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Self-organization of ULF electromagnetic wave structures in the shear flow driven dissipative ionosphere

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Abstract

This work is devoted to investigation of nonlinear dynamics of planetary electromagnetic (EM) ultra-low-frequency wave (ULFW) structures in the rotating dissipative ionosphere in the presence of inhomogeneous zonal wind (shear flow). Planetary EM

- ⁵ ULFW appears as a result of interaction of the ionospheric medium with the spatially inhomogeneous geomagnetic field. The shear flow driven wave perturbations effectively extract energy of the shear flow increasing own amplitude and energy. These perturbations undergo self organization in the form of the nonlinear solitary vortex structures due to nonlinear twisting of the perturbation's front. Depending on the features
- of the velocity profiles of the shear flows the nonlinear vortex structures can be either monopole vortices, or dipole vortex, or vortex streets and vortex chains. From analytical calculation and plots we note that the formation of stationary nonlinear vortex structure requires some threshold value of translation velocity for both non-dissipation and dissipation complex ionospheric plasma. The space and time attenuation specifi-
- cation of the vortices is studied. The characteristic time of vortex longevity in dissipative ionosphere is estimated. The long-lived vortices transfer the trapped medium particles, energy and heat. Thus they represent structural elements of turbulence in the ionosphere.

1 Introduction

- The interaction between the solar wind (SW) and terrestrial magnetosphere is the primary driver of many processes and phenomena occurring in the magnetosphere and consequently, in the ionosphere. However, many of the energy transfer processes have a sporadic/bursty character, and observations have highlighted that the vortex-like plasma flow structures are common in the Earth's magnetosphere. They prevail in the nightside plasma sheet (Hones, 1978, 1981, 1983; Keiling et al., 2009) and at
- the flank magnetospheric low-latitude boundary layers (LLBLs) (Fairfield et al., 2009) and at



Otto and Fairfield, 2000; Hasegawa et al., 2004). Plasma vortex-like flows have also been observed on the middle to high-latitude boundary of the outer radiation belt by the Cluster spacecraft fleet (Zong et al., 2009). The plasma flow vortex found in the magnetotail is characterized by pronounced vortical motion in the plane that is approx-

- imately parallel to the ecliptic plane. Vortex-like plasma flows in the plasma sheet are thought to be important in transportation of the kinetic energy from fast flow or bursty bulk flows (BBFs) in the magnetotail to the near Earth region (Snekvik et al., 2007; Keiling et al., 2009). A sudden braking and/or azimuthal deflection of BBFs may generate the plasma flow vortices at the boundary between the magnetotail plasma sheet and
- the inner magnetosphere as suggested by Hasegawa (1979) and Vasyliunas (1984). Keika et al. (2009) have shown that plasma vortices are formed near the region where the earthward flows slow down and turn in azimuthal directions. The theoretical relation between the field-aligned current (FAC) and plasma vorticity showed that FACs are generated to transport transverse momentum along magnetic field lines (Pritchett
- and Coroniti, 2000). Keiling et al. (2009) presented a scenario in which the plasma flow vortices in the plasma sheet generated the FAC of the substorm current wedge (SCW) at the beginning of the substorm expansion phase and coupled to the ionosphere, causing the ionospheric vortices.

In the present paper, we continue investigation of a special type of internal waves, which appear in the ionosphere under the influence of the spatially inhomogeneous geomagnetic field and the Earth's rotation velocity (Aburjania et al., 2002, 2003, 2004, 2007). In the previous works generation mechanism and self-organization into the vortex structures is studied. Now, we are interested in large-scale (planetary) ultra-lowfrequency (ULF) electromagnetic (EM) vortex structures in the ionospheric medium (consisting of electrons, ions and neutral particles), which have a horizontal linear scale $L_{\rm h}$ of order 10³ km and higher, a vertical scale $L_{\rm y}$ of altitude scale order $H(L_{\rm y} \approx H)$.

Observations (Gershman, 1974; Gossard and Hooke, 1975; Kamide and Chian, 2007) show also, that spatially inhomogeneous zonal winds (shear flows), produced by nonuniform heating of the atmospheric layers by solar radiation, permanently exist



in the atmosphere and ionosphere layers. Herewith, investigation of the problem of generation and evolution of ionospheric EM ULF electromagnetic wave structures taking into account the inhomogeneous zonal wind (shear flow) becomes important.

2 The governing equations

- ⁵ We choose our model as a two-dimensional β-plane with sheared flow. Since the length of planetary waves (λ ≥ 10³ km) is comparable with the Earth's radius *R*, we investigate such notions in approximation of the β-plane, which was specially developed for analysis of large-scale processes (Pedlosky, 1978), in the "standard" coordinate system. In this system, the *x* axis is directed along the parallel to the east, the *y* axis along the meridian to the north and the *z* axis vertically upwards (the local Cartesian system). For simplicity, the equilibrium velocity *V*₀, geomagnetic field *H*₀, perturbed magnetic field *h* and frequency of Earth's rotation Ω₀ are given by formulas *V*₀ = *V*₀(*y*)*e_x*, *H*₀(0,0, −*H*_p cos θ), *h*(0,0, *h_z*), Ω₀(0,0,Ω₀ cos θ). Here and elsewhere *e*(*e_x*, *e_y*, *e_z*) denotes a unit vector, *H_p* = 5 × 10⁻⁵*T* is the value of geomagnetic
- field strength in the pole and we suppose that geomagnetic colatitude θ coincides with a geographical colatitude θ' . In the ionosphere the large-scale motions are quasihorizontal (two-dimensional) (Aburjania et al., 2002, 2003, 2006) and hydrodynamic velocity of the particles $\mathbf{V} = (V_{x,i}, V_{y,i}, 0)$. The fluid is assumed to be incompressible and therefore a stream function $\boldsymbol{\psi}$ can be defined trough $\mathbf{V} = [\nabla \boldsymbol{\psi}, \boldsymbol{e}_z]$. Medium motion is considered near the latitude $\varphi_0 = \pi/2 - \theta_0$.

