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The nonlinear dustgrain-charging on large amplitude electrostatic waves in a dusty plasma with trapped ions

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Abstract. The nonlinear dustgrain-charging and the influence of the ion density and temperature on electrostatic waves in a dusty plasma having trapped ions are investigated by numerical calculation. This work is the first approach to the effect of trapped ions in dusty plasmas. The nonlinear variation of the dust-charge is examined, and it is shown that the characteristics of the dust-charge number sensitively depend on the plasma potential, Mach number, dust mass-to-charge ratio, trapped ion density and temperature. The fast and slow wave modes are shown in this system. An increase of the ion temperature decreases the dust-charging rate and the propagation speed of ion waves. It is found that the existence of electrostatic ion waves sensitively depends on the ion to electron density ratio. New findings of the variable-charge dust grain particles, ion density and temperature in a dusty plasma with trapped ions are predicted.

1 Introduction

There has been a growing interest in investigating new properties of dusty plasmas containing charged, micrometer-sized dust grains which have been observed in space environments (Goertz, 1986; Grün et al., 1984; Melandso, 1992; Tsytovich et al., 1993; Barkan et al., 1995). Recently, it is pointed out that the wave instability having negatively-charged dust grains might be observed at altitudes of 600-700km in the F-region of the polar ionosphere of the Earth (Chow and Rosenberg; 1996). In actual situations, the dust grains have variable charge and mass due to fragmentation and coalescence. However, in studying collective effects involving charged dust grains in dusty plasmas one generally assumes that the dust particles behave like point charges. Multi-fluid descriptions for collective modes in microparticle plasmas were presented by several authors (Goertz, 1989; Verheest, 1992; Srinivas et al., 1996; Nejoh, 1997; 1998). For low frequency nonlinear wave modes, the dust grains can be described as negative ions with large mass and large charge. Ion- and dust-acoustic wave modes in dusty plasmas have been treated by several authors (Tsytovich et.al.,1993; Goertz et. al., 1983; Rao et. al., 1994; Nejoh, 1997a; 1997b; 1998a; 1998b). We have suggested that high-speed streaming particles excite various kind of nonlinear waves in space (Nejoh, 1995; 1996a; 1996b). Dust grain particles are charged due to the local electron and ion currents, and its charge varies as a result of the change of the parameters such as the potential, densities, temperatures, etc. Therefore, since the dust charge-variation affects the characteristics of the collective motion of the plasma, this effect is of crucial importance in understanding dusty plasma waves in comparison with negative ion plasmas, which have the constant charge. However, not many theoretical works on the effect of variable-charge dust grain particles have been done in dusty plasmas. In particular, the effects of the ion density and temperature have not been investigated in dusty plasmas with trapped ions.

The motivation of this articles is as follows. If streaming particles exist in space, we often find that they evolve towards a coherent trapped particle state. This has been confirmed in experiments (Goree, 1992). The onset of an ion trapping is also seen in the formation of double layers (Raadu, 1989) and computer simulation (Borovsky et. al., 1983) in space. When the amplitude of nonlinear waves becomes large, some ions, in general, would be trapped in the electrostatic potential trough of the wave and be carried along with the wave. Ion trapping is essentially a nonlinear phenomenon. There is no doubt that an ion trapping exists in nonlinear wave phenomena. Thus the inclusion of the effect of trapped ions is indispensable to considering nonlinear waves. However, theoretical investigations in relation to the trapped ion effect have not been considered in dusty plasmas. Hence, an aim of this article is to show this effect in order to apply this theory to more extensive nonlinear wave phenomena in space plasmas.

In this paper, we focus our attention on electrostatic waves in an unmagnetized dusty plasma having trapped ions with the finite temperature. It is therefore instructive to examine the effects of the dust-charging, ion density and temperature and trapped ion temperature in dusty plasmas, which, as pointed out earlier, are observed as components of broad regions of space plasmas, from the lower ionosphere to the magnetosphere of the Earth. Our plasma model consists of Boltzmann distributed electrons with finite temperature, non-Boltzmann distributed positive trapped ions with finite temperature, and the negatively-charged, cold dust fluid obeying the nonlinear continuity and momentum equations. We derive a nonlinear equation for variable-charge dust grain particles and the conservation law of energy of electrostatic waves. We show the dependence of the dust grain-charging on the electrostatic potential, ion density and temperature, trapped ions and Mach number. Our results show the existence of super- and sub-sonic electrostatic waves and illustrate the dependence of the dust-charge number on the several parameters. In section 2, we present a new nonlinear equation for variable-charge dust grains and derive the Sagdeev potential from the basic equations. In section 3, we show the numerical results of the nonlinear equations obtained in the preceding section. It is shown that the charging effect of the dust grains drastically changes due to the dust charge, dust mass, ion density and Mach number. Section 4 is devoted to the concluding discussion.

2 Theory

We consider a collisionless, unmagnetized three component plasma consisting of Boltzmann electrons with a constant temperature T_e, non-Boltzmann warm ions having a temperature T_i and negatively charged, heavy, dust particles, and assume that low frequency electrostatic waves propagate in this system. We assume that the phase velocity of the wave is less than the electron thermal velocity, and is more than the ion and dust thermal speeds. The velocity of electrons is smaller than their thermal speed.

The electron number density is assumed to be the Boltzmann distribution as,

$$n_e = n_{e0} exp(e\Phi/T_e), \tag{1}$$

where n_e, n_{e0}, e and Φ are the electron density, background electron density, the magnitude of electron charge and the electrostatic potential.

