Nonlinear Processes in Geophysics (1997) 4: 101–124

Nonlinear Processes in Geophysics

Intermittency, non-Gaussian statistics and fractal scaling of MHD fluctuations in the solar wind

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Received 1 December 1997 - Accepted 21 January 1998

Abstract. This paper gives a review of some recent work on intermittency, non-Gaussain statistics, and fractal scaling of solar wind magnetohydrodynamic turbulence. Model calculations and theories are discussed and put in their context with the in-situ observations of solar wind fluctuations, essentially of the flow velocity and magnetic field. Emphasis is placed more on a comparison of the data with theory than on a complete derivation of the model results, which are treated in a more tutorial fashion. The introduction reminds of some important observations and key aspects of solar wind turbulence. Then structure functions are defined and observational results discussed. The probabiltity density functions provide a direct means to analyse the statistical properties of the fluctuations. Evidence for non-Gaussian statistics is provided. Intermittency and simple scaling models are discussed, which yield algebraic expressions for the scaling exponents of the structure functions. The concept of extended self-similarity is presented and corresponding observational evidence for its existence in the solar wind is provided. Subsequently, an extended structure function model, including the p-model scaling and a scale-dependent cascade, is discussed and compared with selected measurements. The basics of multifractals are presented and applied to solar wind data. The multifractal scaling of the kinetic energy flux as a proxy for the unknown cascading rate is established observationally, and the so-called multifractal spectrum is obtained. Finally, the scaling exponents of the associated correlation functions are derived and analysed. The paper concludes with a discussion of the empirical results and prospects for future research in this field and in solar wind MHD turbulence in general.

1 Introduction

The statistical and scaling properties of solar wind fluctuations have been analysed in considerable detail, and the question of what role intermittency might play in the solar wind MHD turbulence has recently received much attention. This review will address these issues from an observational and theoretical point of view. No claim is made for completeness, and the presentation is not meant to be an exhaustive review of the literature on this subject but is given more in the spirit of an introductory tutorial, in which key observations and fundamental concepts are discussed for the specialist working in the field of space plasma physics and MHD turbulence. Concerning MHD turbulence in the solar wind, comprehensive reviews of observations as well as theories have been given by Marsch (1991a,b), Mangeney et al. (1991), Roberts and Goldstein (1991), Grappin (1994), and lately by Tu and Marsch (1995) and Goldstein et al. (1993). A chapter in Biskamp's (1993) excellent book is devoted to MHD turbulence. A modern treatment of hydrodynamic turbulene with emphasis on direct numerical simulations can be found in the book of Lesieur (1997). The book of Frisch (1995) on turbulence contains an in-depth study of intermittency in hydrodynamics, which is quite relevant also for the following discussion.

Most of the theories assume highly symmetric spatial states of the plasma and invoke isotropy and homogeneity which are rarely, if ever, found in space plasmas such as the solar wind, the upstream region of the Earth's bow shock or the magnetosheath and magnetotail. Mean magnetic fields (Parker spiral in the solar wind) prevail in many space plasmas and break isotropy, or mean flows exist which also define preferred symmetry axes. In many real systems large-scale inhomogeneity renders concepts developed for homogeneous plasmas useless. Traditionally, the quantities of key interest in turbulence theory are two-point correlation functions and structure functions of the vector or scalar fields, which depend on the field differences between two points in space and time and can only be measured properly by at least two probes with variable separation, allowing one to resolve a broad range of spatio-temporal scales of the turbulent fluctuations. Similarly, eddy amplitudes, usually defined by field differences between two locations, require two-point mea-

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surements for their determination which are not available in most space plasma. The fluctuating fields in the solar wind are often large in the moving frame. Their time histories, from a practical point of view, resemble strongly the time evolution of mathematical random varibles. Thus the solar wind may be termed strongly turbulent according to many definitions in use. The radial evolution of observed power spectra clearly indicates nonlinear behaviour and the spectral characteristics indicate intermittency.

In order to address some of the complexities found in real space plasmas we give here a short review of directional vector fluctuations and compressive scalar fluctuations in the solar wind and emphasize some recent findings on turbulent pressure and temperature fluctuations (Marsch and Tu, 1993a,b). The observed small-scale fluctuations appear to be composed of waves and convected pressure-balanced structures, which do not occur in their small-amplitude forms but as short-lived and nonlinearly coupled excitations on a broad range of scales with partial inertial spectral domains. Many examples of fairly clean Alfvénic, but few of pure magnetoacoustic fluctuations, have been found, which to a certain degree resemble the linear modes of MHD theory and, on a statistical basis, still reveal the typical linear eigenvector and polarization relations in their correlation spectra. In addition, temperature fluctuations were detected and found to play a prominent role in the local pressure balance. They often are distinctly anti-correlated with the magnetic field amplitude fluctuations and well correlated with the solar wind speed, a signature which may stem from coronal heating and pressure-equilibration processes between adjacent plasma flow tubes near the Sun. However, radial evolution and coronal boundary conditions of the turbulence are not easily disentangled, because the turbulence is often not fully developed and spatially intermittent. The turbulence properties, such as Alfvénicity and compressibility, are scale dependent and vary in close connection with the interplanetary magnetic field and the stream structure.

In the late eighties and early nineties research on interplanetary MHD waves and fluctuations in the near-Sun wind (Helios mission), out of the ecliptic plane (Ulysses mission) and in the outer heliosphere (Voyager mission) has produced significant advances both from the observational and the theoretical points of view. Our understanding of the nature and dynamics of these fluctuations has progressed significantly because there is agreement now that we dealing with the solar wind magnetofluid as a turbulent nonlinearly evolving system which also shows clear traits of intermittency. The Sun initially generates a broad-band spectrum of fluctuations which strongly develop and are further modified in the radially expanding wind by shear, compression and rarefaction associated with stream interactions and by damping and dissipation of the fluctuation amplitudes owing to large-scale inhomogeneity and microscopic plasma processes. The reviews by Burlaga (1984), and more recently by Roberts and Goldstein (1991) and Tu and Marsch (1995) provide ample empirical testimony to the basic validity of this scenario, and these general ideas are further supported by numerical simulations (Roberts et al., 1991).

However, the observed details of the power and cross-correlation spectra and their variations in space and time are still not understood well and are difficult to model theoretically. In the inner heliosphere single-slope power spectra are the exception and mainly occur in low-speed flows (Denskat and Neubauer, 1982; Bavassano et al., 1982a,b; Tu et al., 1989; Marsch and Tu, 1990a,b). Generally, the spectra are ill-described in terms of a unique spectral index but show considerably varying slopes in the different spectral domains. Grappin et al. (1990, 1991) could show, however, that the fluctuations analysed in terms of Elsässer fields possessed partially the property of self-similarity, yet strict scale invariance did not apply. Therefore the existence of persistent overall power-law spectra in the data had to be excluded. Only the rare spectra of those fluctuations occuring near the heliospheric current sheet typically revealed a Kolmogorov $-5/3$ slope, which was interpreted as an indication of isotropic, developed Gaussian and thus scale-invariant turbulence prevailing there (Tu and Marsch, 1991).

Moreover, since the early investigations by Belcher and Davis (1971) at 1 AU and later by Bavassano et al. (1981) and Denskat and Neubauer (1982) near 0.3 AU it was well known that the magnetic fluctuations, in particular in the Alfvénic domain between some 10^{-4} Hz and 10^{-2} Hz, are anisotropic with minimum variance along the mean field. Recent analyses confirmed anew this property with the combined velocity and magnetic field data from Helios for the inner solar wind (Marsch and Tu, 1990a) and from Voyager (Klein et al., 1991) for the outer heliosphere and from Horbury et al. (1995c) for the polar heliosphere with data from Ulysses. The anisotropies were found to clearly evolve radially whereby the components of the fluctuations in velocity and magnetic field behaved quite differently with respect to the local spiral field direction. Also, Matthaeus et al. (1990) added further evidence for the presence of strong anisotropy by establishing observationally the presence of quasi-two-dimensional, as they called it, and nearly incompressible fluctuations. They studied directly the 2-D correlation function of the fluctuations as a function of distance parallel and perpendicular to the mean field. The observed lack of scale invariance and prevalence of anisotropy strongly objects to the notion that we dealing with isotropic and selfsimilar turbulence.

On the contrary, evidence is increasing after the pioneering work by Burlaga (1991a,b,c; 1992) with the Voyager data, that the turbulence in the solar wind is intermittent. He showed that speed fluctuations bear a multifractal structure in recurrent streams measured between 1 and 6 AU. The large-scale magnetic field fluctuations as observed in the outer heliosphere out to 25 AU were found to have a multifractal appearance (Burlaga, 1991b). Small-scale velocity fluctuations observed by Voyager near 8.5 AU revealed multifractal structure on time scales from 0.8 to 16 hours, a result which is characteristic of intermittency and suggests that solar wind turbulence may consist of a mixture of sheets and space-filling eddies of various sizes.

2 Structure functions

2.1 Definitions of structure functions

Conventional spectral analysis based on auto- and cross-correlation functions is restricted in so far as only second-order moments or two-point correlations are considered. This fully characterizes the turbulence if it is Gaussian and thus if scale-invariance and self-similarity applies. A broad modern introduction into this subject for hydrodynamic (HD) turbulence can by found in the books of Lesicur (1997) and Frisch (1995) and for MHD turbulence of Biskamp (1993). In the case of non-Gaussian statistics, however, higher-order correlations are required to determine fully the features of the fluctuations. A basic tool for theory as well as data analysis is given by the pth order structure function, which e.g. for the plasma flow speed reads

$$S_V^p(\tau) = <\mid V(t+\tau) - V(t) \mid^p> \ . \qquad (1)$$

In data analysis p may range from 1 to at most 10 or so. Higher orders are problematic insofar as long time series are required to guarantee the quantitative reliability of the derived structure function (Horbury et al., 1997a). The flow speed difference characterizes the speed of an "eddy" of the size l, defined as $l = \tau V_{SW}$, by using Taylors hypothesis of fluctuations being frozen in the supersonic/super-Alfvénic solar wind. The fundamental question is whether, and if yes, how the structure function scales with the spatial distance l or the time lag τ. For turbulence with Gaussian statistics a universal scaling law, first established by Kolmogorov (1941), namely

$$S_V^p(\tau) \sim \tau^{s(p)} \ , \qquad (2)$$

should hold with a linear relation $s(p) = p/3$ (for a derivation see e.g. the review by Rose and Sulem, 1978, or Frisch, 1995), implying the Kolmogorov $-5/3$ power spectrum. In the case of spatial intermittency each pth order structure function may have its own index different from $p/3$, and thus it will contain new information about the "local" self-similarity of the fluctuations near the scale under consideration. For example, if the turbulent eddies are not space-filling but occupy at all scales only a fraction of the volume element given by the factor $2^{(D_F-3)}$, then $s(p) = 3 - D_F + p(D_F-2)/3$ with the fractal dimension D_F (Frisch et al., 1978). If observations indicate a departure from the simple law $s = p/3$ then one may conclude that intermittency prevails. The departure from Kolmogorov scaling becomes evident from higher-order correlations, but it is less clear from the two-point correlation which is equivalent to the power spectrum. Fig. 1 shows an example of the pth-order solar wind speed structure function plotted versus time lag τ (top) and the corresponding scaling exponent $s(p)$ plotted versus the moment degree p (bottom) obtained from a Voyager data analysis by Burlaga and Klein (1986). The departure of the measured points from the Kolmogorov (K41) scaling is obvious and clearly indicates intermittency.

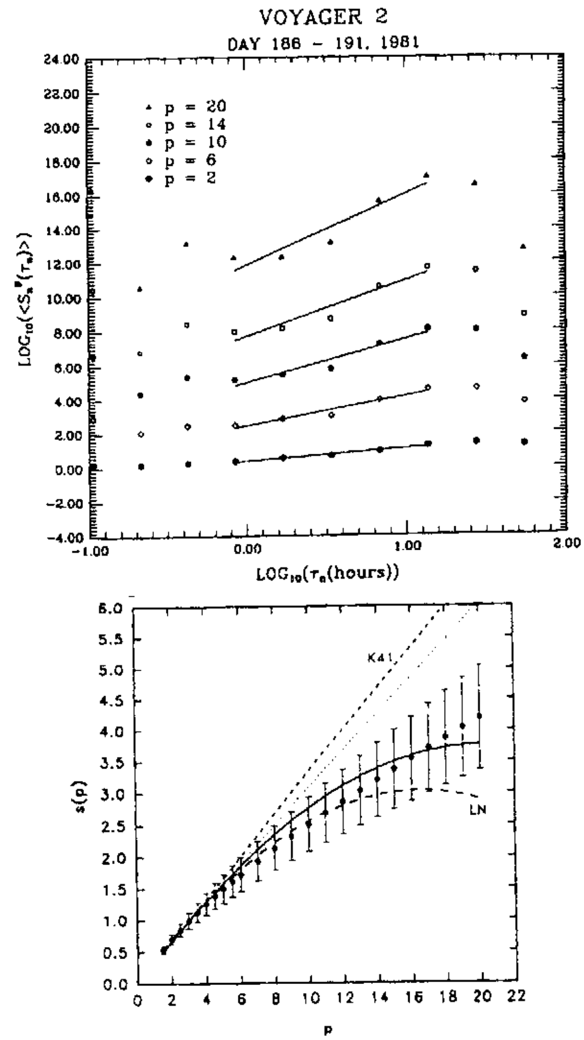

Fig. 1. Structure functions (top) of the solar wind speed, as measured by the Voyager 2 plasma instrument from days 186 to 191 in 1981 at 8.5 AU, versus time scale τ for five values of the order parameter p. Note the straight lines indicating a linear scaling in the interval from 0.85 to 13.6 hours. The corresponding scaling exponent $s(p)$ is shown (bottom) versus p and compared with the Kolmogorov $p/3$ prediction (dashed line). The departure of the observations (dots with error bars) from this line clearly shows that interplanetary turbulence is intermittent (adapted from Burlaga, 1991a).

