Nonlin. Processes Geophys. Discuss., 1, 1–37, 2014
www.nonlin-processes-geophys-discuss.net/1/1/2014/
doi:10.5194/npgd-1-1-2014

Nonlinear Processes in Geophysics
Discussions
Open Access

Field theoretical prediction of a property of the tropical cyclone

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Received: 3 January 2014 – Accepted: 9 January 2014 – Published: 20 January 2014

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Published by Copernicus Publications on behalf of the European Geosciences Union & American Geophysical Union.

Abstract

The large scale atmospheric vortices (tropical cyclones, tornadoes) are complex physical systems combining thermodynamics and fluid-mechanical processes. The late phase of the evolution towards stationarity consists of the vorticity concentration, a well known tendency to self-organization , an universal property of the two-dimensional fluids. It may then be expected that the stationary state of the tropical cyclone has the same nature as the vortices of many other systems in nature: ideal (Euler) fluids, superconductors, Bose–Einsetin condensate, cosmic strings, etc. Indeed it was found that there is a description of the atmospheric vortex in terms of a classical field theory. It is compatible with the more conventional treatment based on conservation laws, but the field theoretical model reveals properties that are almost inaccessible to the conventional formulation: it identifies the stationary states as being close to self-duality. This is of highest importance: the self-duality is known to be the origin of all coherent structures known in natural systems. Therefore the field theoretical (FT) formulation finds that the cuasi-coherent form of the atmospheric vortex (tropical cyclone) at stationarity is an expression of this particular property. In the present work we examine a strong property of the tropical cyclone, which arises in the FT formulation in a natural way: the equality of the masses of the particles associated to the matter field and respectively to the gauge field in the FT model is translated into the equality between the maximum radial extension of the tropical cyclone and the Rossby radius. For the cases where the FT model is a good approximation we calculate characteristic quantities of the tropical cyclone and find good comparison with observational data.

1 Introduction

The generation of a large scale atmospheric vortex of high intensity (tropical cyclone) involves both thermodynamic and fluid–mechanical processes. After cyclogenesis a stationary state can be reached (assuming uniformity of the properties of the environment) and the main spatial characteristics of the vortex only have small changes while the whole structure is drifting. The fact that all tropical cyclones at stationarity have velocity fields with qualitatively the same radial profile suggests that there may be a connection with universal coherent structures (vortices) found in many other systems: ideal fluid, superconductors, topological field theory, cosmic matter, etc. In such systems the vorticity field evolves by self-organization to states that extermize a functional, i.e. they are exceptional within the much wider class of functions that verify the conservation equations for the same system. In the case of the 2-D Euler fluid there is *no* thermodynamic process (no buoyancy, no pressure gradient, no centrifugal force) but the asymptotic organization of flow into coherent structures is a well known and well studied fact (Kraichnan and Montgomery, 1980). The results of Montgomery et al. (1992, 1993), showing the evolution of the 2-D fluid from an initial turbulent state to a highly organized vortical motion, are fully convincing. There is no reason to expect that the thermodynamics (e.g. in the tropical cyclone) will act to suppress or exclude this purely fluid - mechanical self-organization process. Deep in the tropical cyclone dynamics there must exist the tendency of self-organization of the vorticity, similar to the one in the case of the Euler fluid. The description of the tropical cyclone must somehow include this spontaneous self-organization.

The flows of the tropical cyclone are three dimensional but with substantial anisotropy: the azimuthal flow is largely dominant compared with the radial and the vertical flows. Another argument comes from the presence of 2-D vorticity concentration observed in water tank experiments although the flows are three dimensional (Hopfinger and van Heijst, 1993). In numerical simulation of planetary turbulence the vorticity concentration has been observed (McWilliams et al., 1994). We assume that, within the two-dimensional approximation, there is an idealized limit where the dynamical evolution is almost factorised into two distinct, weakly-coupled sub-systems: the thermodynamical processes and the mechanical balance. In this limit there is only a small amount of energy flowing from the thermal sub-system toward the mechanical processes needed for the latter to overcome the loss due to the friction. The loss of

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the mechanical energy by friction in the vortical motion is a small fraction of the total mechanical energy. At stationarity this represents the only interaction between the two subsystems. An useful image of this state would be that of a matrix representing the relative influences of thermodynamic and mechanical variables, factorized into two Jacobi blocks, with only slight interconnection remaining off the diagonal, to connect the two sub-systems.

How useful is such approximation that factorizes the physical system at stationarity into thermal and vorticity-dynamics subsystems? For such ideal state one assumes that the thermal processes are balanced and places emphasis on the vorticity dynamics, seen as the essential factor in establishing the spatio-temporal characteristics of the atmospheric vortex at stationarity (the "shape"). The Taylor–Proudman theorem (Batchelor, 2002) and the typical dimensions of the observed cyclones, with much larger radial extension compared with the height of the perturbation, suggests that a 2-D approximation for the tropical vortex, in what regards the vorticity dynamics, may be reasonable. An indirect confirmation comes from the immediate benefit that follows from the dominance of the two-dimensional nature of the dynamics: the vorticity dynamics in 2-D is radically different than in 3-D: there is inverse cascade and the asymptotic relaxed states of the fluid are highly organized, the vortical structures can be close to stability. If indeed there is an universal vortical structure behind the stationary tropical cyclone then this would only be the result of its dominant two-dimensional geometry.

The intrinsic evolution to vorticity organization exists for the 2-D ideal incompressible (Euler) fluid where the asymptotic states are governed by the self-duality property and satisfy the equation sinh–Poisson equation (also known as elliptic sinh–Gordon equation) $\Delta\psi + \sinh(\psi) = 0$, where ψ is the streamfunction. The self-duality (SD) is a property of the geometric-algebraic structure (a fiber space) attached to the physical problem: the curvature differential two-form is equal to its Hodge dual (Mason and Woodhouse, 1996). Identification of this mathematical structure is highly non-trivial but in practical cases SD is manifested by the possibility of expressing the action functional

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as a sum of square terms plus a term with topological content. The sinh-Poisson equation has been derived from an action that has the SD property. The equation is exactly integrable and the *stable* solutions are necessarily periodic, like – for example, the Mallier–Maslowe chain of vorticies. No other solution is stable, in particular a single monopolar vortex, but it can be very robust and may appear in reality as sufficiently long-lived to be considered stable.

In the case of the 2-D approximation of the atmosphere the sinh–Poisson equation may only be an indicative approximation. The new physical element is the Rossby radius, that changes the physics and the mathematical possibility of relaxed states.

In previous papers we have shown that a description of the stationary state of the 2-D atmospheric vortex as extremum of an action functional is possible. This extends the approach based exclusively on conservation laws (density, momentum, energy, etc.) by considering the states that results from the variational formulation based on a Lagrangian density. The ideal (Euler) fluid is described in 2-D by the equation $d\omega/dt = 0$ where $\boldsymbol{\omega}$ is the vorticity, a vector directed along the perpendicular on the plane. This equation is known to be equivalent with the equations of motion of a discrete set of point-like vortices interacting in plane by a mutual long-range potential (the ln function of the relative distance between point-like vortices). We have formulated a classical field theoretical model of the continuous limit of this system (Spineanu and Vlad, 2003, 2013). It was then possible to derive, in purely analytical terms, the sinh–Poisson equation.

For the 2-D approximation of the planetary atmosphere (and for the 2-D plasma in strong magnetic field: the equations are the same) the dynamics of the vorticity field can be equivalently described by a discrete system of point-like vortices but in this case the potential of interaction in plane is short-range (Morikawa, 1960). The continuum limit of the system of discrete point-like vortices is again a classical field theory. The matter field ϕ which represents the density of the point-like vortices and the gauge field (representing the mutual interaction of the vortices) are elements of the algebra $sl(2, \boldsymbol{C})$, i.e. they are mixed spinors, since they correspond to physical

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elementary vortices. The planetary rotation represents the condensate of vorticity that defines the broken vacuum of the theory and generates, via the Higgs mechanism, the mass of the "photon", i.e. the short range of the interaction, with the spatial decay given by the Rossby radius (respectively the Larmor gyro-radius for plasma). In the following we just remind few elements of the Field Theoretical (FT) formulation for the 2-D atmosphere/plasma. The FT formulation can be found in Spineanu and Vlad (2003, 2005b) and the first application in Spineanu and Vlad (2009).