In order to investigate the effect of trapped ions, we employ the vortex-like ion distribution function (Schamel, 1972; Bujarbarua & Schamel, 1981; Schamel & Bujarbarua, 1983), which solves the ion Vlasov equation. We assume that the ion velocities which appear in the distribution functions are normalized by the ion thermal velocity $v_{i,th}$, and hence the terms of the ion thermal velocity in the distribution functions do not appear explicitly. The electric potential Φ is assumed to be negative, $-\Psi \le \Phi \le 0$, where Ψ denotes the maximum amplitude. Since we consider the case $y \ge 1$ in (7) of Schamel (1972), and use the normalization, the large amplitude waves are investigated in this paper. The author limits his investigation to the case that the wave velocity is less than the ion thermal velocity i.e. $v < v_{i,th} = \sqrt{T_i/m_i}$. Considering the assumptions mentioned above, we can reduce the ion distribution functions to

$$f_f = \frac{1}{\sqrt{2\pi}} exp\left\{-\frac{1}{2}\left(v^2 + 2\Phi\right)\right\}, for\ |v| > \sqrt{-2\Phi},$$

$$f_t = \frac{1}{\sqrt{2\pi}} exp\left\{-\frac{1}{2}\left(\frac{T_i}{T_t}\right)\left(v^2 + 2\Phi\right)\right\}, for\ |v| \le \sqrt{-2\Phi},$$

where $T_i(T_t)$ denotes the free (trapped) ion temperature, and the subscripts f and t represent the free and trapped ion contributions, respectively. The parameter T_i/T_t determines the number of trapped ions. A plateau in the resonant ion region is given by $T_i/T_t = 0$, and $T_i/T_t < 0$ corresponds to a vortex-like trapped ion distribution, which is of interest to us. It is also noted that the ion distribution functions, as is stated above, are continuous in the velocity space and satisfy the regularity requirements for an allowable BGK (Bernstein-Greene-Kruskal) solution. With wave velocities in the ion thermal to ion-acoustic range, $\sqrt{T_i/m_i} \le v \le \sqrt{T_e/m_i}$, the electronic contribution K_e becomes indeed small and negligible, but not the ion one, since the first argument in K_i is O(1) or larger. Hence the trapped ion contribution is important. Integrating the ion distribution functions over the velocity space, we obtain the ion density for the trapped ion distribution as,

$$n_i = n_{i0}\left[\left\{1 - erf\left(\sqrt{-\frac{e\Phi}{T_i}}\right)\right\} exp\left(-\frac{e\Phi}{T_i}\right)\right] + n_{i0}\left[\frac{1}{2}\sqrt{\frac{T_t}{T_i}} exp\left(-\frac{e\Phi}{T_t}\right) erf\left(\sqrt{-\frac{e\Phi}{T_t}}\right)\right], \tag{2}$$

where

$$erf\left(\sqrt{-(T_i/T_t)\Phi}\right) = \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{-(T_i/T_t)\Phi}} exp(-x^2)dx,$$

and n_{i0} denotes the equilibrium ion density. Here the author points out that this point of view is a first approach to the trapped ions in dusty plasmas and there is more need to investigate the K-effect on wave velocities above the ion thermal range.

For one dimensional low frequency acoustic motions, we have the following two equations for the cold dust particles,

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0, \tag{3a}$$

$$\left(\frac{\partial}{\partial t} + v_d \frac{\partial}{\partial x}\right) v_d - \frac{Q_d}{m_d}\frac{\partial \Phi}{\partial x} = 0, \tag{3b}$$

where n_d, v_d and m_d refer to the dust grain density, dust fluid velocity and dust grain mass, respectively. Here the dust-charge variable is defined to $Q_d = eZ_d$, where Z_d is the charge number of dust particles measured in units of e.

The Poisson's equation is given as

$$\frac{\partial^2 \Phi}{\partial x^2} = \frac{e}{\varepsilon_0}(n_e - n_i + Z_d n_d). \tag{4}$$

We assume that the phase velocity of electrostatic ion waves is low in comparison with the electron thermal velocity. Charge neutrality at equilibrium requires that $n_{i0} = n_{e0} + n_{d0} Z_d$, where $n_{i0}(n_{d0}$ denotes the equilibrium ion (dust grain) density. In this system, the ordering, $m_d \gg m_i \gg m_e$ holds, as is obtained in laboratory plasmas. Typical laboratory plasma frequencies are: 10^2Hz, 10^{5-6}Hz, 10^{9-10}Hz, which have roughly the same ordering as the mass ratios. Thus, the inclusion of the mass ratios is equal to considering the motion of dust particles.

We assume that the charging of the dust grain particles arises from plasma currents due to the electrons and the ions reaching the grain surface. In this case, the dust grain-charge variable Q_d is determined by the charge current balance equation:

$$\left(\frac{\partial}{\partial t} + v_d \frac{\partial}{\partial x}\right) Q_d = I_e + I_i. \tag{5}$$

We have the following expressions for the electron and ion currents for spherical grains of radius r:

$$I_e = -e\pi r^2 (8T_e/\pi m_e)^{1/2} n_e(\Phi) exp\left(\frac{eQ_d}{rT_e}\right), \tag{6a}$$

and

$$I_i = e\pi r^2 (8T_i/\pi m_i)^{1/2} n_i(\Phi, T_t)\left(1 - \frac{eQ_d}{rT_i}\right). \tag{6b}$$

We note that the effect of trapped ions is included in the ion current (6b).

We normalize all the physical quantities as follows. The densities are normalized by the background electron density n_{e0}. The space coordinate x, time t, dust velocity and electrostatic potential ϕ are normalized by the electron Debye length $\lambda_d = (\varepsilon_0 T_e/n_0 e^2)^{1/2}$, the inverse ion plasma period $\omega_i^{-1} = (\varepsilon_0 m_i/n_0 e^2)^{1/2}$, the ion sound velocity $C_s = (T_e/m_i)^{1/2}$, and T_e/e, respectively, where m_i, ε_0 and e are the ion mass, the permittivity of vacuum and the magnitude of electron charge, respectively. We study the one-dimensional electrostatic ion waves in the following discussion.