In a magnetofluid described by the MHD equations the basic fields are the flow and Alfvén velocity fields. Therefore, we shall consider the following structure functions:

$$S_{\mathbf{V}}^p(\tau) = <\mid \mathbf{V}(t+\tau) - \mathbf{V}(t) \mid^p> \ , \qquad (3)$$

$$S_{\mathbf{V}_A}^p(\tau) = <\mid \mathbf{V}_A(t+\tau) - \mathbf{V}_A(t) \mid^p> \ . \qquad (4)$$

These pth order structure functions are represented by the pth moments of the velocity differences $\Delta\mathbf{V}$ and $\Delta\mathbf{V}_A$, where the exponent p is a real number. Departures from Gaussian behaviour are present in the intermittent turbulent flow, if $S^p(\tau)$ scales like $\tau^{s(p)}$, where $s(p)$ generally may be a nonlinear function of p. An observation made over the time span

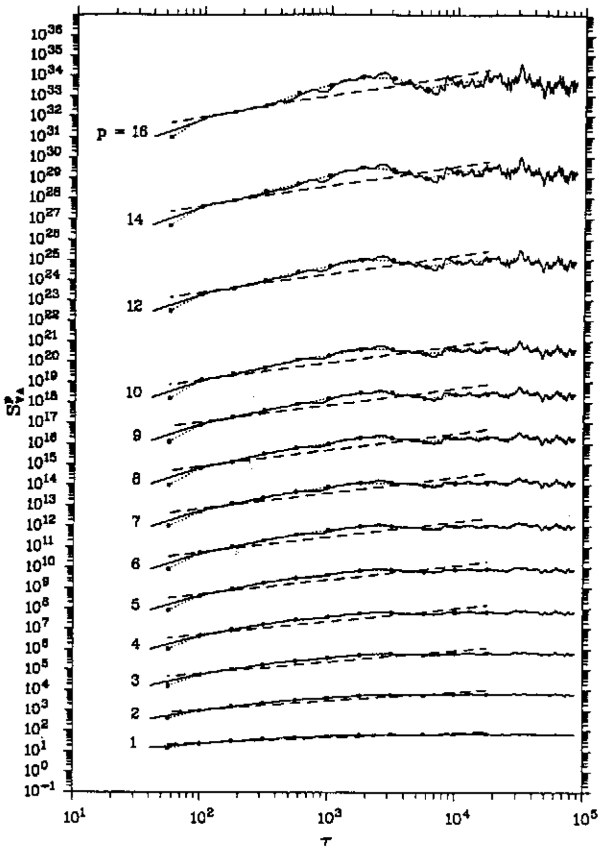

Fig. 2. Structure functions (plotted on top of each other) of the Alfvén velocity plotted versus time lag τ in seconds for various values of the order p. Note that a simple power-law scaling (dashed lines) applies well to the observational data in the range form seconds to hours. The actual slopes of the curves are much flatter than the $p/3$ value, thus indicating clearly intermittency of the magnetic field fluctuations. The data are from Helios 2 and were obtained near 0.87 AU in 1976 (adapted from Marsch and Liu, 1993).

τ picks out eddies of spatial dimension $l = V_{SW}\tau$ which are convected past the observer in the spacecraft frame. Thereby one samples or probes the spatial structure of the magnetofluid in terms of time profiles of the fluctuating wind velocity and the fluctuating magnetic field vector or Alfvén velocity. These observations provide a more complete characterization of the plasma than given by power spectra. The structure function analysis technique was pioneered for experimental measurements of fluid intermittency by Anselmet et al. (1984) and for solar wind intermittency by Burlaga (1993) and Marsch and Liu (1993), from whom we have adapted the structure functions of the Alfvén velocity, which are shown in Fig. 2 and derived from Helios data.

The basic assumption underlying many classical fluid turbulence theories and closure approximations for the two-point correlation functions involved (see e.g. the book by McComb, 1990, for a modern account of fluid turbulence and by Biskamp, 1993, for a recent description of MHD or magnetofluid turbulence) is that the probability functions of the set of random fluid variables are essentially Gaussians or normal distributions. This implies that quantities like the j-component of the velocity difference $\Delta V_j(l) = V_j(x + l) - V_j(x)$ for a given spatial scale l, or $\Delta V_j(\tau) = V_j(t + \tau) - V_j(t)$ for a time lag τ, are approximately normally distributed. While this is true for sufficiently large scales l or τ (e.g. Anselmet et al., 1984, for observations of fluid turbulence, and Vincent and Meneguzzi, 1991, and Biskamp et al., 1990, for numerical simulations), it often is not the case at smaller scales, where the distributions may differ significantly from Gaussian and can be much narrower. This different occurrence rate of fluctuation amplitudes $\Delta V_j(l)$ at small scale l is interpreted as evidence for spatial intermittency and believed to indicate that the behaviour of $\Delta V_j(l)$ is locally determined by coherent rather than stochastic dynamics.

2.2 Normalized structure functions

In the previous section we have discussed intermittency in terms of the non-Gaussian scaling properties of the structure functions for various types of solar wind flow. A more quantitative measurement of intermittency is provided by the scaling properties of the dimensionless structure function (see e.g. Rose and Sulem, 1978):

$$A^p(\tau) = S^p(\tau)/(S^2(\tau))^{p/2} . \quad (5)$$

Clearly, by definition $A^2(\tau)$ is unity. It remains constant for all τ in case of scale-invariant turbulence, but otherwise it may reveal a scaling like

$$A^p(\tau) \sim \tau^{a(p)} , \quad (6)$$

with an exponent $a(p)$ which is generally negative and essentially given by the value $a(p) = s(p) - s(2)p/2$. This negative exponent thus indicates that the smaller scales are increasingly intermittent. This behaviour is illustrated in Fig. 3, where the x-component is pointing radially away from the Sun, z is out of the ecliptic plane and the y-axis completes a right-handed coordinate system. The figure shows the exponent $a(p)$ versus p for the three components of the flow velocity as obtained from Helios plasma data. Note that the dimensionless structure function increases at fixed τ with increasing power index p. Therefore, the higher-order moments, at small scales in particular, exceed substantially their Gaussian values and thus indicate enhanced tails in the probability distribution of the velocity differences as compared to a Gaussian distribution.

2.3 The length function

The length function technique provides a simple way to determine spectral exponents of a power spectrum of a random field $x(t)$, which has been first applied to solar wind data by Burlaga and Klein (1986) and Ruzmaikin et al. (1993). The length of the "curve" of any time series $x(t)$ is defined in the

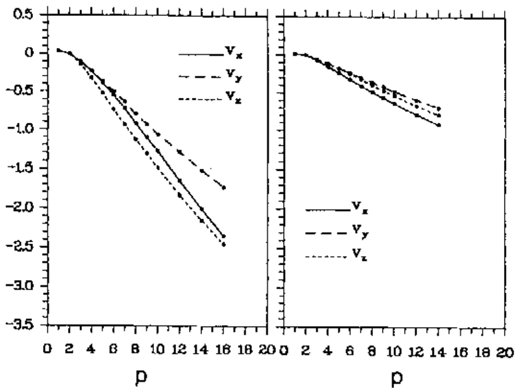

Fig. 3. Scaling exponent $a(p)$ of the dimensionless structure function of the wind flow-velocity components plotted versus the order parameter p for fast (right) and slow (left) solar wind as measured near 0.3 AU by Helios 2 in 1976. Note the distinct differences between fast and slow wind exponents and the spread of the curves showing the anisotropy of the fluctuations (adapted from Marsch and Liu, 1993).

same way as the first order structure function namely as

$$L(\tau) = \sum_{j=1}^{n} \mid x(t_j + \tau) - x(t_j) \mid , \tag{7}$$

where the sum is taken over a large number $n = T/\tau$ of discrete points, and T is the entire time period considered, so that $L(\tau) \propto \mid \Delta x \mid \tau^{-1}$. One then determines the scaling exponent $s(L)$ according to the relation

$$L(\tau) \sim \tau^{s(L)} . \tag{8}$$

Theoretically, the spectral exponent α of the power spectrum of $x(t)$ is linearly related to the exponent $s(2)$ of the second order structure function, that is to say

$$S_x^2(\tau) = <\mid \Delta x \mid^2> = <\mid x(t+\tau) - x(t) \mid^2> , \tag{9}$$

and also to $s(L)$. After Berry (1979) and Monin and Yaglom (1975) the simple expression holds:

$$\alpha = 3 - 2s(L) = 1 + s(2) . \tag{10}$$

In practice, both methods are expected to give slightly different results because of the different approximations made in the estimates of usual power spectra and in the above formulae. They have the advantage that the data are processed in the time domain and the numerical effort remains simple.

2.4 Further results from data analysis

In their analysis of Ulysses magnetic field data, Horbury et al. (1995a,b) investigated the structure function of the magnetic field as obtained by measurements in the range from 4.0 to 1.7 AU, covering a heliographic latitude range from 40° to 80°. They estimated the power spectral index through the index of the second-order structure function and found that the spectral index for a time scale between 10 min and 40 min decreases with decreasing heliocentric distance. They concluded that this change is due to a radial evolution of the fluctuations. Horbury et al. (1996) also calculated the structure functions up to the 15th-order moment with Ulysses data obtained for distances from 4.0 to 2.9 AU and for a range of 44° to 69° in heliolatitude. They presented the gradients of the structure functions in dependence on p and $s(p)$, for both the small scales (80 s - 320 s) and the large scales (1.5 h - 5.5 h). For the small scales they compared their results with the β-model (discussed in Section 4.1) and the Ruzmaikin et al. (1993) model.

These models do not fit the observations well but somehow come close to the observational results. This suggests that the small-scale turbulent fluctuations in the solar wind are indeed intermittent in compliance with the model ideas. For the larger scales, the observed index $s(p)$ is too small to compare favourably with the two abovementioned models. Since the curves of $s(p)$, for the radial component as well as for the tangential (along the cross product of the Sun's rotation vector and the radial vector) component, are more or less linear, Ruzmaikin et al. (1993) concluded that the large-scale fluctuations are not intermittent, and that the spectral slope is really near -1. They therefore suggested that the large-scale fluctuations observed by Ulysses may not be caused by turbulence but instead by sampling effects stemming from the spacecraft tracking across structured flows, which originated from the polar coronal hole while the Sun was rotating.

Tu et al. (1996) pointed out that the spectral index, if directly calculated from power spectra based on the FFT technique, and the index inferred from the structure function may not have the same value, especially for a spectrum with an exponent of -1. When the index of the power spectrum is near or less than unity, both indices, $s(p)$ and $a(p)$, are nearly zero. In this case the structure functions cannot represent the effect of intermittency adequately.

Horbury et al. (1995 a,b,c; 1996; 1997a,b) used the method of structure functions to analyse the Ulysses magnetic field data and explored the properties and the nature of the magnetic fluctuations in the solar polar regions between 1.5 AU to 4 AU. Horbury et al. (1995a) also used the length function to estimate the power index of the fluctuations. Horbury et al. (1997a) found that fluctuations on time scales (as measured on the spacecraft) of about 10 s to 100 s were well described by the p-model of Meneveau and Sreenivasan (1987a,b), which is discussed in Section 4.2. By considering the scaling of the third- and fourth-order structure functions, they showed that the turbulent cascade was probably Kolmogorov-like. The magnetic field data used in this study were taken in high-speed polar flows at around 63° and 3.1 AU from the Sun. In their work they introduced and applied a semi-empirical error criterion to their data which was used to discard unreliable results.

Ruzmaikin et al. (1995) used structure functions to study the Ulysses magnetic field data taken within the polar solar wind flow. They analysed fluctuations at (spacecraft) scales from 1 to 32 minutes and found inertial-range turbulent behaviour. They showed that the turbulence was intermittent, and developed a model of Alfvénic turbulence to explain their results. For the β-model and Kraichnan scaling, they found that $s(2) = 1/2 + 1/2\mu$, where the filling factor $\beta = 2^{-\mu}$. Since the spectral index may be calculated as $\alpha = s(2) + 1$, one has $3/2 = \alpha - 1/2\mu$. They calculated, with the use of the observational data, the reduced spectral index, given by $\alpha' = \alpha - 1/2\mu$, and found that it is near $3/2$, consistent with their model assumption. Based on this result, they tentatively concluded that the analysis agrees with the predictions of the Kraichnan (1965) model of random-phased Alfvén-wave turbulence.

However, with a scaling for fluid-like turbulence one finds that the corrected exponents, α - $1/3\mu$, should be equal to 5/3. Tu et al. (1996) tested this relation with the same data used and presented by Ruzmaikin et al. (1995) and found it to be also approximately true. In conclusion, one cannot decide with the existing observational data which scaling relation, the MHD-like or HD-like, describes appropriately intermittency in solar wind turbulence.

Burlaga (1991a) analyzed the velocity structure function up to the 20th-order moment using the Voyager 2 data obtained at 8.5 AU. He showed that a scaling relation $S^p \sim \tau^{s(p)}$ holds approximately in the τ-range between 0.85 h and 13.6 h. He also found that the curve of the resulting scaling exponent $s(p)$ plotted versus the exponent p was very similar to the curve obtained for the scaling exponent as measured in duct flows of ordinary fluid turbulence and in turbulent jets, although the scales of these two kinds of fluctuations differed by a factor of 10^{11}.

3 Probability density functions

Burlaga and Ogilvie (1970) first analysed the probability density function (PDF) of velocity fluctuations in the solar wind and reported simple exponential distributions of solar wind speed differences. Also, Whang (1977) analysed the probability distribution functions of microscale magnetic fluctuations and found them to be nearly tri-Maxwellian in the three components for a comparatively quiet background field. Recently, Burlaga (1993) and Marsch and Tu (1994) reported non-Gaussian distributions of various averages of field differences and provided evidence for intermittency of the magnetic field and velocity fluctuations. This identification was based on the analysis of the scaling properties of the pth order structure function. Generally, one expects a simple power-law scaling behaviour of the structure function in dependence on the scale variables l or τ. Usually, the scaling exponent $s(p)$ is a low-order polynomial in p in the case of intermittency, associated geometrically with a uniform fractal (s linear in p) or a multifractal (nonlinear in p) scaling; see e.g. the review by Paladin and Vulpiani (1987).

Instead of using the somewhat more abstract structure function of the field differences, we shall now investigate directly the probability distributions of any random-field difference $\Delta x(\tau) = x(t+\tau) - x(t)$ and analyse their shapes and char-

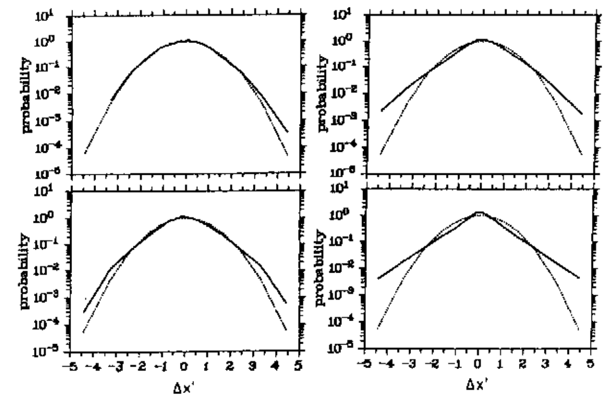

Fig. 4. Probability density functions of the normalized x-component of the wind velocity for four different values of the time lag, τ = 22.5 hours, 2.25 hours, 13.5 minutes, and 81 seconds (frames: top left, bottom left, top right, and bottom right). The plots are double logarithmic, and thus a Gaussian distribution appears as a downward-open parabola. Note the gradual change in the PDFs while the time lag decreases. At the smallest scale the distribution becomes a simple exponential one, signifying intermittency of the fluctuations, which were observed by Helios 2 near 0.3 AU in 1976 in a high-speed solar wind stream (adapted from Marsch and Tu, 1994).

acteristic parameters. Of course, it is well known that a PDF is fully determined if all its moments are given. This is only true in the strict mathematical sense, but not in the practical data analysis, where the moments are represented by ensemble averages which are in turn calculated by averaging over finite time series of data. Furthermore, one can usually only calculate the first few moments, which may not represent the full shape of the PDF adequately. Therefore, it seems interesting to analyze directly the PDF, a quantity which has in the past received much less attention than the power spectra or the structure functions.