The Lagrangean density is

$$\mathcal{L} = -\kappa \varepsilon^{\mu\nu\rho} \operatorname{tr}\left(\partial_\mu \mathbf{A}_\nu \mathbf{A}_\rho + \frac{2}{3}\mathbf{A}_\mu \mathbf{A}_\nu \mathbf{A}_\rho\right) - \operatorname{tr}\left[\left(D^\mu \boldsymbol{\phi}\right)^\dagger \left(D_\mu \boldsymbol{\phi}\right)\right] - \mathbf{V}\left(\boldsymbol{\phi}, \boldsymbol{\phi}^\dagger\right) \tag{1}$$

where κ is a positive constant and

$$\mathbf{V}\left(\boldsymbol{\phi}, \boldsymbol{\phi}^\dagger\right) = \frac{1}{4\kappa^2} \operatorname{tr}\left[\left(\left[\left[\boldsymbol{\phi}, \boldsymbol{\phi}^\dagger\right], \boldsymbol{\phi}\right] - v^2 \boldsymbol{\phi}\right)^\dagger \left(\left[\left[\boldsymbol{\phi}, \boldsymbol{\phi}^\dagger\right], \boldsymbol{\phi}\right] - v^2 \boldsymbol{\phi}\right)\right]. \tag{2}$$

The field variables are $\boldsymbol{\phi}$, $\mathbf{A}^\mu \equiv \left(\mathbf{A}^0, \mathbf{A}^1, \mathbf{A}^2\right)$ and their Hermitean conjugate, $(\ )^\dagger$. The covariant derivative is $D_\mu = \partial_\mu + \left[\mathbf{A}_\mu\right]$, $\mu = 0, 1, 2$ and the metric $g^{00} = -1$, $g^{ik} = \delta^{ik}$. All variables are elements of the algebra $sl(2, \boldsymbol{C})$. A standard Bogomolnyi procedure followed by an algebraic ansatz where $\boldsymbol{\phi}$ only contains the two ladder generators of $sl(2, \boldsymbol{C})$ leads to an equation for the asymptotic states that has no regular real solution. Adopting an algebraic ansatz with only the first ladder generator in $\boldsymbol{\phi}$ leads to a very clear topological theory but the asymptotic equation can only produce stationary rings of vorticity. If we see the field theoretical description of the atmospheric vortex as an extension of the theory for the Euler fluid, then we have to keep the Bogomolnyi procedure, but alter the terms: the action functional becomes as usual a sum of squares plus a residual term. This term is small (being multiplied with the Coriolis frequency $\sim 5 \times 10^{-5}$) and does not have a topological meaning (Spineanu and Vlad, 2005b). The self-duality property is not exact but the resulting equation (Spineanu and Vlad, 2005a)

$$\Delta \psi + \left(\frac{v^2}{\kappa}\right)^2 \sinh(\psi)\,[\cosh(\psi) - 1] = 0 \tag{3}$$

has solutions with the morphology of the tropical cyclone. With the identifications

$$v^2 = f \tag{4}$$

$$\kappa = \sqrt{g\,h_0} \tag{5}$$

we see that the distances should be normalized to the Rossby radius $R_{\text{Rossby}} = \sqrt{g\,h_0}/f$ and the time to f^{-1}, the inverse of the Coriolis frequency. The equation is solved on: (A) a square with half of the length of diagonal L^{diag} (we will also use $L^{\text{sq}} = L^{\text{diag}}/\sqrt{2}$, half of the length of the side); and (B) in azimuthal symmetry on a radial interval L^{rad}. The results coincide for $L^{\text{rad}} = L^{\text{diag}} = \sqrt{2}L^{\text{sq}}$, as described in Spineanu and Vlad (2009). From now on the quantities like L^{diag}, L^{rad}, L^{sq}, ψ, etc. are normalized and when they are dimensional an upperscript phys is used: $\left(L^{\text{diag}}\right)^{\text{phys}} = R_{\text{Rossby}}L^{\text{diag}}$, etc. We note that here R_{Rossby} is defined as a global physical parameter of the tropical cyclone and we do not consider either its spatial variation within a single vortex or the β effect.

Our simplified model for the tropical cyclone now can be formulated in the terms of the two approaches (geophysical and field theoretical). In the present work we underline a result that is derived in the field theoretical description and reveals a strong property in the geophysical picture of the tropical cyclone: the field theoretical result that the mass of the matter field excitation m_H is equal with the mass of the gauge boson m_{gauge} implies that the maximum radial extension of the tropical cyclone must be equal to the Rossby radius.

This is an important and strongly constraining condition on the physical dimensions of a tropical cyclone. According to the simplified, field theoretical (FT) model, if the physical dimensions are so different for different tropical cyclones, this is due to different Rossby radii. Or, the Rossby radius results from the individual history of a particular tropical cyclone, which, after the transient part of growth, should reach a unique shape, given by the solution of the Eq. (3) for $L^{\text{rad}} = 1$. In the FT framework the property $m_\text{H} = m_\text{gauge} \sim \left(L^{\text{rad}}\right)^{-1}$ means that $L^{\text{rad}} = \left(L^{\text{rad}}\right)^{\text{phys}} / R_\text{Rossby} = 1$. This gives a *unique* profile $\psi(r)$ which is obtained by solving the Eq. (3) either on a square region in the plane, or on a radial domain $L^{\text{rad}} = 1$. We remind that the result of solving Eq. (3) is expressed in non-dimensional quantities: distances are normalized to R_Rossby and velocity to $\left(R_\text{Rossby} f\right)$. A physical input coming from observations is necessary to get dimensional quantities. Analysing a database we can find R_Rossby for a particular atmospheric vortex and then calculate the maximal velocity, radius of eye-wall, etc., which must be compared with observational data.

We are interested in three important characteristics of the cyclone: the maximum of the azimuthal velocity v_θ^{max}, the radius where this maximum is found $r_{v_\theta^{\text{max}}}$ and the maximal radial extension of the vortex, R_max. The use of observational data to identify the Rossby radius and then to convert our variables to physical ones, followed by further comparisons, is however a difficult task: our simple model refers to the stationary state of the tropical cyclone, which is difficult to isolate in the full evolution. Second, when two of the three charactersitics mentioned above are fixed, the dispersion of observational data regarding the third one is large. We associate this dispersion with the fact that the state of the tropical cyclone cannot be exactly mapped to the vortex derived in the field theoretical formulation at self-duality and its shape does not correspond to $L^{\text{rad}} = 1$. We then use a range of values around $L^{\text{rad}} = 1$ and try to find the effective $L^{\text{rad}} \in \left[L^{\text{rad}}_{\text{min}}, L^{\text{rad}}_{\text{max}}\right]$ which provides the best fit to the measured data. This means that we assume that the system evolves in close proximity of the stationary Self-Dual state. In short the FT leads us to expect that for whatever physical dimensions of the tropical cyclones, we should find $L^{\text{rad}} = 1$. If we find a different value this means that the special

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state of self-duality, leading to Eq. (3) is not reached and $\left(L^{\mathrm{rad}}\right)^{\mathrm{phys}} \approx R_{\mathrm{Rossby}}$ is not fullfilled. We would like to see to what extent the FT remains an interesting description in the neighborhood of this particular state.

