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# Nonlinear analysis of magnitude and interevent time interval sequences for earthquakes of the Caucasian region

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**Abstract.** It is well known that lithospheric seismic processes are characterized by self-similarity or scale invariance in terms of earthquake-size, time, space and space-time distributions, although precise details of underlying dynamics are not clear. In this study we apply nonlinear dynamics theory tools, such as a correlation dimension, "surrogate" data analysis and positive Lyapunov exponent calculation, to investigate dynamical characteristics of seismicity in the Caucasian region. Inter-event time intervals and magnitude sequences are considered for different area and magnitude windows. We find significant evidence of a low dimensional nonlinear structure of earthquake time distribution, obtained by consideration of time interval sequences between all events encountered, above some threshold magnitude, in the original catalogue. However nonlinear structure is absent in artificially generated sequences of time intervals between independent events as well as time intervals between aftershocks. It seems that this kind of filtration of the original catalogue destroys the existing temporal structure of considered lithospheric processes. Unlike artificial inter-aftershock time interval sequences, obtained by removing independent events from the original series, the time interval sequence between the Racha earthquake aftershocks reveals clear evidence of nonlinear structure. Earthquake magnitude dynamics, for all considered regions and magnitude windows, reveal high dimensional nonlinearity.

## 1 Introduction

In spite of a number of scientific researches of lithospheric processes related to earthquake generation (Kagan and Knopoff, 1981; Nishenko and Buland, 1987; Keilis-Borok, 1992; Korvin, 1992; Turcotte, 1992), dynamical aspects of seismicity still remain almost unknown (Keilis-Borok, 1994; Kagan, 1997). For instance, although it is well known that several earthquake characteristics, such as magnitude, time and spatial distribution, follow a power law, being self-similar or scale invariant (Omori, 1895; Gutenberg, 1944; Kagan, 1994; Turcotte, 1992; Sailhac, 1997), underlying rules of the dynamics of seismic processes are not clear.

In the scientific literature we find two main controversial approaches dealing with the dynamics of earthquake generation. According to Nishenko and Buland (1987) strong earthquakes on a given fault occur quasi-periodically. That means that a seismic process should be not too complex dynamically and may easily be predicted. The second hypothesis considers the dynamics of seismic activity to be extremely complicated, so that the level of "turbulence" of the lithosphere exceeds that of the atmosphere (Kagan, 1992, 1994, 1997). In the latter case, seismicity should be unpredictable, having complex dynamical properties, similar to a random process.

Unfortunately, the present situation in this field does not allow us to identify clearly the dynamical aspects of lithospheric processes responsible for the above mentioned scaling laws even though self-similar fractal properties of earthquake spatial and temporal distribution are well established (Kagan and Knopoff, 1980; Korvin, 1992; Henderson et al., 1994; Smirnov, 1995; Marzocchi et al., 1997; Chen et al., 1998). Further clarification of earthquake generation dynamics is very important, since self-similarity per se is indigenous to systems for which behaviour can be dynamically quite different, ranging from some randomness up to the deterministic chaos.

The main goal of the present work is a qualitative and quantitative evaluation of the dynamics underlying the Caucasian earthquakes' self-similar size and time distribution.

To clarify the main features of the dynamics, it is necessary to use modern practical tools of a nonlinear dynamics theory (Abarbanel et al., 1993; Kantz and Shreiber, 1997).These methods, based on the nonlinear analysis of time series, i.e. uniformly distributed in the real time data sequences, provide us with the quantitative and

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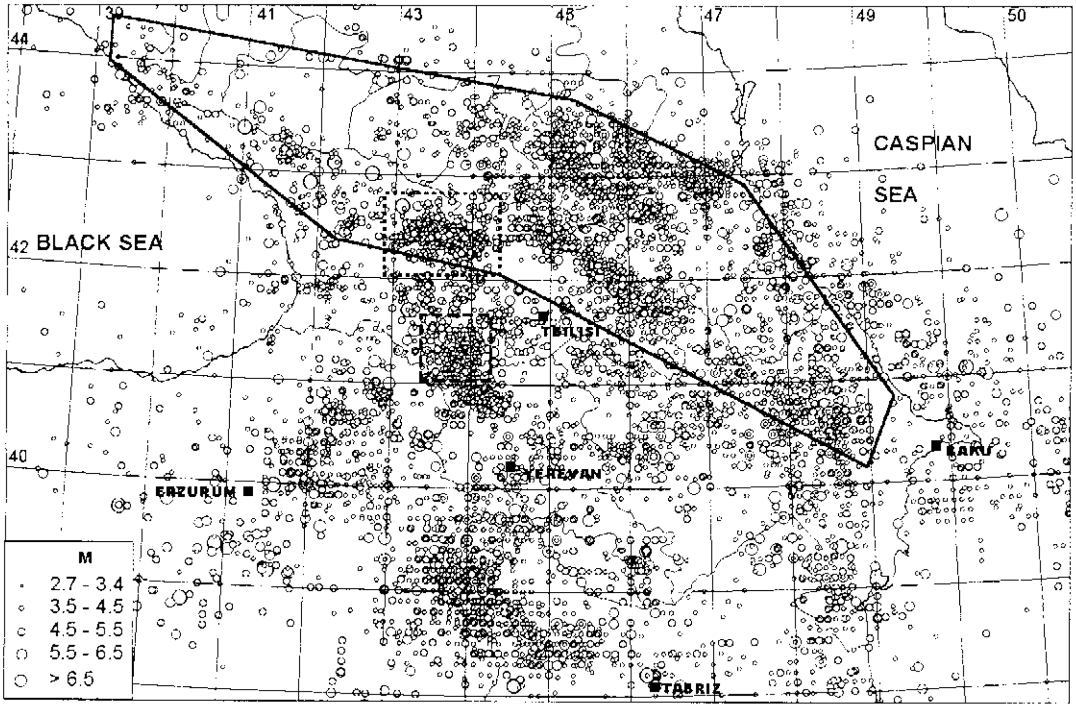



**Fig. 1.** Map of the earthquake epicenter distribution in the Caucasian region from 1962 to 1993. The areas of the present study are outlined: the Greater Caucasus by a solid line, Javakheti region by a dashed line and Racha earthquakes epicentral area by a dotted line.

qualitative tools to distinguish dynamical behaviour of systems (Abarbanel et al., 1993).