To derive the probability distributions from the corresponding time series, one may first calculate the averages and standard deviations $\sigma(\Delta x(\tau))$ of the time series of $\Delta x(\tau)$. Dividing $\Delta x(\tau)$ by σ, a new random time series $\Delta x(\tau)'$ is created, with an r.m.s. value of unity. In such a way all time series are normalized in order to allow for a comparison of their statistical properties. Finally the probability distribution $P(\Delta x')$ is established. Here $P(\Delta x')d(\Delta x')$ is the probability to find that the value for the normalized difference of the random variable $\Delta x(\tau)$ at a given time lag τ ranges between $\Delta x'$ and $\Delta x' + d(\Delta x')$. For a Gaussian we have

$$P(\Delta x') = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\Delta x' - \overline{\Delta x'}\right)^2\right) , \qquad (11)$$

where the mean value is denoted by $\overline{\Delta x'}$. The parameter characterizing the respective distribution is the time scale or time lag τ of the field difference $\Delta x(\tau)'$. The PDFs are best plotted logarithmically versus $\Delta x'$. As can been seen in Fig. 4, the distributions of the x-component of velocity differences evolve systematically with varying time lag τ, and depend clearly on the stream structure of the solar wind. In

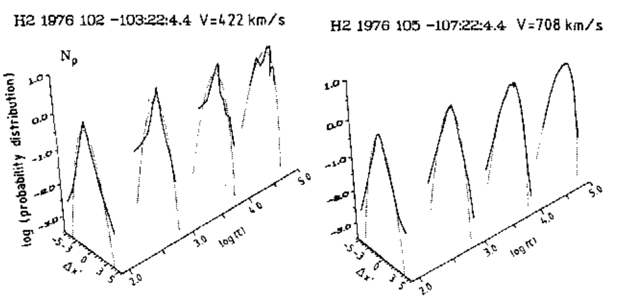

Fig. 5. Probability distributions of the normalized density fluctuations in the solar wind for four different values of the time lag τ. Data are for slow wind (left) and fast wind (right) and were obtained near 0.3 AU by Helios 2 in 1976. Note the spiky PDFs at small scales, clearly indicating intermittency, and the rugged shape at meso scales in slow wind, which reflects non-stationarity of the slow streams and their filamentary nature as compared to the smoother fast streams, where Gaussian statistics applies to the meso scales (adapted from Marsch and Tu, 1994).

the subfigures the equivalent Gaussian distribution is always plotted for comparison. The standard deviation σ is related to the fluctuation amplitude and power spectrum.

Feynman and Ruzmaikin (1994) found that the 1-hour averaged magnetic magnitudes are not distributed normally or log-normally. The 1-hour averaged B_z component was found to have a nonzero kurtosis. They further isolated field variations in the $1/f$ frequency region of the spectrum and showed that for these variations the distributions of the magnitudes also have nonzero skewness and kurtosis, while the magnitudes are not distributed log-normally, and the distributions of the components have a non-zero kurtosis. Thus higher-order spectra are needed for a full characterization of the PDFs of the fluctuations. This is also clearly illustrated in the structure functions of the plasma density, which is shown in Fig. 5. Apparently, the PDF are rather spiky at the small scales and they also have a rugged shape at large scales, indicating that data were sampled from different transient solar wind flow tubes, in which case the ensemble is ill defined and the stationarity assumption breaks down. More data of this kind can be found in Marsch and Tu (1994).

4 Intermittency scalings and models

4.1 The simple β-model

It has been known for a long time that turbulent fluctuations are intermittent at small scales (e.g. Batchelor, 1958). In recent years intermittency in turbulence has been the subject of intensive reasearch, in particular intermittency of the rate of energy dissipation ϵ, which is discussed extensively in Section 7. For a review of this subject see for example the work of Meneveau and Sreenivasan (1991). Here we shall first discuss a simple model which is instructive but has not stood the test of experiments in fluid dynamics. A more complete discussion of other models can be found by the interested reader in the article by Borgas (1992), who gives a critical evaluation and comparison of intermittency models in fluid turbulence.

In the so-called β-model of Frisch et al. (1978), the cascade is assumed to be intermittent in space. Most modern theories consider that at each step of the cascade the offsprings from a decaying eddy receive uneven randomly distributed parts of the mother-eddy energy. In the β-model the offsprings are assumed to be less and less space filling and at each step to occupy only the fraction β of the respective volume of the previous eddy. This fraction is given by $\beta = N/2^3 \leq 1$ where $N = 2^{D_F}$ is the offspring number. At the nth step in the hierarchical chain of decaying eddies one has

$$\beta_n = \beta^n = (2^{D_F-3})^n = (2^n)^{D_F-3} = (L/l_n)^{D_F-3}, \quad (12)$$

where the large scale is denoted by L, and the scale of the nth offspring is given by $l_n = L/2^n$. As usual the simple inverse scaling relation $l \sim k^{-1}$ is assumed. The energy transfer rate at that scale is given by

$$\epsilon_l \sim \beta_l \delta V_l^3 / l \, . \quad (13)$$

The energy spectrum in Fourier space is related to the eddy amplitude by

$$E(k)k \sim \beta_l \delta V_l^2 \, . \quad (14)$$

Under the assumption of scale invariance, i.e. if $\epsilon_\ell = \epsilon =$ const, this relation implies a modified spectral law with an intermittency correction to the Kolmogorov spectral exponent:

$$E(k) \sim \epsilon^{2/3} k^{-5/3} k^{-(3-D_F)/3} \, . \quad (15)$$

Generally, intermittency will therefore lead to steeper energy spectra. Here D_F stands for the fractal dimension. An experimental method to determine the effects of small-scale intermittency is to analyse the scaling properties of the normalized structure function. For this purpose we go again through the same kind of arguments than in the previous paragraphs. Using Taylor's hypothesis, that $l = V_{SW}\tau$, allows one to go from time to space variables. Then the structure function scales like

$$S^p(l) \sim \beta_l \delta V_l^p \, , \quad (16)$$

within the β-model. Solving for the velocity amplitude gives

$$\delta V_l \sim \epsilon^{1/3} l^{1/3} \beta_l^{-1/3} \, . \quad (17)$$

The filling factor β can be expressed as $\beta_l \sim l^{3-D_F}$. Inserting the velocity amplitude results in the relation $S^p(l) \sim \epsilon^{p/3} l^{p/3} \beta_l^{(1-p/3)}$. From this scaling relation we can easily read off the exponent. We find $s(p) = p/3 + (3 - D_F)(1 - p/3)$, and finally we derive from this the exponent $a(p) = (3 - D_F)(1 - p/2)$. Apparently, this exponent does not vanish if the fractal dimension obeys $D_F < 3$. Consequently, if $a(p) \neq 0$ then the scaling of $A^p(\tau)$ indicates intermittency and allows one to determine the fractal dimension D_F. If the eddies are space-filling then $D_F = 3$, and $a(p)$ is identically zero for any p. With this derivation we conclude the section on the simple β-model.

4.2 The p-model

Let us now describe the p-model (Meneveau and Sreenivasan, 1987a,b), in which one considers the cascade process to be represented by a two-scale Cantor set with equal partition intervals and assumes that the space-averaged cascade rate is scale-independent. The largest turbulent eddy may have the scale L and the volumetric energy cascade flux, or in our simple case the specific flux per unit length, ε_L. In the first step of the cascade, the flux density ε_L transfers to two smaller eddies with the same scale length, $l_1 = L/2$, but with different flux portions. The cascade rate of these two segments are $2P_1\varepsilon_L$ and $2P_2\varepsilon_L$, respectively. The P_1 and P_2 are distributed randomly, where $P_1 + P_2 = 1$. This process is repeated for each remaining segment. After n steps, each segment has a length $l_n = L/2^n$. The cascade energy flux per unit length of the ith segment, out of the number $N_n = 2^n = L/l_n$ of the many segments obtained in the nth step, is $\varepsilon_i = \epsilon_n f_i$. Here f_i is the fraction of the flux density received by the ith segment from the total flux density, with $i = 1, 2, ...N_n$. For example, after the second step we have $f_1 = P_1^2$, $f_2 = P_1P_2$, $f_3 = P_2P_1$, and $f_4 = P_2^2$. By definition we have $\sum_{i=1}^{N_n} f_i = 1$. Since the total energy flux in the cascade is assumed to be conserved, we obtain the sum rule

$$\sum_{i=1}^{N_n} \varepsilon_i l_n = \varepsilon_L L \tag{18}$$

Fig. 6 shows artificial numerical results calculated in this way at the 10th step for different values of P_1. Here we show the case $\varepsilon_n l_n = \varepsilon_L L = const$, and we plot the cascade rate or energy flux per unit length, ε_i, for each segment. The integrals over the curves in any of the sub-figures equal unity. This means that the averaged cascade rates are the same in all the sub-figures, though referring to different values of P_1. However, we can see that the spatial distributions of the energy flux density are intermittent and differ substantially. By increasing the value of P_1, the intermittency apparently increases. The total length of the figure is L, which is chosen here arbitrarily as $1024 = 2^{10}$.

Let us now turn to the description of higher-order moments of the fractional cascade rate. The segment average of the cascade rate at the nth step may be defined as

$$\epsilon_n = \sum_{i=1}^{N_n} \varepsilon_i = \epsilon_L L/l_n \tag{19}$$

In the p-model, the fragmentation parameters obey the relation (since $N_n = 2^n$)

$$\sum_{i=1}^{N_n} f_i = (P_1 + P_2)^n = 1 \tag{20}$$

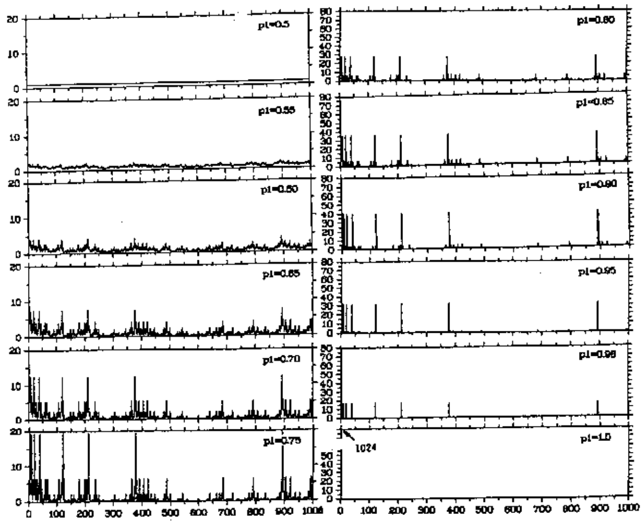

Fig. 6. The distribution of the cascade rate ε as calculated from the p-model at the 10th step of bifurcation of the fractal cascade, with P_1 ranging from 0.5 to 1. This figure illustrates that intermittency increases strongly with growing P_1 (adapted from Tu et al., 1996).

By making use of the binomial expansion and the exponentiation rules we also obtain the relation

$$\sum_{i=1}^{N_n} f_i^q = (P_1^q + P_2^q)^n \tag{21}$$

Therefore, the segment average of the qth power of the cascade function may be written similarly as

$$\sum_{i=1}^{N_n} \varepsilon_i^q = \sum_{i=1}^{N_n} f_i^q \epsilon_n^q \tag{22}$$

By consideration of equations (21) and (22) we find

$$\sum_{i=1}^{N_n} \varepsilon_i^q = \epsilon_n^q \, (P_1^q + P_2^q)^n \tag{23}$$

We now assume that the cascade rate ε_i is related with the velocity fluctuation Δv_i in the ith segment by the following relation

$$\varepsilon_i \sim \frac{\Delta v_i^3}{l_n} \tag{24}$$

It should be pointed out that this assumption cannot be applied to the case where the apparent index of the power spectrum, α', is near or less than one. The second-order structure function may be evaluated by an integration of the power spectrum over the corresponding frequency range, of which the high-frequency limit is determined by the time resolution of the data, while the low-frequency limit is determined by the time lag τ of the velocity difference. In the case of a flat spectrum, the velocity amplitude Δv_i is determined by the high-frequency fluctuations and has nothing to do with τ. Then both $s(p)$ and $a(p)$ are about zero, and hence they do not contain any information on intermittency.

At the nth step of the cascade, the pth order structure function may be evaluated by means of (24) as

$$<|\Delta v|^p>_n = \frac{1}{N_n}\sum_{i=1}^{N_n} |\Delta v_i|^p = \sum_{i=1}^{N_n} \varepsilon_i^{p/3} l_n^{p/3} l_n/L \quad (25)$$

Considering equation (23) and the identity

$$(P_1^q + P_2^q)^n = \left(\frac{l_n}{L}\right)^{-log_2(P_1^q+P_2^q)} \quad (26)$$

we finally get the result for the structure function in form of the scaling law

$$<|\Delta v|^p>_n \sim (\overline{\epsilon_n}L)^{p/3}\left(\frac{l_n}{L}\right)^{1-log_2[P_1^{p/3}+P_2^{p/3}]} \quad (27)$$

Here we have introduced the space-averaged cascade rate $\overline{\epsilon_n} = \epsilon_n l_n/L$ at the nth step. For fully developed turbulence the averaged cascade rate $\overline{\epsilon_n}$ is a constant and equals ϵ_L, and thus does not depend on the scale length l_n. The scaling exponent may be read off from (27), and thus it can be written as

$$s(p) = 1 - log_2[P_1^{p/3} + (1-P_1)^{p/3}] \quad (28)$$

This is the result presented earlier by Meneveau and Sreenivasan (1987a,b, 1991) and Carbone (1994). For the case where there is no intermittency, i.e. $P_1 = 0.5$, we find that $s(p) = p/3$.