2 The geophysical view on the typical dimensions of the atmospheric vortex

It is well known that the 2-D Euler fluid evolves at relaxation to highy organized flow structure (Montgomery et al., 1992). For the 2-D atmopshere the basic aspect that makes the difference with the 2-D ideal (Euler) fluid is the existence of an intrinsic length. This is the Rossby radius and its role is already manifested in the kinetic equations for the equivalent model of discrete point-like vortices (Morikawa, 1960) where the potential of interaction is no more long range (Coulombian $\ln\left(\left|\boldsymbol{r}-\boldsymbol{r}'\right|\right)$, it is $K_0\left(\sigma\left|\boldsymbol{r}-\boldsymbol{r}'\right|\right)$, with $\sigma^2 = f^2/(g h_0)$. Here f is the Coriolis frequency $f = 2\Omega \sin\theta$, Ω is the frequency of planetary rotation and θ is the latitude angle; g is the gravitational acceleration and h_0 is the depth of the fluid (atmosphere) layer. The space parameter is defined (Pedlosky, 1987) as the Rossby radius of deformation.

$$R_{\mathrm{Rossby}} = \frac{(g h_0)^{1/2}}{f} = \sigma^{-1} \tag{6}$$

Besides R_{Rossby} there is another natural space parameter, L, the characteristic horizontal length L of the flow induced by a perturbation of the atmosphere (L is dimensional). These two parameters control the balance of the forces in the fluid dynamics. The relative acceleration of the flow $\mathrm{d}\boldsymbol{u}/\mathrm{d}t$ results from a competition of the forces induced by the horizontal gradient of the pressure $-\boldsymbol{\nabla}_{\mathrm{h}} p$ and the Coriolis force $\boldsymbol{u} \times 2\Omega$. The gradient of pressure exists due to the perturbation of the pressure of the air $p = \rho g z$, created by the perturbation of the *depth* $z = h_0 + \delta z$ of the layer of the fluid,

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$$-\frac{\partial}{\partial x}\delta p = -\rho_0 g \frac{\partial}{\partial x}\delta z \sim -\rho_0 g \frac{\delta z}{L}. \tag{7}$$

We note that, for a perturbation δz of the depth of the fluid layer, if the horizontal extension of the flow L is large, the gradients $-\partial p/\partial x$, $-\partial p/\partial y$ are small ($\sim \delta z/L$) and the Coriolis force is dominant. This term, proportional with $\delta z \sim \psi(x,y)$ leads to the second part of the *potential vorticity*

$$\Pi \equiv \nabla_{\mathrm{h}}^2 \psi - \sigma^2 \psi = \nabla_{\mathrm{h}}^2 \psi - \frac{1}{R_{\mathrm{Rossby}}^2}\psi. \tag{8}$$

In the geostrophic approximation Π verifies the conservation equation $\mathrm{d}\Pi/\mathrm{d}t = 0$, where the convective derivative operator is $\mathrm{d}/\mathrm{d}t = \partial/\partial t + u\partial/\partial x + v\partial/\partial y$. The velocity $\boldsymbol{v} = (u,v)$ is defined in terms of the streamfunction $\psi(x,y)$, $\boldsymbol{v} = -\nabla_{\mathrm{h}}\psi \times \widehat{\boldsymbol{e}}_z$, with $\widehat{\boldsymbol{e}}_z$ the versor perpendicular on the plane and ∇_{h} is the horizontal gradient. The relative vorticity $\nabla_{\perp}^2 \psi$ introduces the horizontal scale of the flow, $\nabla_{\mathrm{h}}^2 \sim L^{-2}$; the contribution to the potential vorticity of the deformation of the free surface (δz, the perturbed heigth of the fluid layer) introduces the Rossby radius $R_{\mathrm{Rossby}} = \sqrt{g h_0}/f$. The importance of the term coming from the deformation of the surface, $\sim \psi$, relative to the vorticity term $\nabla_{\perp}^2 \psi$ is measured by the factor (Pedlosky, 1987)

$$F = \left(\frac{L}{R_{\mathrm{Rossby}}}\right)^2 \tag{9}$$

and two regimes are identified. (1) If the horizontal scale L is small and localised inside the Rossby radius scale

$$L \ll R_{\mathrm{Rossby}} \tag{10}$$

then from the point of view of the vorticity balance the free surface can be considered flat and rigid (i.e. no deformation). In relative terms, a very large Rossby radius means

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that the external origin of rotation is weak (in the equivalent plasma system, a very large Larmor radius means that the applied external magnetic field is weak). The operator $\Delta_h \psi - R_{\text{Rossby}}^{-2} \psi$ approaches $\Delta_h \psi$ and the short range interaction in the system of point-like vortices turns into the long range interaction, $K_0 \rightarrow \ln$. The density and the vorticity decouple and the Ertel's theorem becomes the simple statement of conservation of the vorticity $\mathrm{d}\Delta_h \psi / \mathrm{d}t = 0$, i.e. the Euler equation. (2) If the horizontal extension of the perturbation flow is very large, much larger than the Rossby radius

$$L \gg R_{\text{Rossby}} \tag{11}$$

the *relative vorticity* in the motion $\Delta_h \psi$ is very small and the velocity field appears almost uniform horizontally. For large spatial scales of the flow L, the *relative accelerations* are very weak and the Coriolis acceleration dominates.

Then the basic geophysical analysis finds the Rossby radius of deformation $R_{\text{Rossby}} \approx L$ as the "distance over which the gravitational tendency to render the free surface flat is balanced by the tendency of the Coriolis acceleration to deform the surface" (Pedlosky, 1987).

The fact that the horizontal extension of the tropical cyclone is comparable with the Rossby radius has been noted before (Willoughby, 1988).

3 Field theoretical view on the typical (radial) dimensions

In FT the spatial decay of the interaction is connected with the mass of the particle that carries the interaction. The FT formulation of the atmospheric vortex allows to consider, instead of the typical lengths L and R_{Rossby}, the masses associated with the propagators of the scalar and gauge fields excitations.

In field theory formalism the mass appears as a singularity of the propagator of the field, which is calculated as the two-point correlation of the field values. Alternatively, to identify the mass m_H of the matter $\boldsymbol{\phi}$ field excitation, we need to emphasize from

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the equations of motion derived from the Lagrangian, a structure expressing the main matter field dynamics, as

$$-\partial_i^2 \boldsymbol{\phi} - (m_{\mathrm{H}})^2 \boldsymbol{\phi} \tag{12}$$

and this can be seen in the expression of the action functional, without the need to calculate the propagator (Abers and Lee, 1973). The second order differential operator comes from the kinetic term in the Lagrangian $(D_\mu \phi)^\dagger (D^\mu \boldsymbol{\phi})$ and the last term comes from the part of the potential $V\left(|\boldsymbol{\phi}|^2\right)$ which is quadratic in $\boldsymbol{\phi}$. It is simpler to refer to the Abelian version of the Lagrangian (Dunne, 1995) (in this case we refer to the matter field $\boldsymbol{\phi}$ as the "scalar" field). For the Abelian version, instead of Eq. (2) the potential is (Dunne, 1995, p. 83)

$$V(\phi) = V\left(|\phi|^2\right) = \frac{1}{4\kappa^2}|\phi|^2\left(|\phi|^2 - v^2\right)^2 \tag{13}$$

and this identifies the broken vacuum as

$$|\phi_0|^2 = v^2. \tag{14}$$

In order to find the mass spectrum in the broken vacuum, we have to expand the potential around $|\phi_0|^2 = v^2$ and retain the quadratic terms like in Eq. (12)

$$\begin{aligned} &V\left(\phi_0 + \widetilde{\phi}\right) \\ &= \frac{1}{4\kappa^2}|\phi_0|^4\left(\widetilde{\phi} + \widetilde{\phi}^*\right)^2 + \dots . \end{aligned} \tag{15}$$

The field $\widetilde{\phi}$ is complex $\widetilde{\phi} = \phi_1 + i\phi_2$ which gives $\widetilde{\phi} + \widetilde{\phi}^* = 2\phi_1$ (where $\phi_1 \equiv \mathrm{Re}(\phi)$) and replacing in the expression of the expanded potential V we have

$$V\left(\phi_0 + \widetilde{\phi}\right) = \frac{v^4}{\kappa^2}\phi_1^2 + \dots . \tag{16}$$

