 \quad \eta_x = \psi_z, \quad \eta_z = -\psi_x + ax. \quad (28)$$

Using these relations to substitute η for ψ in Eq. (23) leads to

$$\rho_z \eta_{xt} - \rho_x (\eta_z - ax)_t + \left[-a\rho_t + \frac{1}{2} J \left\{ (\eta_z - ax)^2 + (\eta_x)^2, \rho \right\} \right] = \frac{J\{\rho, z\}}{\beta}. \quad (29)$$

Therefore

$$J\{\eta_t, \rho\} - a\rho_t + \frac{1}{2} J \left\{ (\eta_z - ax)^2 + (\eta_x)^2, \rho \right\} = \frac{J\{\rho, z\}}{\beta}. \quad (30)$$

15 Hence

$$-a\rho_t + J \left\{ \eta_t + \frac{1}{2} [(\eta_z - ax)^2 + (\eta_x)^2] + \frac{z}{\beta}, \rho \right\} = 0. \quad (31)$$

Using Eq. (14) we have

$$-aJ\{\psi, \rho\} + J\left\{\eta_t + \frac{1}{2}[(\eta_z - ax)^2 + (\eta_x)^2] + \frac{z}{\beta}, \rho\right\} = 0. \quad (32)$$

It follows then that

$$-a\psi + \eta_t + \frac{1}{2}[(\eta_z - ax)^2 + (\eta_x)^2] + \frac{z}{\beta} = R(\rho). \quad (33)$$

5 We can eliminate ψ from this equation by differentiating with respect to z and use Eq. (28)

$$-a\eta_x + \left[\eta_t + \frac{1}{2}[(\eta_z - ax)^2 + (\eta_x)^2]\right]_z = -\frac{1}{\beta} + R(\rho)_z. \quad (34)$$

3 Time evolution of stratified flow

10 In this section we shall consider the time evolution of a stratified shearless base flow viz. a flow which satisfies as $t \rightarrow -\infty$

$$\lim_{x \rightarrow -\infty} \rho^0(t, x, z) = \frac{H - z}{H}, \quad \lim_{x \rightarrow -\infty} u = 1, \quad \lim_{x \rightarrow -\infty} v = 0 \quad (35)$$

i.e. the far upstream flow is independent of time and satisfies asymptotically $u = 1$, $v = 0$ and ρ^0 is stratified with height (H is a height at which $\rho^0 \approx 0$). The conditions on u, v imply that asymptotically $\eta^0 = x$.

15 In these limits Eq. (27) is satisfied. Substituting these limiting values in Eq. (26) we obtain that

$$R(\rho) = \frac{z}{\beta} + \frac{1}{2} = \frac{H(1 - \rho)}{\beta} + \frac{1}{2}. \quad (36)$$

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However it is obvious that different profiles of the base flow will yield different $R(\rho)$.

We now consider perturbations from the (shearless) base flow described by Eq. (35) due to shape of the topography viz.

$$\eta = \eta^0 + \epsilon\phi, \quad \rho = \rho^0 + \epsilon\zeta \quad (37)$$

5 From Eqs. (26) and (27) we obtain to first order in ϵ the following equations for ϕ and ζ

$$\frac{\partial\phi}{\partial t} + \frac{\partial\phi}{\partial x} + \frac{H\zeta}{\beta} = 0 \quad (38)$$

$$\frac{\partial\zeta}{\partial t} + \frac{\partial\zeta}{\partial x} - \frac{1}{H} \frac{\partial\phi}{\partial z} = 0. \quad (39)$$

10 To find the general form of the solution of these equations we use Eq. (38) to express ζ in terms of ϕ and substitute in Eq. (39). This yields the following equation for ϕ

$$\frac{\partial^2\phi}{\partial t^2} + 2\frac{\partial^2\phi}{\partial t\partial x} + \frac{\partial^2\phi}{\partial x^2} + \frac{1}{\beta} \frac{\partial\phi}{\partial z} = 0. \quad (40)$$

It is possible to find “elementary solutions” to this equation by separation of variables if we let

$$15 \quad \phi = \rho(t, x)F(z),$$

where c is arbitrary positive constant so that ϕ to represents a perturbation moving forward in time. This leads to

$$\frac{\frac{\partial^2\rho}{\partial t^2} + 2\frac{\partial^2\rho}{\partial t\partial x} + \frac{\partial^2\rho}{\partial x^2}}{\rho} = -\frac{1}{\beta} \frac{F(z)'}{F(z)} = -\omega^2 \quad (41)$$

20 where ω^2 is the separation of variables constant. Primes denote differentiation with respect to the appropriate variable.