Not considering any more detail in the new under review branches of planetary waves (see Aburjania et al., 2002, 2003, 2004, 2007, 2011) we would like to note that beginning with the altitude of 80 km and higher, the upper atmosphere of the Earth is a strongly dissipative medium. Often when modelling large-scale processes for this re-

²⁵ gion of the upper atmosphere, effective coefficient of Rayleigh friction between the ionospheric layers is introduced. The role of the ion friction rapidly increases at the altitudes above 120 km (Kelley, 1989; Kamide and Chian, 2007) and its analytical expression



coincides with the Rayleigh friction formula (Aburjania and Chargazia, 2007). Therefore, often during a study of large-scale $(10^3 - 10^4)$ km, ULF $(10 - 10^{-6})$ s⁻¹ wavy structures in the ionosphere, we will apply the well-known Rayleigh formula to dissipative force $F = -\Lambda V$, assuming the altitudes above (80-130) km $\Lambda \approx 10^{-5}$ s⁻¹ (Dickinson, 1969; Gosard and Hooke, 1975), and the altitudes above 130 km $\Lambda = Nv_{in}/N_n$, where N and N_n denote concentrations of the charged particles and neutral particles, v_{in} is frequency of collision of ions with molecules (Gershman, 1974; Kelley, 1989; Al'perovich and Fedorov, 2007).

The governing equations of the considered problem are the closed system of magnetohydrodynamic equations of the electrically conducting ionosphere (Gershman, 1974; Kelley, 1989; Aburjania et al., 2004, 2007; Al'perovich and Fedorov, 2007). The solution of the temporal evolution of inhomogeneously sheared flow reduces to solution of the set of nonlinear partial differential equations for ψ and magnetic field perturbation, h_z (see Aburjania et al., 2002):

$$\int_{15} \left(\frac{\partial}{\partial t} + V_0(y)\frac{\partial}{\partial x}\right)\Delta\psi + \left(\beta - V_0^{\prime\prime}\right)\frac{\partial\psi}{\partial x} + C_{\rm H}\frac{\partial h}{\partial x} + \Delta\Delta\psi = J(\psi, \Delta\psi),$$

$$\left(\frac{\partial}{\partial t}+V_0(y)\frac{\partial}{\partial x}\right)h-\beta_{\mathsf{H}}\frac{\partial\psi}{\partial x}+\delta\cdot C_{\mathsf{H}}\frac{\partial h}{\partial x}=J(\psi,h).$$

Here

$$\beta = \frac{\partial 2\Omega_{0}}{\partial y} = -\frac{1}{R} \frac{\partial}{\partial \theta} (2\Omega_{0}) = \frac{2\Omega_{0} \sin \theta_{0}}{R},$$

$$p_{H} = \frac{eN}{\rho c} \frac{\partial H_{0z}}{\partial y} = -\frac{N}{N_{n}} \frac{eH_{p}}{MRc} \sin \theta_{0} < 0, \quad V_{0}^{\prime\prime}(y) = \frac{d^{2}V_{0}(y)}{d^{2}y},$$

$$h = \frac{eN}{N_{n}Mc} h_{z}, \quad C_{H} = \frac{c}{4\pi eN} \frac{\partial H_{0z}}{\partial y} = -\frac{cH_{p}}{4\pi eNR} \sin \theta_{0} < 0,$$

$$\Delta = \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}}, \quad J(a,b) = \frac{\partial a}{\partial x} \cdot \frac{\partial b}{\partial y} - \frac{\partial a}{\partial y} \cdot \frac{\partial b}{\partial x}.$$

$$1435$$



(1)

(2)

(3)

 $\rho = N_n M$ is density of neutral particles; m and M are masses of electrons and ions (molecules); e is the magnitude of the electron charge; c is the light speed. Further we consider a motion in neighborhood of fixed latitude ($\theta = \theta_0$). The dimensionless pa-

rameter δ is introduced here for convenience. In the ionospheric E region (80–150) km, where the Hall effect plays an important role, this parameter is equal to unity ($\delta = 1$). In the F region (200–600) km, where the Hall effect is absent, δ turns to zero ($\delta = 0$). The system of Eqs. (1) and (2), at corresponding initial and boundary conditions, describes nonlinear evolution of the spatial two-dimensional large-scale ULF electromagnetic perturbations in sheared incompressible ionospheric E and F regions. 10

From the Eqs. (1) and (2) we determine the temporal evolution of the energy of wavy structures, E(x, y, t)

$$\frac{\partial E}{\partial t} = \int V_0'(y) \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} dx dy - \Lambda \int |\nabla \psi|^2 dx dy.$$
(4)

We note that in the absence of zonal flow ($V_0 = 0$) and Rayleigh friction ($\Lambda = 0$) the 15 wavy structure energy is conserved.

Therefore, the existence of sheared zonal flow can be considered as the presence of an external energy source. One can see, that it (term with $V_0(y)$ in Eq. 4) feeds the medium with external source of energy for generation of the wave structures (development of the shear flow instability). The shear flows can become unstable transiently

- 20 until the condition of the strong relationship between the shear flows and wave perturbations is satisfied (Chagelishvili et al., 1996; Aburjania et al., 2006), e. i. the perturbation falls into amplification region in the wave number space. Leaving this region, e. i. when the perturbation passes to the damping region in the wave vector space,
- it returns an energy to the shear flow and so on (if the nonlinear processes and self-25 organization of the vortex structure will not develop before) (Aburjania et al., 2006). The experimental and observation data shows the same (Gossard and Hooke, 1975; Pedlosky, 1987; Gill, 1982).

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3 Criterion of instability of ULF PEMW wave structures in the ionopashere with the shear flow

The character of the shear flow much defines the evolution of the wave perturbation in the medium. Therefore, the shear flow in the hydrodynamics and in magnetohydrody-

⁵ namics are often unstable (Gossard and Hooke, 1975; Mikhailovskii, 1974; Timofeev, 2000). Existing of the term proportional to $V_0'' = d^2 V_0 / dy^2$ in Eq. (1) is related with the criterion (condition) of instability of the shear flow. In the linear approximation for the small scale perturbations of the form $\psi(x, y, t), h(x, y, t) = (\psi_1(y), h_1(y)) \exp(ik_x x - i\omega)$ from the Eqs. (1) and (2) follows the equation of Ohr–Zomerfeld

$$i_{0} \quad i\frac{\Lambda}{\omega - k_{x}V_{0}} \left(\frac{d^{2}}{dy^{2}} - k_{x}^{2}\right)\psi_{1} + \left(\frac{d^{2}}{dy^{2}} - k_{x}^{2}\right)\psi_{1} - \frac{k_{x}\left(\beta - V_{0}^{\prime\prime}\right)}{\omega - k_{x}V_{0}}\psi_{1} + \frac{k_{x}^{2}C_{H}\beta_{H}}{(\omega - k_{x}V_{0})[\omega - k_{x}(C_{H} + V_{0})]}\psi_{1} = 0.$$