4.3 An MHD-intermittency model

The previous two models were derived for HD turbulence. Carbone (1993) and Grauer et al. (1994) presented a phenomenological model for intermittency in magnetofluids, which was a further development of the model of She and Leveque (1994) adapted to the situation of MHD. It desribes intermittency corrections for the scaling properties of magnetohydrodynamic fluctuations and has the strength that it does not rely on any free or adjustable paramters. The authors use Elsässer (1950) variables,

$$\mathbf{Z}^{\pm} = \mathbf{V} \pm \mathbf{V}_A \quad (29)$$

of which extensive use has been made of in the nineties by various solar wind researchers to analyse Alfvénic turbulence in particular (see the review of Tu and Marsch, 1995), because the Elsässer variables are the natural variables for this type of incompressive fluctuations. But they are also appropiate for the description of magnetosonic waves (Marsch and Mangeney, 1987). Carbone (1993) and Grauer et al. (1994) start from the incompressible MHD equations and define the structure functions

$$S^p_{Z^\pm}(l) = <|\mathbf{Z}^{\pm}(\mathbf{x}+\mathbf{l}) - \mathbf{Z}^{\pm}(\mathbf{x})|^p> \quad (30)$$

which are expected to scale like $l^{\zeta(p)}$ in the inertial range. Marsch and Liu (1993) have derived these structure functions from Helios data and their exponents for a variety of solar wind conditions and found $\zeta(p)$ to be a nonlinear function of the order parameter p. Carbone (1993) was the first to introduce an MHD multifractal-cascade model to derive algebraic expressions for $\zeta(p)$. Grauer and Marliani (1995) presented recently estimates for the structure functions in two-dimensional MHD flows in numerical simulations. Starting point of the Grauer et al. (1994) model is the usual estimate of the energy transfer rate, or equivalently the energy dissipation rate, ε_l, which is given, following Kraichnan (1965) and Dobrowolny et al. (1980) by the simple formula:

$$\varepsilon_l \sim Z_l^2/t_l \quad (31)$$

Here we neglected for the sake of simplicity the $\pm$ sign in the Elsässer variables. The energy transfer time scale is given by the scaling relation $t_l \sim lV_{A0}/Z_l^2$, which inserted above yields

$$S^p(l) \sim (\varepsilon_l V_{A0} l)^{p/4} \quad (32)$$

In the Kraichnan (1965) and Iroshnikov (1964) theory the variations of the cascading rate ε_l with scale are neglected, and thus we can replace it by its scale-independent average $\overline{\varepsilon}$. Hence the classical value of the exponent, $\zeta(p) = p/4$, follows immediately. Recently, Politano and Pouquet (1997) showed that the scaling of the 3rd-order structure function in magnetofluids goes like $S^3(l) \sim l$, a result which can be derived from the MHD analogue to the von Kármán-Howarth equations. In the following we discuss always the K-I-scaling model in parallel with the classical K-scaling, since it plays a prominent role in the current intermittency literature.

The intermittency corrections for $\zeta(p)$ can be derived readily, if we allow for fluctuations of ε_l at small scales, yielding a scaling law for its pth moment according to

$$<\varepsilon_l^p> \sim \overline{\varepsilon}^p (l/L)^{\tau_p} \quad (33)$$

with the large scale L, corresponding to the extent of the system or the energy-containing eddies of the considered magnetofluid, and the intermittency exponent τ_p. For this an expression can be derived from the phenomenological theory of She and Leveque (1994), who considered the ratios of $<\varepsilon_l^p>$ for different p values and constructed a recursion relation for them. The scaling exponent of the structure function then reads

$$\zeta(p) = p/4 + \tau_p \quad (34)$$

where the correction has the obvious property that $\tau_1 = 0$, following from its definition, and that $\tau_0 = 0$, which follows from the assumption that the energy transfer is not supported by structures of a singular measure in the limit of zero dissipation. It can be shown that, for $p \to \infty$, one has $\tau = -p/2 + d - D_F$, where d is the dimension of the magnetofluid, and D_F is the dimension of the most singular structures on which the dissipation takes place, which are currents sheets in MHD (see e.g. the numerical simulations of Grauer and Marliani, 1996). The final result derived by Grauer et al. (1994) is

$$\tau_p = 1 - p/2 - 2^{-p} \quad (35)$$

which has the right asymptotic behaviour and no free paramaters unlike the p-model to be discussed before, and which seems to fit the data well as shown in Fig. 1 of Grauer et al. (1994). The corresponding exponent of the structure function reads

$$\zeta(p) = 1 + p/8 - 2^{-p/4} \quad (36)$$

which gives a small correction to the Kraichnan-Iroshnikov $-3/2$ spectral index for $p = 2$. One has $\alpha = 1 + \zeta(2)$ and obtains $\alpha = 1.543$ for the index of the energy spectrum.

5 Extended self-similarity

5.1 Extended self-similar scaling

Carbone (1993) extended the p-model, which was developed by Menevau and Sreenivasan (1987a,b, 1991) for developed fluid turbulence and which involves a fractal cascade characterized by the bifurcation ratio P_1, to the case of developed MHD turbulence in a magnetofluid. He first derived the equation

$$s(p) = 1 - log_2[P_1^{p/4} + (1 - P_1)^{p/4}] \quad (37)$$

for the scaling exponent. This was later shown to be a special case of the scaling exponent of the extended structure function model established by Tu et al. (1996), if one choses 3/2 for their cascading parameter α (see Section 6.3, equation (59), for more details). Carbone (1994), using the expression for the turbulent energy flux in the solar wind as derived by Tu et al. (1984, 1989) and by Tu (1988), found that from a simple fragmentation process the scaling exponent of the structure function can be written in the form

$$s(p) = 1 - log_2[P_1^{p/2\beta} + (1 - P_1)^{p/2\beta}] \quad (38)$$

The parameter β describes the scaling of the energy transfer rate with the energy spectrum and depends on the nonlinear time scale used. From the transfer function given by Tu et al. (1984), it is found that $\beta = 2$. This value then leads to the form of $s(p)$ as derived by Carbone (1994) for the MHD-extension of the p-model. This exponent is relevant in the presence of a strong magnetic field, in which the decorrelation of the fluctuations can occur on the time scale set by the transit time of an Alfvén wave through a correlation length or wave length. From the transfer function given by Tu (1988), one obtains $\beta = 3/2$. The resulting $s(p)$ is the same as in the p-model, which was originally derived not for MHD but for HD turbulence. By a comparison of the two models with the observational results obtained by Burlaga (1991a), Carbone (1994) found that the HD fluid p-model is relevant for the solar wind as well. Why then can MHD turbulence be described by a model originally devised for fluid turbulence? The reason may be that the decorrelation of the fluctuations by Alfvén waves is absent in the large-scale magnetic fields. See, e.g., the derivation given by Tu (1988), who started from the basic MHD equations when deriving transport equations for the fluctuations.

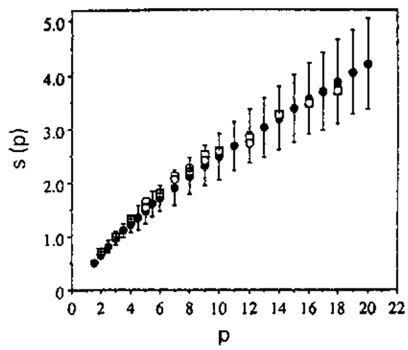

Fig. 7. A comparison of the values of the scaling exponent $s(p)$ of the structure function plotted versus p, along with the relative uncertainties (error bars) of the data, as obtained by Burlaga (1991) in an analysis of the solar wind velocity measured in the interplanetary space near 10 AU far from the Sun (dark points) and by Anselmet et al. (1984) in a laboratory measurement of a turbulent duct flow at a Reynolds numbers of 515 (crossed points) and in a turbulent jet flow with a Reynolds number of 825 (white squares) and 536 (white circles) are shown for comparison. Note the striking coincidence of the points indicating a similar scaling behaviour of the solar wind and the duct flow, i.e. a similarity between a fluid and magnetofluid (adapted from Carbone, 1994).

Burlaga (1991a) found that the scaling exponent of the structure function for MHD turbulence, as obtained in the range from 1 h to 13 h and as observed by Voyager 2 at 8.5 AU, is the same as the exponents obtained for the small-scale turbulence observed in laboratory experiments. Fig. 7 shows a comparison presented by Carbone (1994), who found that the p-model can describe the observed curve with an index $P_1 = 0.72$. We cannot see any difference between measured and predicted values of the exponents in the figure. However, one should further study if there is any difference at all between the intermittency properties of MHD magnetofluid turbulence and HD fluid turbulence. With this question in mind, Carbone et al. (1995) studied the so-called extended self-similarity scaling laws and tried to find experimental evidence for differences between fluid and magnetohydrodynamic turbulence. If for any structure function the scaling law

$$S_l^q \sim l^{s(q)} \quad (39)$$

holds in the inertial domain, then the structure functions are not independent. It is rather so that the qth-order structure function is related to the pth-order one through the relation

$$S_l^{(q)} = A(p,q)[S_l^{(p)}]^{\alpha_p(q)}, \quad (40)$$

with the exponent given by $\alpha_p(q) = s(q)/s(p)$. In fluid turbulence, one has the famous exact Kolmogorov result, given by $s(3) = 1$, and consequently $\alpha_3(q) = s(q)$. The scaling relation (40) has been called extended self-similarity. In MHD flows $s(4) = 1$, and therefore we expect that $\alpha_4(q) = s(q)$. Carbone et al. (1996) presented evidence for the existence of extended self-similarity in solar wind turbulence by calculating the ratio $s(q)/s(4)$ from Voyager data and from Helios

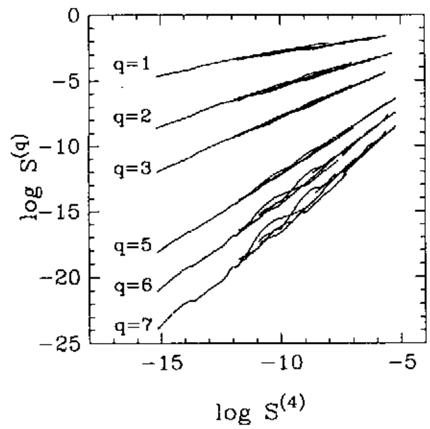

Fig. 8. Evidence for extended self-similarity in solar wind turbulence derived from the scaling of the qth-order velocity structure function, when it is plotted versus the 4th-order structure function, for qs ranging between 1 and 7. The values of $\alpha_4(q)$ can be read off the slopes of the data curves (adapted from Carbone et al., 1996).

data. Their results are shown in Fig. 8, which is a double-logarithmic plot of the relation (40) and thus shows essentially the existence of a linear scaling regime and allows one to infer $\alpha_4(q)$ for different q. The results were then compared with the ones obtained from the laboratory experiments. Carbone et al. (1995) plotted $\alpha_4(q)$ versus q for two kinds of data obtained in MHD and HD fluid measurements. They found that for $q < 7$ one cannot tell a difference between these two groups of data. For for $q > 12$ the values from the two groups are, according to their judgement, clearly different. They suggested that the physical difference seems to be due to the topological properties of the most intermittent structures, which appear to be filaments in HD flows and planar sheets in MHD flows.

However, we must be very careful in drawing any conclusions from calculations of structure functions or statistical moments with exponents higher than 12. From Fig. 7 we can see that the error bars for $s(p)$ are very large for $p > 12$. Horbury et al. (1996) also showed in their Figure 3 that the error bars for $s(p)$ have, at $p = 12$, a comparatively large size of unity and are even larger for $p > 12$. Solar wind fluctuations may in fact not be stationary and homogeneous enough for guaranteeing stability of the statistical ensembles considered. As a consequence, any estimate of very high moments or structure functions with large p may become unreliable. This problem has also been discussed by Dudok de Wit and Krasnosel'skikh (1996) in connection with turbulence measurements made at the Earth's bow shock.

5.2 Cancellation exponents and multifractal scaling

Strongly oscillating signals or fields may have signed measures showing scaling behaviour, which is characterized by a so-called sign-singularity. Carbone and Bruno (1996) calculated the signed measures for turbulent fluctuations in the solar wind and found them to be sign-singular everywhere in the low-frequency-turbulence samples they analysed. The strength of the singularity, which is characterized by a scaling exponent κ and quantified by an exponent called cancellation exponent, was also calculated by them. The results were used to discuss the nature of the solar wind turbulence.

They considered a random field obtained by a calculation of the time derivative of the velocity field, given by $f(t) = dV_{SW}(t)/dt$, in a time interval T, and they considered a hierarchy of subsets T_i of size r, covering the whole original interval of length T. On each subset T_i they defined a signed measure as

$$\mu_r(T_i) = \int_{T_i} f(t)dt / \mid \int_T f(t)dt \mid \tag{41}$$

Then, by introducing the partition function

$$\chi(r) = \sum_i \mid \mu_r(T_i) \mid \tag{42}$$

the cancellation exponent may be defined, for $r \to 0$, as

$$\chi(r) \sim r^{-\kappa} \tag{43}$$

Fig. 9 shows the measure $\mu_r(T_i)$ versus the i-th box of size T_i for different values of the scale r (in hours) calculated from the solar wind velocity observed by the Helios 2 spacecraft in low-speed wind near 1 AU. We have $\kappa = 0.72 \pm 0.01$ in this case. For high-speed winds and the distance range from 0.3 AU to 1 AU, one finds that κ decreases from 0.95 ± 0.02 to 0.85 ± 0.02. They concluded that the results obtained for high-speed streams yield scaling laws which can neither be compared with the usual MHD turbulent scaling laws nor with those derived for fluids, e.g. like the scaling laws for developed turbulence.

Interestingly enough, they found their results can be described by the extended intermittency model as proposed by Tu et al. (1996), which can be applied to turbulence which is not fully developed and is described in much detail in Section 6. Based on the model of Tu et al. (1996), they found that the cancellation exponent is connected with the generalized dimension D_q (which is introduced in Section 7), and with the intrinsic spectral index α (defined in Section 6.3) through the relation

$$\kappa = (n-1)/nD_{1/n} - (1/2(\alpha - 1) - 1/n) \tag{44}$$

where $n = 3$ is for the Kolmogorov scaling and $n = 4$ for the Kraichnan scaling. They pointed out that the cancellation exponent can be simply related to the scaling exponent of the first-order velocity structure functions through the equation, $\kappa = 1 - s(1)$, since $< \delta v_r > \sim r\chi(r)$. The κ-range from 0.85 to 0.95 is fully compatible with a spectral slope in the range $1 \le \alpha \le 1.2$ and with the dimension $0.8 \le D_{1/n} \le 1$ and $P_1 \simeq 0.8$, values which have been observed in high-speed wind and are consistent with the results of Tu et al. (1996). They concluded that this result is an independent confirmation of the fact that, as was already stressed by Tu et al. (1996), the high-speed-stream turbulence is probably not in a fully-developed turbulent state.