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Solving Eq. (41) we obtain the following elementary solution for ϕ

$$\phi_{\omega} = C_{\omega} \exp[\beta\omega^2 z][G(x-t)\cos\omega t + K(x-t)\sin\omega t] \quad (42)$$

where $G(x-t)$, $K(x-t)$ are arbitrary smooth functions and C_{ω} is a constant.

The corresponding solution for ζ can be obtained by substituting this result in Eq. (38)

$$\zeta = C_{\omega} \frac{\beta\omega}{H} \exp[\beta\omega^2 z][G(x-t)\cos\omega t - K(x-t)\sin\omega t]. \quad (43)$$

Hence the general solution for ϕ can be written as

$$\phi = \int_0^{\infty} \exp[\beta\omega^2 z][G_{\omega}(x-t)\cos\omega t + K_{\omega}(x-t)\sin\omega t] d\omega \quad (44)$$

with similar expression for ζ .

Boundary conditions

We consider a flow in an unbounded domain over topography with shape $f(x)$ and maximum height h and impose the following boundary conditions on ρ and ψ in the limits $x = -\infty$ and $t = -\infty$

$$\psi(-\infty, -\infty, z) = z, \quad \rho(-\infty, -\infty, z) = \rho^0(z). \quad (45)$$

(This implies that in this limits $\eta = x$).

At the topography we impose the following boundary condition on ρ at $t = 0$

$$\rho(0, x, \epsilon f(x)) = \rho^0(\epsilon f(x)) = \frac{H - \epsilon f(x)}{H} \quad (46)$$

but

$$\rho(0, x, \epsilon f(x)) \approx \rho^0(0, x, 0) + \epsilon \zeta(0, x, z).$$

Hence at the topography

$$\zeta(0, x, \epsilon f(x)) = -\frac{f(x)}{H}. \quad (47)$$

To derive the corresponding boundary condition for η we first consider the appropriate boundary condition on the stream function ψ along the topography. To this end we assume that the topography is a line on which the stream function is constant and this constant can be chosen to be zero. For the base flow described in Eq. (45) $\psi_0 = z$ and $\psi = \psi_0 + \epsilon\psi_1$ where ψ_1 is the perturbation due to the topography. Hence along the topography

$$0 = \psi_0 + \epsilon\psi_1 = z + \epsilon\psi_1(0, x, \epsilon f(x)) = \epsilon f(x) + \epsilon\psi_1(0, x, \epsilon f(x)). \quad (48)$$

Therefore along the topography we let $\psi_1(0, x, \epsilon f(x)) = -f(x)$. We now observe that by definition $\psi_x = \eta_z$. But $\psi_x = \epsilon\psi_x^1 = -\epsilon f'(x)$, (where primes denote differentiation with respect to x) and $\eta = -x + \epsilon\phi$. Therefore we infer that the boundary condition on η along the topography is

$$\phi_z(0, x, \epsilon f(x)) = -f'(x) \quad (49)$$

(which is consistent with Eq. 39).

As to the boundary condition on $\eta(t, \infty, z)$ we observe that the system Eqs. (26) and (27) contains no dissipation terms and therefore only radiation boundary conditions can be imposed in this limit. (Physically this means that the horizontal group velocity is positive and energy is radiated outward.) Similarly at $z = \infty$ it is customary to impose (following Durran, 1992) radiation boundary conditions. However in view of Eqs. (42) and (43) it is obvious that the perturbation described by these equation is propagating forward in time and this condition is satisfied. A formal verification of this constraint is possible by expressing F, G, K in these equations using Fourier transform representation.