Neglecting the dissipation terms ($\Lambda \rightarrow 0$) from Eq. (5) we get

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$$\psi_1'' - k_x^2 \psi_1 - \frac{k_x \left(\beta - V_0''\right)}{\omega - k_x V_0} \psi_1 + \frac{k_x^2 C_{\mathsf{H}} \beta_{\mathsf{H}}}{(\omega - k_x V_0) [\omega - k_x (C_{\mathsf{H}} + V_0)]} \psi_1 = 0, \tag{6}$$

where $\psi_1'' = d^2 \psi_1 / dy^2$. Equation (6) represents modification of known Rayleigh equation (Timofeev, 2000) (at $\beta_H \rightarrow 0$). For determination shear flow instability criterion in our case Eq. (6) should be multiplied by ψ_1^* , excluding complex-conjugate expression and integrating obtained one between the edges y_1 and y_2 of the plasma flux:

$${}_{20} \int_{y_1}^{y_2} \frac{d}{dy} \left(\psi_1^* \frac{d\psi_1}{dy} - \psi_1 \frac{d\psi_1^*}{dy} \right) dy - \int_{y_1}^{y_2} \left[\frac{1}{\omega - k_x V_0} - \frac{1}{\omega^* - k_x V_0} \right] k_x \left(\beta - V_0'' \right) |\psi_1|^2 dy - \frac{1}{\omega^* - k_x V_0} \left[\frac{1}{\omega^* - k_x V_0} - \frac{1}{\omega^* - k_x V_0} \right] dy$$

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(5)

$$\int_{y_1}^{y_2} \left[\frac{k_x^2 C_{\rm H} \beta_{\rm H}}{(\omega - k_x V_0) [\omega - k_x (C_{\rm H} + V_0)]} - \frac{k_x^2 C_{\rm H} \beta_{\rm H}}{(\omega^* - k_x V_0) [\omega^* - k_x (C_{\rm H} + V_0)]} \right] |\psi_1|^2 \, \mathrm{d}y = 0.$$
(7)

Supposing, the frequency of the perturbation $\omega = \omega_0 + i\gamma$ to be complex, and wave vector k_x – the real, imaginary part of the Eq. (7) can be written as:

$${}_{5} \quad 2\gamma \int_{y_{1}}^{y_{2}} \frac{k_{\chi}}{\left(\omega_{1}^{2} + \gamma^{2}\right)} \left[\beta - V_{0}^{\prime\prime} - \frac{k_{\chi}C_{H}\beta_{H}}{\omega_{2}^{2} + \gamma^{2}}\omega_{3}\right] |\psi_{1}|^{2} dy = 0, \tag{8}$$

where $\omega_1 = \omega_0 - k_x V_0$, $\omega_2 = \omega_0 - k_x (C_H + V_0)$, $\omega_3 = 2\omega_0 - k_x (C_H + V_0)$. In case of $\omega_1, \omega_2, \omega_3, \gamma, |\Psi_1^2| > 0$, from Eq. (8) follows the condition of the linear instability of the shear flow:

$${}_{10} \quad \beta - V_0'' - \frac{k_x C_H \beta_H}{\omega_2^2 + \gamma^2} \omega_3 = 0.$$
(9)

When $\omega_0^{\dagger} = k_x(C_H + V_0)$, $C_H \gg V_0$ and correspondingly, $\omega_2 \approx 0, \omega_3 \approx k_x C_H$, from Eq. (9) we have:

$$\beta + \frac{k_x^2 C_{\rm H}^2 |\beta_{\rm H}|}{\gamma^2} - V_0^{\prime\prime} = 0.$$
 (10)

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For the critical (resonance) level of the ionosphere, where the phase velocity of the slow waves $V_{ph} = \omega^s / k_x$ can coincide with the winds speed, $V_{ph} = V_0(y_r)$ (i.e. $\omega_1 = \omega_0 - k_x v_0 \approx 0, \omega_2 \approx -k_x C_H, \omega_3 \approx -k_x C_H, |\omega_2| \gg \gamma$), equality (9) can be rewritten as:

 $\overline{\beta} - V_0^{\prime\prime}(y) = 0. \tag{11}$



Conditions (9)–(11) can be called as modified Rayleigh instability condition ($V_0'' = 0$) for ULF PEMW at corresponding parameters of the zonal flow, wave and medium. This condition (10) (or 11) for some resonance point $y = y_r$ of the shear flow is the necessary condition for the shear flow instability.

⁵ For the description of the features of unstable oscillations fulfilling the resonance condition with the shear flow $V_{ph} = V_0(y_r)$ and Eq. (9), then Eq. (6) can be written in the following form:

$$\psi_1'' - k_x^2 \psi_1 + \frac{V_0''(y) - \overline{\beta}}{V_0(y_r) - V_0(y)} \psi_1 = 0.$$
(12)

¹⁰ According to oscillatory theory (Timofeev, 2000), finite conditions (12) can have the discrete number of eigen functions $\psi_1^{(n)}$ with corresponding eigen values $k^{(n)}$ and frequencies $\omega^{(n)} = k^{(n)}V_0(y_r)$ if only potential $U_r(y)$ fulfills the condition

$$U_{r}(y) = \frac{V_{0}''(y) - \overline{\beta}}{V_{0}(y_{r}) - V_{0}(y)} > 0.$$
(13)

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Concrete condition for existence of the eigen functions $\psi_1^{(n)}$ depends on the concrete type of the shear flow velocity profile and correspondingly, on the type of $U_r(y)$. Criterion (13) presents necessary condition of the shear flow instability.

It must be mentioned, that for the waves fulfilling the necessary conditions of instability and resonance $V_{ph} = V_0(y_r)$, both – dividend and divisor in Eq. (13) become zero at $y = y_r$. Solving this uncertainty by means of L'Hopital's rule, we get enough condition of the shear flow instability:

$$U_r(y_r) = \frac{V_0''(y) - \overline{\beta}}{V_0(y_r) - V_0(y)}\Big|_{y=y_r} = -\frac{V_0'''(y_r)}{V_0'(y_r)} > 0.$$

(14)

In the earth atmosphere $\beta + k_x^2 C_H^2 |\beta_H| / \gamma^2$ and $\overline{\beta}$ can be greater or less than V_0'' . Thus, in the ionosphere episodic generation of such zonal wind is possible, that at some critical layer $y = y_r$ the conditions (9)–(14) can be fulfilled, which is the reason of instability during some time, after which the zonal wind reorganizes and becomes again stabile.