In order to solve (1)-(4), we introduce the variable $\xi = x - Mt$, which is the moving frame with the velocity M. Then a set of basic equations (1)-(5) reduces to

$$n_e = exp(\phi), \tag{7}$$

$$n_i = \delta_i \left\{\left[1 - erf\left(\sqrt{-\frac{\phi}{\tau_i}}\right)\right] exp\left(-\frac{\phi}{\tau_i}\right)\right\} + \delta_i \left\{\frac{1}{2}\sqrt{\frac{1}{\tau_{ft}}} exp\left(-\frac{\phi}{\tau_t}\right) erf\left(\sqrt{-\frac{\phi}{\tau_t}}\right)\right\}, \tag{8}$$

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial t}(n_d v_d) = 0, \tag{9a}$$

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} - \frac{Z_d}{\mu_d}\frac{\partial \phi}{\partial x} = 0, \tag{9b}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i + Z_d n_d. \tag{10}$$

Here, $\tau_i = T_i/T_e, \tau_t = T_t/T_e, \tau_{ft} = T_i/T_t, \delta_i = n_{i0}/n_{e0}$ and $\mu_d = m_d/m_i$. Here, in the isothermal process, we assume that $\tau_t > 0$. It is significant to consider the dependence of the trapped ion density n_i on the electrostatic potential ϕ and trapped to free ion temperature ratio τ_t in the sense that electrostatic ion waves closely connect with the trapped ion density. The ion density (1) includes the two states of the ion distribution, namely, (i) in the limit of $\tau_{ft} \to 1 (T_i = T_t), n_i$ approaches the Boltzmann distribution, (ii) $\tau_t \to \infty$ means the state for flat-topped distribution. A figure showing n_i vs ϕ for various values of τ_t is shown in an Appendix.

Integrating (9a) and (9b) and using the boundary conditions, $\phi \to 0, n_d \to (\delta_i - 1)/Z_d, n_i \to \delta_i, v_d \to 0$, at $\xi \to \infty$, we obtain the dust grain density as

$$n_d = \frac{\delta_i - 1}{Z_d}\frac{1}{\sqrt{1 + \frac{Z_d}{\mu_d}\frac{2\phi}{M^2}}} \tag{11}$$

At equilibrium, the charge current balance equation (5) reduces to

$$-n_e(\phi) exp(aZ_d) + \sqrt{\frac{\tau_i}{\mu_i}} n_i(\phi, \tau_t, \delta_i)\left(1 - \frac{aZ_d}{\tau_i}\right) = 0. \tag{12}$$

where $\mu_i = m_i/m_e, a = e^2/rT_e$, and the electron (ion) current $I_e(I_i)$ is normalized by $e\pi r^2 (8T_e/\pi m_e)^{1/2}$. In order to solve (12) exactly, we derive a following equation for variable-charge of dust grains as

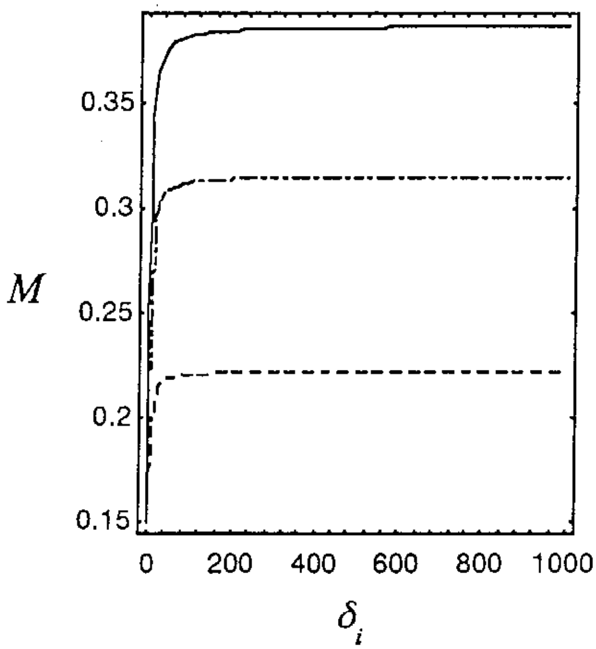

Fig. 1. The Mach number of the slow ion wave modes as a function of the ion-to-electron density ratio δ_i, in the case of $\tau_i = 0.5$ (solid line), 1.0 (dotted line) and 2.0 (dashed line), where $r = 2 \times 0.1\mu$ and $D_d/\mu_d = 10^{-2}$.

$$exp(\phi + aZ_d) =$$
$$\frac{\delta_i(\tau_i - aZ_d)}{\sqrt{\mu_i \tau_i}} \left\{ \left[1 - erf\left(\sqrt{-\frac{\phi}{\tau_i}}\right)\right] exp\left(-\frac{\phi}{\tau_i}\right) \right\}$$
$$+ \frac{\delta_i(\tau_i - aZ_d)}{\sqrt{\mu_i \tau_i}} \frac{1}{2} \sqrt{\frac{1}{\tau_{ft}}} erf\left(\sqrt{-\frac{\phi}{\tau_t}}\right) exp\left(-\frac{\phi}{\tau_t}\right). \quad (13)$$

It is noted that we employed the effect of trapped ions into (13) through the ion density. We regard (13) as a new nonlinear equation for the dust grain surface potential in this system. Equation (13) includes strongly nonlinear terms. Then, we can obtain the solution of (13) by numerical calculation in the next section.

