Nonlinear Processes in Geophysics (1995) 2: 186 - 193

Nonlinear Processes in Geophysics

What can asymptotic expansions tell us about large-scale quasi-geostrophic anticyclonic vortices?

A. Stegner and V. Zeitlin

LMD, B.P. 99, Université P. et M. Curie, 4, pl. Jussieu, 75252 Paris Cedex 05, France

Received 9 November 1994 - Accepted 17 January 1995 - Communicated by D. Sornette

Abstract.

The problem of the large-scale quasi-geostrophic anticyclonic vortices is studied in the framework of the barotropic rotating shallow-water equations on the β-plane. A systematic approach based on the multiple-scale asymptotic expansions is used leading to a hierarchy of governing equations for the large-scale vortices depending on their characteristic size, velocity and a free surface elevation. Among them are the Charney-Obukhov equation, the intermediate geostrophic model equation, the frontal dynamics equation and some new nonlinear quasi-geostrophic equation. We are looking for steady-drifting axisymmetric anticyclonic solutions and find them in a consistent way only in this last equation. These solutions are soliton - like in the sense that the effects of weak non-linearity and dispersion balance each other. The same regimes on the paraboloidal β-plane are studied, all giving a negative result in what concerns the axi-symmetric steady solutions, except for a strong elevation case where any circular profile is found to be steadily propagating within the accuracy of the approximation.

Correspondence to: V. Zeitlin

1 Introduction

Large-scale anticyclonic vortices widely observed in the atmospheres of the rapidly rotating giant planets (see, for example, a review of Nezlin and Sutyrin (1994))is a spectacular phenomenon which at the same time provides an excellent test of our understanding of the atmospheric dynamics. Starting from the pioneering work of Maxworthy and Redekop (1976) this problem continues to be a focus of an intense theoretical activity based on the ideas of the modern non-linear dynamics. The experimental modelling has a rather long history, too (Antipov et al. , 1982; Read and Hide , 1984; Marcus et al. , 1990) and has been able to reproduce, as it is believed (Nezlin and Sutyrin , 1994), the essential properties of the phenomenon in the simplest setup of the barotropic rotating shallow water. This latter fact encouraged further theoretical efforts within a framework of the shallow-water theory, neglecting the convection phenomena altogether. The specific quasi-elastic collision properties of the solitonic vortices of Maxworthy and Redekop (1976) did not seem to be supported by observations and a new model has been proposed by Petviashvili (1980) being still solitonic in spirit. At the same time a series of papers on the regimes close to geostrophy in shallow-water dynamics (the observed vortices as well as the experimental ones are believed to fall into this class of motions) appeared (McWilliams and Gent , 1980; Romanova and Tseitlin , 1984; Williams and Yamagata , 1984; Williams , 1985)[1] bringing a better theoretical understanding of the problem. Probably, the most important achievement of these papers was a discovery of a so-called intermediate geostrophic regime, to be discussed later. Finally, a possibility of having discontinuous in higher derivatives pressure/elevation profiles of arbitrary amplitude (deviating thereby substantially from the previous weak nonlinearity thinking and being closer to the modon philosophy) has been considered in order to explain some features of experimentally observed vortices (Nycander and Sutyrin , 1992).

Although much work has been done, a satisfactory theoretical explanation of the phenomenon is still lacking, in our view, in the framework of the simplest barotropic shallow-water model as there is no convincing theoretical demonstration that localized, steady-propagating exact anticyclonic solutions do exist. In the present paper we do not claim to give an ultimate solution of the problem, rather, we try to approach it systematically by using the multiple-scale asymptotic expansions based on the smallness of the physical parameters present. We report below some quasi- geostrophic regimes relevant to the observed structures and analyze them from the point of view of existence of the localized axi-symmetric steady solutions both in the spherical and in the paraboloidal geometry. We do find some possibilities to have the required behaviour but we also encounter serious problems while interpreting the laboratory experiments. Our philosophy is basically the same as in the paper of Romanova and Tseitlin (1984) (although below we have chosen to work from the very beginning on the β - plane for simplicity) and consists in meticulous application of the perturbative expansions taking into account the fact that there is a number of small parameters in the problem and, hence, a number of perturbative regimes. The governing equations for vortex evolution appear as integrability conditions as it usually happens in multiple-scale asymptotic expansions.

