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Heliospheric pick-up ions influencing thermodynamics and dynamics of the distant solar wind

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Abstract. Neutral interstellar H-atoms penetrate into the inner heliosphere and upon the event of ionization are converted into pick-up ions (PUIs). The magnetized solar wind flow incorporates these ions into the plasma bulk and enforces their co-motion. By nonlinear interactions with wind-entrained Alfvén waves, these ions are then processed in the comoving velocity space. The complete pick-up process is connected with forces acting back to the original solar wind ion flow, thereby decelerating and heating the solar wind plasma. As we show here, the resulting deceleration cannot be treated as a pure loading effect, but requires adequate consideration of the action of the pressure of PUI-scattered waves operating by the PUI pressure gradient. Hereby, it is important to take into proper account the stochastic acceleration which PUIs suffer from at their convection out of the inner heliosphere by quasi-linear interactions with MHD turbulences. Only then can the presently reported VOYAGER observations of solar wind decelerations and heatings in the outer heliosphere be understood in view of the most likely values of interstellar gas parameters, such as an H-atom density of $0.12\,\mathrm{cm}^{-3}$. Solar wind protons (SWPs) appear to be globally heated in their motion to larger solar distances. Ascribing the needed heat transfer to the action of suprathermal PUIs, which drive MHD waves that are partly absorbed by SWPs, in order to establish the observed SWP polytropy, we can obtain a quantitative expression for the solar wind proton pressure as a function of solar distance. This expression clearly shows the change from an adiabatic to a quasi-polytropic SWP behaviour with a decreasing polytropic index at increasing distances. This also allows one to calculate the average percentage of initial pick-up energy fed into the thermal proton energy. In a first order evaluation of this expression, we can estimate that about 10% of the initial PUI injection energy is eventually transfered to SWPs independent of the PUI injection rate.

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1 Introduction to the PUI-induced modulation of the solar wind

We shall study the effect of PUIs which are dynamically involved in the solar wind flow dynamics in the outer heliosphere. Unless the Local Inter-Stellar Medium (LISM) is fully ionized, neutral LISM gases penetrate with the LISM flow into the inner heliosphere and there they become ionized and transformed into PUIs. It is known that the solar wind is decelerated due to both the PUI loading (see e.g. Holzer and Leer, 1973; Fahr, 1973; Ripken and Fahr, 1983; Fahr and Ripken, 1984) and due to the action of the PUI pressure (Isenberg, 1986; Fahr and Fichtner, 1995; Lee, 1997; Whang, 1998; Whang et al., 1999; Fahr and Rucinski, 1999). At the same time the solar wind plasma is also heated by this PUI implantation and thus, its effective sound velocity is increased (see Fahr and Rucinski, 1999). Connected with both phenomena, i.e. deceleration and heating, the solar wind Mach numbers decrease with increasing solar distances. In the following we look into this phenomenon a bit more quantitatively.

The effective Mach number and wind deceleration can be calculated with the help of an expression for the PUI pressure, P_{pui}. To describe the latter, Lee (1997) uses the law of the enthalpy flow conservation, with the enthalpy ε_{pui} of pick-up ions given in the form $\varepsilon_{pui} = \frac{\gamma}{\gamma-1} P_{pui}$, and $\gamma = \frac{5}{3}$ being the polytropic index. He then obtains the result that the PUI pressure is given by $P_{pui} = \left(\frac{1}{7}\right) \rho_{pui} V_w^2$, where V_w is the solar wind velocity and ρ_{pui} denotes the mass density of the PUIs. In a different approach, Whang (1998) uses a polytropic relation exclusively for the SWPs, with an observationally supported, effective polytropic index of $\gamma_w = 1.28$, clearly indicating some heating of the SWP. This heating, however, is unexplained in his physical context. In his theoretical approach, the full amount of PUI-injected energy is reflected exclusively in the conserved enthalpy flow of PUIs while the PUI-induced heating of the solar wind protons, which is to be expected (see e.g. Fahr and Ziemkiewicz, 1988 or Williams et al., 1995), is not taken into account, i.e. no energy transfer from PUIs to SWPs is, in fact, taken into account, thus leaving the solar wind heating unexplained.

In none of the above mentioned approaches is the complete dynamical effect of the PUI pressure adequately taken into account, due to the effect of nonlinear wave-particle interactions between PUIs, and solar wind convected MHD wave turbulences are also not taken into account. It is important to respect that newly injected PUIs are not simply picked-up by the magnetized solar wind and then stored in a tiny toroidal subpart of velocity space. Rather they suffer strong pitch-angle scattering and a less strong energy diffusion due to quasi-linear interactions with comoving turbulences. Thereby, they are effectively redistributed in velocity space (see e.g. Chalov et al., 1995, 1997 or Fichtner et al., 1997). In addition, with the momentum loading force, this redistribution is also connected with an additional net force acting upon the center of mass of the multi-fluid solar wind, which can be identified with the gradient of the PUI pressure; however, a pressure P_{pui} has to be evaluated on the basis of the actual PUI distribution function resulting under quasi-linear wave-PUI interactions.

Starting with the PUI pressure given in the form $P_{pui} = \alpha \rho_{pui} V_w^2$, one can then derive (see Fahr and Fichtner, 1995) the following differential equation for the decelerated solar wind:

$$\frac{d}{dr} V_w = \frac{-m_p \beta_{ex} \frac{1+\alpha}{\rho_w + \rho_{pui}} + \frac{2\alpha}{r} \xi V_w}{1 + \alpha \xi}. \quad (1)$$

Here β_{ex} denotes the local PUI- injection rate, which here is approximated as being exclusively due to charge exchange processes with SWPs (i.e. photoionization is neglected; a near solar minimum condition!) and thus is given by $\beta_{ex} = \sigma_{ex} n_H n_w V_w$. Here σ_{ex} is the charge exchange cross section, and n_H and n_w denote local H-atom and solar wind proton densities. The function $\xi = \rho_{pui} / \left(\rho_w + \rho_{pui}\right)$ denotes the relative abundance of PUIs with respect to all protons. The function $\xi(r)$ used by us here has been calculated by Fahr and Rucinski (1999) using the "hot" kinetic H-atom model developed by Wu and Judge (1979). Integration of the above differential equation then yields:

$$V_w = V_{w0} \exp \left[\int_{r_0}^{r} \frac{\frac{2\alpha}{r} \xi - n_H \sigma_{ex} (1 - \xi)(1 + \alpha)}{1 + \alpha \xi} dr \right]. \quad (2)$$