Nonlinear methods of time series analysis are now used for analysis of different complicated natural systems including the lithosphere (Feder, 1988; Korvin, 1992; Abarbanel et al., 1993).

Besides general scientific interest, a complete investigation of the dynamics of the lithosphere has great practical importance for revealing the inherent mechanism of earthquake generation and possible prediction of an impending catastrophe. For this reason, the number of scientific publications, dedicated to a nonlinear analysis of the seismicity (Pisarenko and Pisarenko, 1991; Korvin, 1992; Turcote, 1992; Keilis-Borok, 1990, 1994; Kagan, 1994, 1997; Marzocchi et al., 1997) has increased significantly over the last few years.

The majority of these investigations are devoted to earthquake spatial distribution (Kagan and Knopoff, 1980; Henderson et al., 1994; Berkovitz and Hadad, 1997); the importance of earthquake time and size distribution is undisputed.

The need for nonlinear analysis of seismicity is especially important for the Caucasian region which remains relatively poorly investigated by modern methods of analysis in spite of some work dedicated to the study of spatial distribution of earthquake foci (Sadovski and Pisarenko, 1991; Smirnov, 1995; Matcharashvili et al., 1996).

To investigate dynamical properties of the Caucasian region earthquake time and size distributions, in the present work, we have carried out the nonlinear time series analysis of inter-event time intervals and magnitude sequences from the corresponding catalogue.

## 2 Aims and methods of investigation

Nonlinear analysis has been performed on the Caucasian earthquake inter-event time interval and magnitude sequences - "time series" (11683 events), as well as for similar time series of the Greater Caucasus (3515 events) and Javakheti region (6694 events) in 1962-1993. We have also investigated inter-event time intervals and magnitude sequences for the Racha earthquake (M=6.9, April 29, 1991) aftershocks recorded in 1991- 1993 (3567 events).

All these time series were taken from the earthquake catalogue for the Caucasus and the adjacent territories of Northern Turkey and Northern Iran for the 1962-1993 time period (Seismological Data Base of Institute of Geophysics, Tbilisi, Georgia).

The map of investigated earthquake epicenters is shown in Figure 1.

Threshold magnitude, calculated for the above mentioned time period (i.e. 1962-1993) by the frequency-magnitude relationship (shown in Figure 2), is 2.7 for the whole Caucasus, as well as for the Greater Caucasus (the area outlined with solid line in Figure 1).

The threshold magnitude for Javakheti region (the area outlined with dashed line in Figure 1) is 1.5, due to the dense local network. For Racha earthquakes epicentral area (outlined with dotted line in Figure 1), the threshold magnitude is 1.7.

For comparison, we have used a set of random numbers generated in the same range as well as the randomized seismic catalogue - the "surrogate time series" set; randomization has been realised followed the methods of Theiler et al., (1992) and Rapp et al., (1993).

For reconstruction of $p$-dimensional phase spaces from the scalar geophysical time series we employed the delay

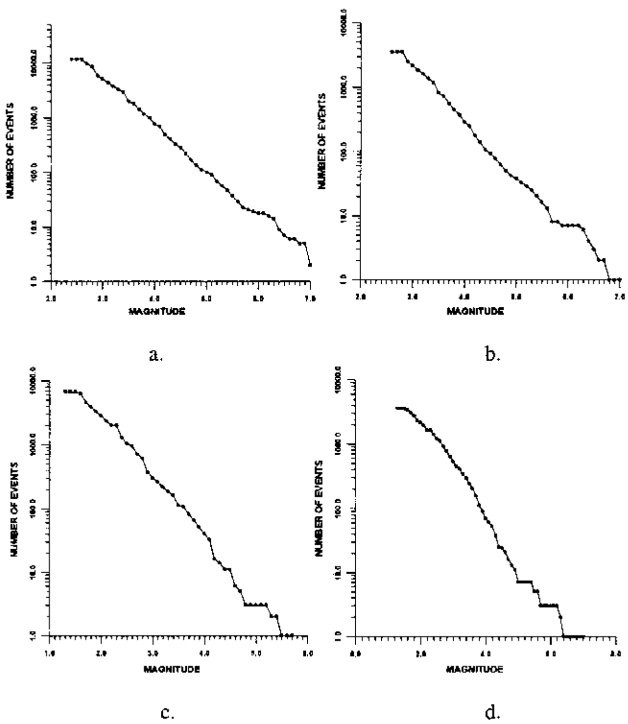



**Fig. 2.** Cumulative frequency-magnitude statistics in the investigated areas. The Caucasus (a), the Greater Caucasus (b), the Javakheti region (c ) and the Racha earthquakes epicentral area (d).

technique (Packard et al., 1980, Takens, 1981). This technique is based upon reconstruction of a set of $r$ vectors in $p$ -dimensional space,

$$r_i = \{x(t_i), x(t_i + \tau), \ldots, x[t_i + (p-1)\tau]\} \tag{1}$$

where $x(t_i)$ are data of a scalar time series and $\tau$ is the delay time. The $p$ -value is known as an embedding dimension. The delay method gives us several additional quantitative tests, namely two- and three-dimensional phase portraits, first return maps, and Poincare sections which encapsulate the essential dynamical properties of the complex dynamical process from which they were extracted.

After reconstruction of the geometrical structure of the process in the phase space, the main problem is quantitative analysis of topological patterns or the complexity of the structure by methods of nonlinear dynamics.

There are several methods for evaluation of complexity of spatial structures in phase space (Eckmann and Ruelle, 1985; Abarbanel et al., 1993) or, what is the same, for evaluation of dimensionality of process.

In this study we have used the well known Grassberger-Procaccia correlation integral calculation algorithm (Grassberger and Procaccia, 1983), in Theiler's modification (Theiler, 1986), as a quantitative method of nonlinear time series analysis. This is the most popular algorithm for evaluation of the fractal dimension of processes.