We work with the well-known system of the barotropic rotating shallow-water equations on the β - plane (the results may be extended to N - layer models or to continuously stratified models along the lines of Romanova and Tseitlin (1985)). We are exploiting the fact that there are three small parameters in the problem: the Rossby number, ϵ ; the geometric parameter, β (the meridional gradient of the Coriolis force) and the nonlinearity parameter, λ (an amplitude of the perturbation). Our starting point is the geostrophy relations which should always hold in the zeroth order of the perturbation theory. Hence, what we are discussing will be called quasi - geostrophy according to the terminology of Romanova and Tseitlin (1984). The quasi - geostrophy in this sense is just given by the relations (1) below. The quasi-geostrophy conditions fix the characteristic scale, r_0 and velocity, v_0 of the vortical structures under investigation, therefore, fixing the relative values of parameters is equal to fixing the scales and velocities with respect to the Rossby radius R_R and Rossby velocity v_R, respectively. A similar study was undertaken long ago by Williams and Yamagata (1984). Below we reproduce some of their results but in addition, due to the different choice of the basic parameters we are able to identify a new regime, apparently missed in previous studies. Our approach also differs from that of Williams and Yamagata (1984) in that our main attention is focused on the existence of exact solutions in both spherical and paraboloidal geometry which allows us to make more detailed statements about solitary vortices.

As a result of an application of the systematic perturbative expansions we obtain the following governing

[1] the second author hereby declares that Zeitlin, Tseitlin(e) and Tseytlin are the different Latin spellings of a same Cyrillic name

equations for the vortex evolution in terms of the free surface elevation:

1. The standard Charney - Obukhov equation for relatively small-scale ($r_0 \sim R_R$), low-amplitude vortices which, as it is well-known, have no monopolar steady solutions.

2. The Petviashvili equation (Petviashvili , 1980) with twist (Romanova and Tseitlin , 1984; Williams and Yamagata , 1984) for large-scale ($r_0 >> R_R$) slowly rotating ($v_0 \sim v_R$) small-amplitude vortices. If the twisting term is merely neglected, as it is sometimes done, the equation possesses only anticyclonic solutions. If this term is consistently removed by introducing a background shear flow with a characteristic velocity of the order of v_R the resulting equation admits only cyclonic steady axisymmetric solutions.

3. A new equation for large-scale ($r_0 >> R_R$), rapidly rotating ($v_0 >> v_R$), small-amplitude vortices which admits strictly anticyclonic axisymmetric steady solutions.

4. The so-called frontal dynamics equation obtained first by Williams and Yamagata (1984) and re-derived later by Cushman-Roisin (1986) for large-scale, rapidly rotating, large- amplitude vortices which leads to a non-compensated nonlinear steepening and eventual breakdown of axisymmetric structures.

If we pass, according to the formalism developed in (Romanova and Tseitlin , 1984; Nycander , 1993) from spherical to paraboloidal geometry which corresponds to the most adapted experimental way to study the barotropic shallow-water system (Antipov et al. , 1982) we find that while p.1 above still holds, the governing equations in pp.2, 3, 4 lose the characteristic scalar non linearity term. As a consequence, they do not have localized steady axi-symmetric solutions anymore, except for the frontal dynamics equation which admits any axisymmetric profile as a solution in this case without any distinction between cyclonic and anti-cyclonic ones.