Integration of the Poisson's equation gives *the Energy Law*,

$$\frac{1}{2}\left(\frac{\partial \phi}{\partial \xi}\right)^2 + V(\phi) = 0, \quad (14)$$

where $V(\phi)$ denotes the Sagdeev potential. The Sagdeev potential becomes

$$V(\phi) = 1 - exp(\phi) + (\delta_i - 1)\frac{\mu_d}{Z_d} M^2 \left(1 - \sqrt{1 + \frac{Z_d}{\mu_d}\frac{2\phi}{M^2}}\right)$$
$$+ \delta_i \tau_i \left[1 - \left(1 - erf\left(\sqrt{-\frac{\phi}{\tau_i}}\right)\right) exp\left(-\frac{\phi}{\tau_i}\right)\right]$$
$$-\delta_i \tau_i \left[\left(1 - \frac{1}{\tau_{fi}}\right)\sqrt{\frac{-\phi}{\pi \tau_i}}\right]$$
$$-\delta_i \tau_i \left[\frac{1}{2}\tau_{ft}^{-3/2} erf\left(\sqrt{-\frac{\phi}{\tau_t}}\right) exp\left(-\frac{\phi}{\tau_t}\right)\right] \quad (15)$$

If there are no dust grains, i.e. the dust-charge number is switched off, $Z_d \to 0$, and electrons are distributed with the Boltzmann distribution, eq.(15) coincides exactly with that obtained in Schamel (1972). The present result includes that obtained earlier.

The oscillatory solution of the nonlinear electrostatic ion waves exists when the following two conditions are satisfied: (i) The pseudopotential $V(\phi)$ has the maximum value if $d^2V(\phi)/d\phi^2 < 0$ at $\phi = 0$.

$$-1 - \frac{\delta_i}{\tau_i} + (\delta_i - 1)\frac{Z_d}{\mu_d M^2} < 0. \quad (16)$$

The speed of the wave is defined by the inequality (16). It should be noted that $V(\phi)$ is real, when $-\mu_d M^2/2Z_d < \phi < 0$. The region of existence of the plasma potential is characterized by that condition. We understand that the regions strongly depend on the dust to ion mass ratio, dust-charge number and the ratio of the ion to electron density and temperature. (ii) Nonlinear ion waves exist only when $V(\phi_M) \geq 0$, where the maximum potential ϕ_M is determined by $\phi_M = -\mu_d M^2/2Z_d$. This implies that the inequality

$$1 - exp\left(\frac{M^2 - \tau_i}{2}\right) erfc\sqrt{\frac{M^2 - \tau_i}{2}}$$
$$-\left(1 - \frac{1}{\tau_{ft}}\right)\sqrt{\frac{M^2 - \tau_i}{2\pi}}$$
$$-\frac{1}{2}\frac{1}{\tau_{ft}^{3/2}} exp\left\{\frac{\tau_{ft}(M^2 - \tau_i)}{2}\right\} erf\sqrt{\frac{\tau_{ft}(M^2 - \tau_i)}{2}}$$
$$+\delta_i(M^2 - \tau_i) + (\delta_i - 1)\frac{\mu_d M^2}{Z_d}$$
$$\times \left(1 - \sqrt{1 + \frac{Z_d(M^2 - \tau_i)}{\mu_d M^2}}\right) \geq 0, \quad (17)$$

holds.

The numerical analysis of the wave velocity gives rise to the existence of the slow and fast ion wave modes. It is noted that the fast wave is an ordinary ion-acoustic wave and the slow one is a slow ion-acoustic wave on which ion holes or clumps rest. We show the speed (Mach number) of the slow ion waves as a function of δ_i in Fig.1, in the case of $\tau_i = 0.5$ (solid line), 1.0 (dotted line) and 2.0 (dashed line), where $r = 2 \times 0.1\mu m$ and $Z_d/\mu_d = 10^{-2}$. On the contrary, we illustrate the fast wave modes in Fig.2. Fig.2 illustrates the dependence of the wave speed on the ion to electron density ratio for $\tau_i = 2.0$ (solid line), 1.5 (dotted line) and 1.0(dashed

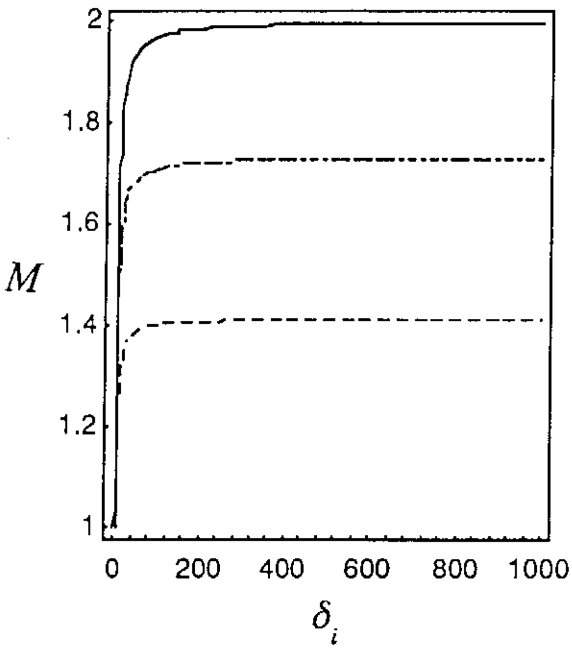

Fig. 2. The dependence of the Mach number of the fast wave modes on the ion-to-electron density ratio δ_i for $\tau_i = 2.0$ (solid line), 1.5 (dotted line) and 1.0 (dashed line), where $r = 2 \times 0.1\mu$ and $Z_d/\mu_d = 10^{-1}$.