Adopting the expression for P_{pui} derived by Fahr and Fichtner (1995), one then obtains with their result, i.e. $\alpha = \left(\frac{1}{3}\right)$:

$$V_w = V_{w0} \exp \left[\int_{r_0}^{r} \frac{2}{3 + \xi} \left(\frac{\xi}{r} - 2 n_H \sigma_{ex} (1 - \xi) \right) dr \right]. \quad (3)$$

With $\xi \Rightarrow 0$ one formally switches off the accelerating effect of the PUI pressure and thus is left with an unrealistically strong solar wind deceleration, as the one expected in papers by Richardson et al. (1995) or Wang et al. (2000) (see Fahr and Rucinski, 2001). This reveals the fact that the wave-induced PUI pressure has to be taken into full account, since it is an essential dynamical ingredient for the modulated two-fluid solar wind.

An accurate expression of P_{pui} can only be derived with the knowledge of the PUI distribution function f_{pui}, resulting under quasi-linear PUI coupling to wind-entrained MHD turbulences. This function is obtained as a solution of the PUI transport equation containing convection, adiabatic deceleration, and energy diffusion by Fermi-2 acceleration.

A fairly realistic expression for P_{pui} can be derived from the results for f_{pui} obtained by Chalov et al. (1995, 1997) as solutions of the complete PUI transport equation. As Fahr and Lay (2000) show, these numerical results are very nicely represented by the following analytical formula:

$$f_{pui} = F \left(x^{-0.33} \right) w^{\beta} \exp \left[-C(x) (w - w_0)^{\kappa} \right], \quad (4)$$

where F is a constant, $x = r/r_E$ is the radial solar distance in units of AU, $w = (v/V_w)^2$ is the squared PUI velocity normalized with V_w, with w_0 being a typical injection value. Furthermore, the quantities β, κ, and C are found as: $\beta = -\frac{1}{6}$; $\kappa = \frac{2}{3}$; and $C(x) = 0.442\ x^{0.2}$. With the above representation of f_{pui} in Eq. (4), one then obtains the PUI density by:

$$n_{pui} = 2\pi F\ x^{-0.33} \left[\frac{3}{2} C(x)^{-2}\ \Gamma(2) \right], \quad (5)$$

and the PUI pressure by:

$$P_{pui} = \frac{2\pi}{3} F\ x^{-0.33} \left(\frac{1}{2} m_p V_w^2 \right) \left[\frac{3}{2} C(x)^{-\frac{7}{2}}\ \Gamma(\frac{7}{2}) \right], \quad (6)$$

where $\Gamma(y)$ is the Gamma function of the argument y. Equations (5) and (6) then permit the following representation for P_{pui}:

$$P_{pui} = \frac{5}{16} \sqrt[2]{\pi} C(x)^{-\frac{3}{2}} \rho_{pui} V_w^2 = \alpha(x) \rho_{pui} V_w^2. \quad (7)$$

In this expression for $P_{pui}(x)$, the function α is found as: $\alpha = \alpha(x) = 1.83\ x^{-0.3}$. This means that α decreases with increasing solar distances, obviously reflecting the fact that at larger distances the adiabatic deceleration starts to slowly overcompensate for the effect of the wave-driven Fermi-2 accelerations. The above formula (7) based on the results by Fahr and Lay (2000) is valid at distances of $x \geq x_c = 15$, where $\alpha = \alpha_c = \alpha(x_c)$ evaluates to $\alpha_c = 0.44$.

In the following calculations, we may assume that the PUI pressure can be represented with sufficient accuracy by Eq. (7), setting $\alpha(x) = \alpha_c$. We then calculate the solar wind deceleration (see Fig. 1) and the effective solar wind Mach number M_w^*, or as in this case the equivalent quantity Π (see Fig. 2), related to the latter by $\Pi \simeq \frac{1}{M_w^{*2}}$, and given by:

$$\Pi(x) = \frac{\alpha_c \rho_{pui} V_w^2}{\frac{1}{2} \rho_w V_w^2} = \frac{2 \alpha_c \xi}{(1 - \xi)}. \quad (8)$$

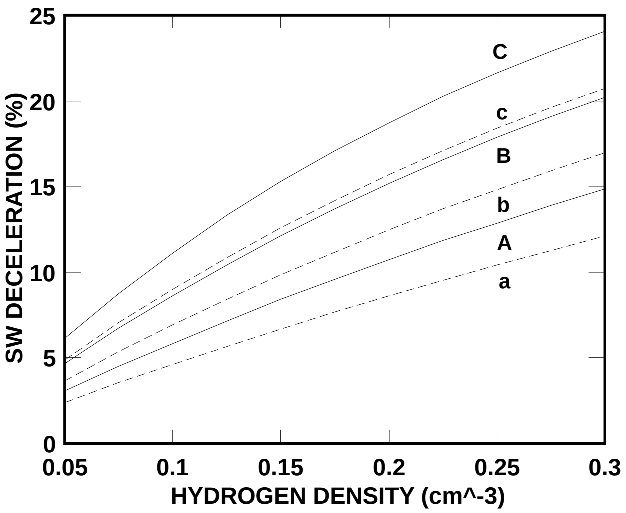

Fig. 1. The solar wind deceleration (in %) with respect to the solar wind velocity $v_{w,0} = 450\,km/s$ (full line, upwind) and $v_{w,0} = 700\,km/s$ (dashed line, crosswind) is shown as a function of the LISM H-atom density $n_{H\infty}$ at various upwind distances, i.e. 40 AU, 60 AU and 80 AU.