4 Shear flow driven nonlinear solitary vortex structures

As it is shown in the previous section, spontaneously generated ULF PEMW waves at different layers of the ionosphere at definite moment of evolution become unstable (at fulfilling instability condition 13 and 14) intensively pump shear flow energy (particularly, during $0 < \tau \le \tau^*$). Getting energy, amplitudes of PEMW grow up (several times) and correspondingly, the nonlinear processes come into play. Herewith, in initial dynamic Eqs. (1)–(5) nonlinear terms become sufficient and the whole system should be analyzed.

- Let us tern to study of the influence of the nonlinear effects on the dynamics of ¹⁵ ULF PEWM studied theoretically by us in the previous works (Aburjania et al., 2002, 2003, 2004, 2007) in non-dissipative ionosphere. The results of observations and missions show (Cmyrev et al., 1991) as it was described above, that the nonlinear solitary vortex structures can be generated at different layers of the atmosphere–ionosphere– magnetosphere. These structures transfer the trapped particles of medium. Therefore, ²⁰ relation of rotational velocity of particles U_C with the motion velocity of the nonlinear
- structures U is defined by $U_C/U \ge 1$ (Kamenkovich and Monin, 1978),

Let us introduce temperature *T* and spatial *L* characteristic scales of nonlinear structures. By virtue of Eq. (1) we can get the following relation between the values: $U_c \sim V, U \sim L/T$. Analogously, for the nonlinear and inertial terms we have: $(V\nabla)V/(\partial V/\partial t) \sim V/(L/T) \sim U_C/U \ge 1$. Thus, nonlinearity plays an important role for

²⁵ $(V \nabla)V/(\partial V/\partial t) \sim V/(L/T) \sim U_C/U ≥ 1$. Thus, nonlinearity plays an important role for the wave processes, fulfilling $U_C ≥ U$. This estimation shows, that the nonlinear terms play an important role in the dynamics of ULF PEMW linear evolution of which is well



described in our previous works (Aburjania et al., 2002, 2003, 2004, 2007). Inequality $U_{\rm C} \ge U$ coincides with anti-twisting condition, only after fulfilling which the initial dynamic Eqs. (1) and (2) have solitary vortex solution (Williams and Yamagata, 1984).

From general theory of the nonlinear waves it is known (Whitham, 1974), that if in

- ⁵ the system the nonlinearity is sufficient then superposition is not applicable in this case and plane wave solution is not correct. Nonlinearity distorts wave profile and then it becomes different from sinusoid. If in nonlinear system dispersion is absent, all small amplitude waves with different wave numbers k spread with same velocities and have possibility to interact with each other for a long time. Thus, even small nonlinearity leads to storing of distortion. Such penlinear distortions, as a rule lead to growth of
- ¹⁰ leads to storing of distortion. Such nonlinear distortions, as a rule, lead to growth of the wave front twisting and its breaking or bow shock formation. At the dispersion, he phase velocities o the waves with different k are not equal, they spread with different velocities and do not interact with each other. Therefore, the wave packet have tendency to sprawl down and at small amplitude dispersion can compete with nonlinearity.
- In consequence, the wave even before its breaking can split into separate wave packets and the bow shock will not be formed. Actually, in the real atmosphere the bow shock will not be formed arbitrarily (Shakina, 1985). First, this means that in atmosphereionosphere media dispersion is strongly pronounced and sufficiently competes with nonlinear distortions. If wave front nonlinear twisting will be compensated by dispersion, then stationary waves can be formed. collitory wattings propagating in medium
- sion, then stationary waves can be formed solitary vortices, propagating in medium without changing its shape.

It must be mentioned also, that the satellite and ground based observations clearly indicate an existence of the zonal winds (flows) at different layers of the ionosphere (Gershman, 1974). At interaction with zonal flows the wave perturbation obtains an additional dispersion and new source of amplification and the nonlinearity affects their dynamics. Thus ionospheric medium with shear flows creates itself favorable conditions for the formation of the nonlinear stationary vortex structures.

Thus, our goal is to find stationary solution of Eqs. (1) ad (2) (in non-dissipative case $\Lambda = 0$) $\overline{\psi} = \psi(\eta, y)$ and $h = h(\eta, y)$ spreading along the parallels (along *x* axis) with



constant velocity U = const without changing the form, where $\eta = x - U\tau$. We consider the case when the localized (even due to one coordinate) structure propagates on the background of mean zonal wind with non-uniform velocity $V_0(y)$.

Transforming the coordinates and taking into account that $\partial/\partial \tau = -U\partial/\partial \eta$, the system (1) and (2) can be written as:

$$-U\frac{\partial}{\partial\eta}\Delta\Psi + \beta\frac{\partial\Psi}{\partial\eta} + C_{\mathsf{H}}\frac{\partial h}{\partial\eta} - J(\Psi, \Delta\Psi) = 0, \tag{15}$$

$$(C_{\rm H} - U)\frac{\partial h}{\partial \eta} - \beta_{\rm H}\frac{\partial \Psi}{\partial \eta} - J(\Psi, h) = 0.$$
⁽¹⁶⁾

Here, the stream function is introduced:

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$$\Psi(\eta, y) = \Phi_0(y) + \overline{\psi}(x, y).$$
(1)

And the velocity potential $\Phi_0(y)$ of the shear flow due to the notations:

$$V_0(y) = -\frac{d\Phi_0(y)}{dy}.$$
 (18)

¹⁵ Solution of Eq. (16) can be given as:

$$h(\eta, y) = \frac{\beta_{\rm H}}{C_{\rm H} - U} \Psi, \tag{19}$$

Further, substituting Eq. (19) into Eq. (15) and with analogous transformation we get Jacobean:

$${}_{20} \quad J\left(\Delta\Psi - \left(\beta + \frac{C_{\rm H}\beta_{\rm H}}{C_{\rm H} - U}\right)y, \Psi - Uy\right) = 0.$$
(20)

General solution of Eq. (20) has a form (Aburjania, 2006):

$$\Delta \Psi - \frac{C_{\rm H} \overline{\beta} - U\beta}{C_{\rm H} - U} y = F(\Psi - Uy),$$
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(21)

7)

where $F(\xi)$ – an arbitrary function of its argument.