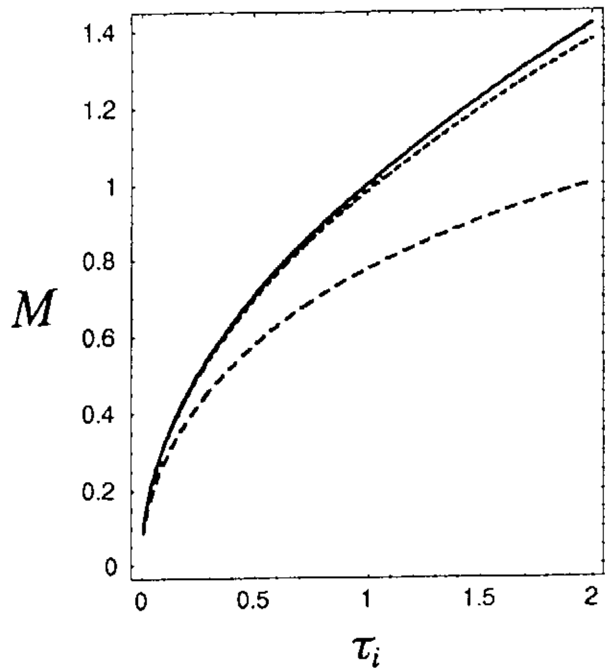

Fig. 3. A $M - \tau_i$ plane for $\delta_i = 2000$ (solid line), 50 (dotted line) and 4.0(dashed line).

line), where $r = 2 \times 0.1\mu m$ and $Z_d/\mu_d = 10^{-1}$. Fig.3 shows a $M - \tau_i$ plane for $\delta_i = 2000$ (solid line), 50 (dotted line) and 4.0 (dashed line). We understand that the speed of the waves increases as the ion density ratio becomes large. The velocity of the wave and, correspondingly, the amplitude of electrostatic ion waves significantly depends on the parameters $Z_d/\mu_d, \delta_i$ and τ_i. It turns out that both fast and slow electrostatic ion wave modes coexist in this system. The criterion of these modes strongly depends on the dust-charge to dust mass and the ion-to-electron density ratios.

3 Numerical results of the dust grain-charge variation and nonlinear ion waves

We examine the numerical analysis of the nonlinear equations obtained in the preceding section. For illustration purposes, we consider a dusty plasma in which most of the background electrons are collected by negatively-charged dust grains. Such a situation, for example, is common to the environment of the Earth. Thus, without loss of generality, we calculate the dust-charging effect concerning electrostatic nonlinear ion waves by numerical calculation. For example a spherical dust grain of radius $2 \times 0.1\mu m$ and mass density $2500\ kg/m^3$ has a mass $8.0 \times 10^{-17} kg$ so that $\mu_d = m_d/m_i = 4.8 \times 10^{10}$, where the ions being protons. In the following discussion, for the illustration purposes, we employ $r = 2 \times 0.1\mu m$, $T_e = 1eV$, $\mu_i = 1836$, $\mu_d = 10^{10}$ and $\alpha = 7.2 \times 10^{-3}$.

First, we study the dependence of the dust-charging effect on the electrostatic potential. If $\phi < 0$, we show, from (12) or (13), a $Z_d - \phi$ plane in Fig.4, in the case of $\tau_i = 0.4$ (solid line) and 0.1 (dashed line), where $\delta_i = 100$ and $\tau_i = 1.0$. The dust-charge number gradually increases as the negative electrostatic potential increases. We understand that the charge number increases as the ion temperature decreases for $\phi < -0.25$. Fig.5 illustrates the dependence of the dust-charge number on the ion-to-electron temperature ratio τ_i , in the case of $\phi = -0.2$ (solid line) and -0.4 (dashed line), where $\delta_i = 100$ and $\tau_t = 1.0$. We find that, in the case of $\phi = -0.2$, an increase of the ion-to-electron temperature ratio decreases the charge number of dust grains within the range of $\tau_i < 0.1$, but an increase of τ_i gradually increases the dust-charge number within the range of $\tau_i > 0.1$. Fig.6 shows the dependence of the dust-charge number on the ratio of the trapped ion to electron temperature τ_t for $\phi = -0.2$ (solid line) and -0.4 (dashed line), where $\delta_i = 100$ and $\tau_i = 0.1$. It turns out that the dust-charge number decreases as the trapped ion temperature increases. The dust-charging ratio of the higher, negative electrostatic potential is higher than that of the lower one. We find that the dust-charge number Z_d lies in $5 \times 10^2 - 10^4$, as is seen in Figs.4-6. When the electroststic potential, the ion and trapped ion temperatures increase, dust grain particles are highly charged.

Second, in the case of the slow mode, we show a 3-dimensional potential structure associated with (14) and

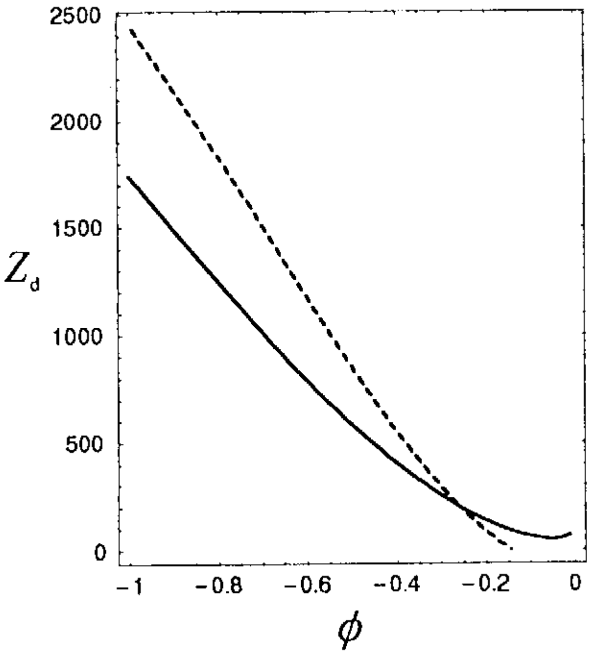

Fig. 4. The variable-charge Z_d as a function of the potential ϕ for $\tau_i = 0.4$ (solid line) and 0.1 (dashed line), where $a = 7.2 \times 10^{-3}, \delta_i = 100$ and $\tau_t = 1.0$.