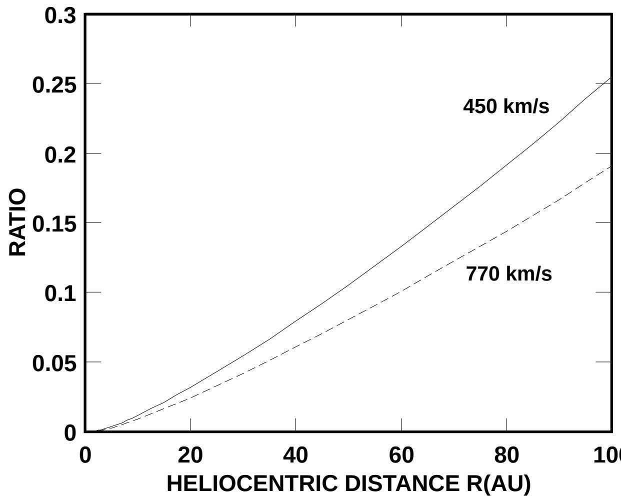

Fig. 2. The ratio $\Pi(r)$ of the pick-up ion thermal pressure and the solar wind kinetic pressure, i.e. SWP ram pressure, is given in upwind direction as a function of the heliocentric distance r in units of AU, calculated for different values of the solar wind velocity $v_{w,0}$.

As is evident in Eq. (8), the quantity Π also represents the ratio of PUI thermal pressure and solar wind kinetic ram pressure. The function ξ results from the accumulated effect of PUI injections due to local charge exchange processes of LISM H-atoms with SWPs in the heliosphere and is explicitly calculated by Fahr and Rucinski (1999, 2001), who describe the penetration of LISM H-atoms into the heliosphere by the so-called "hot" kinetic model developed by Fahr (1971), Thomas (1978) and Wu and Judge (1979).

In Fig. 2, the quantity $\Pi(x) \simeq M_w^{*-2}$ is plotted as a function of the solar distance x. As one may recognize, the effective solar wind Mach number, M_w^*, in the outer region of the heliosphere decreases from very high values in the inner heliosphere towards moderately low values of the order of 2 in the outer heliosphere, i.e. the solar wind due to the modulation, by and mixing with, marginally subsonic PUIs developes with increasing distances from an initially hypersonic flow towards a weakly supersonic flow.

2 Thermodynamics of the PUI-mediated solar wind

As already mentioned above, PUIs are produced by ionization of interstellar neutral atoms in the heliosphere and are convected outwards with the solar wind flow as a separate suprathermal ion fluid. The thermodynamic behaviour of this PUI fluid at its motion outwards to the outer heliosphere, until now has been poorly understood. One clearly expects that PUIs drive waves by virtue of their distribution function, which is unstable with respect to the excitation of wave power (see e.g. Wu and Davidson, 1972; Hartle and Wu, 1973; Lee and Ip, 1987; Freund and Wu, 1989; Fahr and Ziemkiewicz, 1988; Gray et al., 1996), but while doing so they also undergo Fermi-2 energization (energy diffusion) by nonlinear wave-particle interaction with already preexisting, convected wave turbulences (see Chalov et al., 1995, 1997; Fichtner et al., 1996; le Roux and Fichtner, 1997). In the following, we study the branching of the relevant energy flows and for this purpose, we also pay attention to the observational fact that solar wind protons behave non-adiabatic, but polytropic at their expansion to large solar distances (see Whang, 1998 and Whang et al., 1999). This evidently expresses the fact that solar wind protons are globally and continuously heated at their motion to larger solar distances.

This global heating thus cannot be related to sporadic events, such as passages of corotating interaction regions (CIRs) or solar eruptive events (see also Fisk et al., 2000). In contrast, it is, however, highly likely to be indirectly caused by PUIs driving MHD waves cascading to frequencies at which they can be reabsorbed by SWPs. Already Parker (1964) and Coleman (1968) expected that some extended heating due to dissipation of waves might cause a non-adiabatic expansion of the solar wind beyond its critical point. Meanwhile, this non-adiabatic solar wind temperature behaviour is, in fact, clearly recognized in data taken by the VOYAGER-1/2 spacecraft (see Richardson et al., 1995; Whang, 1998; Whang et al., 1999). The dissipation of non-Alfvénic turbulence energy to solar wind protons was then more quantitatively estimated by Matthaeus et al. (1994) to take place with a rate of $q_{turb} \simeq \rho_s u^3/l$, where ρ_s, u, l are the solar wind mass density, the rms turbulent fluctuation speed, and the turbulent correlation scale, respectively. Since observational constraints on u and l, as functions of the solar distance, are missing up to now, it thus remains hard to predict anything more quantitative with respect to the non-adiabaticity of the solar wind expansion.

Furthermore, a permanent dissipation of turbulent wave power at heating the expanding solar wind should quickly lead to a complete consumption of all convected turbulence power, unless some turbulence generating processes are operating. In this respect, Lee and Ip (1987) or Fahr and Ziemkiewicz (1988) have indicated that PUIs implanted into the expanding solar wind by means of their unstable distribution functions generate wave powers which can partly be reabsorbed by SWPs. Using quasi-linear wave-particle interaction theories by Kennel and Engelmann (1966), Gary and Feldman (1978) and Winske and Leroy (1984), the latter authors could show that under optimized conditions, up to 50% of the initial PUI energy can be forwarded to SWPs by means of PUI-driven waves. More recently, Williams et al. (1995) and Gray et al. (1996) have looked into this problem again. Williams et al. (1995) have given representations for the non-adiabatic expansion of the distant solar wind due to dissipation of PUI-driven waves within a simplified energy dissipation concept. Gray et al. (1996), within a hybrid plasma simulation code, study the energy transfer in a homogeneous plasma background from the original unstable PUI ring distribution to the SWP thermal energy degree perpendicular to the magnetic field and find that for vanishing pitch-angle diffusion – at most favourable conditions like "low Beta" plasmas – about 20% of the initial PUI ring energy can be handed over to SWPs.

In all concepts mentioned so far, however, a quantitative number for the average fraction of initial PUI energy transferred under general conditions to the SWPs, while moving towards the heliospheric termination shock, including pitch-angle diffusion and general forms of nonlinear wave-particle couplings, could not be given. We may perhaps have a guide from the observational result presented by Whang (1998) or Whang et al. (1999), showing that the distant SWPs behave polytropic with a best-fitting polytropic index of $\gamma^* = 1.28$. Since γ^* turns out to be substantially smaller than the adiabatic index $\gamma = 5/3 \simeq 1.667$, it is evident that some continuous, i.e. non-CIR-correlated heating of the SWPs takes place, which here we are going to ascribe to the complicated action of PUIs. This SWP heating, since it is global in its nature and independent of latitude, must most certainly be due to wave energy continuously coupled from the PUIs via feeding of wave turbulences to the SWPs, due to nonlinear or quasi-linear wave-particle couplings (e.g. see Fahr and Ziemkiewicz, 1988 or Williams et al., 1995; Gray et al., 1996). It thus represents an energy sink for the PUIs which pump energy into wave turbulences, but at the same time also represents an energy source for SWPs, which absorbs part of these turbulences.