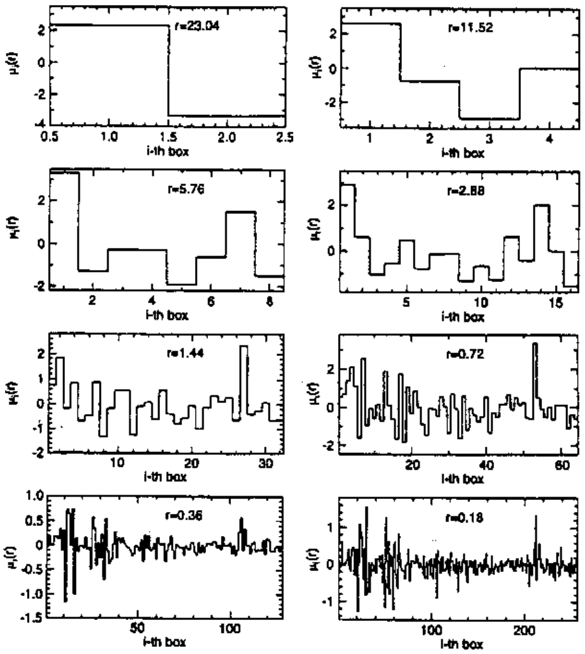

Fig. 9. Plots of the signed measure $\mu_r(T_i)$ of the time-derivatives of the solar wind velocity, which is considered as random variable. The measures are plotted versus the i-th box of size T_i in time for a data sample taken in the low-speed solar wind as observed near 1 AU by the Helios 2 spacecraft. The different values of the time scale r (in hours) are quoted in the various panels. As can be seen, alternation in sign occur at all scales considered (adapted from Carbone and Bruno, 1996).

For low-speed wind streams, when making the assumption of fully-developed turbulence, they found that the values of κ are compatible with the value $s(1)$ being in the range from 0.3 to 0.26 and with P_1 being in the range $0.70-0.74$. Again, this is in good agreement with the Kraichnan scaling law. However, when considering the model of Tu et al. (1996), both relations derived from the above equation, with $n =$ 3 and $n = 4$, are compatible with the results. One cannot decide from the data between the possibility of either a fluid-like or a magnetofluid-like behaviour of the plasma in low-speed streams.

6 An Extended Structure-Function Model

6.1 Introduction

The scaling exponent of the velocity structure function has been considered as a tool for investigating intermittency in fluid turbulence (Anselmet et al., 1984). Burlaga (1991a,b; 1993) studied the structure functions and their exponents, which were calculated for periods ranging from 0.85 hours to 13.6 hours, using data obtained by the Voyager 2 spacecraft at 8.5 AU. Burlaga (1992) also studied the large-scale structure functions of the solar wind for distances from 1 AU to 6 AU. Marsch et al. (1992), Marsch and Liu (1993) and Marsch and Tu (1993) investigated structure functions in the inner heliosphere. Ruzmaikin et al. (1995) analysed the structure function of the magnetic field for in-situ observations made in the south-polar-hole region near 4 AU. They found non-Gaussian statistics of the magnetic fluctuations and pointed out that the observed spectral exponents included effects of intermittency and could not be directly compared with second-order standard spectral theories but must be reduced according to the theory assumed for the energy cascade. All these experimental results indicated the existence of intermittency in the solar wind velocity fluctuations, just like intermittency in fluid turbulence measured in the laboratory (Anselmet et al., 1984).

The observations of intermittency in both laboratory fluids and space magnetofluids are in conflict with the assumption implicit in the theory of Kolmogorov (1941) about an equal distribution of the transfer rate of turbulent energy during the cascade process from large to small eddies. The cascade energy flux seems to be distributed intermittently. Several models have been proposed to describe fluid intermittency. After a detailed comparison, Borgas (1992) concluded that the p-model proposed by Meneveau and Sreenivasan (1987a, b; 1991) represents the currently available fluid measurements better than all other intermittency models, examples of which are the log-normal model (Kolmogorov, 1962), the β-model (Frisch et al., 1978), and the random-β-model (Benzi et al., 1984). The exponents of the structure function presented by Burlaga (1991a) were found to be perfectly consistent with the results from p-model, with $P_1 = 0.7$ (Burlaga, 1993) or $P_1 = 0.72$ (Carbone, 1994). The extension of the β-model to MHD turbulence in the solar wind by Ruzmaikin et al. (1995) characterizes the turbulence by the intrinsic spectral index of the scale-invariant cascade and by the intermittency exponent, a model that was found to describe the magnetic fluctuations in fast high-latitude solar wind streams fairly well.

However, most of the various structure functions calculated from Helios data, except those for the radial components of the velocity in low-speed wind, cannot be explained properly by any previous intermittency model (Marsch and Liu, 1993). Fig. 10 gives an example. The solid curve with error bars shows an observational result calculated from Helios 2 data. The dotted lines present results from the p-model with different values of P_1. We see that none of these theoretical curves can satisfactorily describe the observational results.

The reason for this problem is that all the previous theoretical models for the scaling exponents of the velocity structure functions pertain to developed turbulence. It is assumed in these models that the space-averaged cascade function is constant and does not depend on scale. However, the turbulence in the inner heliosphere is still developing, and therefore this assumption cannot apply. Tu et al. (1984) and Tu (1988) presented models that were successful in describing the power-spectrum evolution in high-speed streams by use of cascading functions derived from dimensional analysis. In those models, the cascade rate changes with scale. This

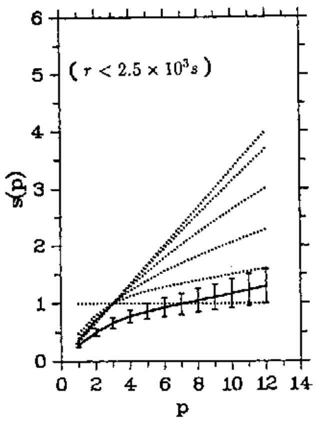

Fig. 10. Comparison between the p-model results (dotted lines) and the Helios observations (continuous line through the error bars). The scaling exponent $s(p)$ of the velocity structure function as derived from high-speed wind data is plotted versus the order parameter p. The model results are for the fractal cascade parameter P_1 ranging from 0.5 to 1.0 in the dotted curves, corresponding to a change in slope from the steepest to the horizontal line. None of the theoretical lines can explain the observed scaling exponent (adapted from Tu et al., 1996).

cascade function is subsequently introduced to the structure function model of Meneveau and Sreenivasan (1987a,b) to account for possible changes of the space-averaged cascade function with scale. In this extended model there are two basic parameters, the intrinsic spectral slope, α, which determines the scaling properties of the space-averaged cascade rate, and the intermittency parameter P_1, which describes the spatial inhomogeneity of the cascade rate. With these two parameters, the extended model can explain all the scaling exponents of the structure functions observed in the inner heliosphere.

6.2 Definitions

The pth order structure function of the velocity component V_i $(i = x, y, z)$ is defined as

$$S^p_{V_i}(\tau) = <\mid V_i(t+\tau) - V_i(t) \mid^p> \tag{45}$$

where τ is a time lag defining the velocity difference, the brackets $<>$ denote the ensemble average that is represented by an average over a certain period of the time t with the time lag τ being fixed. For a vector, the velocity component difference in the above equation is replaced by the vector difference. If the velocity fluctuations can be considered as self-similar, we have the scaling relation

$$S^p_{V_i}(\tau) \sim \tau^{s(p)} \tag{46}$$

Here $s(p)$ is the scaling exponent of the pth order structure function. The normalized structure function is defined as

$$A^p_{V_i}(\tau) = \frac{S^p_{V_i}(\tau)}{(S^2_{V_i}(\tau))^{p/2}} \tag{47}$$

If self-similarity can be established, we have the scaling relation

$$A^p_{V_i}(\tau) \sim \tau^{a(p)} \tag{48}$$

where $a(p)$ is the scaling exponent of the pth order normalized structure function. From the previous equations we have

$$a(p) = s(p) - \frac{p}{2}s(2) \tag{49}$$

These equations for $a(p)$ are equivalent only under the assumption of full self-similarity with Gaussian statistics, in which case the second-order structure function suffices to characterize the fluctuations. Observationally, this assumption is only approximately met by the data for low values of p and in a limited range of time lag τ.

Let us now assume the power spectrum to have a power-law form, $\mathcal{P}(f) \propto f^{-\alpha'}$, and $\alpha' > 1$. Then we can use $\mathcal{P}(f)f$ to evaluate approximately the second order structure function $< (V_i(t+\tau) - V_i(t))^2 >$. Remember that $f \sim 1/\tau$. Therewith we obtain a relation connecting the spectral index α' with the scaling exponent according to the formula

$$\alpha' = 1 + s(2) \tag{50}$$

We shall introduce two power indices, α' and α, where the former is related with the second-order structure function, while the latter controls the cascade function. We will see that only in turbulence without intermittency we have $\alpha' = \alpha$. This is discussed in more detail in the next section.

6.3 Scaling model for the cascade rate

For still developing turbulence the spectral slope is not $-5/3$, and the cascade rate changes with frequency. According to Tu (1988), the averaged cascade rate $\overline{\epsilon_n}$ (see again Section 4.2) can be evaluated by means of the relation $\Delta v_n^2 \sim f\mathcal{P}(f)$, which links the power spectrum with the average velocity fluctuation at scale l_n, which is obtained from equations (24) and (25) by summation: $N_n \Delta v_n^3 := \sum_{i=1}^{N_n} \Delta v_i^3 \sim \epsilon_n l_n = \overline{\epsilon_n} L$
From this relation we obtain

$$\overline{\epsilon_n} \sim f^{5/2}\mathcal{P}(f)^{3/2} \tag{51}$$

where the scaling relation $f \sim l_n^{-1}$ has been used as well. We assume a power law for the spectrum $\mathcal{P}(f)$ with exponent α, i.e. we have

$$\mathcal{P} \sim f^{-\alpha} \sim l_n^{\alpha} \tag{52}$$

If one subjects equations (51) and (52) into (27) one finds a new scaling exponent

$$s(p) = \left(-\frac{5}{2} + \frac{3}{2}\alpha\right)\frac{p}{3} + (1 - log_2[P_1^{p/3} + (1-P_1)^{p/3} \tag{53}$$

The first term of the right hand side is new. It represents the scale dependence of the average cascade function $\overline{\epsilon}(f)$ given by (51). For $\alpha = 5/3$ this term disappears. The second term represents intermittency, as described before by Meneveau

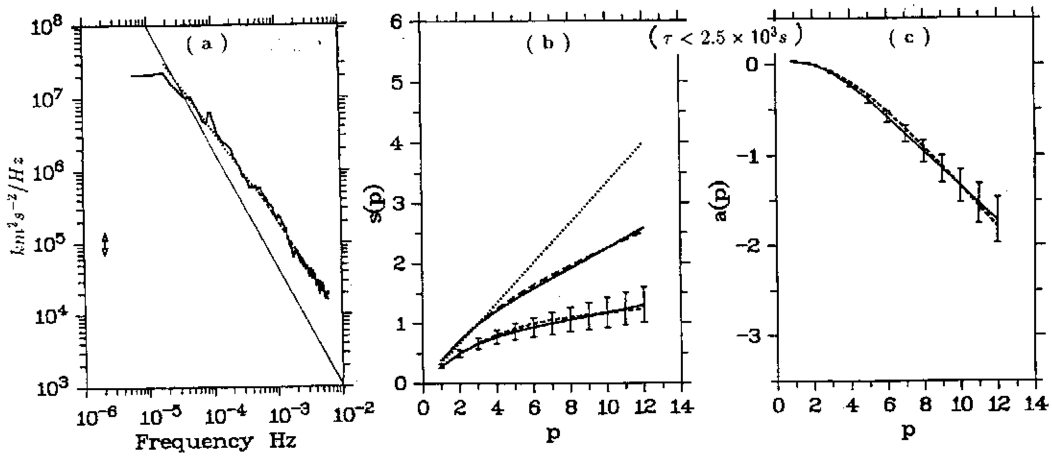

Fig. 11. A comparison of the scaling parameters as derived from the Helios plasma data with the predictions of the extended structure function model. In (a) the power spectrum is shown, in (b) the exponent $s(p)$, and in (c) the exponent of the normalized structure function, $a(p)$. For more details see the corresponding text (adapted from Tu et al., 1996).

and Sreenivasan (1987a,b). This term is zero for $P_1 = 0.5$. For the case without intermittency, $P_1 = 0.5$, and we have $s(p) = (-1+\alpha)(p/2)$. To give an example, for $\alpha = 1.4$ we have $s(p) = 0.2p$. We thus see that the changes induced by a scale-dependence of the cascade rate are large. Using (53) we obtain the scaling exponent for the normalized structure function in the form

$$\begin{aligned} a(p) &= -\left(1 - log_2[P_1^{2/3} + (1-P_1)^{2/3}]\right)\frac{p}{2} \\ &+ 1 - log_2[P_1^{p/3} + (1-P_1)^{p/3}] \end{aligned} \tag{54}$$

We see that the cascade effect has no influence on $a(p)$. The normalized structure functions describe the intermittency more directly than the ones being not normalized. Considering equation (50), the relation between α' and α may be written as follows

$$\alpha' = \alpha + \left(\frac{1}{3} - log_2[P_1^{2/3} + (1-P_1)^{2/3}]\right) \tag{55}$$

We now present results for MHD turbulence with the cascade function proposed by Kraichnan (1965), Dobrowolny et al. (1980) and Tu et al. (1984). The intermittency properties of this kind of turbulence have been studied by Biskamp (1993), Carbone (1993, 1994), and Ruzmaikin (1995). These results are referred to as the MHD case in some papers. However, we prefer to call this turbulence Kraichnan-type, since the corresponding cascade functions can also be applied to the interplanetary medium (Tu, 1988; Carbone, 1994), i.e. to the solar wind magnetofluid. In Kraichnan-type turbulence, the cascade rate may be evaluated by the scaling relation

$$\varepsilon_i \sim \frac{\Delta v_i^4}{l_n V_A} \tag{56}$$

where the nonlinear evolution is based on the Alfvén-wave-decorrelation time. Subjecting equation (56) and equation (26) into (25) yields by summation the result

$$<\mid \Delta v \mid^p>_n \sim (\overline{\epsilon_n} L V_A)^{p/4} \left(\frac{l_n}{L}\right)^{1-log_2[P_1^{p/4}+P_2^{p/4}]} \tag{57}$$

The space-averaged cascade function $\overline{\epsilon_n}$ may now be written (Tu et al., 1984) as

$$\overline{\epsilon_n} \sim f^3(\mathcal{P}(f))^2 \tag{58}$$

Using (52) and inserting (58) into (57), we finally obtain the scaling exponent of the structure function for Kraichnan-type turbulence

$$s(p) = (-3+2\alpha)\frac{p}{4} + 1 - log_2[P_1^{p/4} + (1-P_1)^{p/4}] \tag{59}$$

We see that for $\alpha = 3/2$ the first term on the right-hand side is zero, and equation (59) is then identical with the result of Carbone (1993, 1994). The scaling exponent of the normalized structure function may be written as

$$\begin{aligned} a(p) &= -\left(1 - log_2[P_1^{1/2} + (1-P_1)^{1/2}]\right)\frac{p}{2} \\ &+ 1 - log_2[P_1^{p/4} + (1-P_1)^{p/4}] \end{aligned} \tag{60}$$

Note that by definition $a(2) = 0$. The relation between the intrinsic index α and the apparent index α' is as follows

$$\alpha' = \alpha + \left(\frac{1}{2} - log_2[P_1^{1/2} + (1-P_1)^{1/2}]\right) \tag{61}$$

For $P_1 = 0.5$ we have $\alpha = \alpha'$. Generally $\alpha' > \alpha$. For $P_1 = 1$, we obtain $\alpha' = \alpha + 0.5$.

