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# Evidence of structured Brownian dynamics from temperature time series analysis

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**Abstract.** An analysis of time series of monthly mean temperatures ranging from 1895 to 1989 is performed through application of Singular Spectrum Analysis (SSA) to data of several places in the USA. A common dynamics in the reconstructed spaces is obtained, with the evidence of a non-trivial and structured coupling of two Brownian motions, resembling the so-called Lévy flights. The idea that these two correlated functions are related to the zonal and eddy components of the atmospheric motions is suggested.

## 1 Introduction

Analysis of time series in geo-sciences has been performed since the birth of the so-called chaos theory (Takens, 1981) with the aim of embedding the dynamics of random-like behaviours occurring in nature in a low-dimensional state space. A strong conceptual objection to these reconstruction techniques was soon given by the shortness of the records available, which could invalidate any conjecture on the attractor properties (Smith, 1988; Ruelle, 1990). There is still an open discussion on this subject (Nerenberg and Essex, 1990; Tsonis et al., 1994) and some authors have found techniques useful for the treatment of short and noisy time series, like Singular Spectrum Analysis (SSA) (Broomhead and King, 1986; Vautard and Ghil, 1989; Vautard et al., 1992).

Moreover, the usually short meteo-climatic records exhibit typically, besides more or less broad peaks, a large red-noise component in their power spectra (Ghil and Childress, 1987; Cuomo et al., 1994). This fact seems to exclude the hypothesis of low-dimensional chaos and supports the conjecture of leading Markov dynamics in atmospheric processes, as already suggested about twenty years ago by Leith (1973; 1978) and Hasselmann (1976).

Here we apply SSA to time series of monthly mean temperatures ranging from 1895 through 1989, in randomly selected places of the United States. Results reveal, for all our case-studies, a common dynamics in the reconstructed spaces, showing a non-trivial and structured coupling of two Brownian motions, resembling the so-called Lévy flights, today emerging in many areas of physics, such as statistical mechanics and turbulence theory (Montroll and Shlesinger, 1984; Sagdeev and Zaslavsky, 1992; Shlesinger et al., 1995). Finally, we present preliminar suggestions about the meteo-climatic interpretation of this evidence.

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## 2 The analysis

We analysed the monthly mean temperature time series of different places of the United States (North Coast Drainage of California, Northeast Georgia, Coastal Maine and Southeast Minnesota) from NOAA/NCDC archives as reported in Masters (1995), in order to study their course and possibly to find a dynamics which could explain the inter-annual variability of climate. Our choice was leaded by the opportunity of having spatially distributed records of the same time-length. For reasons of space, in this letter we comment results and show figures related uniquely to one case-study (California). Anyway, the final results appear to be the same for all our case-studies.

By means of SSA, a method adopting the Principal Component Analysis (Jolliffe, 1986), one can find a set of orthogonal directions for a cloud of points, that accounts as much as possible for the data's variance. For a time series, this is a way for discriminating between information and noise in the time-delayed space. This method also resolves the problem of choosing the optimum time delay in the reconstruction techniques. The window width chosen in applying SSA was $\tau_w = 120$ months. This is an acceptable time-range if one considers that anomalies in atmospheric flow patterns, like blocking events, affects climate on the time scales of months and seasons. Moreover, from an estimate of global climatic variability on all time scales, the reconstructed power spectrum shows a gap between the range of Kyears, and the annual and synoptic periods (Mitchell, 1976).

The ordered set of eigenvalues of the covariance matrix is composed of a couple of equal eigenvalues, preceeded by a higher one for coastal regions and followed by a lower one for Minnesota, over a *plateau* of almost vanishing components of the spectrum. This couple is an evidence of a periodical activity (Vautard et al., 1992) and is easily interpreted as the signature of the annual cycle. This brings us to consider the 3D space reconstructed by the autovectors, associated with the eigenvalues above, as the state space in which the maximum of information about the dynamics underlying the time series is given.