Correlation dimension is defined as a set of slopes :

$$d_2 = \lim_{l \to 0} \frac{\log C(l)}{\log l} \tag{2}$$

of the correlation function

$$C(l) = \frac{2}{(N-W)(N-W+1)} \times \\ \times \sum_{k=W}^{N} \sum_{i=1}^{N-k} H(l - \|r_{i+k} - r_i\|) \tag{3}$$

in $p$ -dimensional space. Here, $N$ is the number of attractor points in $R^p$ space, $H$ - is Heaviside function ($H(x)=0$ when $x \le 0$ and $H(x)=1$ when $x>0$), $\| \ \|$ is the Euclidean norm, $l$ - is the distance between points in $R^p$ space, $\{r_i\}$ is the set of vectors in $R^p$ and $W$ - is a constant of the order of a few autocorrelation times of data. This modification of the Grassberger - Procaccia method is very useful to avoid autocorrelation effects in the data (Theiler, 1986; Theiler, 1990; Abarbanel et al., 1993).

The dimensionality of a process is the value of $d_2$ which does not change with increasing $p$. The maximal value of phase space dimension $p$, after which the slope of the correlation function does not change any more, is defined as an embedding dimension. This value corresponds to the dimension of a phase space in which the attractor of a given process is embedded without distortion.

The above mentioned method of surrogate data analysis (Theiler et al., 1992) was used to eliminate linear stochastic processes from the observed time series for an unbiased detection of a nonlinearity; this is an essential criteria for revealing nonrandom dynamics.

For a more precise method of revealing dynamical peculiarities in the process, we have used the largest Lyapunov exponent $\lambda_{max}$ calculating algorithm (Wolf et al., 1985).

We have also used other common qualitative methods of the analysis of chaotic data; the power spectrum and the autocorrelation function calculation.

In the present work, for the calculation of $d_2$, $\lambda_{max}$, surrogate data generation etc., we have used software created at our Institute, written in C++, as well as the licensed versions of J. C. Sprott's Chaos Data Analyzer (CDA) and Chaos Data Analyzer professional version (CDA PRO).

## 3 Results and discussion

### 3.1 Brief discussion of the necessity of nonlinear time series analysis methods for seismic processes

It is well known that real processes, including seismicity, are in effect multivariable, i.e. they are characterised by a number of interdependent variables.

Modelling of such systems is an extremely difficult task. That is why, in order to understand the real temporal and spatial behaviour, an essentially empirical approach to dynamic model reconstruction is used, namely, the parameters are obtained from observations of the system rather than from physical equations. Due to the inherent complexity of seismic processes, data sequences (time series) from real systems are, as a rule, highly irregular. For a dynamical description of such time series, statistical techniques which are linear in principle are insufficient as they do not take into account nonlinear relationships.

Traditional signal-processing procedures which pick out, through Fourier analysis or a similar technique, the component frequencies in the data sequences, provide a limited amount of one dimensional information. The limitations of these methods becomes evident when the signal is more complex than the quasi-periodic one (broadband signals).

A nonlinear dynamical approach gives far more information on complex systems behaviour than can be obtained from linear classical tools (Berge et al., 1984; Theiler, 1990; Abarbanel et al., 1993; Kantz and Shreiber, 1997).

To achieve the general objective of the nonlinear analysis, i.e. a qualitative and, if possible, a quantitative evaluation of dynamics, it is necessary to consider the evolution of the system, given by the sequence of appropriate variables, in phase space; i.e. in the space which is spanned by system variables. Because of the complexity of natural systems, full information about the state of a system is generally lacking. So, from the practical point of view, nonlinear time series analysis procedures are based on concepts of an embedding theorem (Takens, 1981). It is suggested that if we can measure any single suitable variable with sufficient accuracy and for sufficiently long time periods, it is possible to make quantitatively meaningful inferences about the dynamics of an entire system from the behaviour of the single variable (Theiler, 1990; Abarbanel et al., 1993; Kantz and Shreiber, 1997). Moreover, a monovariate reconstruction of multivariate dynamics allows us not only to understand the integral behavior of systems but, in cases where the ability to measure some variables is circumscribed, to recover their dynamics from other measured variables in the phase space (Abarbanel et al., 1994).

Although the monovariate reconstruction is often used for analysis of different real processes, last years multivariate reconstruction of dynamics in phase space, using several data sets simultaneously, has also become popular (Eckman and Ruelle, 1985). Multivariate embedding is in fact more appropriate for complex real dynamics because it reflects a temporal as well as spatial structure in the system behaviour. Also, the multivariate approach can distinguish states in phase space which appear similar to univariate probes due to projection effects (Smith, 1992; Kantz and Shreiber, 1997).

Therefore, as a first step toward evaluation of Caucasian earthquake dynamical properties, we have used a method of monovariate reconstruction in spite of the fact that the processes are multivariable in their origin.

We begin our analysis using traditional linear methods such as power spectrum and autocorrelation analysis; after this we apply a nonlinear approach.

Our computations of univariate time series show that, in all cases examined, the autocorrelation function of magnitude sequences decays drastically and tends to zero as the delay time increases. This means that the self resemblance of magnitude sequences in the time domain decreases very rapidly; this indicates weak correlation between the data.

This peculiarity, together with the broad power spectrum and the absence of a clear attractor in phase space (two-and three-dimensional phase portraits, Poincare sections and return maps, not shown here) qualitatively excludes dynamical simplicity (Abarbanel et al., 1993; Sailhac, 1997) of underlying earthquake size generation and indicates complexity of this process.

For a detailed qualitative and quantitative study of earthquake generation dynamics according to the familiar nonlinear time series analysis, we have transposed the dynamics given by monovariate magnitude and interevent time interval sequences into phase space. Then we have proceeded with the determination of some characteristic quantitative properties or dimensions of structure in phase space (Abarbanel et al, 1993; Kantz and Shreiber, 1997; and similar references). As mentioned above, for this purpose we have used a correlation integral calculation algorithm. Calculation of a correlation dimension must be done for different phase spaces in order to avoid distortion of the embedded dynamics due to discrepancies between numbers of freedom of the system and phase space dimension. If the system, from which the time series originates, is dynamically relatively simple, its dimension will reach some limiting value. It is the dimension of a structure in phase space which indicates the minimum number of independent variables to be considered in the description of the underlying dynamics.

It is well known that, generally, dimensionality of real physical processes is not an integer; it is a fractal. If the limiting dimension does not exist, the sequence of fractal dimensions for different phase spaces can be used as a distinguishing statistic. Of course this is only justified in cases where nonlinear structure in data sequences is established, i.e. if the observed dynamics are not caused by different types of noises (Rapp et al., 1993 Kantz and Shreiber, 1997).