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There is a single *real* field with mass

$$m_{\mathrm{H}} = \frac{v^2}{\kappa}. \tag{17}$$

For the gauge field, again taking the Abelian form for simplicity, the following part in the Lagrangian, which can lead to the identification of a mass for the gauge field, is

$$-\kappa\, \varepsilon^{\mu\nu\rho} \left(\partial_\mu A_\nu\right) A_\rho - |\phi_0|^2 A_\mu A^\mu. \tag{18}$$

The first term is the Chern–Simons term in the Lagrangian and the second term comes from the square of the covariant derivative (in the kinetic term $\left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right)$), after taking the scalar field in the *vacuum* state, $\phi \to \phi_0$. This gives a mass

$$m_{\mathrm{gauge}} = \frac{1}{\kappa} |\phi_0|^2 = \frac{v^2}{\kappa}. \tag{19}$$

It results

$$m_{\mathrm{H}} = m_{\mathrm{gauge}} = \frac{v^2}{\kappa}. \tag{20}$$

The identification of the mass spectrum of the field particles for the action functional Eq. (1) with Eq. (2) is complicated by the non-trivial algebraic content of the theory. As above, the masses of the excitations around the broken vacuum ϕ_0 are obtained by expanding $V\left(\phi_0 + \widetilde{\phi}\right)$. As shown by Dunne (1995), retaining the quadratic terms in the expansion leads to a matrix and the mass spectrum is determined from the eigenvalues of this matrix. The relationship between the masses of the scalar field (Higgs) particle and of the gauge particle is the same $m_{\mathrm{H}} = m_{\mathrm{gauge}} = v^2/\kappa$.

We *note* that the mass of the vector potential is related to the condensate of the vorticity, which is the vacuum of the theory ($|\phi_0|^2 = v^2$): the Coriolis frequency of the planetary rotation exists even in the absence of any flow in the atmosphere. Alternatively we can say that whatever is the perturbation of velocity/vorticity, at large distances we only remain with the planetary rotation. This generates the vacuum non-zero density of the matter field. The mass of the gauge field excitation (vector boson of the gauge field) is the inverse of the radius of decay of the interaction between elements of the scalar field, i.e. the Rossby radius.

4 Comments on the relationship between the two views on the characteristic parameters of the atmospheric vortex

The two descriptions refer to the same physical reality. In the geophysical formulation the state where the two characterstic lengths are comparable, $L \approx R_{\text{Rossby}}$ has been identified as having particular properties. In the field theoretical formulation the equality $m_{\text{H}} = m_{\text{gauge}}$ (which through the mapping corresponds to $L = R_{\text{Rossby}}$) indicates a state with exceptional properties, the self-duality. Now we should recall that the fundamental property that is behind the high organization of the vorticity in the Euler asymptotic states is the *self-duality*, which is only revealed by the FT formulation. It is an admitted fact that any coherent structure known to date (solitons, instantons, topological field configurations, etc.) owes its existence to the self-duality (Mason and Woodhouse, 1996). Therefore, the well known experimental observation of vorticity organization into coherent structure of the flow in the Euler fluid naturally suggested to look for self-duality and the FT formulation confirmed that indeed the SD exists.

In the case of the atmospheric vortex (as for 2-D magnetized plasma) the SD state is only an approximation but we are still led to follow the suggestion that the existence of a quasi-stationary, quasi-coherent vortex like the idealized tropical cyclone is due to this approximative self-duality. Then the particular relationships: $L \approx R_{\text{Rossby}}$ and $m_{\text{H}} = m_{\text{gauge}}$ are associated to self-duality and the atmospheric vortex that verifies this condition is quasi-coherent. This is the reason that the tropical cyclone has the highest state of organization and the highest stability.

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Solving the Eq. (3) for $L \neq 1$ means that we consider that the departures from the self-duality state can still be reflected by the Lagrangian dynamics and this can be obtained from the same equation but for unbalanced lengths $L \neq R_{\text{Rossby}}$, which may be supposed that reflects different masses for the matter and gauge fields. We do not have a demonstration for this. We just note that this point of view is similar to the procedure adopted by Jacobs and Rebbi (1979) to calculate the energy of interaction of an ensemble of Abrikosov Nielsen Olesen vortices in superfluids in close proximity of the self-dual state; also, it is similar to the assumption adopted by Manton (1993) in the calculation of the motion of the vortices, near self-duality, as geodesic motion on the manifold consisting of the moduli space of a set of vortices which are solutions of the Abelian–Higgs model.

5 Numerical studies close to the equality of the two radial lengths

Using a large number of solutions of Eq. (3) we have identified systematic relationships between the three characteristics, with only the parameter L^{rad} (Spineanu and Vlad, 2009). The differential equation has been solved both on a plane square and on the radius in cylindrical symmetry, for an interval of $L^{\text{rad}} = \sqrt{2} L^{\text{sq}}$ around 1. The results allow to find two relationships between the tropical cyclone parameters: the radius of the circle where the azimuthal velocity is maximum, $r_{v_\theta^{\max}}$, the magnitude of the maximum of the azimuthal velocity $v_\theta^{\max}$ and the maximum radial extension of the cyclone, $R_{\max}$.

$$v_\theta^{\max}\left(L^{\text{sq}}\right) = 2.6461 \times \exp\left(\frac{1}{L^{\text{sq}}}\right) - 2.7748 \tag{21}$$

A simple approximation is

$$v_\theta^{\max}\left(L^{\text{sq}}\right) \approx e\left[\exp\left(\frac{1}{L^{\text{sq}}}\right) - 1\right] \tag{22}$$

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where $e \equiv \exp(1)$.

The other relation is

$$\frac{r_{v_\theta^{\max}}}{L^{\mathrm{sq}}}\left(L^{\mathrm{sq}}\right) = 0.395892 + 0.386360\left[-\exp\left(-\frac{L^{\mathrm{sq}}}{\sqrt{2}}\right)\right] \tag{23}$$

with a simple approximation

$$\frac{r_{v_\theta^{\max}}}{\sqrt{2}L^{\mathrm{sq}}} \approx \frac{1}{4}\left[1-\exp\left(-\frac{\sqrt{2}L^{\mathrm{sq}}}{2}\right)\right]. \tag{24}$$

[Note that, compared with a previous work (Spineanu and Vlad, 2009), we have eliminated the factor 1/2 in front of the nonlinear term in Eq. (3). In general an arbitrary factor λ can be used as long as the scaling of the coordinates is made $x' = x/\sqrt{\lambda}$, but in the present case taking $\lambda = 1$ makes easier the comparison with observations. Another difference is a better procedure of fit that have led to an improved calculation of the coefficients in the two equations above].

The two relationships Eqs. (21) and (23) will first be used in conjunction with the observational data which we take from the paper of Shea and Gray (1973). The objective is to examine consequences of the relationship discussed in this work: $L^{\mathrm{rad}} \approx 1$, or $\left(L^{\mathrm{rad}}\right)^{\mathrm{phys}} \approx R_{\mathrm{Rossby}}$. We expect to find a clusterization of observational data around those results that take into account this relationship.

Shea and Gray have organized a large set of observations in a graphical representation of the relationship between the radius where the maximum azimuthal velocity is measured and the magnitude of the maximum velocity, in our notations $\left(r_{v_\theta^{\max}}, v_\theta^{\max}\right)^{\mathrm{phys}}$. This is Fig. 45 of their paper. A line represents the best fit and we will refer to its points as "SG" data in the following. The figure also shows a substantial dispersion of the observed points. The best-fit line limits the maximum velocity that we can use for comparisons to a range between 70 and 115 knots (36 to 59 $\mathrm{m\,s^{-1}}$).

Assuming that the set of points of the fitting line in SG is parameterized by R_{Rossby} we find for each pair $\left(r_{v_\theta^{\max}}, v_\theta^{\max}\right)^{\text{phys}}$ the corresponding R_{Rossby} using the following procedure.