We will investigate bellow the solitary vortex structures in the ionosphere for different profiles of the shear flow velocity (Eq. 18).

5 4.1 The monopole vortex solutions

Let us consider the case, when the shear flow velocity is given by harmonic function

$$\Phi_0(y) = a_0 \sin(\varpi_0 y), \tag{22}$$

where a_0 characterizes amplitude, $æ_0$ – transversal scale of the shear flow. In this case, choosing the linear *F* function:

 $F = \kappa^2 (\Psi - Uy), \tag{23}$

where $\kappa^2 = (C_H \overline{\beta} - U\beta)/[U(C_H - U)] = \text{const} > 0$, Eq. (21) can be transformed into:

$$\Delta \Psi - \kappa^2 \Psi = \left(\kappa^2 + \varpi_0^2\right) \Phi_0,\tag{24}$$

solution of which obtains the form

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15

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 $\Psi = b_1 K_0(\kappa r) - \Phi_0(y), \tag{25}$

where K_0 – Mcdonalds function of 0th order; $r = (\eta^2 + y^2)^{1/2}$ and $b_1 = \text{const} - \text{ampli-tude of structure.}$

On the basis of solution (25) the medium velocity components V_x and V_z can be determined:

$$V_x = \frac{b_1 \kappa}{r} \cdot y \cdot K_1(\kappa r) + \mathfrak{B}_0 a_0 \cos(\mathfrak{B}_0 y), \quad V_y = -\frac{b_1 \kappa}{r} \cdot \eta \cdot K_1(\kappa r).$$
(26)



Parameter b_1 can be estimated from the boundary conditions. Thus, solution (26) depends on three parameters: from structure velocity U, characteristic transversal scale $æ_0$ and amplitude of the shear flow. The solution is strongly localized, being solitary monopole on the background of zonal harmonic wind, which behaves asymptotically $\sim \exp(-\kappa r)/\sqrt{r}$ at $r \rightarrow \infty$.

4.2 The stationary vortex streets in the nondissipative ionosphere

Vortex streets of various shapes can be generated in conventional liquid and plasma media with a sheared flow as a result of the nonlinear saturation of the Kelvin–Helmholtz instability (Gossard and Hooke, 1975; Kamide and Chian, 2007).

¹⁰ Thus, we will seek the solution of the nonlinear dynamic Eqs. (1) and (2) (in nondissipative stage, when $\Lambda \approx 0$) in the form $\psi = \psi_0(\eta, y)$, $h = h(\eta, y)$, where $\eta = x - U\tau$, i.e. the stationary solitary structures, propagating along *x* axis (along the parallels) with velocity *U* = const without changing its' shape. In accordance to Aburjania and Chargazia (2007), system of Eq. (1), (2) has the solution

¹⁵
$$h(\eta, y) = \frac{\beta_{\mathrm{H}}}{C_{\mathrm{H}} - U} \Psi,$$

$$\Delta \psi_0 - v'_0(y) - \frac{C_{\mathsf{H}}\beta' - U\beta}{C_{\mathsf{H}} - U}y = F\left(\psi_0 - \int^y v_0(y) \mathrm{d}y - Uy\right),$$

with $F(\xi)$ being an arbitrary function of its argument and $\Delta = \partial^2 / \partial \eta^2 + \partial^2 / \partial y^2$. Vortex streets have complicated topology and can occur when the function $F(\xi)$ in Eq. (28) is nonlinear (Petviashvili and Pokhotelov, 1992; Aburjania, 2006).

In Eq. (28) we assume that a nonlinear structure propagates with the velocity U satisfying the condition

 $U = \frac{\beta'}{\beta} C_{\mathsf{H}}.$



(27)

(28)

(29)

For this case, choosing *F* to be a nonlinear function, $F(\xi) = \psi_0^0 \kappa^2 (\exp(-2\xi/\psi_0^0))$ (Petviashvili and Pokhotelov, 1992; Aburjania, 2006), we can reduce Eq. (28) to

$$\Delta(\psi_0 - Uy) = \psi_0^0 k^2 \exp\left[-2(\psi_0 - Uy)/\psi_0^0\right].$$
(30)

Then we introduce the new stream function

$$\Psi_0(\eta, y) = \Phi_0(y) + \psi_0(x, y), \tag{31}$$

and the velocity potential $\Phi_0(y)$ of the background sheared zonal flow,

$$V_0(y) = \frac{d\Phi_0(y)}{dy}.$$

The stream function of the background sheared flow $\Phi_0(y)$ can be chosen to have the form

$$\Phi_0(y) = Uy + \psi_0^0 \ln(\mathfrak{X}_0 y).$$

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Here, ψ_0^0 is the amplitude of the vortex structure, $2\pi/\kappa$ kappa is its characteristic size, and $2\pi/\varpi_0$ is the nonuniformity parameter of the background sheared flow.

Taking into account Eq. (31) and using stream function (33), we can write vortex Eq. (30) as

$$^{20} \quad \Delta \psi_0 = \psi_0^0 \mathfrak{B}_0^2 \left[\frac{\kappa^2}{\mathfrak{B}_0^2} e^{-2\psi_0/\psi_0^0} - 1 \right].$$

This equation has the solution (Mallier and Maslowe, 1993)

$$\psi_0(\eta, y) = \psi_0^0 \ln \left[\frac{ch(\kappa y) + \sqrt{1 - \varpi_0^2} \cos(\kappa \eta)}{ch(\varpi_0 y)} \right],$$
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(32)

(33)

(34)

(35)

which describes a street of oppositely circulating vortices. Substituting solution (35) and stream function (33) into Eq. (31), we arrive at the final solution

$$\Psi_0(\eta, y) = Uy + \psi_0^0 \ln[ch(\kappa y) + \sqrt{1 - \omega_0^2} \cos(\kappa \eta)].$$
(36)

Equations (35), (33), and (32) yield the following expressions for the velocity components of the medium and sheared flow:

$$V_{x}(\eta, y) = U + \psi_{0}^{0} \kappa \frac{sh(\kappa y)}{ch(\kappa y) + \sqrt{1 - \varpi_{0}^{2}} \cos(\kappa \eta)},$$

$$V_{y}(\eta, y) = \psi_{0}^{0} \kappa \frac{\sqrt{1 - \varpi_{0}^{2}} \sin(\kappa \eta)}{ch(\kappa y) + \sqrt{1 - \varpi_{0}^{2}} \cos(\kappa \eta)},$$

$$V_{0}(y) = U + \psi_{0}^{0} \varpi_{0} th(\varpi_{0} y).$$
(37)
(37)
(37)
(37)
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(37)

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$$V_0(y) = U + \psi_0^0 \mathfrak{B}_0 \operatorname{th}(\mathfrak{B}_0 y)$$

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For $\varpi_0 = 1$, solution (37) describes a background flow of the type of sheared zonal flow with velocity (Eq. 39). For $\varpi_n^2 < 1$, a street of cyclonic-type vortices forms in the middle of the zonal flow with velocity (Eq. 39) (Fig. 1). A solution like that described by Eq. (37) and (38), with closed current lines in the form of cat's eyes, was for the first time obtained by Kelvin.