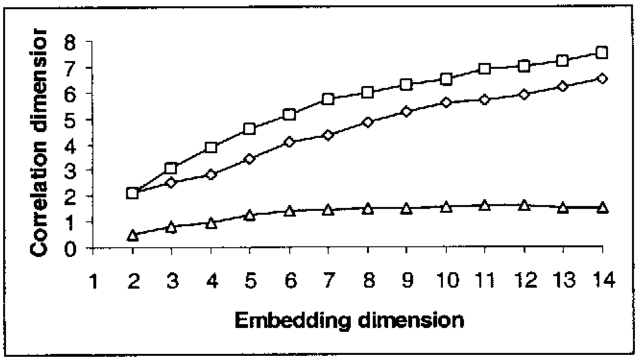



**Fig. 3.** Typical plot of correlation dimension $d_2$ versus embedding dimension $p$ for the Caucasus, the Greater Caucasus, and the Javakheti region, magnitude (middle curve) and inter-event time intervals (lower curve) sequences. The upper curve represents the random numbers set.

## 3.2 Variation of correlation dimension of earthquake size and temporal distribution

In Figure 3, we show a typical plot of the variation of correlation dimension versus embedding dimension for the whole Caucasus, the Greater Caucasus region and the Javakheti area earthquake magnitude sequences (middle curve). Slopes for each embedding dimension were obtained from log C( r ) - log( r) plots (not shown here) for linear intervals of appropriate time series, using standard least squares regression.

Our computations show that patterns of earthquake magnitude generation dynamics for all the regions considered are not typical of low dimensional processes. Namely, there were not sufficiently long scaling intervals (Theiler et al., 1992; Rapp et al., 1993; Yedynak et al., 1994; Kantz and Shreiber, 1997) with identical slopes on the log C(r) - log r curves. It follows from Figure 3 (middle curve) that there is no asymptote in the $d_2$ versus $p$ relationship for these magnitude sequences.

As in the original time series, which included all independent events and aftershock magnitudes encountered in the catalogue above threshold magnitude, artificially designed sequences of magnitudes, obtained by removing aftershocks from the original magnitude sequences, are high-dimensional.

Aftershock magnitude sequences, obtained from the original magnitude series by removing the independent events, also reveal high dimensionality of the seismic process for all regions.

In general, because of the limited size of the real seismic catalogues, the time series of magnitudes are too small to carry out an exact quantitative analysis of high dimensional lithospheric processes. Indeed, it is known that for exact reconstruction of the attractor in phase space and exact calculation of its dimension, the length of time series ($N$) should be at least of order of $N \approx 10^{\frac{d}{2}}$ (Eckmann and Ruelle, 1992; Abarbanel et al., 1993); i.e. unrealistically long for existing geophysical time series. In our case, the maximum length of the time series was about 12000; this is only sufficient for calculation of dynamics with correlation dimension < 8. As the analysis of magnitude sequences for the Caucasian region does not show saturation at this value of correlation dimension on $d_2$ versus $p$ plots, we can assume that the fractal dimension of these time series is larger than 8. Thus, as already noted, the analyzed process reveals dynamical properties quite different from the deterministic chaos which is characterised by low dimensionality, namely $d_2 \leq 5$.

Consequently, our results give evidence of high dimensionality ($d_2 > 8$), for the dynamical properties of earthquake size distribution. Hence, unlike the Caucasian earthquake spatial distribution (fractal dimension of order of 1-1.5 for epicenters) (Sadovski and Pisarenko, 1991; Pisarenko and Pisarenko, 1991; Smirnov, 1995), dynamics of earthquake size distribution is more complex (Abarbanel et al., 1993; Sailhac, 1997), i.e. depends upon more independent variables.

The observed high dimensionality of earthquake size distribution is in good agreement with existing data (Korvin, 1992; Kagan, 1994; Marzocchi et al., 1997), thus excluding low dimensionality and quasiperiodicity of self similar earthquake size (magnitude) generation.

Contrary to magnitude sequences, the correlation dimension $d_2$ of earthquake temporal distribution given by interevent time intervals sequences saturates close to 1.5 in all cases (Figure 4), thus revealing surprising properties of low-dimensional nonlinearity. This is a nonrandom, somewhat similar to, the determined chaos type of a dynamical behaviour (Theiler and Prichard, 1997).

It must be emphasised that the creation of ordered sets for time series analysis, by plotting interevent time interval size versus the interval number in the sequence of measured data, gives an indication of variations in the rate of the dynamics (Rapp et al., 1993; Kantz and Shreiber, 1997). The necessity for such an approach is clear because in certain scientific areas "equidistant" time series, i.e. sequences of data measured for uniform time intervals are not available; consequently the data, measured from a dynamical process, are often collected as interevent time intervals, either because that form is more convenient or because it is more representative of the process than an equidistant time series (Rapp et al, 1993; Castro and Sauer, 1997). Moreover, according to Rapp and his colleagues, event interval sequences allow us to construct an approximate continuous function in time and to use analysis procedures that presuppose the existence of a continuous function defined at uniformly spaced time intervals (Rapp et al., 1993). Recently, from interspike interval sequences, R. Castro and T. Sauer have argued, that there are no theoretical or practical objections to computing a correlation dimension

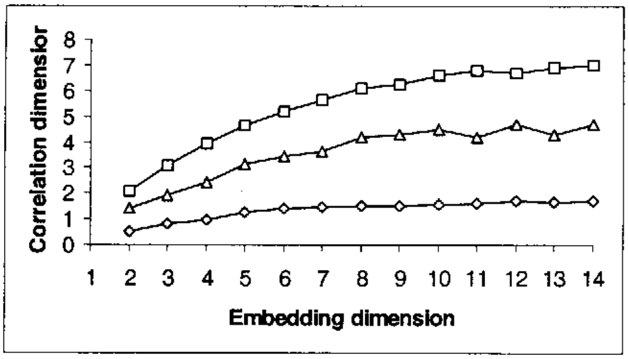



**Fig. 4.** Typical plots of correlation dimension $d_2$ versus embedding dimension $p$ for the Caucasus, the Greater Caucasus and Javakheti region: inter-event time interval sequences (lower curve), phase randomized interevent time interval surrogates (upper curve) and Gaussian scaled random phase surrogates (middle curve).

