Nonlinear Processes in Geophysics (1996) 3: 231–235

Nonlinear Processes in Geophysics

Geophysical turbulence data and turbulence theory

Charles W. Van Atta

Scripps Institution of Oceanography and Applied Mechanics and Engineering Sciences
University of California, San Diego
La Jolla, CA 92093 USA

Received 05 January 1996 - Accepted 24 April 1996

Abstract. Many vital insights into the nature of turbuence in fluids have originated from experimental data obtained in geophysical flows. Geophysical data have often helped to stimulate the creation of new turbulence theory, while theory has in many cases motivated the experimental efforts. The present brief review discusses several key examples of this interaction between experiment and theory, citing mainly work which is of particular interest to the author. No attempt is made to provide a complete listing of the extensive and currently rapidly developing literature for some of the problems discussed.

1 Introduction

New experimental results for turbulent flows can stimulate the creation of new theory when 1) the experimental results conflict with accepted existing theory, or 2) the experiments uncover or suggest some effect or simplification not previously considered by the theory. Turbulence theory has a complementary impact on experimental studies . The study of turbulence in fluids is a continuing saga of this interaction between experimental results and theory. Geophysical turbulence data has such a profound influence upon turbulence theory because, in terrestrial flows, Reynolds numbers sufficiently large to provide conclusive tests of many of the most cogent theoretical results are readily obtained only in the geophysical flows found in the earth's oceans and atmosphere. The present account reviews some examples of this process, highly restricted to only a few questions in which the author has had particular interest. The focus here is on the impact of geophysical data on theory, with no discussion of the considerable impact of laboratory experimental results on turbulence theory.