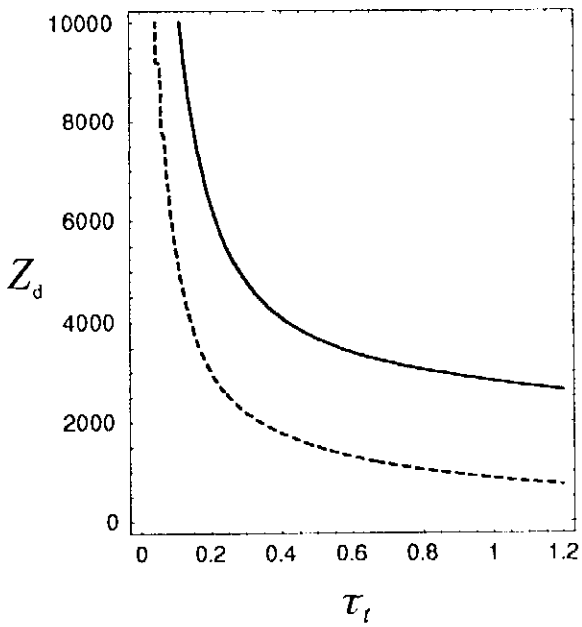

Fig. 6. The dependence of the dust-charge number Z_d on the ratio of the trapped ion-to-electron temperature τ_t for $\phi = 0.2$ (solid line) and -0.4 (dashed line), where $a = 7.2 \times 10^{-3}, \delta_i = 100$ and $\tau_i = 0.1$.

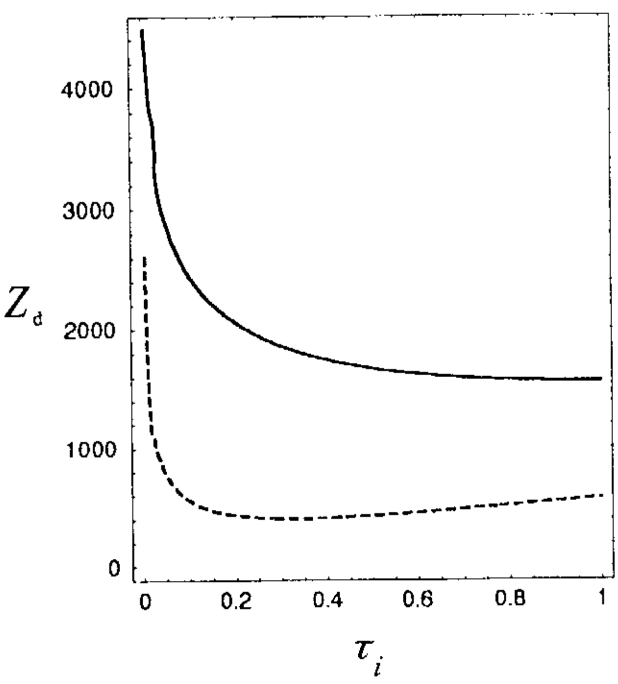

Fig. 5. The dependence of the dust-charge number Z_d on the ratio of the trapped ion-to-electron temperature τ_i for $\phi = -0.2$ (solid line) and -0.4 (dashed line), where $a = 7.2 \times 10^{-3}, \delta_i = 100$ and $\tau_t = 1.0$.

(15) in Fig.7, where $\mu_d/Z_d = 10^2, M = 0.1, \tau_i = 0.2$ and $\tau_t = 2.0$, and illustrate a 2-dimensional structure in Fig.8 in the case where $\delta_i = 100$ (dashed line), 500 (dotted line) and 1000 (solid line). In the case of the fast mode, we illustrate a bird's-eye view of (15) in Fig.9, where $\mu_d/Z_d = 10^2, M = 1.2, \tau_i = 0.2$ and $\tau_t = 30$. Fig.10 shows a $V(\phi) - \phi$ plane for the cases of $\delta_i = 100$ (dashed line), 500 (dotted line) and 1000 (solid line). We understand that the existence of nonlinear electrostatic ion waves drastically changes due to the ratio of the dust mass-to-charge ratio μ_d/Z_d, ion-to-electron density ratio δ_i, Mach number and the trapped ion-to-electron temperature ratio τ_t. Thus, we understand that the dust-charge number and the region for existence of nonlinear electrostatic ion waves sensitively depend on the significant parameters mentioned above.

4 Discussion

In this article, we have shown the effect of the dust-charging of electrostatic ion waves in a dusty plasma whose constituents are Boltzmann electrons, non-Boltzmann trapped ions, and a cold dust fluid consisting of negatively-charged, micrometer-sized dust particles. Such plasmas may exist in space environments. We find the remarkable properties of the variable-charge of dust grain particles and electrostatic ion waves obtained here as follows.