6.4 Observational results

The scaling exponents calculated from the extended structure function model have been experimentally verified in the paper of Tu et al. (1996), who used Helios plasma data. The results are shown in Fig. 11, which consists of three subfigures. In (a) the power spectrum of solar wind velocity fluctuations is plotted versus frequency in a standard format. The Fast-Fourier-Transform technique was used with a 15-point centered smoothing to evaluate the spectrum. The Kolmogorov $-5/3$ slope is shown for comparison with the inferred spectral index of -1.4, which was derived from the linear fit to the data (dotted line). In (b) the scaling exponent $s(p)$ of the corresponding structure function is shown as obtained from the observations (solid line with error bars), from the extended structure function model (short dashed line) and from the p-model (long dashed line). The solid line without error bars relates to the observations if corrected by the effects of a scale-dependent cascade. The model parameters are $P_1 = 0.77$ and $\alpha = 1.45$. In (c) the scaling exponent $a(p)$ of the normalized structure function versus the order parameter p for the observations (continuous line with bars) and the p-model (dashed line), which explains the normalized data well, but not the original structure function scaling. This comparison between data and theory reveals a remarkable agreement and indicates that the basic concept of an energy-dependent cascade seems to be correct for solar wind turbulence which is not fully developed.

In two recent papers Horbury et al. (1997a,b) studied the magnetic fluctuations on scales ranging from 20 s to 300 s. They found that the intermittency model proposed by Tu et al. (1996) provides a good description of fluctuations in the transition region regime of the spectra observed by Ulysses. They calculated structure functions up to a value of $p = 6$ with the Ulysses magnetic field data obtained in the polar high-speed solar wind at a solar distance of 1.9 AU and at 71° in heliolatitude and for time scales between 20 s and 300 s. At this radial distance the inertial range of the turbulence extended to around 100 s as measured on the spacecraft. So, part of these time scales are in the transition regime from the inertial range to the large-scale range with a -1 spectral slope of the fluctuations. They found that the Tu et al. model can describe the shape of $s(p)$ well, see Fig. 12. In contrast, the procedure used by Horbury et al. (1996), who employed the original p-model of Meneveau and Sreenivasan (1987) but allowed for the intermittency parameter P_1 to change arbitrarily, cannot reproduce the measured shape of $s(p)$.

This result provides important new support for the model of Tu et al. (1996), who used Helios data with time scales longer than 81 s, while in the previous work time scales ranged from 20 s to 300 s. All these results suggest that the assumptions made in that model are essentially valid. However, it is difficult with the data provided in the Horbury et al. (1996) paper to tell the difference between the Kolmogorov and Kraichnan variant of the Tu et al. (1996) model. By best fitting their data with the parameters of that model, they found that for time scales ranging from 100 s to 300 s the intermittency parameter P_1 is around 0.9 (for a Kolmogorov-type) and 0.85 (for a Kraichnan-type cascade), see Fig. 13. These numbers are higher than the value, being around 0.75, which was obtained for high-speed wind near 1 AU and for a longer time scale, 2.5×10^3 s, by Tu et al. (1996). This is consistent with the trend observed by Marsch et al. (1996) in which the intermittency of solar wind fluctuations is stronger at the smaller scales.

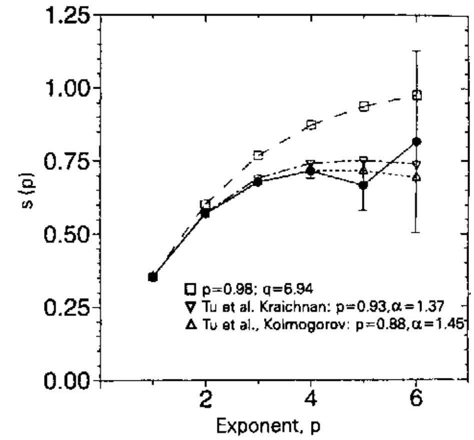

Fig. 12. Observed exponent $s(p)$ (circles) for time scales from 60 s to 120 s (within the regime in the power spectrum where the slope changes), with the best-fit curves of the intermittency model developed by Tu et al. (1996), with a Kraichnan-like cascade and with a Kolmogorov-like cascade, and of the extended p-model with arbitrary values of the exponent q (adapted from Horbury et al., 1996b).

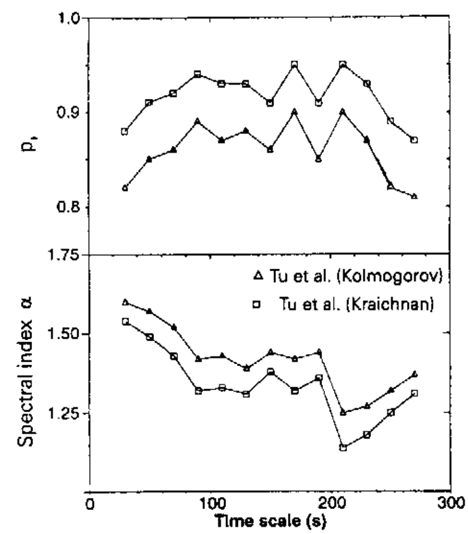

Fig. 13. The scale dependence of the best-fit intermittency parameter P_1 and of the intrinsic spectral index α of the intermittency model by Tu et al. (1996). The parameters are plotted versus time for Ulysses magnetic field data taken in the fast polar solar wind at a distance of 1.9 AU and a heliolatitude of 71°. The boxes correspond to a Kraichnan-type and triangles to a Kolmogorov-type cascade (adapted from Horbury et al., 1996).

7 Multifractal scaling and dissipation-scale intermittency

7.1 Introduction

In conventional studies of magnetohydrodynamic fluctuations in the solar wind one has used the techniques of spectral analysis, in which the fields are decomposed in a large number of Fourier modes. The corresponding power spectral densities of the flow and magnetic field could usually be represented by multiple or sometimes even single power-laws in wave vector or frequency, ranging over several orders of magnitude. We advice the reader to read the extensive reviews on this vast subject by Marsch (1991a,b), Tu and Marsch (1995) and Goldstein et al. (1993). However, as shown in the previous sections there is increasing evidence from theory and data analysis carried out in the recent past that the magnetohydrodynamic turbulence in the solar wind is not adequately and fully described by spectral analysis. Burlaga and Klein (1986) and Burlaga (1991a,b, 1992) showed that the interplanetary magnetic field and flow speed exhibit properties of fractal curves and first established the multifractal structure of the speed and magnetic field in recurrent solar wind streams. Fractal properties were also identified by Polygiannakis and Moussas (1994), using plasma and field data from various spacecraft.

There is mounting evidence for intermittent dissipation of the fluctuations. For example, Burlaga (1991a, b; 1993), Marsch et al. (1992), and Marsch and Liu (1993) demonstrated intermittency of the fluctuations by a detailed analysis of their scaling properties, using the structure functions of the scalar and vector fields in the wind. These findings were corroborated by direct investigation of the field differences or eddy amplitudes and their probability distribution functions (Burlaga, 1993; Feynman and Ruzmaikin, 1994; Marsch and Tu, 1994). As shown in Sections 2 and 3, these probability functions have variable shapes in dependence on the spatial or temporal scale considered, and often they are distinctly non-Gaussian at small scales with sizable skewness and flatness. Sometimes they resemble simple exponentials or exhibit spiky maxima (see e.g. Fig. 4, bottom right frame). All this indicates that higher-order statistical moments are required to characterize entirely the fluctuations.

Apparently, the time series of the observed fields in the solar wind look random and appear to represent fully developed turbulence at a first glance. Yet, spectral analysis of this turbulent behaviour cannot conclusively tell the difference between space-filling or spatially-intermittent dynamics of the observed fluctuations. The theory of nonlinear dynamical systems (see e.g. Eckman and Ruelle, 1985) and the geometrical concepts of anomalous scaling in multifractal objects (see e.g. Paladin and Vulpiani, 1987) provide useful tools for the analysis of nonperiodic time series with multifractal scaling, and thus enable one to scrutinize more deeply the characteristics of deterministic or random field signals and to detect intermittency of such fluctuations, which at a gross inspection seem to be self-similar with multiple-exponent or continuous single-exponent power-law spectrum, yet which really have an underlying fractal cascade that is only revealed by scaling analysis and higher-order statistical investigations. In particular, the generalized dimension, D_q to be defined below, and the related multifractal spectrum enable us to analyse scale-dependent intermittency in any situation, whereas the structure function obtained by time averaging fails to characterize intermittency in cases where the exponents of the energy power spectra are close to unity.

7.2 Basic definitions and data preparation

For the study of developed turbulence it is necessary to find an adequate theoretical description of the dissipation of the kinetic energy, which in classical HD is known for a long time to be intermittent (Batchelor, 1953). The phenomenology of small-scale turbulence and intermittency at the dissipation scale in fluids has recently been reviewed by Sreenivasan and Antonia (1997). The situation in magnetofluids is complicated by the additional degrees of freedom in the magnetic field and by its different dynamics as described by the dissipative MHD induction equation. Ohmic dissipation by collisions is well understood, but in space plasmas the low densities render the collisional transport theory inapplicable, and thus the classical formula of neither the kinematic viscosity nor the magnetic diffusivity can be employed. For the solar wind situation see e.g. the critical evaluation given by Montgomery (1983). There is no doubt about the need to study and understand the basic plasma physics processes on a kinetic level of description (Marsch, 1991a,b), if one wants to understand the turbulence dissipation processes in the wind.

Because of this, no simple closure relations, such as given by the viscous stress tensor in the Navier-Stokes equation, are known for the MHD of space plasmas. Tu (1988), in his turbulence heating model of the solar wind, has estimated the heating rate from the nonlinear cascade of the high-frequency Alfvénic fluctuations. The associated spectral flux function was derived on the basis of traditional (Kolmogorov, 1941; Kraichnan, 1965; Dobrowolny et al., 1980) dimensional analysis of the nonlinear advective terms in the MHD equations. In the spirit of this turbulence phenomenology, we may think of the local dissipation of turbulence as being equal to the work done on the velocity fluctuation at smale scales by the associated fluctuating inertial force. That is to say, given a turbulent eddy of temporal size Δt with a fluctuation amplitude estimated by the flow velocity difference over this time lag, the turbulent energy transfer rate may be estimated as

$$\epsilon_{\Delta t}(t) = | V_j(t+\Delta t) - V_j(t) |^3 / (\Delta t V_{SW}) \qquad (62)$$

The subscript j denotes the cartesian vector component of the flow velocity and is later omitted. At scales close to the dissipative ones (gyro-kinetic scales in the solar wind) this work should on average match the dissipation rate for developed turbulence with continuous broad-band power spectra. We therefore take $\epsilon_{\Delta t}$ as instantaneous dissipation rate based on an eddy of scale Δt or spatial scale $\Delta l = V_{SW}\Delta t$. Here the solar wind speed is denoted again by V_{SW}. In the Helios data analysis, Marsch et al. (1996) chose the z-component of the wind velocity and took Δt to be 81 s or a small multiple of this basic measurement-cycle time. It must be emphasized that 81 s is almost two orders of magnitude larger than the proton gyration period at 1 AU, and consequently the scales considered in the following are essentially still within the inertial range. Unfortunately, even with modern plasma instruments one does not come near the time resolution required to resolve the gyro-kinetic scale in the solar wind. Thus as a note of caution, it should be stressed that the above expression is only a proxy for the real dissipation rate, which may in fact differ substantially from $\epsilon_{\Delta t}$. For a detailed discussion of the dissipation rate and its relation with the energy flux in the K41 turbulence phenomenology see the chapter 6 of the book by Frisch (1995).

The relation (62) is expected to be valid on dimensional grounds and widely used in the literature on the scaling of the intermittent dissipation fields in fluid turbulence (Meneveau and Sreenivasan, 1987a,b). The quantity $\epsilon_{\Delta t}$ is the basic quantity we are going to consider in this paper. Since Δt is fixed we may skip this subscript to ease the notation in the following. Then $\epsilon(t)$ is assumed to be the random dissipation field, the scaling properties of which with time are the central issue of this section. For later purposes, it is convenient to normalize (62) to its mean value and to consider discrete time series with the time step running through a large number of points N, covering a given time interval worth of data. We therefore define

$$\varepsilon(i) = \epsilon_{\Delta t}(t_i) / \sum_{i=1}^{N} \epsilon_{\Delta t}(t_i) \tag{63}$$

as the normalized dissipation rate with

$$\sum_{i=1}^{N} \varepsilon(i) = 1 \tag{64}$$

Note that (63) may be thought of as a probability measure for the realization of the average dissipation at moment t_i in the discrete time series under consideration. There is a certain resemblance with the third-order velocity structure function $S_j^3(\tau)$ in Section 6.2, which is obtained, besides a factor, from (62) through averaging over time t, for a given small scale $\tau = \Delta t$. In order to give the reader a visual impression about the intermittent nature of $\varepsilon(t)$ we plot this quantity in Fig. 14 for $\Delta t = 81$ s (top) and $\Delta t = 8 \times 81$ s (bottom panel). This time variability of ε in solar wind flow is reminiscent of the time traces of turbulent dissipation in laboratory fluids and atmospheric boundary flows reported e.g. in the reviews by Meneveau and Sreenivasan (1987a,b, 1991) and Sreenivasan and Antonia (1997). Also, note the similarity in appearance with the fractal cascade of the p-model shown in Fig. 6.