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For low lying topography (viz $\epsilon \ll 1$) it is customary to replace the boundary conditions Eqs. (46) and (47) by

$$\zeta(0, x, 0) = -\frac{f(x)}{H}, \quad \phi_z(0, x, 0) = -f'(x). \quad (50)$$

Example If $f(x)$ is given by a “witch of Agnesi” curve then

$$f(x) = \frac{a^2}{(a^2 + x^2)}, \quad f'(x) = -\frac{2a^2x}{(x^2 + a^2)^2}. \quad (51)$$

Let the initial perturbation in ρ be

$$\zeta(0, x, z) = e^{\beta\lambda^2 z},$$

where λ is a constant. From Eq. (50) we infer that the general expression for ζ is given by Eq. (43) with $\omega = \lambda$. Hence at $t = 0$ we must have

$$G(x) = -\frac{f(x)}{\beta\lambda}.$$

Similarly the boundary condition on ϕ yields

$$K(x) = -\frac{f'(x)}{\beta\lambda^2}.$$

Figures 1 and 2 exhibit cross sections of the perturbation at $z = 2$ and $x = 20$ at different times with $C_\omega = 0.1$, $a = 2$, and $\lambda = 1$.

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Time dependent Long's equation

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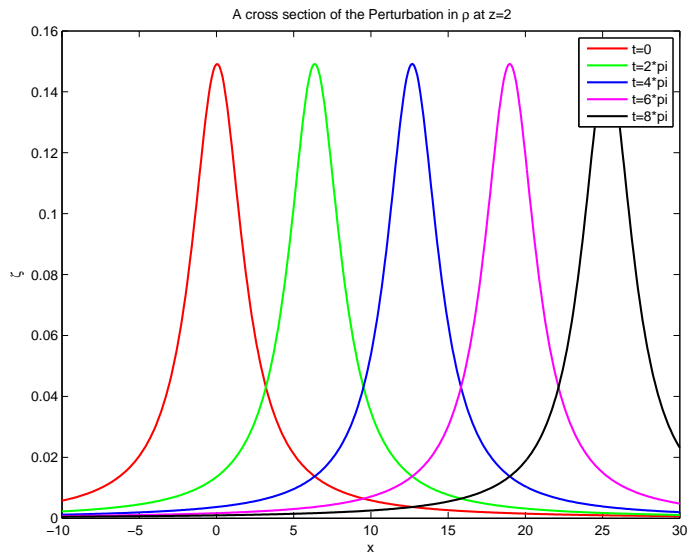


Figure 1. A cross section of the perturbation in ρ at $z = 2$.

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Time dependent Long's equation

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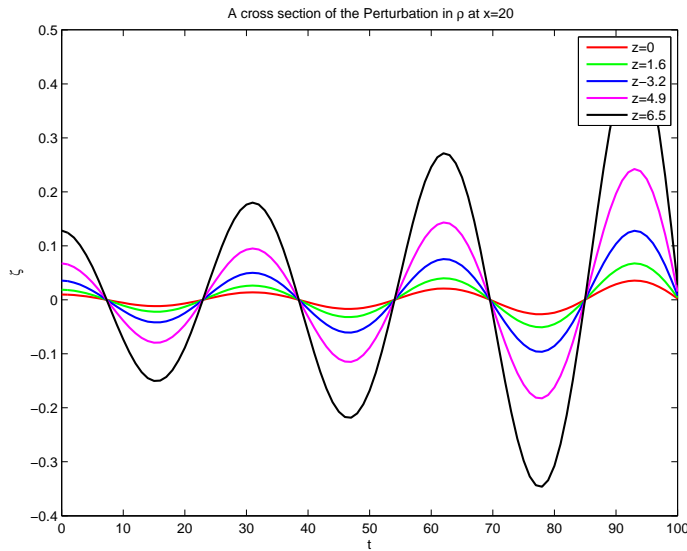


Figure 2. A cross section of the perturbation in ρ at $x = 20$.

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