The energization of the SWPs may most likely be ascribed to a process similar to the Fermi-2 acceleration process considered for more energetic ions such as PUIs or ACRs (Anomalous Cosmic Rays), i.e. diffusion in energy space due to scatterings between counterpropagating hydromagnetic waves. Of course, it must also be taken into account that PUIs as well undergo this type of Fermi-2 acceleration process. This, for instance, is clearly manifest as an evident and ubiquitous heliospheric phenomenon both in view of theory and observations (see, e.g. Fisk et al., 2000). But it should be kept in mind that these PUIs at driving wave turbulences also experience genuine energy losses which a complete PUI thermodynamics has to include. These PUI energy losses are primarily due to the generation of wave power, which under stationary conditions may be absorbed by protons. In addition, some loss of PUI energy in a more hydrodynamic view is also connected with work done by the PUIs through their pressure at driving the effective solar wind to keep it at an effective bulk velocity V_w jointly shared by PUIs and SWPs (see e.g. Chalov and Fahr, 1997 or Fahr and Rucinski, 1999).

Here we begin our consideration of the PUI-SWP two fluid thermodynamics from the kinetic result obtained by Chalov and Fahr (1965), yielding a function f_{pui} which leads to the PUI pressure in the form of its third moment by the expression (see Eq. 7):

$$P_{pui}(r) = \alpha(r)\rho_{pui}(r)V_w^2(r). \tag{9}$$

As we have already stated before, this expression in the outer heliosphere (i.e. beyond 10 AU) can be well approximated by setting $\alpha(r) \simeq \alpha_c = 0.44$. Realizing, in addition, that the solar wind velocity V_w between 10 AU and 90 AU only drops by less than 10% (see e.g. Richardson et al., 1995 and Fig. 1) may then allow one to approximate P_{pui} by:

$$P_{pui}(r) \simeq \alpha_c \rho_{pui}(r) V_{w_0}^2, \tag{10}$$

where V_{w0} denotes the solar wind velocity at 10 AU. This relation indicates that PUI essentially behave in a isothermal manner at their motion to larger distances since they fulfill a polytropic relation of the form: $P_{pui}/\rho_{pui}^{\gamma_i} \simeq C_{pui} = \alpha_c V_{wo}^2$ with the polytropic index given by $\gamma_i = 1$. The isothermal PUI temperature thus follows from Eq. (10) that:

$$\frac{\partial P_{pui}}{\partial \rho_{pui}} = \frac{P_{pui}}{\rho_{pui}} = \alpha_c V_{w0}^2 = KT_{pui}/m_p. \tag{11}$$

This result is also supported in a perfect way by the analogous calculations carried out by Fichtner et al. (1996), who find that the PUI distribution beyond 10 AU takes an asymptotically constant profile yielding the fact that: $rf_{pui}(r, v) =$ const. From this fact one also concludes directly that the ratio of the two moments, P_{pui} and ρ_{pui}, presents a constant, just as expressed in Eq. (11).

In hydrodynamical terms, this obviously means that PUIs, when expanding with the solar wind, experience just enough heating to keep their temperature T_{pui} constant at the expansion of the solar wind to larger distances. This phenomenon must thus be reflected in a fine-tuned strength of the energy input terms on the RHS of the equation of conservation of the PUI enthalpy flow given by:

$$div\left(\frac{\gamma}{\gamma-1}C_{pui}\rho_{pui}\overrightarrow{V}_w\right) - (\overrightarrow{V}_w \circ \nabla)(C_{pui}\rho_{pui}) = \beta_{ex}(\frac{1}{2}m_p V_w^2) + Q_{pui}\ , \tag{12}$$

where β_{ex} is again the PUI injection rate, $E_i = \frac{1}{2} m_p V_w^2$ is the initial PUI injection energy seen in the solar wind rest frame, and Q_{pui} denotes the net energy input into the PUI fluid due to nonlinear wave-particle interactions, including losses due to wave-driving and gains due to Fermi-2 accelerations. We now want to find the form of the term Q_{pui} which can satisfy the above differential Eq. (12). Keeping in mind that the mass flow conservation of PUIs requires that β_{ex} is representable by:

$$m_p \beta_{ex} = div(\rho_{pui} \overrightarrow{V_w}), \tag{13}$$

we then obtain:

$$div\left((\frac{\gamma}{\gamma-1} - \frac{V_w^2}{2C_{pui}}) \rho_{pui} \overrightarrow{V}_w \right) - (\overrightarrow{V}_w \circ \nabla) \rho_{pui} = Q_{pui}/C_{pui} \tag{14}$$

and find the following result:

$$Q_{pui} = (\frac{\gamma}{\gamma-1} - \frac{V_w^2}{2C_{pui}}) div(P_{pui} \overrightarrow{V}_w) - (\overrightarrow{V}_w \circ \nabla) P_{pui}. \tag{15}$$

With the above expression (15) we have now found the exact form of that net energy input Q_{pui} which just leads to an isothermal behaviour of PUIs.

Before we study the thermodynamics of the solar wind protons separately, we take a look into the required thermodynamics of the joint PUI-SWP two-fluid system which expresses itself in the following form:

$$div\left(\frac{\gamma}{\gamma-1} (P_{pui} + P_w) \overrightarrow{V}_w \right) - (\overrightarrow{V}_w \circ \nabla)(P_{pui} + P_w) = \beta_{ex}(\frac{1}{2} m_p V_w^2 - K T_w) + Q_{pui} + Q_w, \tag{16}$$

where K is the Boltzmann constant and T_w is the temperature of SWPs.