The vortex structures move with velocity (Eq. 29). If we take into account that $\beta_{\rm H} < 0, \beta' = \beta - |\beta_{\rm H}| < 0$ as far as $|\beta_{\rm H}| > \beta > 0, C_{\rm H} < 0$ from expression Eq. (29) follows U > 0. For E region the characteristic parameters $N/N_{\rm p} = 5 \times 10^{-7}$, $\Omega_{\rm H} = eH_{\rm p}/(Mc) \sim$ 10^3 s^{-1} , $R = 6.4 \times 10^6 \text{ m}$, $2\Omega_0 \cong 10^{-4} \text{ rad s}^{-1}$, we get that:

$$\begin{split} \beta &= 2\Omega_0 \sin\theta_0 / R \sim 0.8 \times 10^{-11} \,\mathrm{m}^{-1} \,\mathrm{s}^{-1} |C_{\rm H}| \approx 10 \,\mathrm{km} \,\mathrm{s}^{-1} \\ |\beta_{\rm H}| &= (N/(N_{\rm n}R))\Omega_{\rm H} \sin\theta_0 \approx 4 \times 10^{-11} \,\mathrm{m}^{-1} \mathrm{s}^{-1}. \end{split}$$



Thus, the vortices move with velocity $U \approx 4|C_H| > |C_H|$ along the parallels to the east. Therefore, this velocity is greater than the phase one of the corresponding linear periodic waves $U > |C_H| \approx 10 \text{ km s}^{-1}$. So, the vortices don't come into resonance with the linear waves and don't loose energy on their excitation (Stepanyants and Fabrikant, 1992).

For estimation of the linear scale of the vortex structures let's remember the general formal relation between the dispersion equation of the linear waves and with so-called modified dispersion equation of the nonlinear structures (Petviashvili and Pokhotelov, 1992; Aburjania, 2006). This is coupling of the phase velocity of linear wave $V_p = \omega/k$ with motion velocity of the nonlinear structures $U: -\omega/k \rightarrow U$; relation of the wave vector k of the linear disturbances with the characteristic linear scale of the vortex $d: -k \rightarrow d^{-1}$. Taking into account this fact for characteristic scale of the fast vortex structures we get:

$$d^{\rm f} = \left(\frac{|C_{\rm H}|}{\beta}\right)^{1/2}.$$
(40)

$$d^{s} = \left(\frac{U}{\beta}\right)^{1/2}.$$
(41)

Substituting in these expressions the typical for the Earth's ionosphere numerical values $|C_{\rm H}| \approx 10 \,{\rm km \, s^{-1}}$, $\beta \approx 10^{-11} \,{\rm m^{-1} \, s^{-1}}$, we find for fast structures $d^{\rm f} \approx 10^{4} \,{\rm km}$. For slow Rossby-type vortices $U \approx 10 \,{\rm m \, s^{-1}}$ and we can obtain $d^{\rm s} \approx 10^{3} \,{\rm km}$.

For magnetic field perturbation from Eqs. (27) and (36), we can obtain the following estimation:

 $|h| \approx |\beta_{\mathsf{H}}| \cdot d,$

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(42)

valid for both the fast and slow modes. For the ionospheric conditions $|\beta_{\rm H}| \approx 4 \times 10^{-11} \,{\rm m}^{-1} \,{\rm s}^{-1}$, thus using the estimations to carried out above, we may conclude that fast vortical motion generate magnetic pulsations $h^{\rm f} \approx 10^{-4} T$, while in case slow Rossby-type vortical motions $-h^{\rm s} \approx 10^{-5} T$.

Note that nonlinear stationary Eq. (28) also has an analytic solution in the form of a Larichev–Reznik cyclone-anticyclone dipole pair and other class of solitary solutions by different profiles of background shear flows (Petviashvili and Pokhotelov, 1992; Jovanovich et al., 2002; Aburjania, 2006; Aburjania et al., 2003, 2004, 2007).

5 Attenuation of the vortex streets in the dissipative ionosphere

In the dissipative approximation ($\Lambda \neq 0$), we switch to the above self-similar variables (η and y) and take into account the relationship $\partial/\partial \tau = -U\partial/\partial \eta$, which then holds. As a result, we can write Eqs. (1) and (2) as

$$-U\frac{\partial}{\partial\eta}\Delta\Psi + \beta\frac{\partial\Psi}{\partial\eta} + C_{\mathsf{H}}\frac{\partial h}{\partial\eta} + \Lambda\Delta\psi - J(\Psi, \Delta\Psi) = 0, \tag{43}$$

$$(C_{\rm H} - U)\frac{\partial h}{\partial \eta} - \beta_{\rm H}\frac{\partial \Psi}{\partial \eta} - J(\Psi, h) = 0.$$
(44)

Equation (44) has the solution

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$$h(\eta, y) = \frac{\beta_{\rm H}}{C_{\rm H} - U} \Psi.$$
(45)

Substituting solution (45) into Eq. (43), take into account the expression (29) and rearranging the term, we arrive at a single nonlinear equation:

 $\left(D_{\eta} + \frac{\Lambda}{U}\right) \Delta \Psi = 0, \tag{46}$



where

$$D_{\eta} = \frac{\partial}{\partial \eta} + \frac{1}{U} \left(\frac{\partial \Psi}{\partial \eta} \frac{\partial}{\partial y} - \frac{\partial \Psi}{\partial y} \frac{\partial}{\partial \eta} \right).$$

The Eq. (46) yield a solution as

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$$\Psi = \Psi_0 \cdot \exp\left(-\frac{\Lambda}{U}\eta\right)$$
.