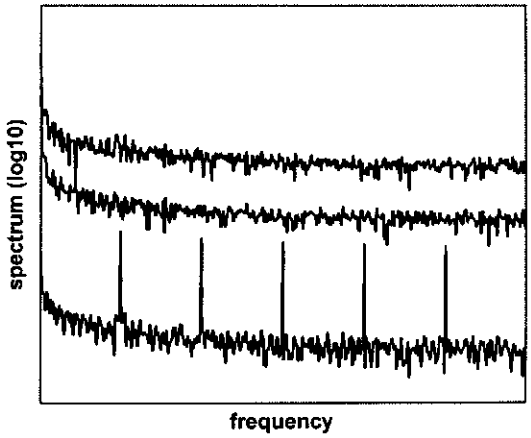


**Fig.1** Power spectra of the first three eigenfunctions obtained from the time series. The lowest graph, related to the θ coordinate, shows peaks of periodicity; the other two (associated with the r and z components) show only red noise.

The first eigenfunction $f_1$, associated with the single high eigenvalue, has been revealed to be a Brownian function, as will be shown later. A plot of the motion in the reconstructed space shows helicoidal dynamics with an orbital period of 12 time-steps and whose pitch is driven by $f_1$. The symmetry found suggests to study the dynamics in a cylindrical coordinate system $(\theta, r, z)$ with origin in the centre of the reconstructed cloud:

$$r(t) = \sqrt{f_2^2(t) + f_3^2(t)}$$

$$\theta(t) = \text{arctg}\frac{f_3(t)}{f_2(t)} \qquad (1)$$

$$z(t) = f_1(t)$$

where $f_i(t)$ are the eigenfunctions of the covariance matrix. This simple geometrical transformation leads to the confinement of the orbital cycle to the θ coordinate, leaving the non-annual dynamics to the r-z plane. This is evident from Fig.1, where a nice theorem of SSA, stating that the power spectrum of a time series can be decomposed in the sum of the eigenfunction power spectra (Vautard et al., 1992), is applied. Moreover, the spectrum of the r-component has the same behaviour of the z-component, inducing therefore the idea of the presence of a further Brownian motion in the dynamics of our system. A proof of this is given performing a log-log plot of power spectrum vs. frequency for both the a-periodic components. The spectral density S(ω) scales with frequency ω according to a power law of the kind $\omega^{-\alpha}$, with $1.9 \leq \alpha \leq 2.1$ for the various time series. This range of values implies the possibility of having both pure Brownian motions ($\alpha = 2$) or fractional Brownian motions ($\alpha \neq 2$) (Falconer, 1990). From the plot of the above functions, shown in Fig.2, there seem to appear recurrent opposite local trends in the Brownian walks; therefore it is interesting to investigate the dynamics in the r-z plane.

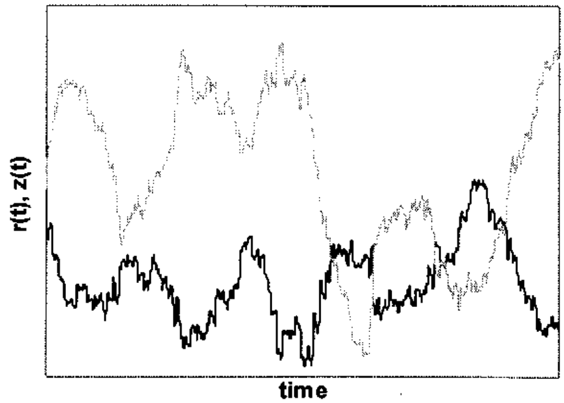


**Fig.2.** Time evolution of the eigenfunctions representing the radius r(t) (black line) and the shifted pitch z(t) (gray line) of the helicoidal motion in the reconstructed space. Here z(t) and r(t) are shifted in order to start from a common point.

Generally speaking, a 2D dynamics driven by two independent Brownian functions shows an isotropic walk without any kind of geometrical structure. In our case, instead, the composition of r- and z-components gives rise to a random walk along preferred directions (Fig.3), typical of the so-called Lévy flights. In fact, one can recognize in the last figure a motion along straight lines interrupted by jumps. Furthermore, the slope of the straight lines supports the idea of a frequent anti-correlation between r and z local trends. A more evident verification of the confinement of the motion directions is given in Fig.4, where a cone of not allowed directions is clearly visible.