(1) Fast (supersonic) and slow (subsonic) ion waves

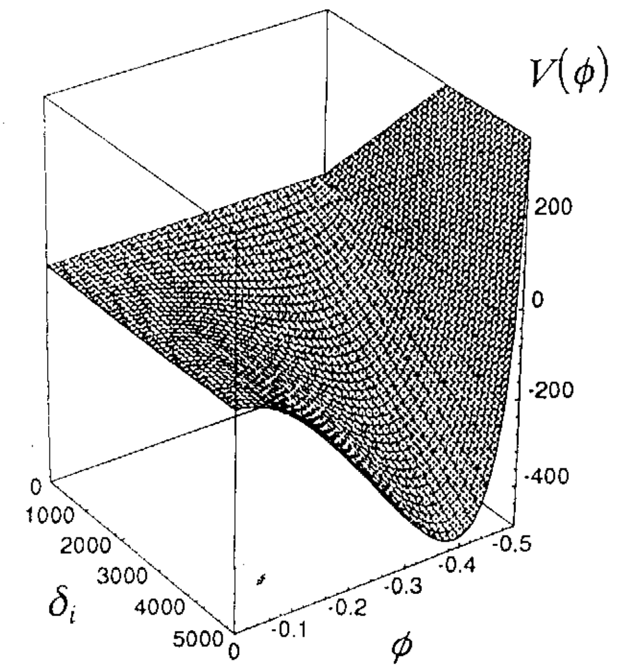

Fig. 7. A 3-dimensional structure of the Sagdeev potential for the slow wave mode, where $a = 7.2 \times 10^{-3}, \mu_d/Z_d = 10^2, M = 0.1, \tau_i = 0.2$ and $\tau_t = 2.0$.

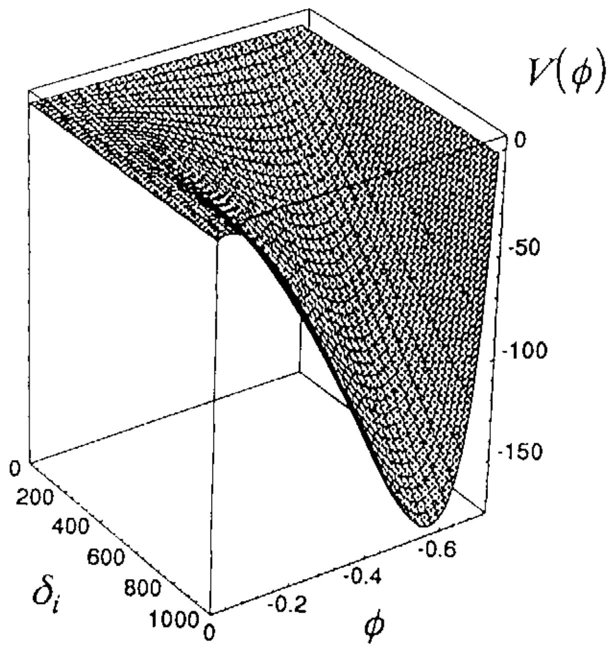

Fig. 9. A bird's-eye view for the fast wave mode, where $a = 7.2 \times 10^{-3}, \mu_d/Z_d = 10^2, M = 1.2, \tau_i = 0.2$ and $\tau_t = 30$.

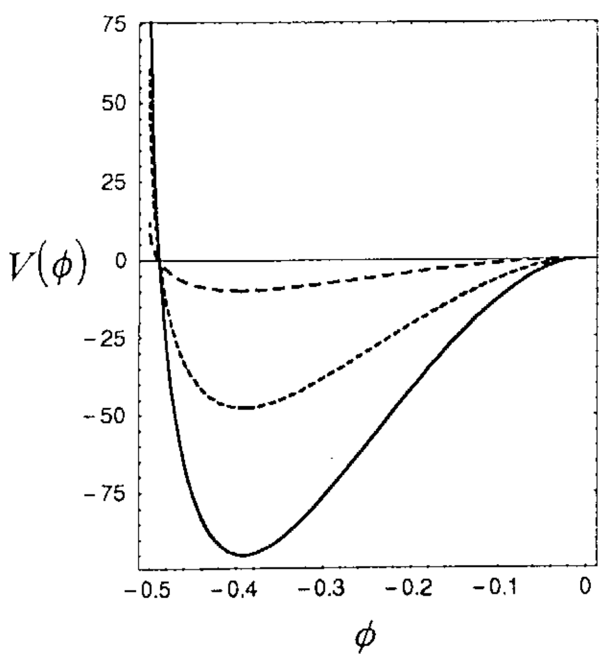

Fig. 8. A 2-dimensional potential structure in the cases of $\delta_i = 100$ (dashed line), 500 (dotted line) and 1000 (solid line) in Fig.7.

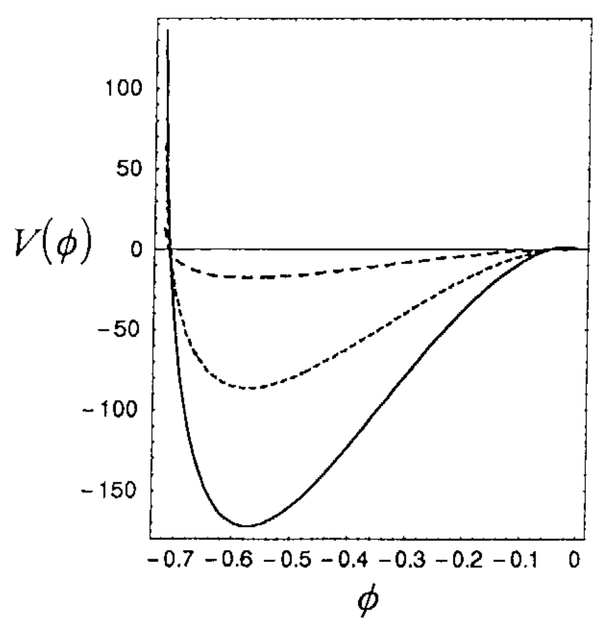

Fig. 10. A $V(\phi) - \phi$ plane for the cases of $\delta_i = 100$ (dashed line), 500 (dotted line) and 1000 (solid line) in Fig.9.

can propagate in this system. Dependence of the Mach number on the ion-to-electron density ratio drastically changes due to the dust charge-to-mass ratio, trapped ion and free ion temperatures.