The intermittent nature of the dissipation in the solar wind becomes particularly conspicuous at small scales (top part of Fig. 14), and gradually fades away as the eddy size is increasing (bottom part of Fig. 14). This figure nicely illustrates that the conceptually appealing view of the transfer

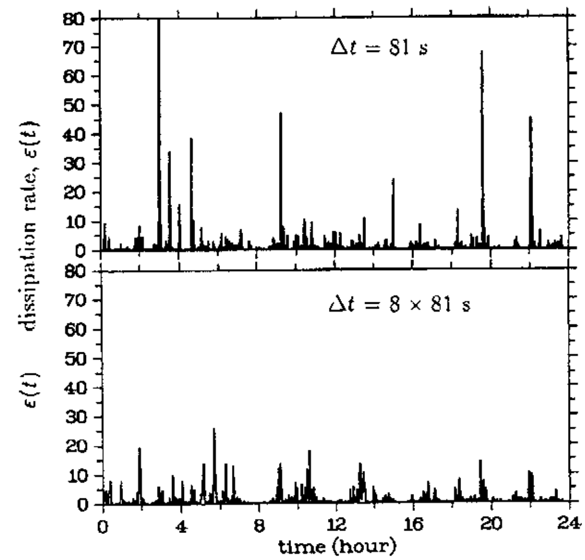

Fig. 14. The time trace of the dissipation rate ε is shown for two values of the time difference Δt, 81 s at the top and 648 s at the bottom. The data set extends over one day. Note the intermittent pulses of varying height and the erratic behaviour of the dissipation rate at small scales, whereas at larger scales the extreme peaks are gone and the variations are more moderate (adapted from Marsch et al., 1996).

of turbulent kinetic energy to the dissipation range through a spatially homogeneous cascade is an unrealistic picture of the real situation in a turbulent magnetofluid such as the solar wind. This statement is consolidated by the behaviour of the higher-order moments and the scaling properties of ε.

7.3 Multifractals - theoretical definitions and concepts

The key issue in this section is how the cascading or dissipation rate behaves under a scale transformation, and what its scaling exponents are. Here we largely follow the work by Meneveau and Sreenivasan (1987a,b; 1991), who described the multifractal nature of the energy dissipation in turbulent fluids, concepts that are adopted here for the velocity field and its dissipation in a turbulent magnetofluid. For a more general introduction to fractals, their definition and scaling properties we refer to the book by Feder (1988), who gives a clear account of the quantities discussed here.

The main question concerning the flux of turbulent kinetic energy then is: Given the seemingly random and temporally intermittent function ε such as shown in Fig. 14, how can one characterize it best, and describe its statistics and scaling properties? The basic ingredients of this description are the concepts of a cascade and multiplicative process, which implies that large eddies or magnetofluid parcels are transformed, stretched, and folded and break down into decreasingly smaller ones. This process defines eddies of different generations. The cascade will of course stop at the dissipation scale of the magnetofluid. The generation step is denoted by n, the corresponding nth eddy may have a linear spatial scale l and temporal scale τ. In the solar wind case we can only measure the scaling behaviour in one dimension of real space along the flow direction of the wind. The spatial and temporal scales are then related by Taylor's frozen-in flow assumption, $l = \tau V_{SW}$ or $x = tV_{SW}$, with the wind velocity pointing in the radial direction away from the Sun.

Of particular interest is the smoothed integral of $\varepsilon(x)$ over a piece of volume or box of size l in space, which we denote by E_l and which is defined as:

$$E_l(x) = \int_x^{x+l} dy\, \varepsilon(y) \tag{65}$$

We omit the argument x in the following, whenever it can be done without loss of clarity. Note that we loosely refer to box also in one dimension, where it simply means an interval on the real number axis. The local scaling behaviour of (65) may be expressed by the formula

$$E_l \sim l^{\alpha} \tag{66}$$

with the scaling exponent α that characterizes the strength of the singularity. This exponent should not be confused with the spectral exponent used earlier. In the multifractal framework it is now supposed that α takes on different values in the different boxes covering the multiple interwoven fractal subsets, which are embedded in real space and support the dissipation field. The number of boxes needed to cover the subsets of typical size l, for which the scaling exponent ranges between α and $\alpha + d\alpha$, is assumed to scale for small l like $N(\alpha) \sim l^{-f(\alpha)}$. Here the corresponding fractal dimension is denoted by $f(\alpha)$ in order to indicate its functional dependence on α. It may be thought of as the local dimension of the region, in which the integrated dissipation rate scales with the diameter l of the integration volume in proportion to l^{α}. A concise and clear description of multifractals is given in the review by Tel (1988). To characterize the geometry of the filamentary and porous subspace in which the dissipation actually takes place, one needs to establish from data analysis the $f - \alpha$ curve. This is one main objective of the subsequent sections.

We now divide the space in boxes of size l, and subsequently sum over all boxes, thus defining an average over E_l denoted by brackets:

$$< E >_n = \sum_{i=1}^{N_n} E_i = E_L = \sum_{i=1}^{N_n} E_L f_i \tag{67}$$

Here E_i denotes the value of E_l in the ith box, and f_i denotes its energy fraction of E_L. The sum is taken over all eddies or boxes associated with the nth stage of the cascade. We may normalize E_l by this average value such that it has a mean value of unity. The index n is meant to remind us of the step number in the cascade. Consider the case where the domain of large scale L, corresponding roughly to the integral length scale of the turbulent magnetofluid, and of a total dissipation rate E_L is repeatedly divided by a factor of 2, such that at the nth step the eddy size will be given by $l_n = L/2^n$. The number of eddies then is $N_n = 2^n$. The central issue is the scaling behaviour of (65). This is usually characterized by the generalized dimension D_q (see e.g. Hentschel and Procaccia, 1983) and the scaling relation for the qth order moment of E_l, which allows one to analyse higher order statistics of the cascading rate. This scaling relation is written as

$$< E^q >_n = \sum_{i=1}^{N_n} E_i^q = E_L^q (l_n/L)^{(q-1)D_q} \tag{68}$$

For convenience, a factor $q-1$ has been separated from the scaling exponent, to compensate for the trivial case, $q = 1$, which is equivalent to the definition (67). The exponent q may vary between plus and minus "infinity". The generalized dimension D_q is obtained from taking the logarithm of (68). The result is

$$D_q = \log_2(< E^q >_n / E_L^q)^{1/(q-1)} / \log_2(l_n/L) \tag{69}$$

Therefore the dimension can be determined experimentally by plotting the quantities in (68) double-logarithmically for various q values, and by determining the slope of these curves from linear least-squares fits, which according to (69) gives directly the required dimension D_q.

The generalized dimension can be calculated theoretically for the simple p-model or multifractal cascade model by Meneveau and Sreenivasan (1987b). According to this model, which was already described in detail in Section 6.3, the flux of kinetic energy E_L to smaller scales is randomly divided at each step into nonequal fractions P_1 and P_2, with $P_1 + P_2 = 1$, whereby the cascade terminates at the dissipation scale. At the nth stage the scale will be $l = L/2^n$ and the eddy E_l will have received the fraction f_i of the original flux E_L, where f_i stands for the factors obtained by the binomial expansion given in equation (20). Correspondingly, we may define sums over the qth moment of f_i as in equation (21).

These equations describe the multiplicative process mathematically, in which through fragmentation of a large piece into increasingly smaller ones each fragment receives a fraction of the previous piece in such a way that its size given by f_i can be described as a product of multipliers associated with all its predecessors at earlier generations. For the p-model the qth moment (68) can actually be evaluated with the result

$$< E^q >_n = \sum_{i=1}^{N_n} E_L^q f_i^q = E_L^q (P_1^q + P_2^q)^n \tag{70}$$

By insertion of (70) in (69) the generalized fractal dimension is readily obtained as

$$D_q = \log_2(P_1^q + P_2^q)/(1-q) \tag{71}$$

Note that for equipartition, $P_1 = P_2 = 1/2$, we have the simple result $D_q = 1$ for all q. Also, for $q = 0$ we have $D_0 = 1$. We now concentrate on the determination of the fractal dimension $f(\alpha)$. Using the scaling relation (66) in (68) allows us to rewrite the latter equation in the form

$$< (l/L)^{q\alpha} > = (l/L)^{(q-1)D_q} \tag{72}$$

where we have omitted the index n for simplicity here.

7.4 The multifractal spectrum

Unless we have an explicit form for the factors f_i in the average (70), like in the p-model given by (20), we cannot further evaluate equation (72). We therefore suggest, following Halsey et al. (1986), that the average may be taken instead over the scaling exponent α itself, with a positive weight function $\rho(\alpha)$ normalized to unity, i.e. for any observable $\mathcal{O}(\alpha)$ we have the mean or expectation value

$$\ll \mathcal{O} \gg = \int d\alpha \rho(\alpha)\mathcal{O}(\alpha) \tag{73}$$

In particular $\mathcal{O} = 1$ yields unity, and any constant can be pulled out of the brackets. The number $N(\alpha)$ of boxes, which are required to cover the sets on which the scaling exponent α attains a value between α and $\alpha + d\alpha$, is assumed for $l/L \to 0$ to scale like: $N(\alpha) \sim (l/L)^{-f(\alpha)}$. Thus the exponent $f(\alpha)$ is a variable fractal dimension and supposed to reflect the differing dimensions of the support on which E_l scales according to (66) with a singularity of strength α. The average value of any observable $\mathcal{O}$ is defined by box summation and, at the nth step of the cascade, obtained as

$$< \mathcal{O} > = \sum_{i=1}^{N_n} \mathcal{O}_i := \ll \mathcal{O}N \gg = \int d\alpha \rho(\alpha)\mathcal{O}(\alpha)N(\alpha) \tag{74}$$

In particular we have the normalization

$$< 1 > = N_n = \int d\alpha \rho(\alpha)(l/L)^{-f(\alpha)} \tag{75}$$

showing that $f(\alpha) \equiv 1$ is a trivial solution with a non-fractal dimension of unity, consistent with $D_q = 1$ in (72) for $q = 0$. By exploiting (74) and (75) we can now recast the α-independent right-hand side of (72) formally as an average and thus obtain the identity:

$$\int d\alpha \rho(\alpha)((l/L)^{q\alpha - f(\alpha)} - (l/L)^{(q-1)D_q}) = 0 \tag{76}$$

from which we find a simple equation connecting the scaling exponents

$$q\alpha - f(\alpha) = (q-1)D_q \tag{77}$$

Thus given q, then $f(\alpha)$ is a linear function of α with slope q and ordinate intersection determined by the generalized dimension. Apparently, the left-hand side of (76) must be independent of q. Therefore, by taking the derivative of (76) with respect to q and by exploiting (77) we obtain the relation

$$\int d\alpha \rho(\alpha) \left[\alpha - \frac{d}{dq}((q-1)D_q)\right] = 0 \tag{78}$$

Using (74) in (78), we finally obtain an equation for the expectation value of α as a function of q in the form

$$\ll \alpha \gg = \alpha_q = \frac{d}{dq}((q-1)D_q) \tag{79}$$

which defines a functional dependence between α and q. This can be inserted again in (77) to obtain $f_q = f(\alpha_q)$. As a result, the parameter q, which by partial differentiation of (77) with respect to α obeys

$$\frac{\partial f(\alpha)}{\partial \alpha} = q \tag{80}$$

selects a specific pair α_q and f_q in (77). This describes the multifractal spectrum in parameterized form, from which a nonlinear function $f(\alpha)$ is obtained by elimination of q. This has by construction tangents with slope q at the particular value $\alpha = \ll \alpha \gg$. Assume that α and $f(\alpha)$ are given, then q follows from (80) and subsequently D_q from (77). Conversely, if q and the function D_q are given, then α follows from the known derivative (79), and subsequently $f(\alpha)$ from (77). Therefore, both pairs of parameter and function can be used alternatively to characterize the multifractal. The spectrum $f(\alpha)$ as a function of α is thus defined as the envelope of the continuous family of lines given by (77) in dependence on q. Note that our derivation here is rigorous and somewhat different from the traditional approach (Meneveau and Sreenivasan, 1987a, 1991), in which the integrand in (76) is approximated by its maximum value, using the method of steepest descend. Interestingly enough, their extreme α value is exactly given by our expectation value (79).

Let us consider some important limiting cases of the above relations. For $q = 0$ we find $f_0 = D_0$, which is the fractal dimension of the support. It follows that $f_0 > f_q$ for $q \neq 0$, since the multifractal as the union of all the fractals by which it is composed must have the largest dimension. Thus f_q has the maximum value D_0 at α_0. A well readable general description of the properties of multifractals and the importance of the multifractal spectrum $f(\alpha)$ for their characterization has been provided by Tel (1988), who also discusses various limiting cases and shows in his Figs. 15 and 16 mathematical examples for multifractals and their corresponding dimensions. A simple fractal obeys $f(\alpha) = \alpha = D_0$, i.e. the spectrum consists in this special case of the diagonal in the $f - \alpha$ plane, which apparently satisfies equation (77). Any deviation from this line indicates multifractal properties of the variable under consideration. For $q = 0$ the relation $f_0 = D_0$ is recovered. Furthermore, since f_q is finite, the limiting case $D_{\pm\infty} = \alpha_{\pm\infty}$ is obtained, which is illustrated in Fig.17 of Tel (1988). Finally, the value $q = 1$ gives $D_1 = f_1 = \alpha_1$, meaning that the f_q and α_q curves touch each other at this point. In the subsequent data analysis sections these fractal scaling exponents and the singularity spectrum $f(\alpha)$ are determined from solar wind in situ measurements.

There is a close connection between the multifractal formalism used here and the nonlinear behaviour of the scaling exponents of the velocity structure function (Frisch and Parisi, 1983). Without derivation we simply quote here the result, which after Meneveau and Sreenivasan (1987a) reads

$$s(p) = (p/3 - 1)D_{p/3} + 1 \tag{81}$$

This result is easily obtained by a comparison of the scaling properties of the structure function and the dissipation rate, which are by the definition (62) closely related. After (65) one has $E_l \sim l\varepsilon_l$, and therefore from (68) one finds

$$< E^q >_n \sim \sum_{i=1}^{N_n} \varepsilon_i^q l_n^q \sim (l_n/L)^{(q-1)D_q} \tag{82}$$

On the other hand the pth-order structure function scales like

$$S^p(l_n) \sim \sum_{i=1}^{N_n} \varepsilon_i^{p/3} l_n^{p/3} l_n/L \sim (l_n/L)^{s(p)} \tag{83}$$

By a comparison of the scaling exponents of these last two equations the result (81) follows immediately. If $D_q = 1$ for all q in equation (81), then $s(p) = p/3$, which is the Kolmogorov (1941) result. Therefore, only if the generalized dimension varies with q does one have intermittency.

7.5 Observations of multifractal scalings

Burlaga (1991b) calculated the generalized dimension D_q and plotted it versus q, for $-10 \leq q \leq 10$, using the data of the magnetic field strength observed by Voyager 2 in the distance range from 23.3 AU to 27.8 AU in the years 1987 to 1988 near solar maximum. The data correspond to fluctuations with time scales from 16 hours to 21 days. It was found that multifractal structures exist in the large-scale interplanetary magnetic field. Burlaga (1991c) further demonstrated the existence of multifractal structure in the velocity fluctuations associated with the large recurrent streams as observed at 1 AU (at temporal scales ranging from 4 to 32 hours) and at 6 AU (in the scale range from 2 to 16 hours). For the fluctuations occurring on these large scales he also analysed the radial evolution of the intermittency from 1 AU to 6 AU.