2 Isotropy, Energy Spectra, and Structure Functions

Symmetry breaking associated with the preferred direction of the vertical coordinate z in the atmosphere and oceans, and the presence of rotation and vertical stratification precludes any global large scale isotropy in geophysical flows. However, for the smaller scales of motion the turbulence can be locally isotropic. While making meteorological measurements aboard ships using tethered balloons, Taylor (1927) noticed that, when subjected to buffeting by atmospheric turbulence, the horizontal, lateral, and vertical excursions of his balloons were roughly equal. This led him to the concept of globally isotropic turbulence, for which the mean square fluctuating velocities are equal in all three coordinate directions, the application by Taylor (1935) of the principle of invariance with respect to rotations , and the elegant employment of isotropic tensors in the theory of isotropic turbulence by Karman and Howarth (1938). These developements also led to the very useful concept of local isotropy , in which the exact relations given by local isotropy theory apply only over restricted finite ranges of length scales or wavenumbers, and the global isotropy restriction of equality of the mean square velocities in different directions is not invoked. In the 1940's , the statistical theory was further developed in terms of structure functions, moments of differences in velocity measured at two different spatial locations separated by a distance r, by Kolmogorov (1941) and in spectral terms by Obukhov (1941). These results, which include the inertial subrange $r^{2/3}$ behavior of the second-order structure function and the corresponding $k^{-5/3}$ (k is wavenumber) behavior of the energy spectrum , are sometimes referred to as the K41 theory. Obukhov's spectral formulation was rediscovered by Onsager (1945), von Weizsacker (1948), and Heisenberg (1948). Obukhov also made the first attempt to use geophysical (atmospheric) data for verification of the theory. In several experimental papers from his group published between 1941 and 1960 the 2/3 and -5/3 power laws were confirmed, but the instrumentation had insufficient resolution to resolve dissipation range behavior. Definitive, quantitative verification of full inertial and dissipative range K41 scaling was found in tidal channel measurements by Grant et al. (1962) using an instrumented ship-towed underwater body , and then in the atmospheric boundary layer by Pond et al. (1966). The tidal channel measurements also showed that the variance of ϵ_r, the rate of dissipation of turbulent kinetic energy ϵ averaged over a spatial region of characteristic length scale r, was relatively large. Universal spectral scaling was obtained employing ϵ_r , thus precluding successful universal scaling when employing the long term average value of ϵ. To address similar observations by Gurvich of strong variability of the dimensional coefficient in the -5/3 law , Kolmogorov (1962), and Obukhov (1962) proposed refined similarity hypotheses based on ϵ_r , instead of simply on the overall value of ϵ. Statistical quantities involving moments of ϵ_r thus acquired an additional r-dependence through the r-dependence of the pdf of ϵ_r. Assuming a lognormal form for the pdf of ϵ_r, with a variance which depended on r, as suggested by Kolmogorov (1962) , Yaglom, A.M. (1966), and Gurvich and Yaglom (1967), yielded small modifications to the -5/3 spectral and +2/3 structure function power laws. The modifications became increasingly larger as the order p of the structure function $< (u(x+r) - u(x))^p >$ increased for $p > 3$, and were strongly dependent on the assumed form of the pdf of ϵ_r for large values of ϵ_r. Experimentally measured large deviations from the K41 scaling were first found , in geophysical data for higher-order structure functions measured in the atmospheric boundary layer over the ocean, by Van Atta and Chen (1970), and by Van Atta and Park (1972). As p increased for $3 < p \leq 9$ their measured values of ζ_p, the power law exponent for the pth order structure functions, increased with p, but at a significantly slower rate than the values of $\zeta_p = p/3$ predicted by a straightforward extension of K41. Qualitatively similar behavior has been found in all successive experiments in both geophysical and laboratory flows, and in direct numerical simulations, as discussed below. Higher-order structure function behavior is naturally strongly dependent on the tails of the pdf's. Gagne et al (1990) compared their data taken in the very large Modane wind tunnel with $R_\lambda = 2720$, where R_λ is the turbulence Reynolds number based on the Taylor microscale λ, with the atmospheric data of Van Atta and Park (1972) for $R_\lambda = 3000$, and with the laboratory data of Anselmet et al. (1984) for R_λ in the range 515-852 . Numerical values of the slopes of exponential fits to the positive and negative tails of the pdf's were very similar for the atmospheric and Modane data sets, defining unique power law functions of r/η, where η is the Kolmogorov scale. The slopes were smaller for the Anselmet et al. (1984) data, increasing with increasing R_λ, and following the same power law dependence on r/η. This comparative behavior suggests a significant R_λ dependence of the ζ_p, and that an R_λ at least as large as several thousand is required to achieve conditions which might be characteristic of the asymptotic limit of infinite R_λ. This suggestion is apparently not supported by the recent atmospheric boundary layer data of Schertzer et al. (1995), who find, for $p \leq 7$, ζ_p that are the same for atmospheric and laboratory flows. However, for $p > 7$ their ζ_p are found to depend on the length of the time series they used. Thus, while they argue that the behavior they observed is compatible with divergence of moments for empirical data, proposed in Schertzer and Lovejoy (1987), the dependence on sample size could be a consequence of nonstationarity . Instead of focussing on the behavior of individual structure functions, Benzi et al. (1993) have examined the comparative behavior of structure functions of different orders . Structure functions obey *scaling* when each structure function has a power law dependence on r. Plotting the structure function of order m versus that of order n will then also yield a power law, and the structure functions then obey *relative scaling* . If two different normalized structure functions, each made dimensionless by dividing by the appropriate powers of the third-order structure function, obey relative scaling , this is called *normalized relative scaling.* Benzi et al. (1993) find that their measured structure functions obey relative scaling, persisting even for small values of r in the viscous range, a behavior to which they give the name *extended self similarity.* Their data suggests that the improvement in scaling behavior is due to the similar dependence of all structure functions in the dissipative range on p-dependent powers of a universal function $f(r/\eta)$, which itself does not depend on p. L'vov and Procaccia (1994) argue that this *extended universality* stems from the structure of the Navier-Stokes equations and from the property of the locality of interactions, and examine the possibility that experimentally observed deviations from K41 behavior are due to the finite values of the Reynolds numbers and anisotropy of the turbulence. Segel et al. (1996) have demonstrated extended self-similarity analytically in the context of the Kraichnan (1968) model of a passive scalar advected by a very rapidly varying velocity field.