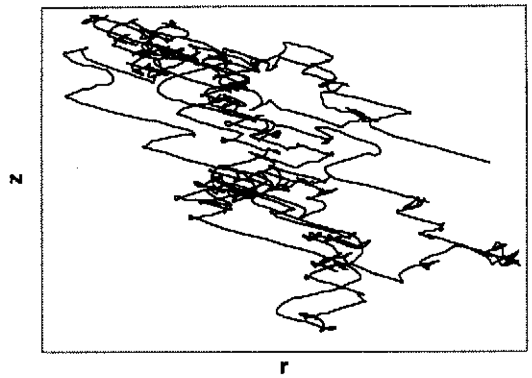


**Fig.3** Coupling of the two Brownian motions in the r-z plane.

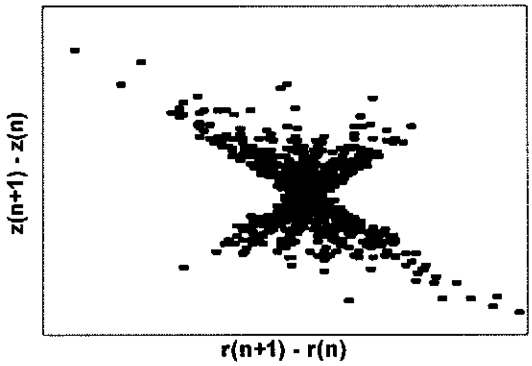


**Fig.4** $\Delta r$-$\Delta z$ diagram of the displacement directions. The cone structure is a clear evidence of the anisotropy of the walk in comparison with a 2D isotropic uncorrelated Brownian motion.

## 3 Meteorological considerations

At this point, a brief meteorological discussion is required in order to give a possible explanation to the dynamics found from the time series analysed. It is well known that monthly mean temperatures at mid-latitudes, such as other meteorological variables, depend, apart from the annual cycle and the geographical position, also from the so-called index cycle, i.e. the alternative occurrence of zonal (high index) and wavy (low index) flow structures in the planetary waves (Wiin-Nielsen and Chen, 1993). For example, a low index circulation, classical of blocking phenomena, is responsible for temperature anomalies due to persistent weather conditions for several days. The sum of zonal flow energy and its deviation (eddy energy) is proved to be constant (Wiin-Nielsen and Chen, 1993). There are, however, exchanges between these two components, as shown by Lorenz (1955). These transfer processes are responsible for the a-periodic component of the monthly mean temperatures. For example, a toy-model for monthly mean temperature evolution can be worked out from the following equation:

$$dT/dt = F(\Omega, \eta(t), \xi(t)), \qquad (2)$$

where $\Omega$ is the annual frequency, $\eta$ and $\xi$ are two correlated functions related to the zonal and eddy flow components, respectively.

The hypothesis suggested by our work is that the two functions above could be thought of as correlated Brownian ones. Furthermore, it should imply that a fingerprint of chaos, whose existence has not been underlined in time-series studies and in low-order non-linear models (Wiin-Nielsen, 1994), is hidden in the red-noise component of the spectra, recurrent in climatic time series, as an interaction of distinct Brownian walks. Lévy flights, for example, are the solution of the equation of motion of a particle embedded in a 2D conservative periodic potential V(x,y), giving rise to Hamiltonian chaos (Klafter and Zumofen, 1994). Therefore, the dynamics found in our reconstructed state space could be justified by assuming the $\theta$-component as the annual orbital variable, and the r and z eigenfunctions of eqs.(1) as the $\eta$ and $\xi$ Brownian functions responsible for the quasi-periodic behaviours of averaged states of the atmosphere, well represented by our original time series.

## 4 Conclusions

In conclusion, the red-noise component of the analysed temperature records has revealed a random motion in the reconstructed space, but it has been pointed out that the randomness found shows defined structures, which can be thought as determined by motion in a periodic potential similar to those typical of Lévy-flight processes.

Obviously, further work has to be done, but this result could give new ideas on the study of the problem of meteo-climatic predictability, because of the analogy with other processes studied in physics and the existence of techniques already available in the literature.

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