The fact that this PUI-SWP two-fluid system is lacking any external energy sources besides the evident energy sinks and sources connected with the removal of thermal SWP-energy, i.e. KT_w, and the gain of the PUI injection energy, and E_i, per creation of a PUI, then leads to the obvious conclusion that the energy inputs, Q_w and Q_{pui}, to the SWP and the PUI fluids connected with nonlinear wave-particle interactions have to cancel each other (i.e. no net energy gain or loss of the wave fields!). This then evidently requires that:

$$Q_w = -Q_{pui}. \tag{17}$$

Based on this result and on the expression we have derived for Q_{pui} in Eq. (15), we then obtain the single-fluid thermodynamics of SWPs given by the following equation:

$$div\left(\frac{\gamma}{\gamma-1} P_w \overrightarrow{V}_w \right) - (\overrightarrow{V}_w \circ \nabla) P_w = -\beta_{ex}(KT_w) - (\frac{\gamma}{\gamma-1} - \frac{V_w^2}{2C_{pui}}) div(P_{pui} \overrightarrow{V_w}) + (\overrightarrow{V_w} \cdot \nabla) P_{pui}. \tag{18}$$

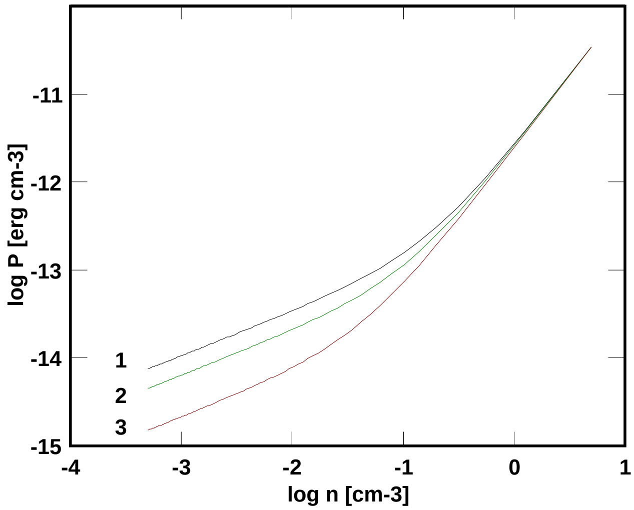

Fig. 3. Plotted is the logarithm of the solar wind thermal pressure P_w versus the logarithm of the SWP density n_w at $\Lambda = 1 \cdot 10^{-3}$ and $T_{w,0} = 5 \cdot 10^4 K$ for different values of $\Delta\alpha = \alpha_1 - \alpha_2$; i.e. for: 1:$\Delta\alpha = 50$, 2:$\Delta\alpha = 30$, 3:$\Delta\alpha = 10$.

We now want to obtain from the above equation a solution for the solar wind pressure P_w as a function of the distance r and for that purpose, we arrange Eq. (18) into the following, more appropriate form:

$$div\left(\frac{\gamma}{\gamma-1} P_w \overrightarrow{V}_w \right) - (\overrightarrow{V}_w \circ \nabla) P_w = -KT_w div \left(n_w \overrightarrow{V_w} \right) + (\frac{\gamma}{\gamma-1} m_p C_{pui} - \frac{m_p v_w^2}{2}) div(n_{swp} \overrightarrow{V}_w) + m_p C_{pui} (\overrightarrow{V}_w \circ \nabla) n_{pui}. \tag{19}$$

We shall now evaluate this equation for a spherically symmetric solar wind flow, assuming that $\frac{2v_w}{r} \gg \frac{dv_w}{dr}$ can be used as a satisfactory approximation, and obtain:

$$\frac{\gamma}{\gamma-1}\left[\frac{dP_w}{dr} + \frac{2P_w}{r} \right] - \frac{dP_w}{dr} = \left[-kT_w + \frac{\gamma}{\gamma-1} m_p C_{pui} - \frac{m_p V_w^2}{2} \right] \left[\frac{dn_w}{dr} + \frac{2n_w}{r} \right] - m_p C_{pui} \left[\frac{dn_w}{dr} + \frac{2}{r}(n_w + n_{pui}) \right]. \tag{20}$$

This equation can be simplified into the following form:

$$\frac{1}{\gamma-1} \frac{dP_w}{dr} + \frac{2\gamma}{\gamma-1} \frac{P_w}{r} = \left[-KT_w + \frac{1}{\gamma-1} m_p C_{pui} - \frac{m_p V_w^2}{2} \right] (\frac{dn_w}{dr} + \frac{2n_w}{r}) - m_p C_{pui} \left[\frac{2}{r} n_{pui} \right]. \tag{21}$$

Keeping in mind that $KT_w \ll m_p C_{pui} = KT_{pui}$, and that the PUI density is related to the total proton density by

$n_{pui} = n - n_w$, with the total solar proton density n simply given by: $n = n_0(r/r_0)^{-2}$, then yields the following equation:

$$\frac{dP_w}{dr} + 2\gamma\frac{P_w}{r} = -\left[KT_{pui} - (\gamma-1)\frac{m_p V_w^2}{2}\right]\frac{\beta_{ex}}{V_w}$$
$$-KT_{pui}(\gamma-1)\left[\frac{2}{r}(n - n_w)\right], \qquad (22)$$

which finally, together with the PUI injection rate $\beta_{ex} = n_w n_H \sigma_{ex} V_w$ yields:

$$\frac{dP_w}{dr} + 2\gamma\frac{P_w}{r} = \left[\frac{2}{r}(\gamma-1)KT_{pui} - (KT_{pui}\right.$$
$$\left. +(\gamma-1)\frac{m_p V_w^2}{2})n_H\sigma_{ex}\right]n_w$$
$$-KT_{pui}(\gamma-1)\frac{2n_0}{r_0}(\frac{r_0}{r})^3. \qquad (23)$$

This differential equation formally is of the following form:

$$\frac{dP_w}{dr} + g_1(r)P_w = g_2(r) \qquad (24)$$

and thus has the solution:

$$P_w = \exp(-2\gamma\int_{r_0}^{r}\frac{dr}{r})$$
$$\left\{P_{w,0} + \int_{r_0}^{r}\exp(+2\gamma\int_{r_0}^{r'}\frac{dr}{r})g_2(r')dr'\right\}. \qquad (25)$$

As we are going to show in the Appendix of this paper, this expression (25) can be simplified and finally, can be evaluated to yield:

$$P_w = x^{-2\gamma}\cdot\left\{P_{w,0} + \Lambda(KT_{pui} - (\gamma-1)\frac{m_p V_w^2}{2})\right.$$
$$\left. n_0\frac{1}{2\gamma-1}(x^{2\gamma-1} - 1)\right\}. \qquad (26)$$