We start by taking a value of the normalized radii in the range $r_{v_\theta^{\max}} \in [0.1, 0.25]$ and we solve Eq. (23) for $\left(L^{\text{sq}}\right)$. It results $L^{\text{sq}} \in [0.655, 1.13]$. Each $\left(L^{\text{sq}}\right)$ is then inserted into the Eq. (21) and the resulting velocity $\left(v_\theta^{\max}\right)$ is compared with the data from SG. For this we need to return to dimensional variables i.e. to multiply with $R_{\text{Rossby}} f$, $v_\theta^{\max} \rightarrow \left(v_\theta^{\max}\right)^{\text{phys}}$. We do that iteratively until the equality is obtained

$$\left(v_\theta^{\max}\right)^{\text{phys}} - v^{\text{SG}} = 0. \tag{25}$$

This equation for R_{Rossby} leads to $R_{\text{Rossby}}\,(\text{m}) \in \left[106 \times 10^3, 190 \times 10^3\right]$.

Using the SG data we can obtain a qualitative image of the relationships between the physical parameters $\left(v_\theta^{\max}, r_{v_\theta^{\max}}\right)^{\text{phys}}$ and R_{Rossby} with the maximal radial extension $\left(L^{\text{rad}}\right)^{\text{phys}} = L^{\text{sq}} \sqrt{2} \times R_{\text{Rossby}}$ (already included in Eq. 23).

As shown in the previous work (Spineanu and Vlad, 2009) the two Eqs. (21) and (23) are able to correctly reproduce physical characteristics of the tropical cyclone when the physical input is close to the stationary state, which is the only state that can be described by the Eq. (3). The radial profile of the azimuthal velocity $v_\theta(r)$ obtained from integration of Eq. (3) also reproduces the Holland empiric formula, for the cases where data are available. To obtain a more general (even if approximative) idea about the quality of calculated $v_\theta(r)$ to reproduce observed profiles we have compared a large set of radial integration results for various L^{rad} with the empiric formula of Hsu and Babin (2005), $v_\theta^{\text{HB}}(r) = v_\theta^{\max}\left(r_{v_\theta^{\max}}/r\right)^x$. We find that $x \approx 0.7$, derived by Hsu and Babin from observations , also provides a good fit to our calculated profiles. However we also note that the departure between the calculated and observed (fitted with the

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formula) profiles mainly comes from the faster decay with r of our profiles, at large r. This means that the Eq. (3) generates maximal extension of the vortex that is somhow shorter than that observed in reality. This is compatible with the interpretation that the peripheric part of the tropical cyclone is dominated by thermodynamic processes, which are absent from the FT description. For the outer part of the tropical cyclone, Emanuel (2004) considers local balance between subsidence warming and radiative cooling. The radial distribution of the azimuthal velocity results from the equality betwen the Eckman suction and the subsidence rate. This strong thermodynamics aspect goes beyond FT model (which relies on vorticity organisation) and is the main obstacle in verifying the FT result that $\left(L^{\mathrm{rad}}\right)^{\mathrm{phys}} = R_{\mathrm{Rossby}}$. We will then look for estimation of an "effective" maximal radial extension – like the radius where the azimuthal velocity is 12 $(\mathrm{m\,s^{-1}})$ – and we will evaluate the ability of the FT model to describe the atmospheric vortex according to its ability to reproduce this value.

The database QuikSCAT of Chavas and Emanuel (2010) (denoted CE in the following) is organized as a collection of sets of several quantities measured for a single observation on a tropical cyclone, in particular the maximum velocity, the radius where the azimuthal velocity is 12 $(\mathrm{m\,s^{-1}})$, the maximum radial extension. The latter is obtained by extrapolation as mentioned above. For almost all tropical cyclones in the CE database there are sequences of observations at successive times, which we can use to qualitatively identify stationarity plateaux, if any. Our results can only be compared with such cases.

The data from SG and CE are used according to the following procedure.

We take from CE, for a particular case (a line in the file) the maximum velocity $\left(v_{\theta}^{\max}\right)_{\mathrm{CE}}^{\mathrm{phys}}$ $(\mathrm{m\,s^{-1}})$. Using the fitting curve of SG we obtain $\left(r_{v_{\theta}^{\max}}\right)_{\mathrm{SG}}^{\mathrm{phys}}$ (m). Now we turn to the two Eqs. (21)–(23) and define an algebraic equation whose solution is R_{Rossby} corresponding to that particular observation. We start by assuming a value for R_{Rossby} and with it we normalize

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$$\left(v_\theta^{\max}\right)_{\mathrm{CE}}^{\mathrm{phys}}\left(\mathrm{m\,s^{-1}}\right) \rightarrow \left(v_\theta^{\max}\right)_{\mathrm{CE}} = \frac{\left(v_\theta^{\max}\right)_{\mathrm{CE}}^{\mathrm{phys}}}{R_{\mathrm{Rossby}}\, f} \tag{26}$$

$$\left(r_{v_\theta^{\max}}\right)_{\mathrm{SG}}^{\mathrm{phys}}(\mathrm{m}) \rightarrow \left(r_{v_\theta^{\max}}\right)_{\mathrm{SG}} = \frac{\left(r_{v_\theta^{\max}}\right)_{\mathrm{SG}}^{\mathrm{phys}}}{R_{\mathrm{Rossby}}}. \tag{27}$$

Next we ask that the velocity from CE, normalized, equals the velocity of Eq. (21)

$$\left(v_\theta^{\max}\right)_{\mathrm{CE}} = v_\theta^{\max}\left(L^{\mathrm{sq}}\right) = 2.6461 \times \exp\left(\frac{1}{L^{\mathrm{sq}}}\right) - 2.7748. \tag{28}$$

This is an algebraic equation for L^{sq}. The result is inserted in Eq. (23) to determine $r_{v_\theta^{\max}}$, normalized. This must be compared with the *normalized* value of the radius of maximum velocity $\left(r_{v_\theta^{\max}}\right)_{\mathrm{SG}}$ obtained from SG, i.e. Eq. (27). If they are different then we will change R_{Rossby} and reiterate the sequence until the equality is obtained. Therefore the equation to be solved is

$$r_{v_\theta^{\max}}\left(L^{\mathrm{sq}}\right)\Big|_{\mathrm{Eq(23)}} = \frac{\left(r_{v_\theta^{\max}}\right)_{\mathrm{SG}}^{\mathrm{phys}}}{R_{\mathrm{Rossby}}}$$

for the unknown R_{Rossby}. Assuming that a solution exists, we will have found $\left(L^{\mathrm{sq}}\right)^{\mathrm{sol}}$ (non-dimensional) and $\left(R_{\mathrm{Rossby}}\right)^{\mathrm{sol}}$. Knowledge of these solutions allows to convert Eqs. (21) and (23) into dimensionful (physical) quantities that can be compared with observations, other than those that have been involved in the procedure described above. In particular, r_{12}, the radius where the azimuthal velocity is $12\,(\mathrm{m\,s^{-1}})$, (from the database Chavas Emanuel). We have chosen in CE a set of cases that seem to present stationarity and carried out calculations. We only illustrate the procedure in the following three cases.

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5.1 Case 1

The position in CE database is Line *440 BERTHA*. The latitude is $\theta = 29.65$ and the Coriolis parameter is

$$f = 2\Omega \sin\theta = 7.1951 \times 10^{-5}\ (\mathrm{s}).$$

From CE we take $\left(v_\theta^{\max}\right)_{\mathrm{CE}}^{\mathrm{phys}} = 40.098\,(\mathrm{m\,s^{-1}})$. It results

$$R_{\mathrm{Rossby}} = 178\,862\,(\mathrm{m})$$
$$L^{\mathrm{sq}} = 1.2427.$$

Now we can make further comparison with observations, in particular with the radius of, v_{12}, i.e. $v_\theta = 12\,(\mathrm{m\,s^{-1}})$, which in CE is $\left(r_{v_\theta=12}\right)_{\mathrm{CE}}^{\mathrm{phys}} = 166\,411\,(\mathrm{m})$. Since now we know R_{Rossby} we normalize the velocity with $R_{\mathrm{Rossby}} \times f = 12.86\,(\mathrm{m\,s^{-1}})$,

$$\frac{v_{12}}{12.86} = \frac{12}{12.86} = 0.9331.$$

We return to solve Eq. (3) for $L^{\mathrm{rad}} = L^{\mathrm{sq}}\sqrt{2} = 1.7574$ and find the radial profile of the (normalized) velocity, $v_\theta(r)$. On this profile, $v_\theta = 0.9331$ is found at $r_{v=0.9331} \approx 1.26$ which means

$$\left(r_{v=0.9331}\right)^{\mathrm{phys}} = 178\,862 \times 1.26\,(\mathrm{m}) = 225\,370\,(\mathrm{m}). \tag{29}$$

This compares well with the data from CE $\left(r_{v_\theta=12}\right)_{\mathrm{CE}}^{\mathrm{phys}} = 225\,129\,(\mathrm{m})$.