2 Quasi-geostrophic regimes on the β - plane

Introducing the characteristic horizontal scales and velocities of vortex structures, r_0, v_0, the vertical scale H_0: $H = H_0(1+\lambda h)$, where H is a surface elevation, using $t_0 = r_0/v_0$ as a time scale, remembering that by definition the Rossby number is $\epsilon = v_0/2\Omega r_0$, non-dimensionalizing and taking into account the quasi-geostrophy conditions (Romanova and Tseitlin , 1984)

$$\epsilon << 1, \quad \frac{\lambda g H_0}{2\Omega r_0 v_0} = \frac{\lambda}{\epsilon}\left(\frac{R_R}{r_0}\right)^2 = \mathcal{O}(1) \tag{1}$$

we arrive to a following system of equations:

$$\begin{aligned} \epsilon Du - v(1+\beta y) &= -h_x \\ \epsilon Dv + u(1+\beta y) &= -h_y \\ \lambda Dh + (u_x+v_y)(1+\lambda h) &= 0. \end{aligned} \tag{2}$$

Here x, y are the longitudinal and latidudinal coordinates on the β - plane, corresponding subscripts denote the partial derivatives, u, v are the horizontal velocity components, D denotes the horizontal Lagrangian derivative: $D = \partial_t + u\partial_x + v\partial_y$ and all numerical factors of order unity are omitted (absorbed into dependent and independent variables). In what follows we shall apply the perturbation theory to (2) but in order to do it systematically we have to specify the ratios of the parameters ϵ, β, λ. We note first that (1) together with the definition of Rossby velocity as the phase velocity of long Rossby waves are equivalent to

$$r_0 = \mathcal{O}\left(\left(\frac{\lambda}{\epsilon}\right)^{1/2} R_R\right), \quad v_0 = \mathcal{O}\left(\frac{\lambda}{\beta}\, v_R\right) \tag{3}$$

and, hence, the parameters' ratios determine the characteristic scales and velocities of vortices. For example, in the "standard" case when all the small parameters are of the same order, the characteristic scales and velocities are the Rossby ones. Introducing $\epsilon = \lambda = \beta$ (i.e. neglecting possible numerical factors of order unity which may be easily reconstructed) and developing all dynamical variables in perturbation series:

$$\begin{aligned} u &= u^{(0)} + \epsilon u^{(1)} + \epsilon^2 u^{(2)} + \ldots \\ v &= v^{(0)} + \epsilon v^{(1)} + \epsilon^2 v^{(2)} + \ldots \\ h &= h^{(0)} + \epsilon h^{(1)} + \epsilon^2 h^{(2)} + \ldots \end{aligned} \quad (4)$$

we get the geostrophic balance equations

$$v^{(0)} = h_x^{(0)};\ \ u^{(0)} = -h_y^{(0)} \quad (5)$$

in the zeroth order in ϵ and the classical Charney - Obukhov equation in the first order:

$$h_t^{(0)} - \Delta h_t^{(0)} - J(h^{(0)}, \Delta h^{(0)}) - h_x^{(0)} = 0. \quad (6)$$

Here $J(...,...)$ denotes the Jacobian. As it is well-known, this equation does not admit monopolar steady solutions. Note that, as it follows from (1), the caracteristic scale of vortices in this case is of order R_R. So this is consistent with observations and experiment where the characteristic scale of the steady vortices is much greater than R_R. To get such a regime we need to have $\lambda >> \epsilon$. There are two possibilities here: $\lambda = \mathcal{O}(1)$ or $\epsilon << \lambda << 1$ and we shall consider both of them.

2.1 The intermediate geostrophic regime

Let us take $\epsilon \sim \lambda^2$. There is still a freedom in the choice of β. Taking $\beta \sim \lambda$ we recover the intermediate geostrophic regime (Romanova and Tseitlin , 1984; Williams and Yamagata , 1984), i.e. the one for large-scale, but relatively slowly rotating ($v_0 \sim v_r$) structures. We shall not repeat the perturbative calculatons in this case - they are the same as in the above - mentioned papers. Note only that instead of ϵ, it is λ now which appears in the perturbative expansions (4). In the zeroth order in λ we have the geostrophic balance (5) and in the first order we get an equation for a dispersionless propagation of the long Rossby waves

$$h_t^{(0)} - h_x^{(0)} = 0 \quad (7)$$

meaning that any perturbation will just drift westward in this approximation. Finally, in the second order we get a dynamical equation for $h^{(1)}$:

$$\begin{aligned} h_t^{(1)} \ - \ h_x^{(1)} &= -h_\tau^{(0)} + \Delta h_t^{(0)} + h^{(0)} h_x^{(0)} + \\ & J(h^{(0)}, \Delta h^{(0)}) - 2y h_x^{(0)} \end{aligned} \quad (8)$$

where we have introduced a slow time scale $\tau = \lambda t$. Note the appearance of the scalar nonlinearity and the twisting term explicitly dependent on y in the r.h.s. In order to suppress unbounded in t solutions (or, in other words, to suppress the long Rossby waves radiation from the perturbation $h^{(0)}$) we have to put the r.h.s. of this equation to be equal to zero (integrability condition) and, thus, arrive to the following governing equation - the Petviashvili equation with twist (Petviashvili , 1980; Romanova and Tseitlin , 1984; Williams and Yamagata , 1984)

$$h_\tau^{(0)} - \Delta h_x^{(0)} - h^{(0)} h_x^{(0)} - J(h^{(0)}, \Delta h^{(0)}) + 2y h_x^{(0)} = 0 \quad (9)$$

(we changed the t - derivative for the x - derivative in the Laplacian term using the fact that $h^{(0)}$ is a solution of (7). If we just drop out the twist the resulting equation will have an anticyclonic exact solution. However, if we try to eliminate this term in a consistent manner the situation changes drastically. Indeed, equation (7) admits any steady drifting profile as a solution to which we may add an arbitrary function of y. If we choose this function as to compensate the twist by the scalar nonlinearity contribution we immediately see that this leads to a change of sign of the Laplacian term with respect to the original equation (9) due to the Jacobian term. The fact that there exist localized, steady-drifting axisymmetric solutions is based on the observation that for those latter the equation takes a form (we discuss the equation resulting from the elimination of twist and drop the superscript "0" for brevity)

$$(ch - \frac{h^2}{2} + h'' + \frac{h}{r})_x = 0 \quad (10)$$

Here r - is the radial coordinate, a prime denotes the corresponding derivative, c is a first order correction to the propagation speed along the x - axis, $h = h(x + t + c\tau)$. This equation, after integrating it once in x becomes an equation of a material point moving in the potential

$$V(h) = -\frac{h^3}{6} + \frac{ch^2}{2} \tag{11}$$

(we put an integration constant to be equal to zero as we are looking only for solutions having zero asymptotics at infinity) with a specific time - dependent friction ("time" $\equiv r$). The localized solution is close to the separatrix of the frictionless equation and approaches it asymptotically as $r \to \infty$ (see, e.g. Finkelstein et al. (1951)). The potential (11) is not symmetric with respect to transformation $h \to -h$ which is the reason for the cyclone - anticyclone asymmetry. Indeed, as $V(h)$ is positive for large negative h and vice-versa the separatrix solution starts from some negative h at $r = 0$ and ends up either with $h = 0$ for negative c, or with some positive h for positive c. This latter case does not satisfy boundary conditions and should be discarded. Hence, the exact axisymmetric localized solutions of the Petviashvili equation with twist exist only on the background of the fine-tuned shear flow and are cyclones.

2.2 The non-linear quasi-geostrophic regime

Now what happens if within the same assumption $\epsilon \sim \lambda^2$ we choose another possibility for large-scale vortices, namely, $\beta \sim \epsilon$? In this case we have to develop in λ and in the zeroth order we have a geostrophic balance (5) for zeroth order fields. In the first order we have a geostrophic balance for the first order fields

$$v^{(1)} = h_x^{(1)}; \quad u^{(1)} = -h_y^{(1)} \tag{12}$$

and a "dynamical equation" $h_t^{(0)} = 0$ from the third equation in (2). This simply means that our time-scale is badly chosen. Introducing $\tau = \lambda t$ and continuing perturbative calculations we get

$$\begin{aligned} v^{(2)} &= h_x^{(2)} - v^{(0)}y + u^{(0)}u_x^{(0)} + v^{(0)}u_y^{(0)} \\ u^{(2)} &= -h_y^{(2)} - u^{(0)}y - u^{(0)}v_x^{(0)} - v^{(0)}v_y^{(0)} \end{aligned} \tag{13}$$