First we now intend to investigate the polytropic behaviour of the PUI-heated solar wind and for that purpose, we study the expression derivable for the local polytropic SWP index γ_w:

$$\gamma_w = \frac{\rho_w}{P_w}\frac{dP_w}{d\rho_w}. \qquad (27)$$

To evaluate expression (27) we first take the derivative of P_w with respect to r given in the form:

$$\frac{dP_w}{dr} = \frac{1}{r_0}\left\{-\frac{2\gamma P_w}{x} + P_{w,0}\Lambda(\alpha_1 - \alpha_2)x^{-2}\right\}, \qquad (28)$$

where α_1 and α_2 are defined by:

$$\alpha_1 = \frac{n_0 KT_{pui}}{P_{w,0}} = (T_{pui}/T_0)$$

and

$$\alpha_2 = (\gamma-1)\frac{n_0 m_p V_w^2}{2P_{w,0}}. \qquad (29)$$

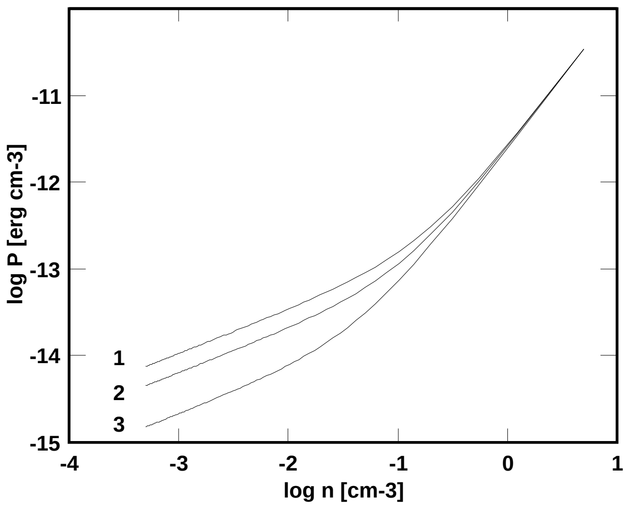

Fig. 4. Plotted is the logarithm of the solar wind pressure P_w versus the logarithm of the SWP density n_w at $\Delta\alpha = 50$ and $T_{w,0} = 5\cdot 10^4 K$ for different values of Λ, i.e. for: $1{:}\Lambda = 1\cdot 10^{-3}$, $2{:}\Lambda = 2\cdot 10^{-3}$, $3{:}\Lambda = 3\cdot 10^{-3}$.

With Eq. (28) and the evident relation:

$$\frac{d\rho_w}{dr} = -2\frac{\rho_w}{r}, \qquad (30)$$

we then obtain from relation (27):

$$\gamma_w(x) = \frac{\rho_w}{P_w}\frac{dP_w}{d\rho_w} = \gamma - \frac{P_{w,0}}{P_w}\frac{\Lambda}{4}(\alpha_1 - \alpha_2)x^{-1}. \qquad (31)$$

In the following we shall demonstrate results of the thermodynamic behaviour of PUI-heated SWPs by plotting in Fig. 3 the quantities $Log(P_w)$ versus $Log(\rho_w)$ with $\Delta\alpha = \alpha_1 - \alpha_2$, $P_{w,0}$, and Λ, respectively, as open parameters. In this figure, the parameter $\Delta\alpha$ is varied with the following values selected: $\Delta\alpha_1 = 50$; $\Delta\alpha_2 = 30$; $\Delta\alpha_3 = 10$. As is evident in this figure, the SWP pressure drops the least with SWP density, or solar distance x, the higher the value is for $\Delta\alpha$, i.e. the more efficient is the PUI-induced heating of the SWPs.

The SWP pressure at larger solar distances reacts even more sensitively to a variation in the quantity $\Lambda = n_H\sigma_{ex}r_0$. Ascribing to this variation in Λ ($\Lambda_1 = 1\cdot 10^{-3}$, $\Lambda_2 = 2\cdot 10^{-3}$, $\Lambda_3 = 3\cdot 10^{-3}$) a corresponding variation in the H-atom density n_{H0} outside of the solar system, i.e. the interstellar H-atom density, Fig. 4 then reveals that at higher values of n_{H0}, the non-adiabatic behaviour of P_w already starts further inwards at smaller solar distances of x.

Furthermore, in Fig. 5, we show the polytropic index $\gamma_w(x)$ given in Eq. (31) as a function of x for different values of Λ. As one can already see from Eqs. (26) and (31), the function $\gamma_w(x)$ reduces from its initial value of $\gamma_s \simeq \gamma = 5/3$ to an asymptotic value of $\gamma_w(x\to\infty) = \gamma_{w,\infty}$, which neither depends on $\Delta\alpha$ nor Λ. The range of solar distances where γ_w turns out to be between, say, 1.4 and 1.2, i.e. clearly below the adiabatic value, is, however, fairly sensitive to both Λ and $\Delta\alpha$. With parameter values $\Lambda = 3\cdot 10^{-3}$

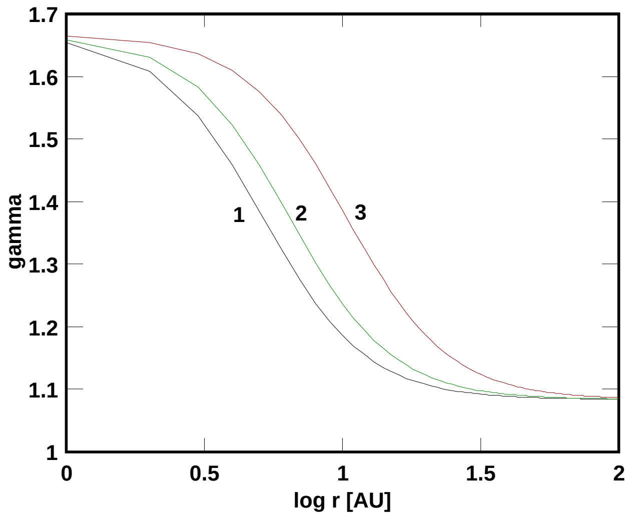

Fig. 5. Plotted is the polytropic index γ_w versus the logarithm of the solar distance r at $\Delta\alpha = 50$ and $T_{w,0} = 5 \cdot 10^4 K$ for different values of Λ, i.e. for: 1:$\Lambda = 1 \cdot 10^{-3}$, 2: $\Lambda = 2 \cdot 10^{-3}$, 3:$\Lambda = 3 \cdot 10^{-3}$.

and $\Delta\alpha = 50$, one would obtain polytropic indices below 1.3 all the way from 5 AU outwards, as was observed by VOYAGER-2 (see Whang, 1999).