Here the zero^s order Ψ_0 is identified with solution (36) (Fig. 1). The incorporation of dissipation effects has modified the solution of the dynamical non-linear differential equation. It can be seen from Eq. (47) that friction (or collision) is responsible for exponential decay of stationary nonlinear vortex structures in space. This street of vortices can be studied by plotting the stream line function $\Psi(\eta, y)$ (Eqs. 47 and 36). We have free parameters Ψ_0^0 , κ and ϖ_0 , and the velocity of movement of the structures *U* will be determine by (29).

Figure 2a shows the $\kappa = 0.5$ case and $\Psi_0^0 = 1, U = 0.1, \varpi_0 = 0.2$, while Fig. 2c shows the $\kappa = 1$ case. Three dimensional plots for the same parameters are shown in Fig. 2b and d. At decrease of the linear scales of the vortices (with increase of κ) the number of the vortices will increase in the given area of the medium and their amplitudes will decrease (Fig. 2c and d). The reduction in κ causes a reduction in number of vortices, e.g., the $\kappa = 1$ stream function plots six vortices (Fig. 2c and d). We, therefore, note that the number of vortices increases with increasing κ , e.g. the formation of nonlinear structures is attributed to low frequency mode.

At decrease of the linear scale of the background wind inhomogeneity (increasing $æ_0$) the linear scales, amplitudes and steepness of peaks of the vortices decrease accordingly (Fig. 3).

The street of vortices is in almost stationary frame of reference, it disappears for higher frame velocity (U > 1), i.e. the contribution of logarithmic and hyperbolic trigonometric functions are no longer overcome by the contribution of linear term viz. Uy in

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(47)

Eq. (36) and, therefore, vortex formation is replaced by straight stream lines (Fig. 4). Due to increase of the translation velocity (U) of the structures and the background flows the scales and amplitudes of the generated vortices will decrease. In case of comparably high velocity background wind (U > 1) the vortex will not be generated at all and only the background flow will remain in the medium (Fig. 5). Further, due to the nonlinear term, the velocity of dispersive waves must be greater than the phase velocity of a wave which resulted in a bending of the wave front and hence vortices start to form.

The street of vortex disappears in the space for high dissipation rate Λ (or collision frequency) (Fig. 6). We credence that the dissipation effect has not permitted the vortex formation, but the topography of stream line function has been modified (Fig. 6).

6 Relaxation of the vortex structures in the ionosphere

The real mechanism of dissipation in the atmosphere against the background of baroclinic, nonlinear and dispersive effects generates in the ionosphere moving spatial structures representing the equilibrium stationary solutions (35) and (36) of the governing magneto-hydrodynamic Eqs. (1) and (2). For qualitative estimation of the evolution and the temporal relaxation of stationary vortex structures in the ionosphere, built in previous paragraphs, the dynamic Eqs. (1) and (2) can be approximately written as the following Helmholtz's vortex transfer equation:

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$$\frac{\partial}{\partial t} \nabla \times \mathbf{V} = \mathbf{P} - \Lambda(\nabla \times V),$$

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which describes the generation of nonzero vorticity $\nabla \times \mathbf{V} ((\nabla \times \mathbf{V})_z = \Delta \Psi)$ in the ionosphere under the action baroclinic vector **P** (source function) taking in to account the temperature contrasts in the form of advection of warm and cold, medium dispersion and influence of small nonlinearity. According to the observations (Gill, 1982; Pedlosky, 1987), vector **P** for low-frequency disturbances is a slowly varying function of time. In



(48)

this case the vortex Eq. (29), with the initial conditions of Cauchy $\nabla \times \mathbf{V}|_{t=0} = 0$ (at the initial moment in the atmosphere there no vortices) has the bounded solution:

$$\nabla \times \mathbf{V} = \frac{\mathbf{P}}{\Lambda} \mathbf{1} - \mathbf{e}^{-\Lambda t}.$$
(49)

Dissipative effects have an accumulative nature and its action becomes perceptible only after a certain interval. From Eq. (49) it follow that vorticity will increase linearly with time only at small time intervals (*t* ≪ 1/Λ) under the action of baroclinicity and some other effects. After a certain time, when the dissipation effect reaches a specific value, vortex growth speed decreases (the vorticity growth rate decreases) and for the large intervals of time (*t* ≫ 1/Λ) it tends to constant (equilibrium) value P/Λ (Fig. 7). The value of dimensional time *T* = 1/Λ ≈ 10⁵ s ≥ 24 h can be called a relaxation time of non-stationary vortex street. Indeed, for the lower atmosphere relaxation time is of the order of twenty-four hours (Gossard and Hooke, 1975; Pedlosky, 1982) and consequently here large scale vortices must be long-lived. Stationary solution describes the equilibrium between baroclinicity and the dissipation effects (**P** = Λ(∇ × **V**)). As a result, the dissipative structure is generated in the ionosphere in the form of Stationary Street of cyclones and anticyclones.

7 Conclusions

Based on the nonlinear dynamical equations, considered in Aburjania et al. (2011), the linear and nonlinear interaction of planetary electromagnetic ultra-low-frequency fast and slow wavy structures with zonal shear flow in the Earth's dissipative ionosphere is investigated. Along with the prevalent effect of Hall conductivity for such waves, the latitudinal inhomogeneity of both the angular velocity of the Earth's rotation and the geomagnetic field becomes essential. Due to spatial inhomogeneity of the Earth's rotation

velocity fast and slow waves can be coupled. Such coupling results in an appearance of strong dispersion of these waves. Note that, without this coupling the fast branches



in the both ionospheric E and F regions lose the dispersion property for both large and short wave-length perturbations.