In a later paper Burlaga (1992) demonstrated again the existence of multifractal structures in the large-scale fluctuations of the magnetic field strength (scales between 2 and 32 hours), of the temperature and density (scales from 2 to 16 hours) as observed at 1 AU. These large fluctuations are related with the recurrent stream structures and the heliospheric current sheet. Burlaga (1993) found that the large-scale fluctuations of the solar wind bulk speed, as observed at 1 AU near solar maximum in 1979, exhibited multifractal scaling over the time range from 8 hours to 2.7 days, and that this scaling could be described by the p-model with an exponent of $P_1 = 0.7$.

Marsch et al. (1996) have recently analyzed the Helios plasma data and also found clear evidence for multifractal scaling of the velocity fluctuations and the energy dissipation rate $\varepsilon(t)$ defined by (63)–(65). They considered the z-component of the wind velocity only, arguing that this is least affected by stream dynamics and compressive effects in the turbulent wind and most strongly related with the Alfvénic component (out of the ecliptic plane) of the fluctuations. They found a regime of linear scaling of the $q-1$ root of the time-averaged energy flux or cascade rate as defined in (68), and could derive the generalized dimension from a double-logarithmic plot of this quantity versus time (or space according to Taylor's hypothesis). It was also found that, for

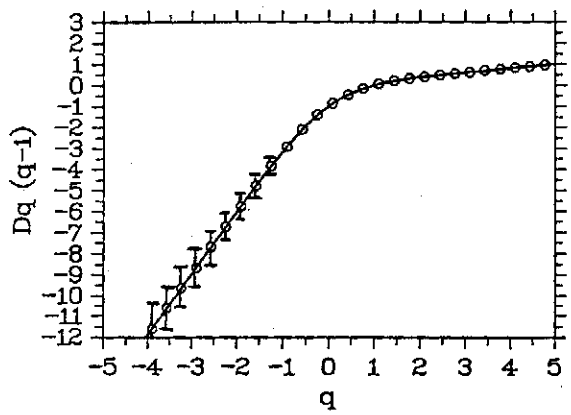

Fig. 15. Scaling exponent, $\tau_q = (q-1)D_q$, corresponding to the generalized dimension D_q of the qth moment of the box-averaged smoothed dissipation rate, E_l, plotted versus q for the data shown already in Fig. 14 with the time difference being 81 s. The data can be fitted well by the p-model with $P_1 = 0.87$ (adapted from Marsch et al., 1996).

the smallest possible time difference of $\Delta t = 81$ s in the Helios data, the measured D_q could well be represented by the formula (71) obtained from the p-model described in Section 4.2, with the parameter $P_1 = 0.87$. Despite its name, the generalized dimension D_q is not a real dimension in the geometrical sense. It seems more instructive to use the scaling parameter $\tau_q = (q-1)D_q$ itself, which is directly derived from the relation (68). This τ_q is plotted versus the cumulant exponent q in Fig. 15, which shows the measured points together with the p-model result as a continuous line that apparently fits the data quite well. By the definition (69), the exponent D_q equals -1 at $q = 0$ and 0 at $q = 1$. In the presentation of Fig. 15 the asymptotic limits of D_q are also obvious, since D_q is simply given by the slopes of the straight lines for $q \to \pm\infty$. For a homogeneous system and scale-invariant cascade rate $\bar{\varepsilon}$, equation (65) yields $E_l(x) = \bar{\varepsilon}l$, and thus (69) implies $D_q = 1$ for all q. Correspondingly, the exponent τ_q should be equal to $q-1$. The clear deviation of the data from this line in Fig. 15 indicates intermittency and multifractal scaling of the cascading rate in the solar wind.

As discussed in Section 7.4, the parameter pair D_q and q is equivalent to the multifractal spectrum, given by α and $f(\alpha)$. According to equation (79), α_q is obtained by numerical differentiation of the curve shown in Fig. 15, and then $f(\alpha)$ follows from (77). Fig. 16 shows the inferred multifractal spectrum. Obviously, the spectrum deviates strongly from a straight line expected for the homogeneous case, thus indicating the multifractal geometrical nature of the averaged dissipation field $E_l(x)$ in the solar wind. The multifractal spectrum shows the characteristic parabolic shape found in many other nonlinear physical systems (see e.g. Meneveau and Sreenivasan, 1987a,b, 1991, in particular for hydrodynamical turbulence) and in mathematics (e.g. Tel, 1988; Halsey et al., 1986). Only Burlaga (1991b) has calculated the multifractal spectrum for a magnetofluid from solar wind

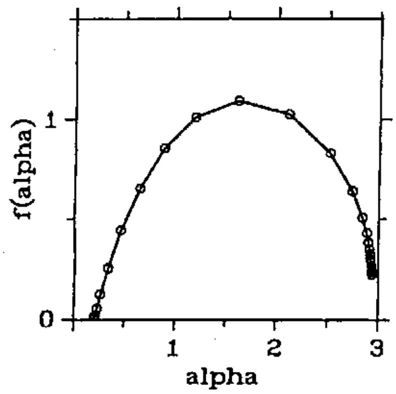

Fig. 16. The multifractal spectrum $f(\alpha)$ plotted versus the singularity exponent α for the data of Figs. 14 and 15 (adapted from Marsch et al., 1996).

data, although not for the cascade rate but for the magnetic field on large scales ranging from 16 hours to 21 days and in the outer heliosphere. Burlaga (1992) also found a similar behaviour in the fluctuations of such scalar quantities as the proton temperature and density in the solar wind. The significance of these observations is discussed in more detail in that reference.

7.6 Scaling of the ε-autocorrelation function

We finally investigate directly the scaling behaviour of the dissipation rate $\varepsilon(l)$. Remember that the spatial scale l relates to the time scale τ through the simple Doppler-shift formula for the wind streaming past the spacecraft, i.e. $l = V_{SW}\tau$. Therefore we use l here and plot experimental data versus τ. The autocorrelation function scales like

$$< \varepsilon(x)\varepsilon(x+l) > \sim < \varepsilon_l^2 > = \varepsilon_L^2 (l/L)^{-\mu} \tag{84}$$

From the definition (65) one has the scaling relation $E_l \sim \varepsilon_l l$, and thus (68) yields the scaling

$$< \varepsilon_l^2 > \sim \varepsilon_L^2 (l/L)^{D_2 - 1} \tag{85}$$

from which by comparison with (84) we can readily read off the exponent

$$\mu = 1 - D_2 \tag{86}$$

The exponent μ can also be evaluated in the p-model by help of (23) and (26) as

$$\mu = 1 + log_2[P_1^2 + P_2^2] \tag{87}$$

In the β-model μ can be evaluated as being equal to 2 times the exponent of the normalized structure function, $a(p)$ (Ruzmaikin et al., 1995).

Marsch et al. (1996) estimated from Helios data the scaling exponent μ. In their Fig.8 the observed autocorrelation function of the dissipation field $\varepsilon(\tau)$ is displayed double-logarithmically versus τ. The scaling exponent μ follows from the slope. The exponents are the same within the error bars and have the value $\mu = 0.38 \pm 0.19$ and $\mu = 0.37 \pm 0.08$, respectively, where this estimate is based upon values for τ ranging between 8×10^2s and 5×10^3s. Apparently, there is no agreement of this μ-value with the number obtained by inserting D_2 from Fig. 15 in equation (86). We do not have an obvious reason for this result.

8 Discussion and conclusions

The main objective of this paper was to review the work that has been done to study the occurrence of intermittency and fractal scalings in solar wind turbulence, and to analyse the non-Gaussian higher-order statistics and scaling properties of MHD fluctuations in the interplanetary medium. The emphasis was put on a presentation of selected key measurements and experimental results from in-situ observations as well as on a tutorial outline of some intermittency models and on their comparison with the data, which were obtained from the Helios, Voyager and Ulysses spacecraft.

In the introduction, we have briefly described the wealth of detailed results obtained on solar wind fluctuations and turbulence, which on the one hand show clear evidence for the non-linear dynamics and spectral evolution as expected for homogeneous turbulence, but on the other hand are certainly also influenced by the effects of spatial inhomogeneity and anisotropy, and by compression and temporal variability. The data time series considered are often too short to allow a reliable determination of the higher-order structure functions or tails in the probability distributions and their derived moments. Also, the property of stationarity of a given data set is difficult to ensure in the variable solar wind and is only approximately guaranteed. Its characteristics render many of the assumptions made in turbulence and intermittency models only approximately correct for the solar wind and partly applicable to it. Consequently, the predictions made by the comparatively simple models, e.g. concerning the scaling laws of the velocity increments or the geometrical nature of the most singular dissipative structures, can only with restrictions be relevant to the measurements. Yet, the models seem to represent some of the observations surprisingly well. No specific theories have been developed to describe turbulence in a collisionless space plasma, such as the solar wind. Therefore, from a practical point of view we have to be satisfied with using the available models and relating them to the observations.

In summarizing the obervations we can state the following. Various studies have been carried out with plasma and field data gathered on different spacecraft and at various times in the solar cycle and in many locations of the heliosphere. The magnetic field components and magnitude and the proton velocity, density and temperature have been analyzed mostly. Generally, power spectra and structure functions show what might be called a quasi-inertial range, extending usually over several orders of magnitude in time or space scales. The fluctuations in this range are not fully self-similar and usually reveal several domains with different scaling exponents. The turbulence observed near the heliospheric current sheet and in the ecliptic plane appears to be most developed and often reveals Kolmogorov-type $-5/3$ power spectra or the famous $1/3$ scaling of the velocity increments with the length scale l or time lag τ. On the contrary, the turbulence measured out of the ecliptic plane, in fast solar wind streams originating from coronal holes on the Sun, shows commonly a high degree of Alfvénic correlations and coherent wave motions with flatter spectra and structure functions, which indicate reduced nonlinear evolution and may still carry large-scale signatures of its sources on the Sun. Using the technique of coronal radio sounding, Karl et al. (1997) have recently shown that density fluctuations in the source regions of the fast and slow solar indicate non-Gaussian turbulence and intermittency close to Sun.

At those scales, which are small but still larger than the kinetic scales where collisionless dissipation is supposed to take place, ample evidence was found for intermittent behaviour, being visible in particular in non-Gaussian PDFs and the scale dependence of the normalized structure functions. Their scaling behaviour cannot be explained by intermittency effects alone, which are understood to originate for self-similar and thus scale-invariant fluctuations, but requires accounting for an explicit scale dependence of the energy flux or cascading function. This fact is considered in the extended structure function model (presented in Section 6), which seems to be consistent with the scaling behaviour found in the Helios velocity fluctuations and Ulysses magnetic field fluctuations. Generally speaking, the p-model (Meneveau and Sreenivasan, 1987a,b) or the extended structure function model (Tu et al., 1996) decribe the data well. There is convincing evidence found in the solar wind data in support of the existence of a fractal cascade. The p-model model is intuitively simple but non-trivial and explains the data satisfactorily, given the experimental and theoretical limitations discussed above. For the velocity fluctuations, the model parameter P_1 is found to range between 0.7 and 0.8, values which are consistent with fluid measurements in the laboratory.

The range where simple scaling laws apply may be further extended, if the structure functions are plotted against each other, such as done in Fig. 8. It shows that solar wind turbulence, if analysed this way, posseses the property of extended self-similarity, applying to a larger range of scales (see also the Fig. 5 of Grappin et al., 1990) even if the power spectra or structure functions themselves cannot be fitted by a single spectral index or scaling exponent. In conclusion, clear evidence has been established for intermittency at inertial-range scales.

There is another realm of intermittency at the smallest (kinetic) scales, where the dissipation may take place on geometrically singular structures with a spatial support of a reduced or fractal dimension. Examples are small and thin vorticity and current sheets which tend to develope in turbulent magnetofluids (Biskamp et al., 1990; Grauer and Marliani, 1995; 1996). In Section 7 we have discussed multifractal scaling and intermittency in the energy flux or cascade rate. The time trace of this quantity shows intermittent pulses of varying height, reminiscent of the fractal cascade in the p-model. Its scaling behaviour can be described by the concepts of multifractals. The associated multifractal spectrum has been evaluated for the first time. It shows the characteristic parabolic shape found in HD turbulence and other nonlinear systems. Whereas in collisional neutral fluids the dissipation is of viscous nature at the molecular level, the dissipation in the solar wind as a collisionless quasineutral magnetofluid is of kinetic nature at the level of the individual motions of ions and electrons in their self-generated and turbulent high-frequency wave fields. The dissipation processes in the solar wind are not well understood (see the reviews of Marsch, 1991a,b), and thus it seems surprising at a first glance that concepts developed for HD are also valid for MHD. That this is the case emphasizes the universality of the scaling laws and indicates their independence of the microphysics of the dissipation.

The existing models do not permit us to discriminate reliably between the various scaling exponents on the basis of the first four to six statistical moments alone, since they only differ substantially where the data become much less reliable. This fact places tight constraints on the possibility to verify or falsify intermittency models or test their relevance to solar wind data. Unfortunately, the scaling exponents, $s(p)$, derived in various models, only differ substantially at high-enough values of the order parameter, which is for $p > 10$, a range where the time-limited data do usually not permit a reliable determination of the statistical properties required (Horbury et al., 1996). Taking longer time series does not solve the problem, because long data sets do often not fulfill the requirements in order to be considered meaningfully as a statistical ensemble (see e.g. Matthaeus et al., 1986), or they do not guarantee stationarity and/or are influenced by coherent structures such as wind streams or shock waves (see e.g. Dudok de Wit and Krasnosel'skikh, 1996). Any further sophistication in models is hard to justify, because the data will not allow one to judge this properly or to chose between models on the basis of observationally marginal differences. Probably, just because of the above mentioned reason, the question is still debated which the relevant turbulence phenomenology for the solar wind is, the one initiated by Kolmogorov or the one suggested by Kraichnan.

To conclude, the work that has been done in the past years on intermittency and scaling of solar wind fluctuations has cleary shown the need to go beyond standard spectral analysis, with e.g. Fast Fourier Transform (FFT) techniques, if one wants to understand the statistics and the turbulent energy transfer and dissipation. The simple models proposed in the HD literature seem to be, with minor modifications accounting for the existence of a mean magnetic field, quite relevant to the solar wind as well. They appear to grasp the essence of intermittency in a turbulent magnetofluid, despite their neglection of the observed anisotropy in the fields and of compressive processes and inhomogeneity of the background medium. To make further progress in this field more complete measurements of the components of wind velocity and density and the interplanetary magnetic field are required, simultaneously from at least two points in space with variable distance and in fast cadence, in order to provide the measurements of the field differences required in the structure functions. Also, better measurements and adequate models for the fluctuations at the gyro-kinetic scales are direly needed, if real progress is to be made in the understanding of the microphysics of dissipation in collisionless magnetofluids.

Acknowledgements. The work of C.-Y. Tu was partly supported by the University of Peking and the National Natural Science Foundation of China.

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