The Kolmogorov (1962) lognormal hypothesis yields pathological structure function behavior for large p, and many other pdf's, fractal scaling recipes for the spatial distribution of ϵ, and other scaling scenarios have been hypothesized to model the measured dependence of ζ_p on p . These models include those of Frisch et al (1978), Kida (1991), Saito (1992), Kailasnath et al (1992), She and Leveque (1994), Schmitt et al. (1994), Dubrulle (1994), She and Waymire (1995), Nelkin (1995), and Chen and Cao (1996). Many of the scaling theories are expected to apply strictly only for very large values of

R_λ, and the Reynolds number does not appear in the theoretical predictions for the ζ_p. Another theoretical approach is to calculate the pdf of Δu from its governing equation, as derived from the Navier-Stokes equations by Lundgren (1972) and Monin (1967) , and then to calculate the structure function moments directly using this pdf. However, the integro-differential equation for the pdf of Δu is not closed , since the equation for the n-point pdf (where n=1,2,...) includes also the (n+1)-point pdf (similar to the BBGKY heirarchy of equations for the pdf in the kinetic theory of gases. Lundgren (1972) proposed some model closures of this equation . A great number of other closures were proposed in later years (see, e.g., the surveys by Kollmann and Pope (1990,1991), for some examples) and their applications to higher-order structure functions are a subject of currrent research (see, e.g. Pedrizzetti and Novikov (1994)). A more rigorous approach is to calculate the behavior of higher-order structure functions directly from the Navier-Stokes equations. This has been done using DNS (direct numerical simulations) for modest values of $R_\lambda \leq 200$ by Cao et al. (1996) and Boratov and Pelz (1996). Their computed values of relative scaling exponents are numerically close to those measured by Benzi et al. (1993) and those predicted by She and Leveque (1994).

It thus appears that results from experiments, modelling, and DNS for some aspects of higher-order structure function behavior may be converging. Some questions remain about the behavior of higher-order structure functions for large Reynolds number . Due to the uncontrollability of geophysical flows , lack of sufficient stationarity, etc. , data obtained in geophysical flows may not provide the decisive evidence for the behavior of higher-order moments that was obtained for lower-order structure functions and spectra. It is desirable to continue large Reynolds number laboratory studies of both shear flows and unsheared grid generated turbulence. The latter flow provides an important benchmark as there are no effects of mean shear (which is zero) and the turbulence is locally isotropic over a large range of scales. As discussed by Uberoi (1957) and Durbin and Speziale (1991), mean shear can significantly decrease the degree of local isotropy , even in the limit of infinite Reynolds numbers (for some flows). Systematic, closely coordinated DNS and experimental studies of the influence of varying mean shear and Reynolds number would be valuable.

3 Higher-order spectra

The synergistic interaction of geophysical turbulence experiments and turbulence theory is also evident in studies of higher-order spectra. Initial studies focused on the discrepancy between measured higher-order spectra and K41 dimensional analysis. Successive developments have dealt with other fundamental theoretical issues.