**Table 1.** Values of the correlation dimension ($d_2$) and the largest Lyapunov exponent ($\lambda_{max}$) of the interevent time interval sequence, for the Caucasian region.

| Parameter | The whole Caucasus | The Greater Caucasus | Javakheti area | Racha earthquake epicentral area |
|---|---|---|---|---|
| $d_2$ | 1.52 ± 0.42 | 1.70 ± 0.51 | 1.42 ± 0.13 | 3.4 ±0.55 |
| $\lambda_{max}$ (bits / events) | 0.241 ±0.018 | 0.539 ± 0.041 | 0.784 ± 0.026 | 0.129 ± 0.038 |

using interevent time intervals measured from experimental data sequences (Castro and Sauer, 1997).

In our case, for a detailed analysis of the dynamics of earthquake time distribution, it is appropriate to examine time series of interevent (inter-earthquake) time intervals, $\Delta t_{n+1,n}$ (in sec), where n is the earthquake's ordinal number in the appropriate catalogue.

As shown in Figure 4 (lower curve) and Table 1, all the interevent time interval sequences indicate a low correlation dimension $d_2$, which is always low and significantly smaller than the above mentioned threshold of dimensionality. As Table 1 shows, we have also obtained positive $\lambda_{max}$ for all the interevent time interval sequences at the appropriate embedding dimension ($d_2+1$).

### 3.3 Testing of nonlinear structure in the data sequences

The above results, 6 or 7 years ago, could have been regarded as evidence of chaotic determinism in time series but, nowadays, such findings must be interpreted with greater care as it is well known that linear stochastic processes can also mimic low-dimensional dynamics (Theiler et al., 1992; Rapp et al., 1993). In other words, the saturation of a correlation dimension and the existence of positive Lyapunov exponents cannot always be considered as proof of deterministic chaos (Rapp et al., 1993; Kantz and Shreiber, 1997).

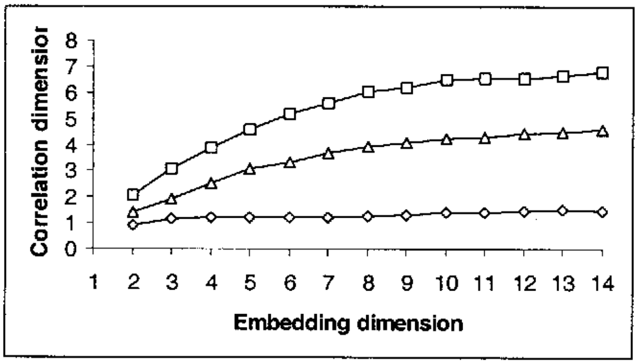
Since linear correlations lead to many spurious conclusions in nonlinear time series analyses, we have used the surrogate data approach to test the null hypothesis that interevent time interval time series are generated by a linear stochastic process, i.e. they are indistinguishable from coloured noise having the same power spectrum and autocorrelation function (Theiler et al., 1992). The method compares the original time series with an artificially generated random series or surrogate data that mimics linear properties of the original data sequences. In this study we have generated and used random phase surrogate sets, as well as Gaussian scaled random phase surrogate sets (Theiler et al., 1992; Rapp et al., 1993).

Random phase surrogate sets (obtained by destroying the nonlinear structure through randomization of the phases of a Fourier transform of the original time series and following invert transformation) were used to test the null hypothesis that the time series are linearly correlated with Gaussian noise (Theiler et al., 1992).

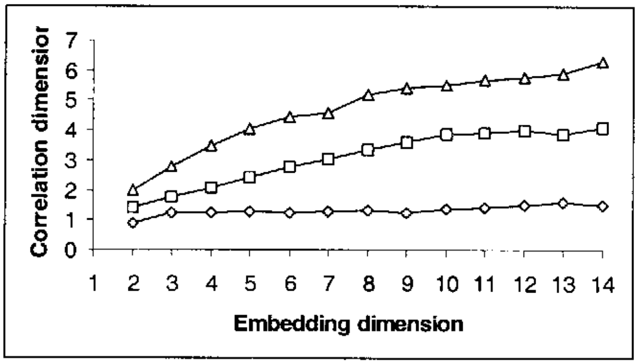
Gaussian scaled random phase surrogate set were generated in a three step procedure. At first was generated a Gaussian set of random numbers, which has the same rank structure as original time series. After that, the phase randomized surrogates of this Gaussian distributed set was constructed. Finally, the original time series were reordered so that, rank structure of the original time series agrees with the rank structure of the phase randomized Gaussian set. The Gausian scaled random phase surrogate sets addresses a null hypothesis that the original time series is linearly correlated noise that has been transformed by a static, monotone nonlinearity (Rapp et al., 1993, 1994).

Generally these two methods of generation of surrogates are based on shuffling of the original data set but, in the case of Gaussian scaled random phase surrogates, the controlled shuffles (Rapp et al., 1994) can give more precise and reliable results than the unstructured shuffles of the random phase surrogates.

Commonly, for testing the null hypothesis, $d_2$ is used as the discriminating metric. There are several ways to measure the difference between the discriminating metric measure of the original (given by $M_{orig}$) and the surrogate (given by $M_{surr}$), time series of interevent time interval sequences.
The most commonly used measure of the significance of the difference between the original time series and the surrogate data is given by the criterion $S = |\langle M_{surr}\rangle - M_{orig}|/$




a.




b.

**Fig. 5.** Correlation dimension $d_2$ versus embedding dimension $p$ of inter-event time interval sequences for the Caucasus (a) (the magnitude window $2.7 \le M_s \le 3.7$), and the Javakheti area (b) (the magnitude window $1.5 \le M_s \le 2.5$). Lower, middle and upper curves correspond to inter-event time intervals, their phase randomized and Gaussian scaled random phases surrogates respectively.

$\sigma_{surr}$, where $\sigma_{surr}$ denotes standard deviation of $M_{surr}$. The details of the procedure, as well as an analytic expression for $\Delta S$, - the uncertainty in S, are described in Theiler et al., (1992).

Alternatively, we have used the Monte Carlo probability, defined as $P_M$ = (number of cases $M \le M_{orig}$)/(number of cases) where $P_M$ is an empirical measure of the probability that a value of $M_{surr}$ will be less than $M_{orig}$. It is particularly appropriate when the number of surrogates is small, or when the distribution of values of M obtained with surrogates is non-Gaussian (Rapp et al., 1994).