3 Pick-up ion energy transfer to solar wind protons

In the preceding section, we have used the hypothesis that waves driven by PUIs energize solar wind protons most probably by Fermi-2 acceleration processes and thereby, eventually transfer a specific fraction of their initial pick-up energy per PUI, i.e. of $E_i = \frac{1}{2} m_p V_w^2$, to the solar wind background, i.e. to the SWPs. We shall study which fraction of this initial PUI energy is eventually transfered to the SWPs when they finally leave the inner heliosphere passing over the heliospheric termination shock. The net PUI-induced wave energy input to SWPs per unit volume and time according to Eq. (15) is given by:

$$Q_w = -Q_{pui} = -(\frac{\gamma}{\gamma - 1} - \frac{V_w^2}{2C_{pui}}) div(P_{pui} \overrightarrow{V}_w) + (\overrightarrow{V}_w \circ \nabla) P_{pui}. \tag{32}$$

We may evaluate this expression here assuming, as already done before, that $KT_{pui} = m_p C_{pui}$ and the solar wind velocity V_w are constants. Then, the above expression evaluates to:

$$Q_w = -(\frac{\gamma}{\gamma - 1} KT_{pui} - \frac{m_p V_w^2}{2}) div(n_{pui} \overrightarrow{V_w}) - KT_{pui} (\overrightarrow{V_w} \circ \nabla) n_{pui}. \tag{33}$$

Keeping in mind that:

$$div(n_{pui} \overrightarrow{V_w}) = n_H n_w \sigma_{ex} V_w, \tag{34}$$

and that:

$$n_{pui} = n - n_w, \tag{35}$$

with the relation (see Eq.A7):

$$n_w = n_0 x^{-2} \Big[1 - \exp\big(- \Lambda(x - 1) \big) \Big], \tag{36}$$

then allows one to transform Eq. (33) into:

$$Q_w = -\Big(\frac{\gamma}{\gamma - 1} KT_{pui} - \frac{m_p V_w^2}{2} \Big) \cdot \Lambda \frac{n_0 V_w}{r_0} \Big[x^{-2} \big(1 - \Lambda(x - 1) \big) \Big] + KT_{pui} \frac{V_w n_0}{r_0} \Lambda (x^{-2} + 2x^{-3}), \tag{37}$$

which in view of the fact that within our integration limits of $x \le 100$ the quantity $\Lambda x \ll 1$ can be further simplified to:

$$Q_w = -\Lambda \frac{n_0 V_w}{r_0} \cdot \left\{ (\frac{2\gamma - 1}{\gamma - 1} KT_{pui} - \frac{m_p V_w^2}{2}) x^{-2} + 2KT_{pui} x^{-3} \right\}. \tag{38}$$

With this expression for the PUI-induced energy input, one is then led to an energy input per unit time into a sector of the inner heliosphere distending with a space angle $d\Omega$ from $r = r_0$ (i.e. inner boundary where no PUIs are present) to $r = r_s = 100 r_0$ (i.e. heliospheric shock location) given by:

$$Y_w = d\Omega \int_{r_0}^{r_s} r^2 Q_w \, dr, \tag{39}$$

which, with the use of Eq. (38), takes the following form:

$$Y_w = -r_0^3 d\Omega \int_1^x x^2 \Lambda \frac{n_0 V_w}{r_0} \cdot \left\{ (\frac{2\gamma - 1}{\gamma - 1} KT_{pui} - \frac{m_p V_w^2}{2}) x^{-2} + 2KT_{pui} x^{-3} \right\} dx, \tag{40}$$

and thus can be simplified to:

$$Y_w = -\Lambda r_0^2 n_0 V_w d\Omega \cdot \int_1^x \left\{ (\frac{2\gamma - 1}{\gamma - 1} KT_{pui} - \frac{m_p V_w^2}{2}) + 2KT_{pui} x^{-1} \right\} dx. \tag{41}$$

This finally can be evaluated to yield:

$$Q_w = -\Lambda r_0^2 n_0 v_w d\Omega \cdot \left\{ (\frac{2\gamma - 1}{\gamma - 1} KT_{pui} - \frac{m_p V_w^2}{2})(x - 1) + 2KT_{pui} \ln(x) \right\}. \tag{42}$$

For the outer boundary $x_s \simeq 100$ of the integration (i.e. the location of the termination shock), this expression finally simplifies to:

$$Y_w = -\Lambda r_0^2 n_0 V_w d\Omega \cdot \left[(\frac{2\gamma - 1}{\gamma - 1} KT_{pui} - \frac{m_p V_w^2}{2}) x_s + 9.2 KT_{pui} \right], \tag{43}$$

where the last term in view of $x_s \gg 9.2$ can also be neglected for the estimate aimed at here.

Now we want to compare this expression for Y_w with the total energy input Y_{ex} into the same inner heliospheric solar wind sector per unit of time, due to the total loading of the solar wind with freshly implanted PUIs of energy $E_i = (1/2)m_p V_w^2$ at a local implantation rate β_{ex} within the same space sector as considered above. For Y_{ex} one thus obtains the following expression:

$$Y_{ex} = d\Omega \int_{r_0}^{r_s} r^2 \beta_{ex}(r) \left(\frac{1}{2} m_p V_w^2\right) dr \ , \tag{44}$$

Keeping in mind that the local PUI production rate can be expressed by $\beta_{ex} = div(\xi n \overrightarrow{V}_w)$, then allows one to arrive at:

$$Y_{ex} = \Lambda n_0 r_0^2 V_w [\frac{m_p}{2} V_w^2] d\Omega (x_s - 1). \tag{45}$$