Using the zeroth and the first order quasi-geostrophic relations (5), (12) and divergence

$$u_x^{(2)} + v_y^{(2)} = -h_x^{(0)} - J(h^{(0)}, \Delta h^{(0)}) \tag{14}$$

calculated from (13) we get a dynamical equation for $h^{(0)}$:

$$h_\tau^{(0)} - J(h^{(0)}, \Delta h^{(0)}) - h_x^{(0)} = 0, \tag{15}$$

a "dispersionless β - plane vorticity equation", which is already non-linear, in comparison with (7), but the non-linearity vanishes for axisymmetric profiles and we again have a mere drift of these latter. In the next, third order after a similar but a bit more cumbersome calculation we get an evolution equation for $h^{(1)}$

$$\begin{aligned} h_\tau^{(1)} - J(h^{(0)}, \Delta h^{(1)}) - J(h^{(1)}, \Delta h^{(0)}) - h_x^{(1)} = -h_\sigma^{(0)} &+ \\ \Delta h_x^{(0)} + h^{(0)}h_x^{(0)} + J(\frac{h^{(0)2}}{2}, \Delta h^{(0)}) - J(h^{(0)}, \frac{(\nabla h^0)^2}{2}) & \quad (1 \end{aligned}$$

where we have introduced a next slow time-scale $\sigma = \lambda^2 t$. An integrability condition is

$$\begin{aligned} -h_\sigma^{(0)} + \Delta h_x^{(0)} + h^{(0)}h_x^{(0)} + J(\frac{h^{(0)2}}{2}, \Delta h^{(0)}) &+ \\ J(h^{(0)}, \frac{(\nabla h)^2}{2}) &= 0(17) \end{aligned}$$

Note that the first three terms of this equation are the same as in the Petviashvili equation. The Jacobian terms are different but they vanish identically for axisymmetric profiles. For steady translating solutions we get

$$(h'' + \frac{h'}{r} + \frac{h^2}{2} - ch)_x = 0 \tag{18}$$

where we, as before, have dropped superscripts and introduced a radial coordinate. This is the same equation as the one resulting from Petviashvili equation without twist and the same reasoning as above leads to a conclusion that there exist anticyclonic solutions decaying at infinity for $c > 0$, i.e. moving slightly faster ($v_{drift} = (1+\lambda c)v_R$) than the long Rossby waves. Hence, the cyclone - anticyclone asymmetry manifests itself very clearly: one can balance (weak) non-linearity and dispersion only for anticyclones.

Note, however, that although we have got an exact solution of (17) it is clearly not an exact solution of the full shallow water equations. What we can only guarantee is that such a profile, once created, will have a characteristic life-time much greater than $\epsilon^{-1}T$ and much less than $\epsilon^{-2}T$), where T is the vortex turnover time, $T \sim t_0$. To draw any conclusion about the solution's behaviour on slower time-scales one has to consider the next orders of perturbation theory. However, already at the order ϵ^2 we run out of the domain of applicability of the beta-plane equations (2) since corrections due to sphericity of the planet are of the same order. As these latter introduce an explicit y - dependence into the governing equations, which seems to be incompatible with the axial symmetry of solutions, we never go beyond the $\mathcal{O}(\epsilon)$ velocity fields in the present study.