For the maximum radial extension we find

$$R_{\max} = L^{\mathrm{sq}}\sqrt{2} \times R_{\mathrm{Rossby}} = 314\,340\,(\mathrm{m}) \tag{30}$$

which is again small compared with the data from CE $R_{\max}^{\mathrm{CE}} = 391\,874\,(\mathrm{m})$.

Note similarity with *625 MARTY*.

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5.2 Case 2

This is *480 OMAR*. The latitude is $\theta = 16.44$ and the Coriolis frequency is $f = 2\Omega \sin\theta = 4.1162 \times 10^{-5}\,\mathrm{s}^{-1}$.

From CE we take $\left(v_\theta^{\max}\right)_{\mathrm{CE}}^{\mathrm{phys}} = 46.27\,(\mathrm{m\,s^{-1}})$. It results following the procedure described above

$$R_{\mathrm{Rossby}} = 202\,193\,(\mathrm{m})$$
$$L^{\mathrm{sq}} = 0.87.$$

Now we can make further comparison with observations, in particular with the radius of, v_{12}, i.e. $v_\theta = 12\,(\mathrm{m\,s^{-1}})$, which in CE is $\left(r_{v_\theta=12}\right)_{\mathrm{CE}}^{\mathrm{phys}} = 187\,613\,(\mathrm{m})$. Since now we know R_{Rossby} we normalize the velocity with $R_{\mathrm{Rossby}} \times f = 8.32\,(\mathrm{m\,s^{-1}})$,

$$\frac{v_{12}}{8.32} = \frac{12}{8.32} = 1.4418.$$

We return to solve Eq. (3) for $L^{\mathrm{rad}} = L^{\mathrm{sq}}\sqrt{2} = 1.2304$ and find the radial profile of the (normalized) velocity, $v_\theta(r)$. On this profile, $v_\theta = 1.4418$ is found at $r_{v=1.4418} \approx 0.9$ which means

$$\left(r_{v=0.9331}\right)^{\mathrm{phys}} = 202\,193 \times 0.9\,(\mathrm{m}) = 181\,970\,(\mathrm{m}). \tag{31}$$

This compares well with the data from CE $\left(r_{v_\theta=12}\right)_{\mathrm{CE}}^{\mathrm{phys}} = 187\,613\,(\mathrm{m})$.

For the maximum radial extension we find $R_{\max} = L^{\mathrm{sq}}\sqrt{2} \times R_{\mathrm{Rossby}} = 248\,770\,(\mathrm{m})$ which is again small compared with the data from CE $R_{\max}^{\mathrm{CE}} = 423\,035\,(\mathrm{m})$.

We note however that for similar data (line 509 Aletta of CE) with $\left(v_\theta^{\max}\right)_{\mathrm{CE}}^{\mathrm{phys}} = 46.205\,(\mathrm{m\,s^{-1}})$ at latitude 14.68, we have $f = 3.7129 \times 10^{-5}\,(\mathrm{s^{-1}})$ and obtain $R_{\mathrm{Rossby}} = 213\,070\,(\mathrm{m})$ and $L^{\mathrm{sq}} = 0.8458$. After similar calculations we get

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$\left(r_{v_\theta=12}\right)^{\text{phys}} = 191\,763\,(\text{m})$ while $\left(r_{v_\theta=12}\right)^{\text{phys}}_{\text{CE}} = 122\,620\,(\text{m})$. The difference is substantial. While the calculation, for close magnitudes, gives close results, the reality (i.e. the observation) may be rather different: close magnitudes of $v_\theta^{\max}$ and of $f(\theta)$ can give very different r_{12}'s.

5.3 Case 3

This is the line *299 KARL* in the CE database. The input is $\left(v_\theta^{\max}\right)^{\text{phys}}_{\text{CE}} = 48.92\,(\text{m}\,\text{s}^{-1})$ Since the *latitude is* 15°, we have a Coriolis frequency (taking $\Omega = 7.2722 \times 10^{-5}\,(\text{s}^{-1})$)

$$f = 2\Omega \sin\theta = 3.7644 \times 10^{-5} \left(\text{s}^{-1}\right).$$

Using $\left(v_\theta^{\max}\right)^{\text{phys}}_{\text{CE}}$ we start a search of R_{Rossby}. For every step, using the current guess for $\left(R_{\text{Rossby}}\right)^{(k)}$ we convert to non-dimensional velocity

$$\frac{\left(v_\theta^{\max}\right)^{\text{CE}}}{R^{(k)}_{\text{Rossby}} \times f}$$

and impose to be equal to Eq. (21), which determines $\left(L^{\text{sq}}\right)^{(k)}$. With $\left(v_\theta^{\max}\right)^{\text{phys}}_{\text{CE}}$ we calculate by spline interpolation on the *Shea Gray* data, $\left(r_{v_\theta^{\max}}\right)_{\text{SG}}$ (m) and normalize

$$\frac{\left(r_{v_\theta^{\max}}\right)_{\text{SG}}}{R^{(k)}_{\text{Rossby}}}.$$

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This is compared with $r_{v_\theta^{max}}$ from Eq. (23) where $\left(L^{sq}\right)^{(k)}$ has been inserted. The comparison is used as equation and an iterative procedure (the NAG subroutine *c05awf* is employed) leads to the solution. It resulted

$$R_{\text{Rossby}} = 196\,554\,(\text{m}),\ L^{sq} = 0.7891. \quad (32)$$

This corresponds to $L^{rad} = 1.1126$. We now want to estimate the radius where the azimuthal velocity takes value $12\,(\text{m}\,\text{s}^{-1})$, using the approach based on FT. We first normalize the velocity, since R_{Rossby} is known

$$R_{\text{Rossby}} \times f = 196\,554 \times 3.7644 \times 10^{-5} = 7.39\,\left(\text{m}\,\text{s}^{-1}\right)$$

$$v_{12} = \frac{12\left(\text{m}\,\text{s}^{-1}\right)}{R_{\text{Rossby}} \times f} = 1.6218.$$

We find the radial profile of the azimuthal velocity by performing the radial integration of Eq. (3) with $L^{rad} = L^{sq}\sqrt{2} = 1.1126$. On the plot (r, v) the velocity $v_{12} = 1.6218$ is obtained at the radius $r_{v=1.6218} \approx 0.85$. Now we can return to dimensionful quantities

$$\left(r_{v=1.6218}\right)^{phys} = R_{\text{Rossby}} \times 0.85 = 167\,070\,(\text{m}). \quad (33)$$

This is smaller than the value found in CE, $(r_{12})_{CE} = 206\,639\,(\text{m})$, a possible reflection of the weak ability of FT to describe the peripheric region of the vortex, where thermodynamics is stronger. The estimation for the maximum radial extension is

$$R_{max} = L^{sq}\sqrt{2} \times R_{\text{Rossby}} = 218\,690\,(\text{m}).$$

6 Conclusions

On very general basis we have derived the equality between the maximum radius R_{max} of the tropical cyclone and the Rossby radius R_{Rossby}. This has been done for a simplified model of the atmospheric vortex: it is assumed stationarity, the predominance of

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the intrinsic self-organization of vorticity (over the source/loss processes) and the validity of the two-dimensional approximation. These simplifications may only be acceptable in the large time regime of a tropical cyclone (after cyclogenesis) and have been previously adopted in various studies. To evaluate the validity of our result (the equality $R_{\text{Rossby}} = R_{\text{max}}$) we have to confront our theoretical results with the observations. We have to use the Eqs. (21) and (23) and choose from data bases the cases with acceptable signatures of stationarity, the only regimes we can afford with our model. We show that the results are in good agreement.