The ratio Θ of the above energy inputs Y_w and Y_{ex} taken from Eqs. (43) and (45) is thus given by:

$$\begin{aligned}\Theta &= \frac{Y_w}{Y_{ex}} \\ &= \frac{-\Lambda r_0^2 n_0 V_w d\Omega (\frac{2\gamma-1}{\gamma-1} K T_{pui} - \frac{m_p V_w^2}{2}) x_s}{\Lambda n_0 r_0^2 V_w d\Omega [\frac{m_p}{2} V_w^2] x_s} \\ &= 1 - \frac{\frac{2\gamma-1}{\gamma-1} K T_{pui}}{\frac{m_p}{2} V_w^2} = 1 - \frac{\frac{2\gamma-1}{\gamma-1}}{\frac{1}{2} M_{pui}^2},\end{aligned} \tag{46}$$

where M_{pui} is the PUI Mach number defined by:

$$M_{pui}^2 = \frac{\rho_{pui} V_w^2}{P_{pui}} \ . \tag{47}$$

The above expression when evaluated for $\gamma = 5/3$ then tells us that the above result can only describe reasonably well the PUI-SWP two-fluid thermodynamics, if the PUI Mach number fulfills the following relation:

$$M_{pui} \geq \sqrt[2]{7} = 2.65. \tag{48}$$

As one can see in the result presented for Θ in Eq. (46), the effectivity of the energy transfer from PUIs to SWPs shows that the value of Λ, i.e. of n_H, does not play any role in this context. What counts, however, are the values of α_1 and of α_2, as one can see when rewriting Eq. (46) in the following form:

$$\Theta = 1 - \frac{\frac{2\gamma-1}{\gamma-1} K T_{pui}}{\frac{m_p}{2} V_w^2} = \frac{\alpha_2 - (2\gamma - 1)\alpha_1}{\alpha_2}. \tag{49}$$

As one can conclude from the above relation, it is necessary for an energy transfer from PUIs to SWPs that $\alpha_2 \geq (7/3)\alpha_1$. For instance, for values like $\alpha_2 = (8/3)\alpha_1; = (9/3)\alpha_1; = (10/3)\alpha_1$, one could expect to have energy transfer ratios of $\Theta = 0.125; = 0.222; = 0.3$.

4 Concluding remarks

In concluding our views on PUI-mediated winds, we can state that, whenever the solar wind system moves through a fractionally ionized interstellar medium, PUIs are automatically produced by ionization of neutral interstellar H-atoms that penetrate into the supersonic region of the heliosphere. These PUIs upon momentum-sharing with the solar wind at the PUI loading process, decelerate the wind. In addition, the original solar wind is substantially modulated in its dynamics and thermodynamics when PUIs, as a separate suprathermal ion population, are mixed up with SWPs and at the same time are tied to a joint bulk velocity V_w. According to the calculations presented in the preceding sections of this paper, the solar wind is decelerated by 10 to 20%, depending on the density of the interstellar H-atoms.

The solar wind protons, in addition, are polytropically heated by nonlinear wave-particle interactions induced by PUI-driven hydromagnetic waves, leading to a quasi-polytropic SWP behaviour with distance-dependent polytropic SWP indices $\gamma_w(x) \leq (5/3)$. A polytropic solar wind behaviour with indices $\gamma_w \simeq 1.28$ in regions between 10 and 40 AU, as obtained in our calculations, is, in fact, confirmed by solar wind proton temperature measurements carried out with VOYAGER-2 (see Whang, 1999). By means of this nonlinear PUI-wave-SWP energy coupling, about 10 to 20% of the intitial PUI injection energy E_i is transfered to solar wind protons. The effective Mach numbers of the solar wind flow are substantially reduced to values of about 2 to 3, which are mainly associated with the solar wind PUI Mach number M_{pui} given in Eq. (37) and are limited to $M_{pui} \geq 2.65$. The two-fluid plasma mixture composed of SWPs and PUIs in many respects behaves like a mixture of a heavy and a light gas, except that the moment transfer terms are not of the same type as the classical ones valid under collision-dominated conditions (see e.g. Braginski, 1965; Burgers, 1969), but are by their nature wave-particle coupling terms.

Appendix

The expression (25) first can be simplified to:

$$P_w = \left(\frac{r}{r_0}\right)^{-2\gamma} (P_{w,0} + \int_{r_0}^{r} \left(\frac{r}{r_0}\right)^{2\gamma} g_2(r\acute{}) dr\acute{}). \tag{A1}$$

Representing the function $g_2(r)$ in the form:

$$g_2(r) = g_{21}(r) + g_{22}(r) + g_{23}(r), \tag{A2}$$

then leads to the following form of a solution for P_w:

$$P_w = \left(\frac{r}{r_0}\right)^{-2\gamma} (P_{w,0} + I_{21} + I_{22} + I_{23}) \ , \tag{A3}$$

where the integrals I_{21}, I_{22}, I_{23} are given by:

$$I_{21} = 2n_0(KT_{pui})(\gamma - 1) \cdot \int_1^x (x\acute{})^{2\gamma-3} \exp(-\Lambda(x\acute{} - 1)) dx\acute{}, \tag{A4}$$

$$I_{22} = \Lambda(KT_{pui} - (\gamma - 1)\frac{m_p v_w^2}{2})n_0 \cdot \int_1^x x^{2\gamma-2} \exp(-\Lambda(x' - 1))dx', \quad \text{(A5)}$$

$$I_{23} = -KT_{pui}(\gamma - 1)2n_0 \int_1^x (x')^{2\gamma-3} dx'. \quad \text{(A6)}$$

To derive the above integrals in these forms, we assume the SWP density n_w, as given by (see Fahr and Rucinski, 1999):

$$n_w = n_0 x^{-2} \exp(-\Lambda(x - 1)). \quad \text{(A7)}$$

Furthermore, it is assumed that the H-atom density in the outer heliosphere is essentially constant, i.e. $n_H \simeq n_{H0}$, and the following abbreviations were used: $x = r/r_o$ and: $\Lambda = n_{H0}\sigma_{ex}r_o$.

Keeping in mind that $\Lambda = n_{H0}\sigma_{ex}r_0$ is of the order of 10^{-3}, may permit us to set the above integrals: $\exp(-\Lambda(x - 1)) \simeq 1$. In this physically reasonable approximation, one then obtains the following solution for P_w:

$$P_w = x^{-2\gamma} \cdot \left\{ P_{w,0} + \Lambda(KT_{pui} - \frac{m_p V_w^2}{2})n_0 \frac{\gamma - 1}{2\gamma - 1}(x^{2\gamma-1} - 1) \right\}. \quad \text{(A8)}$$

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