For rejecting the null hypothesis, we have used the Barnard and Hope nonparametric test (Rapp et al., 1994). With this criterion, the null hypothesis is rejected by a confidence level $p_c = 1/(N_{surr}+1)$, if $M_{orig} < M_{surr}$ for all the surrogates.

In the case where the inter-event interval sequences and surrogates have significantly different metrics for a chosen discriminating statistic, we can conclude that the time series is not correlated noise and that the corresponding processes are characterized by the inherent nonlinear structure.

For each of our data sequences, we have generated 200 random phase surrogates.

The criterion S, according to Theiler et al. (1992), for the whole Caucasus, the Greater Caucasus region and Javakheti area inter-earthquake time interval sequences has significant values: 49.3 ± 0.4, 52.2 ± 0.2, 58.4 ± 0.3 respectively. It was found that, in all the cases considered here, $M_{surr}$ values are greater than $M_{orig}$, giving $P_M \sim 0$ with a confidence level $p_c < 0.005$. Hence we can reject the null hypothesis that earthquake inter-event time interval sequences correspond to linearly correlated Gaussian noise.

To study further the earthquake time distribution dynamics, we generated 200 Gaussian scaled random phase surrogates. These surrogates, as mentioned above, enable us to test the null hypothesis that the original time series is linearly correlated noise that has been transformed by a static, monotone nonlinearity. The values of S for the Caucasian, the Greater Caucasus and the Javakheti area inter-earthquake time interval sequences are 14.5 ± 0.2, 9.7 ± 0.1, 12.4 ± 0.1 respectively. These values are still significant, although there is a decrease in S compared to the random phase surrogates. For all cases, $M_{surr}$ is greater than $M_{orig}$ leading to confident rejection of the null hypothesis. The value of $P_M \sim 0$ and $p_c < 0.005$.

The general result of both type of surrogates, random phase and Gaussian scaled random phase surrogate time series, is a clear indication of the presence of nonlinear structure in the earthquake temporal distribution, given by inter-earthquake time interval sequences.

As stated previously, we have investigated time interval sequences for all events in the catalogue for corresponding threshold magnitudes, obtained from a frequency magnitude relationship. In order to reveal the contribution of events with particular magnitudes to nonlinear structure of temporal distribution properties, we have carried out an analysis of inter-event time interval sequences for different magnitude windows. So, for the whole Caucasus region, where we have the longest time series, we have carried out a nonlinear analysis of inter-event time interval sequences between earthquakes inside two magnitude windows: $2.7 \le M_s \le 3.7$ (number of events n = 9894), and $3.7 \le M_s \le 4.7$. (n = 1567). Further, for the Javakhety area, where we have the lowest threshold magnitude due to the dense local network, we considered inter-event time interval sequences for magnitude windows $1.5 \le M_s \le 2.5$ ( n = 5775 ), and $2.5 \le M_s \le 3.5$ (n = 830).

The results obtained from this analysis indicate that the nonlinear structure found is almost entirely due to inter-event intervals between relatively low (for the catalogue) magnitude earthquakes. Indeed, all the whole Caucasian region earthquakes with threshold magnitude 2.7 and inter-event time interval sequences, inside a $2.7 \le M_s \le 3.7$ window (Figure 5a), are characterized by a low-dimensional structure : $d_2$=1.28, S = 57.2 ± 0.2, $P_M \sim 0$. $p_c < 0.005$ and S = 9.5± 0.1, $p_c < 0.005$, for random phase and Gaussian scaled random phase surrogate tests, respectively. The same procedures show the absence of clear structure inside a $3.7 \le M_s \le 4.7$ window (not shown here). For the latter case, inter-event time interval sequences also have a low correlation dimension but this set is indistingui-

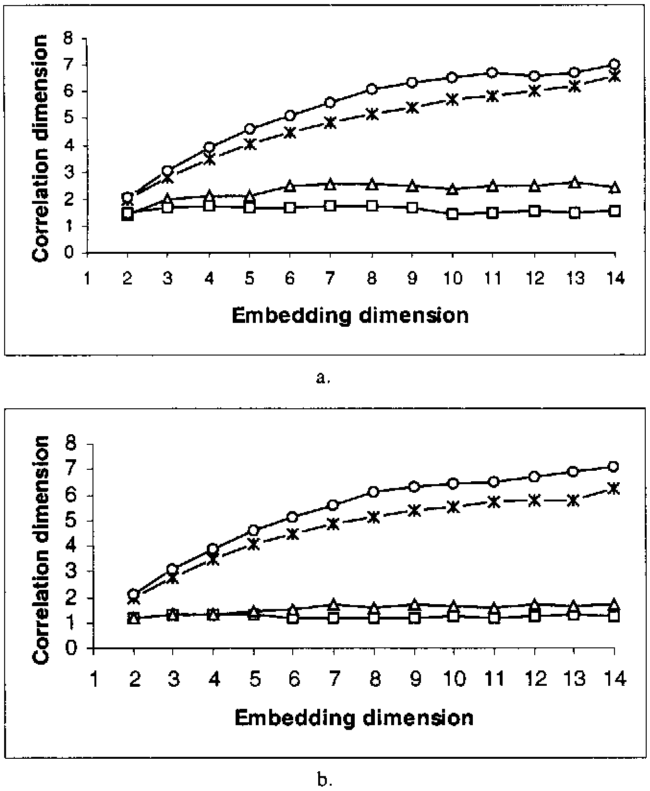



**Fig. 6.** Correlation dimension $d_2$ versus embedding dimension $p$ of a sequence of time intervals between independent events (circles) and inter-aftershocks time interval sequences (squares) for the whole Caucasus (a) and the Javakheti area (b). Triangles and asterisks correspond to phase randomized and Gaussian scaled random phase surrogates of inter-aftershock time interval sequences respectively.

shable from correlated noise, perhaps due to the smaller number of data.

For the Javakheti area inter-event time interval sequences with threshold magnitude 1.5, the situation is the same; significant nonlinear structure is found for low magnitude earthquakes $1.5 \le M_s \le 2.5$ ( Figure 5b). Here, $d_2 = 1.17 \pm 0.09$, $S = 53.2 \pm 0.1$, $P_M \sim 0$. $p_c < 0.005$ and $S = 11.0 \pm 0.15$, $p_c < 0.005$ for both type of surrogates, respectively. Again, no clear structure is found for the window $2.5 \le M_s \le 3.5$ (not shown).