The property of a large scale stationary atmospheric vortex $R_{\text{max}} = R_{\text{Rossby}}$ has been derived from the mapping that connects the vorticity distribution to the extremum of an action functional. The field theoretical formulation of the continuum version of the point – like vortex system has been taken as an approximative representation of the atmospheric vortex. The equality of the masses of the matter field particle and of the gauge particle translates through the mapping into the equality of the radial extension of the vortex with the Rossby radius. In numerical calculations this relationship is used either directly or implicitely, i.e. confronting with observation some important characteristics of the tropical cyclone (maximum velocity, radius of the maximum velocity, maximum radial extension). The results confirm in general this relationship. In cases that can be used within our approximations, the FT model (implicitely $R_{\text{max}} = R_{\text{Rossby}}$) reproduces reasonably well the observation.

There are also cases where the results of the model are sensibly different from observations and this can come from various sources. The stationarity of the tropical cyclone may be a short period or even inexistent. The thermodynamics and the vorticity dynamics at stationarity cannot be decoupled in sufficient measure such that the intrinsic self-organization tendency of the vorticity to be manifested. Finally, we note the extreme sensitivity (already mentioned previously Spineanu and Vlad, 2009) of the results to even small variation of the input data. This is clearly seen in the two Eqs. (21) and (23) where the dependence on the parameter L^{sq} is exponential.

The fact that the model receives confirmation by a reasonably good comparison between its calculated results and the observation also validates the main result derived in this work, i.e. $R_{\mathrm{max}} = R_{\mathrm{Rossby}}$. However, we see here a much more important message. It suggests that the process of self-organization of the vorticity, a part of the dynamics of the tropical cyclone that is distinct of any thermodynamic process, appears as an important factor determining the spatial distribution of the main flow variables. It seems to become a necessity to combine the spontaneous self-organization with the thermodynamics of the atmospheric vortex. This is an important area of investigation.

Acknowledgements. This work has been partly supported by the Grant ERC-Like 4/2012 of UEFISCDI. The authors aknowledge useful discussions with Jun-Ichi Yano, Robert S. Plant and Emmanuel Vincent. The fruitful exchange of ideas within the COST collaboration ES0905 is highly appreciated.

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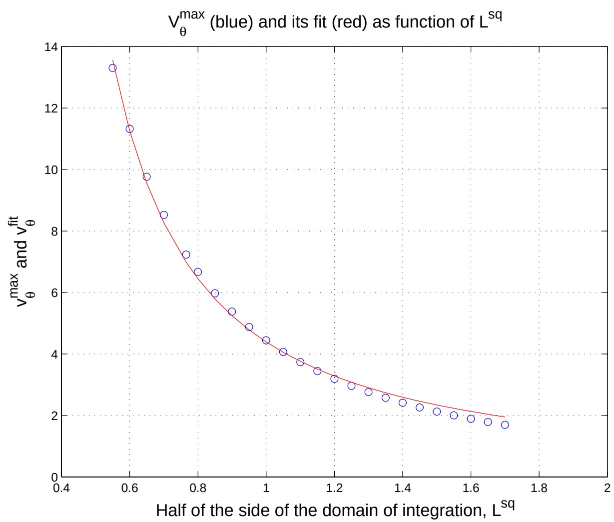

Fig. 1. The analytical fit of the maximum velocity resulting from solving the Eq. (3).

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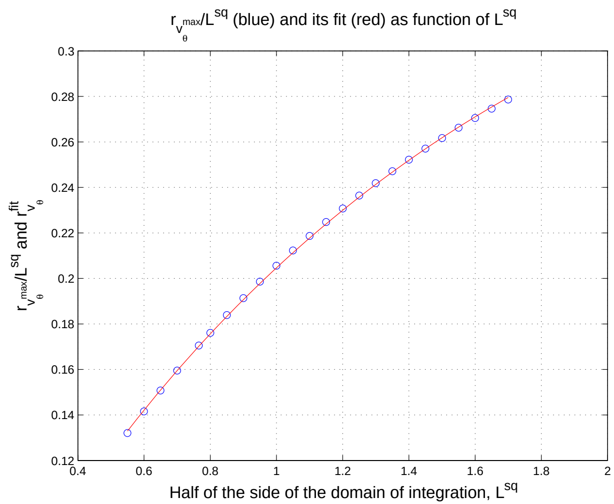

Fig. 2. The analytical fit of the ratio: radius where the maximum velocity is found over the length L^{sq}. This is inferred from a large set of solutions of the Eq. (3) for various space domains $\left(L^{sq} \text{ or } L^{rad} = \sqrt{2}L^{sq}\right)$.

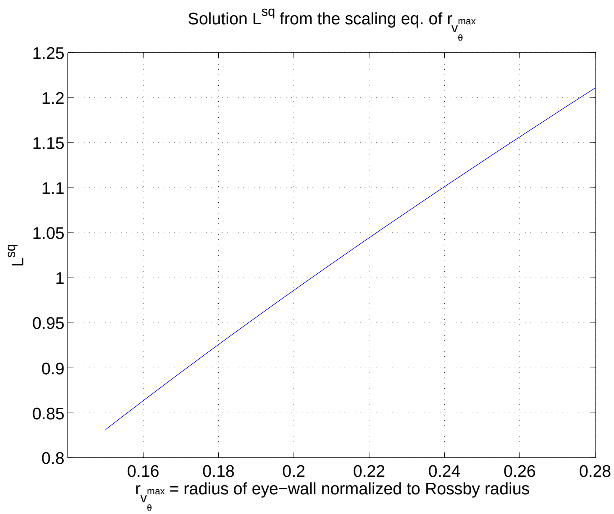

Fig. 3. The length of the half-side of the square domain of integration in plane L^{sq}, for a fixed value of $r_{v_\theta}^{\max}$. Both are non-dimensional (normalized to R_{Rossby}).

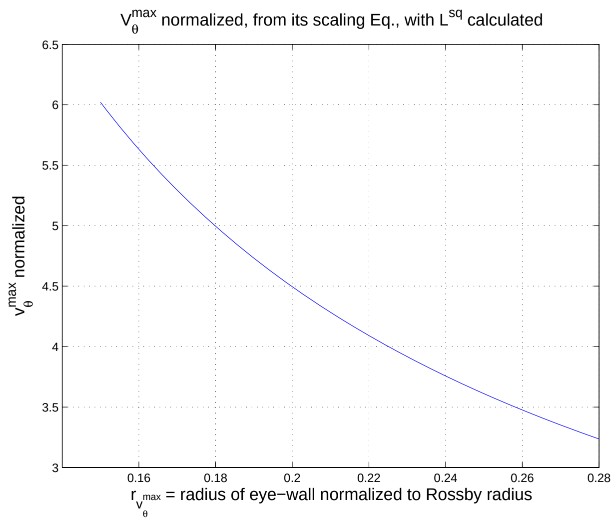

Fig. 4. The maximum velocity v_θ^{max} as results form the analytical formula. The parameter L^{sq} is determined previously for given values of $r_{v_\theta^{max}}$ (see Fig. 3).

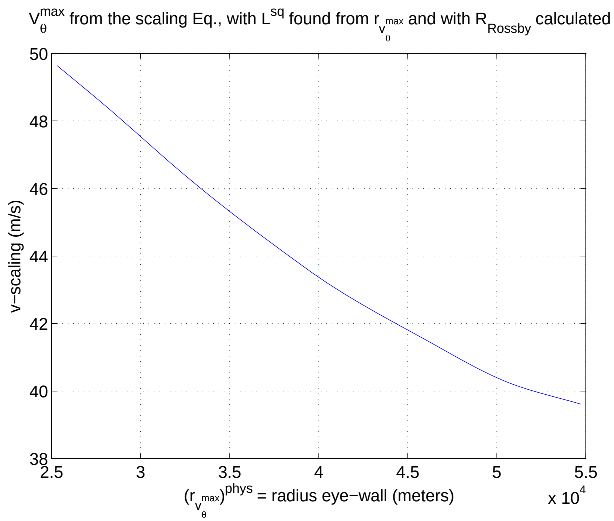

Fig. 5. The maximum velocity v_θ^{max} (dimansional, $\mathrm{m\,s^{-1}}$) after the Rossby radius is calculated using input from the Shea Gray data.