As noted by Dutton and Deaven (1972) , in analogy with K41 inertial subrange scaling, dimensional analyis predicts that spectra of the nth power of the velocity fluctuation will scale as $E_n(k) = C_n \epsilon^{2n/3} k^{-(2n+3)/3}$. However, in their atmospheric turbulence data obtained at four different altitudes with instrumented aircraft, a different behavior was observed. Rather than increasing in slope with increasing order, in the inertial range the higher-order spectra either retained an approximately $k^{-5/3}$ power law behavior or decreased somewhat in slope as n increased. This puzzle stimulated Van Atta and Wyngaard (1975) to propose an alternative extension of Kolmogorov's ideas. The key factor in their argument is an order-dependent dissipation term ϵ_n, which appears in the dynamical equation for $< u^n >$. In a separate analytical and numerical study , they also derived expressions for higher-order inertial range spectra based on the assumption of a Gaussian velocity distribution . Results of their dimensional arguments and Gaussian analyses showed good agreement with the measured spectral slopes and energy levels of Dutton and Deaven (1972) and with those of the higher-order spectra computed from the same open ocean atmospheric boundary later data that was employed earlier by Van Atta and Chen (1970) and Van Atta and Park (1972) in their studies of higher-order structure functions. In studies of higher-order spectra of a different sort, bispectral analysis of velocity fluctuations in the atmospheric boundary layer by Lii et al (1976) and Van Atta (1979) indicated a k^{-3} behavior in the inertial subrange, as predicted by extension of K41 scaling. More recently, Zhou et al (1993) extended ideas of Kraichnan (1965) to predict that higher-order spectral moments can scale as $k^{-5/3}$ for any order without the assumption of Gaussianity, in agreement with the results of the ϵ_n-based dimensional analysis of Van Atta and Wyngaard (1975), which also did not assume Gaussianity . In a different theoretical context Nelkin and Tabor (1989) noted that a consequence of the random sweeping hypothesis is that the spectrum of the kinetic energy u^2 scales as $k^{-5/3}$. If, on the other hand, the u^2 spectrum satisfies the $E_2 = C_2 \epsilon^{4/3} k^{-7/3}$ scaling predicted by the expression for E_n derived in the spirit of K41 , then the renormalization group theory prediction of no sweeping is recovered. Since the experiments clearly indicate that the spectrum of u^2 goes as $k^{-5/3}$ at high Reynolds numbers, they provide positive support for the sweeping hypothesis.

4 Skewness and flatness of velocity derivatives

Geophysical data for moments of velocity gradients have stimulated a great deal of theoretical work , as the numerical values of the normalized moments, such as skewness and flatness factors (denoted by S and F) measured for large R_λ differ greatly from those obtained for lower R_λ in laboratory experiments.

The K41 equilibrium hypothesis predicts that for sufficiently large R_λ, dimensionless ratios of mean powers of velocity derivatives, like skewness and flatness factors, should be absolute constants independent of Reynolds number (see, e.g., Batchelor (1953)). Except for the very low Reynolds numbers associated with the final period of decay , measured skewness and flatness factors of velocity derivatives increase with increasing R_λ (see the data compiled by Van Atta and Antonia (1980)). Intermittency corrections in the spirit of K62 predict that S and K will increase with increasing R_λ, as seen in the experimental data (for scalar variables , see, e.g. Van Atta (1973)). Many other models , e.g., scaling intermittency models, would also predict such a dependence. Another approach to the problem is the use of vortex-based physical models of turbulence fine scales using spatial ensembles of small-scale structures represented by local solutions of the Navier-Stokes equations. Pullin and Saffman (1993) have used the statistical methodology of Townsend (1951) combined with the Lundgren (1982) spiral vortex model to calculate higher-order moments of velocity derivatives for homogeneous turbulence. Lundgren (1982) showed that an ensemble of such spiral vortex structures produces a $k^{-5/3}$ range in the energy spectrum of velocity, so its use lends dynamical credentials to the calculation. Numerical values of two dimensionless groups in the model are fixed by requiring agreement with experimental estimates of the Kolmogorov constant and with the skewness S. When the vortex lateral scale R is assumed to be the geometric mean of the Tayor microscale λ and the smallest physical scale of the model $(\nu/a)^{1/2}$, where ν is the kinematic viscosity of the fluid and a is the rate of strain, the flatness factor F is found to be proportional to ${R_\lambda}^{1/4}$. From figure 4 of Pullin and Saffman (1993) , one notes that there is effectively a discontinuous upward jump in the F data at $R_\lambda = 700$. The ${R_\lambda}^{1/4}$ dependence is in good agreement with the experimental data and that of DNS (for the presently available DNS results the maximum R_λ is about $2x10^2$). For $R_\lambda > 300$ the experimental data lie in a " scatter" band lying significantly above a continuation of the ${R_\lambda}^{1/4}$ fit to the data for the lower values of R_λ, but a separate ${R_\lambda}^{1/4}$ fit to this data also looks quite reasonable. The model thus furnishes an interesting predictive physical theory for intermittency effects based on the essential mechanism of balance between vorticity production through amplification of the vorticity by local rate-of-strain and vorticity dissipation by viscosity. In contrast with the above findings, recent data of Tabeling et al (1996) show a decrease in S and K with increasing R_λ for their largest values of R_λ, so this question cannot yet be considered to be closed. The recent review of Sreenivasan and Antonia (1997) contains an updated discussion and list of references on this subject.