It is show, that at interaction with the inhomogeneous local wind the EM ULF wave perturbations can sufficiently increase own amplitude and energy and in their dynamics

- the nonlinear effects will be appeared. Dynamical competition of the nonlinear and the dispersion effects at the different layers of the ionosphere creates a favorable condition for self-organization of the EM ULF disturbances into nonlinear vortex structures. The self-localization of the planetary ULF waves into the long-lived solitary vortex streets in the non-dissipative ionosphere is proved in the basis of the analytical solution of the self-localization of the self-localization of the self-localization of the planetary ULF waves into the long-lived solitary vortex streets in the non-dissipative ionosphere is proved in the basis of the analytical solution of the self-localization of the self-localization
- ¹⁰ the governed nonlinear dynamic equations. The exact stationary solution of these nonlinear equations has an asymptote $\psi \sim \exp(-\kappa r)$ at $r \to \infty$, so the wave is strongly localized along the Earth surface. The translation velocity U of ULF EM vortices is very crucial which in turn depends on parameters β and $\beta_{\rm H}$. From analytical calculation and plots we note that the formation of stationary nonlinear vortex street require some
- ¹⁵ threshold value of translation velocity *U* (Eq. 29) for both nondissipative and dissipative complex ionospheric plasma. For some large value of the background wind's spreading velocity ($U \ge 10$) the vortex structures may not be raised at all and only the background wind will be preserved in the medium (Fig. 4). Number of vortices in generated nonlinear structures and a value of amplitudes of these vortices essentially depend on the
- size of the background wind's inhomogeneity decreasing the latter generated vortex's size and amplitude will automatically decrease (Fig. 4). It's shown that the space and time attenuation can not resist the formation of the vortex structures, but affect the topographic features of the structures (Figs. 5 and 6). The generated nonlinear vortex structures are enough long-live (> 24 h) in dissipative ionosphere.
- Depending on the type of velocity profile of the zonal shear flow (wind), the generated nonlinear long-lived vortex structures maybe represent monopole solitary anticyclone or cyclone, the cyclone – anticyclone pair, connected in a certain manner and/or the pure dipole cyclone – anticyclone structure of equal intensity, and/or the vortex street, or the vortex chains, rotating in the opposite direction and moving along the latitudinal



circles (along the parallels) against a background of the mean zonal wind (see also Jovanovich et al., 2002; Aburjania et al., 2003, 2006, 2007).

The nonlinear large-scale vortices generate the stronger pulses of the geomagnetic field than the corresponding linear waves. Thus, the fast vortices generate the magnetic field $h^{f} \approx 10^{5} nT$, and the slow vortices form magnetic field $h^{s} \approx 10^{4} nT$. The formation of such intensive perturbations could be related to the specific properties of the considering low frequency planetary structures. Indeed, they trap the environmental particles, and the charged particles in *E* and *F* regions of the ionosphere are completely or partially frozen into the geomagnetic field. That's why, the formation of these structures indicates at the significant densification of the magnetic force lines and, respectively,

- the intensification of the disturbances of the geomagnetic field in their location. Since, the number of the capture parcels is the order of the passed-by (transient), the perturbation of the magnetic field in the stronger faster vortices would be the same order as of the background field. On the earth surface located R_0 (~ (1 ÷ 3) × 10² km) below the
- ¹⁵ region of the researching wave structure, the level of the geomagnetic pulses would be less by $\exp(-R_0/\lambda_0)$ factor. λ_0 is the characteristic length of the electromagnetic perturbations. Since $\lambda_0 \sim (10 \div 10^2)R_0 \gg R_0$ the magnetic effect on the earth would be less then in *E* and *F* regions, but in spite of this they are easily registered too.

Hence, inhomogeneity of the Earth's rotation along the meridian, geomagnetic field and zonal prevailing flow (wind) can be considered among the real sources generating planetary ULF vortex structures of an electromagnetic nature in the ionosphere.

Author contribution. Statement of the problem, elaboration of the physical model and the governing equations for the ionospheric plasma medium with shear flows belongs to G. Aburjania and G. Zimbardo. K. Chargazia and O. Kharshiladze were involved in all stages of problem
 solving – estimation of the difficulty of the mathematical model and obtaining the analytical solutions in case of the shear flows.

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Figure 1. Relief and level lines in the rest frame of the vortices $\Psi(\eta, y) - Uy$, calculated from Eq. (55) for $\psi_0^0 = 1$, k = 1, $\varpi_0 = 0.5$ (the longitudinal vortex street).





Figure 2. The level lines and relief of the stream function of vortex solution (36) in the moving system of coordinates to the parameters: (a) $\psi_0^0 = 1$, U = 0.1, $\varpi_0 = 0.2$, $\kappa = 0.5$; (b) $\psi_0^0 = 1$, U = 0.1, $\varpi_0 = 0.2$, $\kappa = 1$; (c) $\psi_0^0 = 1$, U = 0.1, $\varpi_0 = 0.2$, $\kappa = 0.5$; (d) $\psi_0^0 = 1$, U = 0.1, $\varpi_0 = 0.2$, $\kappa = 1$.





Figure 3. The level lines and relief of the stream function of vortex solution (36) in the moving system of coordinates to the parameters: (a) $\psi_0^0 = 1$, U = 0.1, $\varpi_0 = 0.5$, $\kappa = 1$; (b) $\psi_0^0 = 1$, U = 0.1, $\varpi_0 = 0.5$, $\kappa = 1$; (c) $\psi_0^0 = 1$, U = 0.1, $\varpi_0 = 0.9$, $\kappa = 1$; (d) $\psi_0^0 = 1$, U = 0.1, $\varpi_0 = 0.9$, $\kappa = 1$.





Figure 4. The level lines of the stream function of vortex solution (36) in the moving system of coordinates to the parameters: (a) $\psi_0^0 = 1$, U = 0.6, $\varpi_0 = 0.2$, $\kappa = 0.5$; (b) $\psi_0^0 = 1$, U = 1.5, $\varpi_0 = 0.2, \ \kappa = 0.5; \ (c) \ \psi_0^0 = 1, \ U = 5, \ \varpi_0 = 0.2, \ \kappa = 0.5; \ (d) \ \psi_0^0 = 1, \ U = 10, \ \varpi_0 = 0.2, \ \kappa = 0.5.$

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Figure 5. Spatial damping of the vortex structures (the level lines and relief of the stream function), calculated from Eq. (66) to the parameters: **(a)** $\psi_0^0 = 1$, U = 0.1, $\varpi_0 = 0.2$, $\kappa = 0.3$, $\Lambda = 0.000$; **(b)** $\psi_0^0 = 1$, U = 0.1, $\varpi_0 = 0.2$, $\kappa = 0.3$, $\Lambda = 0.0025$; **(c)** $\psi_0^0 = 1$, U = 0.1, $\varpi_0 = 0.2$, $\kappa = 0.3$, $\Lambda = 0.01$.





Figure 6. Relaxation of the vorticity of perturbations, calculated from Eq. (49) to the parameters: (a) $\Lambda = 0.8$, $P/\Lambda = 10$; (b) $\Lambda = 0.2$, $P/\Lambda = 10$.