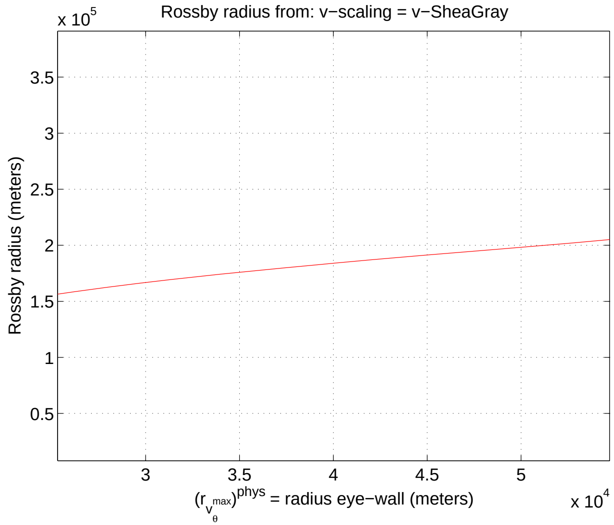

Fig. 6. The Rossby radius as results form imposing equality of the maximum velocities: from the analytical formula and from Shea Gray data.

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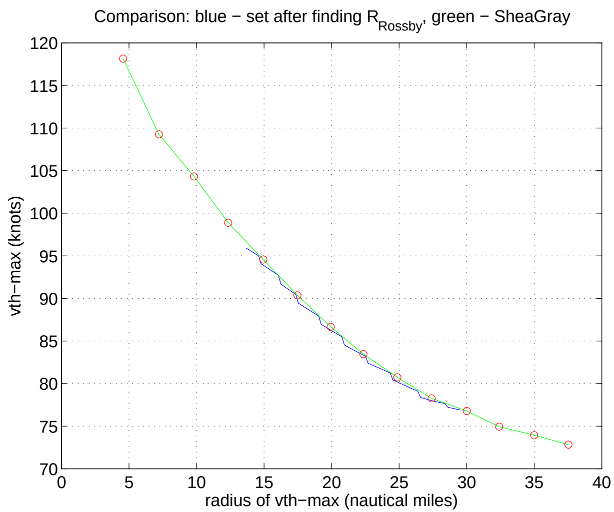

Fig. 7. The range of Shea Gray data effectively used: the green line represents all SG data and the blue line is the sub-domain used for calculation of the Rossby radius.

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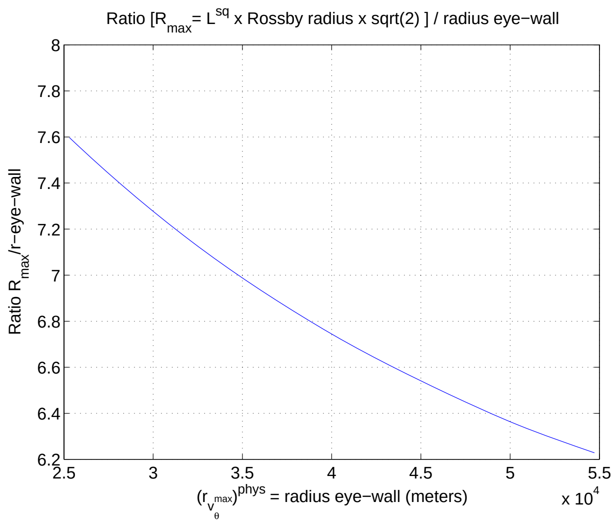

Fig. 8. The ratio of the maximum radial extension ($L^{sq}\sqrt{2} \times R_{\text{Rossby}}$) and the radius of the maximum azimuthal velocity. This is in general a number substantially greater than unity. After the determination of R_{Rossby} we find a range [6 . . . 8].

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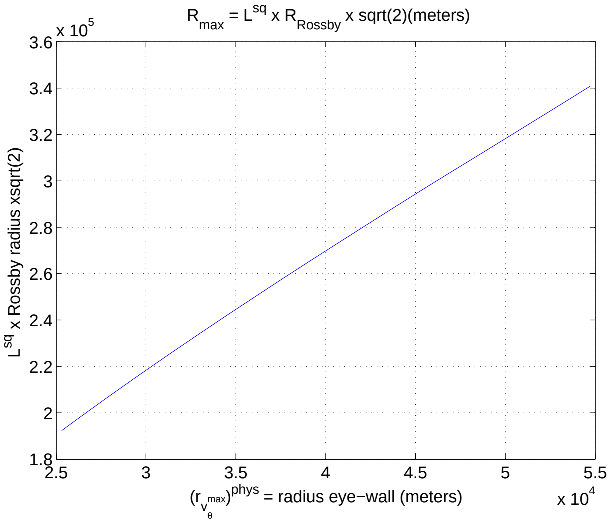

Fig. 9. The maximum radial extension $R_{max} = L^{sq}\sqrt{2} \times R_{Rossby}$ (m) versus the radius of the eye-wall $\left(r_{v_\theta^{max}}\right)^{phys}$ expressed in maters.

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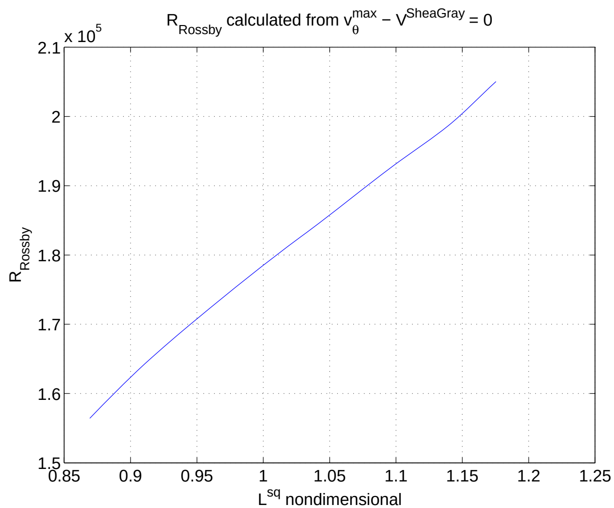

Fig. 10. The Rossby radius obtained by imposing the equality of the velocity as given by the analytical fit with the velocity from the Shea Gray data.

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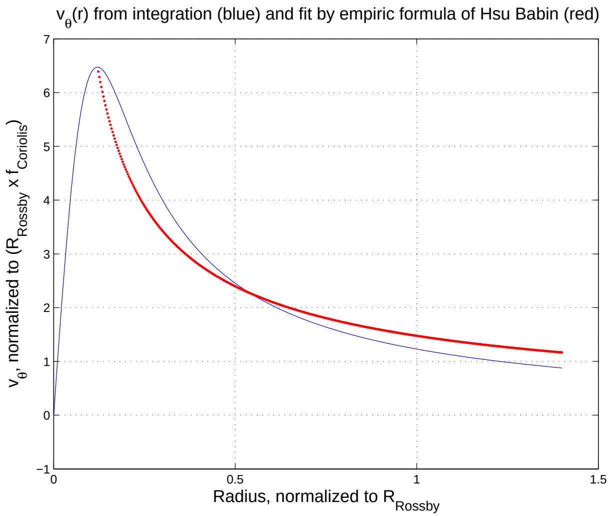

Fig. 11. The profiles of the azimuthal velocity as results form integration of Eq. (3) (blue) and from the empirical formula of Hsu-Babin (red) (i.e. from observations). The figure shows that the radial decay of the theoretical $v_\theta(r)$ at large r gives systematically a shorter maximum radial extension of the tropical cyclone.

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