5 Final Comments

I hope that the above brief account gives the reader some useful insight into the impact of geophysical data on turbulence theory, and that it may encourage others to continue the interactive process by devising new experiments and complementary theories .

Acknowledgements. I thank Dr. Daniel Schertzer and Dr. Edith Falgarone for inviting me to take part in the very stimulating workshop, Astrophysical and Geophysical Fluids: The Impact of Data on Turbulence Theories, held at the Ecole Normale Supérieure, Paris, in November, 1995. Thanks also to Prof. Akiva M. Yaglom, especially for his comments on historical matters, and another, anonymous, reviewer for a variety of useful remarks.

References

Anselmet, F., Gagne, Y. , Hopfinger , E. J. ,and Antonia, R. A., High-order structure functions in turbulent shear flows, *J. Fluid Mech., 140*, 63-89, 1984.

Batchelor, G. K., The theory of homogeneous turbulence, Cambridge University Press, 117-119, 1953

Benzi, R., Ciliberto, S., Tripiccione, R., Baudet, C., Massaioli, F., and Succi, S. , Extended self similarity in turbulent flows , *Phys. Rev. E, 48* , R29, 1993.

Boratav, O.N. and Pelz, R.M., Structures and structure functions in the inertial range of turbulence, preprint, 1996

Cao, N., Chen, S., She, Z-S, Scalings and Relative Scalings in the Navier-Stokes Turbulence, *Phys. Rev. Lett.* , in press, 1996

Chen, S., and Cao, N., A note on inertial range scaling in turbulence, submitted to Phys. Rev. E, 1995

Dubrulle, B. , Intermittency in fully developed turbulence:Log-Poisson statistics and generalized scale covariance, *Phys. Rev. Lett.,73, 7*, 959-962, 1994

Durbin, P.A. , and Speziale, C.G., Local anisotropy in strained turbulence at high Reynolds numbers, *J. Fluids Engr., 113*, 707-709, 1991

Dutton, J. A. and Deaven, D. G., Some properties of atmospheric turbulence. *Statistical Models and Turbulence , Lecture Notes in Physics, 12* ed. M.Rosenblatt and C.W. Van Atta,402-426, 1972 Springer

Frisch, U., Sulem, P-L, and Nelkin, M., A simple dynamical model of intermittent fully developed turbulence , *J. Fluid Mech., 87*,719-736, 1978

Gagne, Y., Hopfinger,E.J., and Frisch, U., A new universal scaling for fully developed turbulence: the distribution of velocity increments, *New Trends in Nonlinear Dynamics and Pattern-Forming Phenomena*, ed. P. Coullet and P. Huerre, 315-319, Plenum 1990

Grant, H. L., Stewart, R. W., and Molliet, A., Turbulence spectra from a tidal channel, *J. Fluid Mech., 12*, 241-268, 1962

Gurvich, A.S., and Yaglom, A.M., Breakdown of eddies and probability distributions for small-scale turbulence, *Phys. Fluids Supp., S59-S64*, 1967

Heisenberg, W., On the theory of statistical and isotropic turbulence, *Proc. Roy. Soc. (London),A195, No.1042*, 402-406, 1948

Kailasnath, P., Sreenivasan, K. R., and Stolovitsky, G., Probability Density of Velocity Increments in Turbulent Flows, *Phys. Rev. Lett.* , *68*, 2766-2769, 1992