(2) Dependence of the charge-variation of dust particles on the trapped ion temperature and ion density is found for the first time in dusty plasmas. The effect of the trapped ion temperature decreases the dust-charge number. Since the effect of trapped ions affects the characteristics of the collective motion of the plasma, this effect is important in understanding nonlinear waves propagating in dusty plasmas. The dust-charging effect is of crucial importance in the sense that the dust-charge number drastically changes due to the parameters such as the floating potential of dust particles, plasma potential, ion-to-electron density ratio, dust-to-ion mass ratio, trapped ion temperature and Mach number.

(3) The region for existence of nonlinear ion waves varies due to the dust mass-to-charge ratio, and the ion-to-electron density ratio and the floating potential of dust grains. In particular, it is found that the existence of the fast wave mode depends on the trapped ions with quite high temperature compared with free ion temperature.

As is mentioned in this article, the inclusion of trapped ions is required to consider nonlinear dust waves. If such waves are present in dusty plasmas, which are observed in the ionosphere and magnetosphere of the Earth and other space environments, the investigation of their peculiar features will contribute to the future development in dusty plasmas. It is predicted that the wave instability having negatively-charged dust grains might be observed at altitudes of 600-700 km in the F-region of the polar ionosphere of the Earth. In this situation, our results are important in understanding the charging mechanism of the streaming of dust grain particles and confirming the existence of arbitrary amplitude electrostatic ion waves in dusty plasmas with trapped ions. With wave velocities in the ion thermal to ion-acoustic range, $\sqrt{T_i/m_i} \leq v \leq \sqrt{t_e/m_i}$, the electronic contribution K_e becomes indeed small and negligible, but not the ion one, since the first argument in K_i is O(1) or larger. Although the author limits his investigation to the case $v < \sqrt{T_i/m_i}$, the trapped ion contribution is very important. The author points out that this work is the first approach to the trapped ions and there is more need to investigate the K-effect on wave velocities (in Schamel, 1972) above the ion thermal range.

Appendix: The profile of the trapped ion density

It is significant to consider the dependence of the trapped ion density n_i on the electrostatic potential ϕ and trapped ion to electron temperature ratio τ_t in the sense that electrostatic ion waves closely connect with the trapped ion density. In order to investigate the rela-

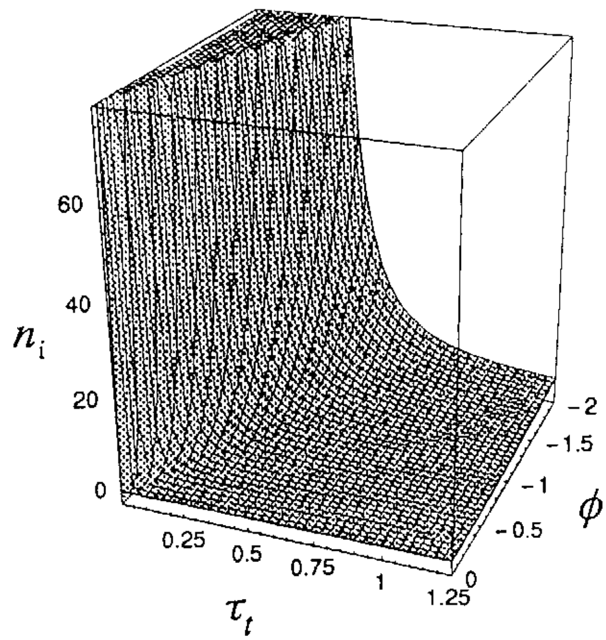

Fig. 11. A bird's-eye view of the trapped ion density n_i depending on the electrostatic potential ϕ and the trapped ion-to-electron temperature ratio τ_t.

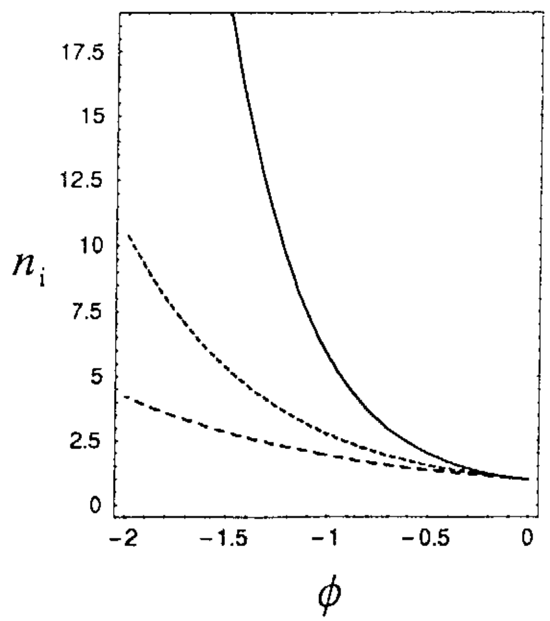

Fig. 12. A $V(\phi)-\phi$ plane for the cases of $\delta_i = 100$ (dashed line), 500 (dotted line) and 1000 (solid line) in Fig.11.

tionship between the ion density and plasma potential (trapped ion temperature), we show the trapped ion density n_i as the functions of ϕ and τ_t associated with Eq.(8). We illustrate a bird's-eye view in Fig.11, and a $n_i - \phi$ plane in Fig.12, for $\tau_t = 0.4$ (solid line), 0.7 (dotted line) and 1.2 (dashed line). As is seen in Figs.11 and 12, we find the following. In the case where $\tau_t > 1$, the trapped ion density n_i lies in the small range even if the electrostatic potential ϕ grows. In the case where $\tau_t < 1, n_i$ increases exponentially as ϕ increases. The lower τ_t becomes, the grower n_i becomes exponentially. It turns out that, when τ_t becomes lower, this tendency becomes remarkable.

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