2.3 The strong nonlinearity regime

Let us consider, finally, the case $\lambda = \mathcal{O}(1)$. In the zeroth order we have the geostrophic balance and the following equation for the elevation variable: $h_t^{(0)} = 0$. Introducing a new time scale $\tau = \epsilon t$ we get the same equation as (13), but now for the first-order in ϵ fields, and also a dynamical equation for $h^{(0)}$

$$h_\tau^{(0)} - (1+h^{(0)})h_x^{(0)} - (1+h^{(0)})J(h^{(0)}, \Delta h^{(0)}) - J(h^{(0)}, \frac{(\nabla h^{(0)})^2}{2}) = 0. \qquad (19)$$

This equation (a so called frontal dynamics equation) appeared first in (Williams and Yamagata , 1984) and was re-derived later by Cushman-Roisin (1986). For axisymmetric profiles it becomes a simple wave equation

$$h_\tau^{(0)} - (1+h^{(0)})h_x^{(0)} = 0 \qquad (20)$$

and gives a non-compensated nonlinear steepening and breakdown of the wave.

To conclude, we see that the only regime being able to give the large-scale long-living axisymmetric anticyclons within the quasi-geostrophic shallow-water theory on the β - plane is the non-linear quasi- geostrophic one.

3 Quasi-geostrophic regimes on the paraboloidal β - plane

If one considers a rotating shallow-water layer on the surface of a paraboloid (Nezlin and Sutyrin , 1994), (Antipov et al. , 1982), (Nycander , 1993) the corresponding β - plane equations differ from the standard ones (2) due to the y - dependence of the normal acceleration (a so-called "γ - effect") and to the fact that, unlike the sphere, there are two independent curvature radii. The equations read (Nycander , 1993):

$$\begin{aligned} \epsilon Du - v(1+\beta b_2 y) + (1+\beta(b_1-b_2)y)h_x &= 0 \\ \epsilon Dv + u(1+\beta y) + h_y - \beta b_2 h &= 0 \qquad (21) \\ \lambda Dh + [(1+\beta b_1 y)u_x + (1+\beta b_2 y)v_y &- \\ \beta b_1 v](1+\lambda h) &= 0. \end{aligned}$$

Here the two β - factors $b_1 \neq b_2$ correspond to the two above-mentioned independent curvature components (Nycander , 1993). All the calculations of the previous section may be repeated taking into account the additional contributions. The results are as follows: the standard case remains unchanged - we get the Charney - Obukhov equation with b_2 playing the role of β. As to the other regimes discussed in the previous section, a curious cancellation takes place due to the γ - effect and they lose the scalar nonlinearity term, as it was first noticed by Nycander (1993). This has drastic consequences for the intermediate geostrophic regime as there is no mean to compensate an explicit y - dependence in the equation anymore. So any initially localized solution will be destroyed by the twist. What is more important, equation (17) loses the scalar nonlinearity, too. Hence, for axisymmetric structures it will give a non-compensated dispersive destruction of a vortex. On the contrary, in the frontal dynamics case the disappearance of the scalar nonlinearity means that there is no nonlinearity at all for any axi-symmetric profile (see (20)). Hence, any such profile drifts steadily within the accuracy of the approximation, i.e. at least at time-scales of order $\epsilon^{-1}T$. There is no distinction between positive and negative signs of vorticity at this stage and, hence, no cyclone - anticyclone asymmetry.

Thus, due to specific properties of the paraboloidal geometry there is no quasi-geostrophic regime where the effects of weak non-linearity and dispersion would be compensated for axi-symmetric profiles. For strong nonlinearities we find a degenerate situation where in order to get a non-trivial equation (with a hope to distinguish between positive and negative vorticities) we have to go beyond the beta-plane approximation where (21) is not valid anymore.

We have indeed calculated up to the order $\epsilon^2, \epsilon\beta, \beta^2$ and obtained the y -dependent terms incompatible with axial symmetry in the equation for h^0. The detailed analysis at this order will be presented elsewhere. As to non-axisymmetric profiles, a question arises whether a Jacobian cubic nonlinearity present both in the frontal dynamics and non-linear geostrophic dynamics allows for any exact solution of these equations. There are some indications that large-scale non-axisymmetric solutions might exist (Nycander and Sutyrin , 1992) but a systematic perturbative analysis, especially in the presence of discontinuities where matched asymptotic expansions are needed, has not been applied yet.