Karman, T. von and Howarth , L., On the Statistical Theory of Isotropic Turbulence, *Proc. Roy. Soc.A ,164*, 192-215, 1938

Kida, S. , Log-stable distribution and intermittency in turbulence, *J. Phys. Soc. Japan, 60, 1*, 5-8, 1991

Kollmann, W. , and Pope, S.B. , *Theor. Comp. Fluid Dyn.,1*, 249-285, 1990

Kollmann, W. , and Pope, S.B. , *Theor. Comp. Fluid Dyn.,2*, 255-270, 1991

Kolmogorov, A.N. , Decay of isotropic turbulence in incompressible viscous fluids, *Dokl. Akad. Nauk SSSR, 31,No.6*, 538-541, 1941.

Kolmogorov, A.N. , Energy dissipation in locally isotropic turblence, *Dokl. Akad. Nauk SSSR, 32,No.1*, 19-21, 1941.

Kolmogorov, A.N. , A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number , *J.Fluid Mech.* , *13* , 82-85, 1962

Kraichnan, R. H., Inertial-range spectrum of hydromagnetic turbulence, *Phys.Fluids A,2*, 1385-1387, 1965

Kraichnan, R.H., Small scale structure of a scalar field convected by turbulence, *Phys. Fluids, 11*, 945-953, 1968

Lii, K. S., Rosenblatt, M., and Van Atta, C. W., Bispectral measurements in turbulence, *J. Fluid Mech.,77* , 45-62, 1976

Lundgren, T., A closure hypothesis for the hierarchy of equations for turbulent probability distribution functions,*Statistical Models and Turbulence , Lecture Notes in Physics, 12* ed. M.Rosenblatt and C.W. Van Atta,70-100, 1972 Springer

Lundgren, T. , Strained spiral vortex model for turbulent fine structure, *Phys. Fluids,25*, 2193, 1982

L'vov, V.S. and Procaccia, I., Extended universality in moderate-Reynolds number flows, *Phys. Rev. E, 49*, 4044-4051, 1994

Monin, A.S. , Equation for finite-dimensional probability distributions of a turbulent field, *Dokl. Akad. Nauk USSR, 177, No.5*, 1036-1038, 1967

Nelkin, M. , Inertial range scaling of intense events in turbulence, *Phys. Rev. E,52*, R4610-4611, 1995

Nelkin, M. and Tabor, M. , Time correlations and random sweeping in isotropic turbulence, *Phys. Fluids A,2*,81, 1989

Obukhov, A. M., Spectral energy distribution in a turbulent flow, *Dokl. Akad. Nauk SSSR,32, No.1*, 22-24 , 1941

Obukhov, A.M., Spectral energy distribution in a turbulent flow, *Izv. Akad. Nauk SSSR, Ser. Geogr. i Geofiz.* , *5, No. 4-5*, 453-466, 1941

Obukhov , A.M., Some specific features of atmospheric turbulence, *J. Fluid Mech.,13*, 77-81, 1962

Onsager, L. , The distribution of energy in turbulence (abstr.), *Phys. Rev., 68, No. 11-12*, 286, 1945

Onsager, L. , Statistical Hydrodynamics, Nuovo Cimento, *6, Suppl. No. 2*, 279-287, 1949

Pedrizzetti, G. and Novikov, E.A. , On Markov modelling of turbulence , *J. Fluid Mech., 280*, 69-93, 1994

Pond, S.S., Smith, S.D., Hamblin, P.F., and Burling, R. W.,Spectra of velocity and temperature fluctuations in the atmospheric boundary layer over sea , *J. Atmospheric Sci., 20*, 319-324, 1966

Pullin, D.I., and Saffman, P.G., On the Lundgren-Townsend model of turbulent fine scales, *Phys. Fluids A, 5*, 126-145, 1993

Saito, Y. , Log-gamma distribution model of intermittency in turbulence, *J. Phys. Soc. Japan, 61*, 403, 1992