For a better understanding of the role of aftershock time distribution in the nonlinear structure, the original catalogue has been "filtered" to some extent. Artificially generated sequences of aftershocks, obtained by removing independent events from the original series, were analysed. Inter-event time sequences between events as recognised independent, obtained by removing aftershocks from original series, were analysed separately.

It was found that, for all the regions considered, sequences of time intervals between independent events are characterised by a high correlation dimension (see upper curves of Fig. 6 a, b).

On the other hand, inter-aftershock time interval sequences for the whole Caucasus and Javakhti area reveal a low correlation dimension, significantly different from linearly correlated noise but indistinguishable from the linearly correlated noise transformed by a static monotone nonlinearity (see lower curves of Figures 6 a and b). This means that the existence of a nonlinear structure for filtered time series must eventually be rejected. Despite the absence of structure in separated independent event and aftershock time distribution dynamics, these results, nevertheless, give some indication that aftershocks contribute more to the regularity of earthquake temporal distribution than time distribution of independent events characterized by clear high dimensionality. Of course, this does not mean that a nonlinear structure is entirely conditioned by peculiarities of inter-aftershock time interval distribution.

Taking into account the well known causal relationship in the aftershock time distribution, our results lead to the conclusion that the insignificance of a low dimensional nonlinear structure in the inter-aftershock time interval sequence may be caused by complicated filtering effects during generation of artificial aftershock sequences. Therefore, it is necessary to rule out effects related to filtration because it is quite possible that the artificially generated catalogue does not adequately reflect the general properties of earthquake temporal distribution and leads to distortion of the original features of earthquake clustering.

Such caution is justified by well known effects of different types of filtering procedures on raw data sets, namely the occurrence of spurious dynamical characteristics (Theiler et al., 1992; Rapp et al., 1993; Kantz and Shreiber, 1997). With this in mind, we have further considered the Racha earthquake epicentral area aftershocks original data set, their magnitudes and inter-event time interval sequences with threshold magnitude 1.7.

As shown in Figure 7 (upper curve) the aftershock magnitude sequence is a high dimensional whereas the inter-event time interval series (Table 1 and lower curve of Figure 7) reveal a low dimensional nonlinear structure. The criteria for significance of differences were significant both for the random phase ($S = 34.2 \pm 0.2$ $P_M \sim 0$. $p_c < 0.005$) and the Gaussian scaled random phase ($S = 7.82 \pm 0.1$, $p_c < 0.005$) surrogates.

In the case of the Racha earthquake aftershock sequences, a low dimensional structure is also related to the specific time distribution of small events. Indeed, the inter-aftershock time interval distribution above $M = 2.5$ ($n = 1100$), does not have a low correlation dimension.

At the same time, contrary to the raw data sequences, inter-aftershock time interval sequences for the low magnitude windows $1.7 \le M_s \le 2.7$ ($n = 2464$) and $2.7 \le M_s \le 3.7$ ($n = 901$) do not reveal a nonlinear structure although the correlation dimension in both cases is about - 3. This absence of a structure, in principle, seems quite reasonable so far as data included in the time series, belong to the same event, the Racha earthquake. Hence the data must be highly interconnected and any perturbation or filtration in the data sequences could lead to distortion of the underlying dynamics. Thus it is reasonable to assume that low dimensional nonlinearity is mostly related to the clusterization of small earthquakes and their aftershocks.

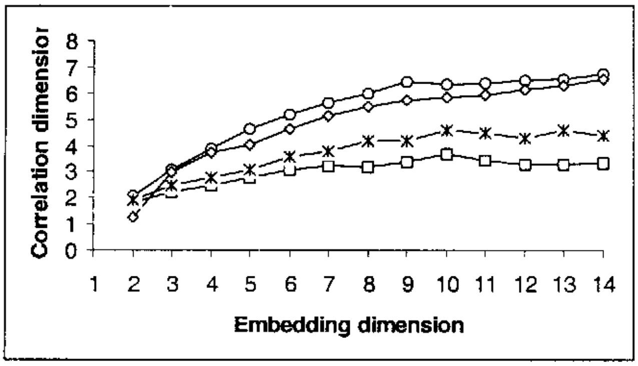
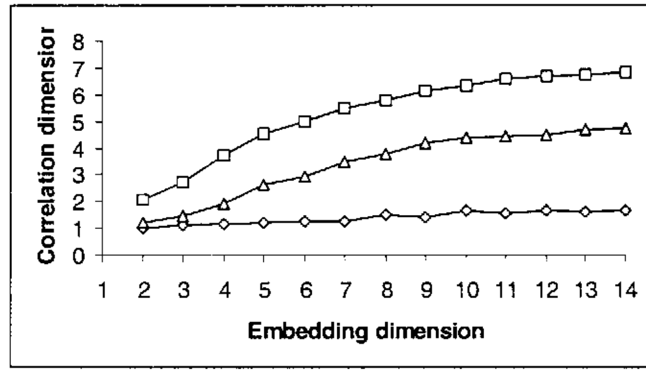



**Fig. 7.** Correlation dimension [redacted] versus embedding dimension $p$ for the Racha earthquake aftershock magnitudes (circles) and interevent time interval sequences (squares). Diamonds and asterisks correspond to phase randomized and Gaussian scaled random phase surrogates of inter-event time interval sequences, respectively.

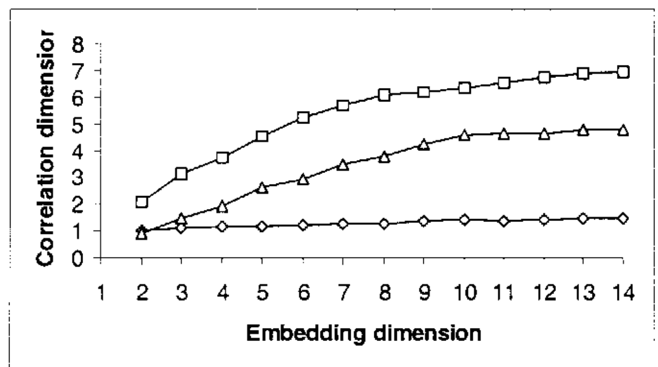
## 3.4 Stability of results

Finally, it was necessary to test the stability of our results with respect to choice of the investigated spatial region. So we carried out an additional analysis of earthquakes in the Javakhety region. A set of magnitude and inter-event time interval sequences was obtained by increasing and decreasing the size of the tested area (linear sizes were changed by 10 and 20 km). As shown in Figures 8 and 9, these changes do not alter the general characteristics of the earthquake temporal distribution – the correlation dimension is always low (about 1.5). Again, the correlation dimension of magnitude sequences does not change, it remains high dimensional for spatial variation of the size of the test area. We have also carried out an analysis of the stability of our results with regard to the uncertainty in the magnitude. Namely, we suppose that earthquakes with magnitudes slightly below the threshold really have magnitudes above the cut-off. So far as the magnitude. uncertainty is 0.2 for our catalogue we have randomly added different numbers of earthquakes: 1000, 500, 200 events from the magnitude range {M-0.2, M} and removed from the range {M, M+0.2} 500, 250, 100 events, respectively.