4 Discussion

Thus, we have investigated the problem of the large-scale anticyclonic vortices in the framework of the rotating shallow-water model both on the sphere and on the paraboloid. We identify only one, namely non-linear quasi-geostrophic, regime where axisymmetric steady-propagating vortices exist as exact solutions of integrability conditions for multiple-scale perturbative expansions in spherical geometry. The cyclone - anticyclone asymmetry finds its natural explanation within this regime.

On the contrary, in the paraboloidal geometry the γ - effect prevents a mutual compensation of the weak nonlinearity and dispersion at least for axisymmetric structures. Nevertheless, strongly non-linear axi-symmetric vortices of both signs are shown to drift without change of form for long enough times.

If we want now to compare our results with experiment (the most comprehensive review is given by Nezlin and Snezhkin (1993)) we see that there are certain problems here. First of all, the life-times of the observed vortices lie within the interval $[\epsilon^{-1}T, \epsilon^{-2}T]$. Second, the elevations may be substantial. One may, thus, believe that the strong nonlinearity regime on the paraboloid is consistent with experiment. But in this case the observed cyclone - anticyclone asymmetry remains unexplained in the framework of asymptotic expansions. The center-of-mass arguments together with estimates of the higher- order terms in velocity field (Nycander , 1993) indicate that these latter may be responsible for the asymmetry. However, after having calculated all the terms of this order, i.e. $\mathcal{O}(\epsilon^2), \mathcal{O}(\epsilon\beta), \mathcal{O}(\beta^2)$, in the elevation equation we do not see how to separate a constructive influence of such terms from a destructive one, due to their explicit coordinate dependence. We hope that a more detailed study of non-axisymmetric structures, which is in progress now, may clarify the situation along with a further experimental study of the deviations from the axial symmetry of vortices. As to the direct application of these results to the planetary vortices, two comments are in order. First, in Nature these vortices exist on the background of a shear flow and are elongated along the shear axis. So the first task is to include these two (probably, related) effects into consideration. Second, the asymptotyc analysis based on the simplest β - plane shallow water model cannot guarantee, as it was mentioned, the vortex lifitemes greater than $\epsilon^{-2}T$. So the reason of the miraculous longevity of the Great Red Spot which is many orders of magnitude greater than this last estimate is still to be understood.

Acknowledgements

We thank M.V. Nezlin, J. Nycander and G.G. Sutyrin for discussions.

References

M.V. Nezlin and G.G. Sutyrin, 1994, Surveys in Geophys. **15**, 63 - 99.

T. Maxworthy and L.G. Redekop, 1976, Icarus **29**, 261 - 273.

S.V. Antipov , M.V. Nezlin, E. N. Snezhkin and A. S. Trubnikov 1982, Sov. Phys. JETP **55**, 85 - 95.

P.L. Read and R. Hide, 1984, Nature **308**, 45 -48.

P. Marcus *et al*, 1990 Nature **343**, 517 - 518.

V.I. Petviashvili, 1980, JETP Letters **32**, 632 - 635.

J.C. McWilliams and P.R. Gent, 1980, J. Atmos. Sci. **37**, 1657 - 1678.

N.N. Romanova and V. Yu. Tseitlin, 1984, Izvestija Atm.Ocean. Physics **20**, 85 - 91.

G.P. Williams and T. Yamagata, 1984, J. Atmos. Sci. **41**, 453 - 478.

G.P. Williams, 1985, J. Atmos. Sci. **42**, 1237 - 1243.

J. Nycander and G.G. Sutyrin, 1992, Dyn. Atmos. Oceans **16**, 473 - 498.

N.N. Romanova and V.Yu. Tseitlin, 1985, Izvestija Atm. Ocean. Phys **21**, 627 - 630.

B. Cushman-Roisin, 1986, J. Phys. Oceanogr. **16**, 132 - 143.

J. Nycander, 1993, J.Fluid Mech. **254**, 561 - 577.

R. Finkelstein *et al* Phys. Rev., 1951, **83**, 326 - 333.

M.V. Nezlin and E.N. Snezhkin "Rossby Vortices, Spiral Structures, Solitons ", Springer, 1993.