Schertzer, D., and Lovejoy, S., Physical Modeling and Analysis of Rain and Clouds by Anisotropic Scaling Multiplicative Processes, *J. Geophys. Res., 92, D8*, 9693-9714, 1987

Schertzer, D., Lovejoy, S., and Schmitt , F., Structures in Turbulence and Multifractal Universality, in Small-scale structures in 3D hydro and MHD turbulence, ed. M. Meneguzzi, A. Pouquet, and P.L Sulem, Springer, *Lecture Notes in Physics* , *462*,137-144, 1995

Schmitt, F., Schertzer, D., Lovejoy, S. and Brunet, Y., Empirical study of multifractal phase transitions in atmospheric turbulence, *Nonlinear Processes in Geophysics, 1*,95-104, 1994

Segel, D. , L'vov, V.S., and Procaccia, I., Extended self-similarity in turbulent systems: An analytically soluble example, *Phys. Rev. Lett., 76*, 1828-1831, 1996

She, Z. S., and Leveque, E. , Universal scaling law in fully developed turbulence, *Phys. Rev. Lett., 72*, 336, 1994

She, Z.-S., and Waymire, E.D., Quantized energy cascade and log-Poisson statistics in fully developed turbulence, *Phys. Rev. Lett., 74*, 262-265, 1995

Sreenivasan, K.R., and Antonia, R.A., The phenomenology of small-scale turbulence, *Annual Review of Fluid Mechanics*, 1997, in press

Tabeling, P., Zocchi, G., Belin, F., Maurer, J. and Williame, H., Probability density functions, skewness and flatness in large Reynolds number turbulence, *Phys. Rev. E, 53*, 1613-1621, 1996

Taylor, G. I., Turbulence, *Quart. J. Roy. Met. Soc., 53*, 201-211, 1927

Taylor, G. I., Statistical Theory of Turbulence: Part 1, *Proc. Roy. Soc. A, CLI*, 421-424, 1935

Townsend , A. A., On the fine-scale structure of turbulence,*Proc. R. Soc. London SER. A,208*,534, 1951

Uberoi, M. S, Equipartition of energy and local isotropy in turbulent flows, *J. Appl. Phys.* , *28* , 1165-1170, 1957

Van Atta, C. W., Influence of fluctuations in local dissipation rates on some statistical properties of turbulent scalar fields, *Izv. Acad. Sci. USSR Atmos. and Oceanic Phys.* , *10:7*, 712-719, 1974, English version in Trans. Am. Geophys. Union, *Izv. Acad. Sci. USSR, Atmospheric and Oceanic Physics,10:7*, 443-446, 1974

Van Atta, C. W., Inertial range bispectra in turbulence, *Phys. Fluids, 22*,1440-1442, 1979

Van Atta, C. W., and Antonia, R. A. , Reynolds number dependence of skewness and flatness factors of turbulent velocity derivatives, *Phys. Fluids, 23*, 252-257, 1980.

Van Atta, C. W., and Chen, W. Y., Structure functions of turbulence in the atmospheric boundary layer over the ocean, *J.Fluid Mech., 44*, 145-159, 1970.

Van Atta, C.W. and Park, J., Statistical self-similarity and inertial subrange turbulence,*in Statistical Models and Turbulence , Lecture Notes in Physics, 12*, ed. M.Rosenblatt and C.W. Van Atta, 402-426, 1972 Springer

Van Atta, C. W., and Wyngaard, J. C., On higher-order spectra of turbulence , *J. Fluid Mech., 72*, 673-694, 1975.

von Weizsacker, C.F. , Das Spektrum der Turbulenz bei grossen Reynolds'schen Zahlen, *Z. Physik, 124*, No.7, 614-627, 1948

Yaglom, A.M., The influence of fluctuations in energy disipation on the shape of turbulence characteristics in the inertial interval, *Sov. Phys. Dokl.,11*, 26-29, 1966

Zhou,Y., Praskovsky, A., and Vahala, G., A non-Gaussian phenomenological model for higher-order spectra in turbulence, *Physics Letters A, 178*, 138-142, 1993