Time series obtained from such modified catalogues confirm evidence for the low dimensionality of time interval sequences (Figure 10) and high dimensionality of magnitude sequences. These experiments show that our results are consistent with respect to magnitude uncertainty. Thus, the results of this investigation give evidence of a nonlinear structure in a raw catalogue, underlying the self- similar scaling laws of earthquake time distribution. From this result it becomes more understandable that the fractal dimension of earthquake temporal clustering (about 0.3 for different magnitudes), in one dimensional time interval set obtained by non dynamical fractal dimension analysis, is significantly different from a uniform Poisson distribution (Chen et al., 1998). At the same time, there is insufficient evidence to establish a chaotic determinism in the considered process.




a.




b.

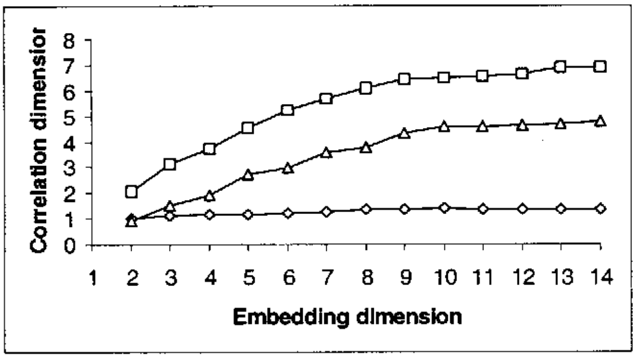
**Fig. 8** Correlation dimension versus embedding dimension for the Javakheti enlarged area inter-event time interval sequences: a) 10 km enlargement. b) 20 km enlargement. Lower curves: original sequences, middle and upper curves: Gaussian scaled random phases and phase randomized inter-event time interval surrogates, respectively.

In other words, these results suggest the possibility of using non statistical (based on a nonlinear approach) methods for prediction of earthquake time distribution, although further investigation is needed to reveal the precise nature of the low dimensional structure.

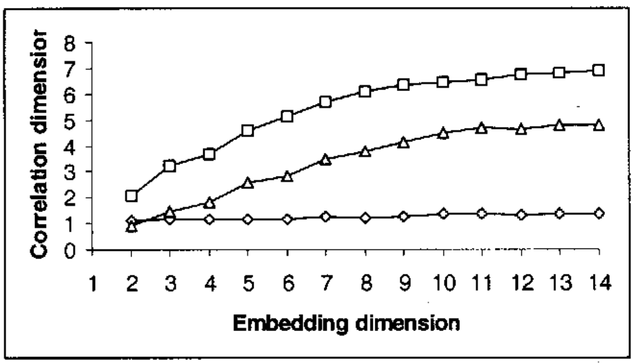
It must be emphasised that the existence of hidden order, discovered in earthquake time distribution, cannot be considered as an argument for earthquake quasiperiodicity; a quasiperiodicity was proposed for a time distribution of strong events on a given fault while, as it was shown, a nonlinear structure is mainly caused by the temporal distribution of low magnitude events. On the other hand, our results provide additional evidence that the earthquake generation process cannot be considered as random and unpredictable, at least in space and time distribution domains.

## 4 Conclusions

We have investigated the dynamical properties of the earthquake size and time distribution. It is shown that the correlation dimension of earthquake magnitude generation dynamics is a high dimensional process for all three regions considered in the Caucasus.




a.




b.

**Fig. 9** Correlation dimension versus embedding dimension for the Javakheti narrowed area inter-event time interval sequences: a) 10 km narrowing, b) 20 km narrowing. Lower curves: original sequences, middle and upper curves: Gaussian scaled random phases and phase randomized inter-event time intervals surrogates respectively.

Specific properties of the earthquake time distribution were analysed using two modern tests of surrogate data sets.

It was shown that the time distribution of earthquakes for all the regions considered reveals clear evidence of a nonlinear structure which is mostly caused by weak event temporal distribution properties. However we cannot identify this process as low dimensional chaos; the problem needs additional study.

The nonlinear structure of time interval sequences, observed in the original raw data catalogues, disappears in cases of filtered series, apparently due to the distortion of the dynamical properties of the time series by the filtering procedure.

Thus it becomes clear that well-known scaling laws of the earthquake time and, size distribution have different underlying dynamics and from the point of view of possible prediction of hazardous events, must be considered separately.

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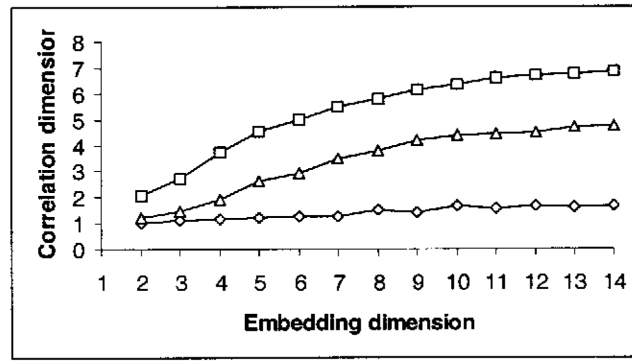


**Fig.10** Typical plot of correlation dimension versus embedding dimension for the Javakheti inter-event time interval sequences replacing 500 events above threshold in the uncertain zone by 1000 events below threshold, 250 event above threshold in the uncertain zone by 500 events below threshold and 100 events above threshold in the uncertain zone by 200 events below threshold. Lower curve - original sequences, middle and upper curves - Gaussian scaled random phases and phase randomized inter-event time interval surrogates respectively.

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