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Effect of thermal pressure on upward plasma fluxes due to ponderomotive force of Alfvén waves

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Abstract. We consider the action of the ponderomotive force of low-frequency Alfvén waves on the distribution of the background plasma. It is assumed that the ponderomotive force for traveling waves arises as a result of the background inhomogeneity of medium under study. Expressions for the ponderomotive force obtained in this paper differ from previous analogous results. The induced magnetic moment of medium is taken into account. It is shown that the well-known Pitayevsky's formula for the magnetic moment is not complete. The role of the induced nonlinear thermal pressure in the evolution of the background plasma is considered. We give estimations for plasma displacement due to the long- and short-acting nonlinear wave perturbations. Some discussion of the ponderomotive action of standing waves is provided.

1 Introduction

A study of the ponderomotive forces induced by the geomagnetic pulsations in the magnetospheric plasma has attracted a great attention in past years (e.g. Allan, 1992, 1994; Guglielmi et al., 1993, 1995; Witt et al., 1995; Pokhotelov et al., 1996; Feygin et al., 1998). These forces can arise as a result of nonlinear interaction of pulsations with an inhomogeneous background plasma. Ponderomotive forces can play an important role in the modification of the magnetospheric plasma. It can be expected that the longitudinal component (along the background magnetic field) of the ponderomotive force could significantly influence on the distribution of the magnetospheric plasma along the magnetic field lines, despite of the relatively small amplitudes of these pulsations in the magnetosphere (of the order of nT and less). For example, the ponderomotive action of geomagnetic pulsations of the Pc 1 type in the Earth's magnetosphere has been considered in Guglielmi et al. (1993, 1995), Pokhotelov et al. (1996), and Feygin et al. (1998). It is known that the Pc 1 pulsations have the maximum of the magnetic amplitude in the vicinity of the magnetic equator. Therefore, it should be expected that the most essential ponderomotive effect on the state of the background plasma could occur in this region. In papers by Guglielmi et al. (1993, 1995), it has been demonstrated that when the amplitude of the Pc 1 pulsations exceeds some critical value the pronounced maximum of the plasma density is formed in the vicinity of the equator where the dipole magnetic field has a minimum. Ponderomotive effects in the two-ion plasma (H^+ and heavy ions, e.g., He^+) have been studied in Feygin et al. (1998). We also note that observations of the plasma density in the plasmapause region carried out by the paired satellites DE 1 and DE 2 recording the magnitude of the plasma density at high and low altitudes simultaneously have revealed the existence of the local minimum of the cold plasma at the magnetic equator (Olsen, 1992).

However, the Pc 1 pulsations are the narrow packets of the electromagnetic Alfvén ion cyclotron waves driven by the proton cyclotron instability in Earth's radiation belts. These packets oscillate along the magnetic field lines between the conjugate points. It is obvious that such packets can only result in the finite displacement of plasma due to ponderomotive action of the forward and backward fronts of the packet. Therefore, it is difficult to wait that this type of pulsations can considerably modify the magnetospheric plasma.

To cause a considerable modification of the magnetospheric plasma, a wave must be sufficiently long along the magnetic field lines and operate a rather long time. Suitable candidates for such waves are the PC 4–5 pulsations in the frequency range 1.5–10 mHz. These pulsations appear to be a fundamental odd mode of the field line resonances with a magnetic node at the equator (Singer and Kivelson, 1979).

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The amplitudes of these pulsations can attain from several nT to several dozens nT. In the conjugate points, they are observed simultaneously. Their amplitude at the level of the Earth is 3–6 times larger than that in the equatorial plane of the magnetosphere. A study of the transverse component of the Pc 4–5 pulsations with the satellite OGO 5 has shown that they are mainly observed at magnetic latitudes above 10 degrees (Kokubun et al., 1976). Consequently, the equatorial plane is a nodal point for these pulsations. These results have been conformed by observations of the long-period waves with satellites GOES-8 and GOES-10. Both satellite measurements were performed near the magnetic equator, where the magnetic amplitude of the odd harmonics of the standing Alfvén waves has a minimum. The magnetic latitude of GOES-8 was 5 degrees higher than that of GOES-10. The spectrum amplitude maximum of the transverse waves observed with GOES-8 was 0.25 nT above the measurement of the second satellite. This fact indicates an increase of the amplitude of these pulsations with magnetic latitude (Zolotukhina, 2009).

In the present paper, we consider the effect of the ponderomotive force of low-frequency Alfvén waves propagating along the geomagnetic field lines on the background plasma distribution. We assume that the ponderomotive force arises due to the background density and magnetic field inhomogeneities. Our expression for the ponderomotive force differs from analogous results obtained in Guglielmi et al. (1993, 1995). We take into account, as in Guglielmi et al. (1993, 1995), the contribution to the ponderomotive force from the magnetic moment of medium induced by the high-frequency electromagnetic field (Pitayevsky, 1960). In particular, we show that the well-known Pitayevsky's formula for the induced magnetic moment is not complete. This formula takes into account only one part of the quasi-stationary nonlinear current and does not include the other part connected with the quasi-stationary nonlinear velocities. We discuss the influence of the induced nonlinear thermal pressure on the evolution of the background plasma. We give estimations for the plasma displacement due to the long- and short-acting nonlinear wave perturbations.

The paper is organized as follows. In Sect. 2, the basic equations are given. In Sect. 3, we calculate the total magnetic moment induced by the circularly polarized electromagnetic waves traveling along the background magnetic field. The expression for the ponderomotive force of these waves is deduced in Sect. 4. In Sect. 5, we discuss the role of the induced nonlinear thermal pressure in the evolution of the background plasma. The plasma displacement due to the short-acting waves is given in Sect. 6. The ponderomotive action of standing waves is shortly discussed in Sect. 7. In Sect. 8, we summarize our results.

2 Basic equations

We consider the low-frequency finite amplitude electromagnetic waves propagating or standing along the geomagnetic field lines. We use the ideal magnetohydrodynamics and Maxwell's equations which have the following form:

$$m_{\mathrm{i}}\frac{d\boldsymbol{v}_{\mathrm{i}}}{dt}=-\frac{\nabla p_{\mathrm{i}}}{n_{\mathrm{i}}}+m_{\mathrm{i}}\boldsymbol{g}+e(\boldsymbol{E}_0+\boldsymbol{E})+\frac{e}{c}\boldsymbol{v}_{\mathrm{i}}\times(\boldsymbol{B}_0+\boldsymbol{B}) \qquad (1)$$

and

$$\boldsymbol{0}=-\frac{\nabla p_{\mathrm{e}}}{n_{\mathrm{e}}}-e(\boldsymbol{E}_0+\boldsymbol{E})-\frac{e}{c}\boldsymbol{v}_{\mathrm{e}}\times(\boldsymbol{B}_0+\boldsymbol{B}) \qquad (2)$$

are the momentum equations,

$$\frac{\partial n_j}{\partial t}+\nabla\cdot n_j\boldsymbol{v}_j=0 \qquad (3)$$

is the continuity equation,

$$\frac{\partial p_j}{\partial t}+\boldsymbol{v}_j\cdot\nabla p_j+\gamma p_j\nabla\cdot\boldsymbol{v}_j=0 \qquad (4)$$

is the pressure equation. Electrodynamic Maxwell's equations are

$$\nabla\times\boldsymbol{E}=-\frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t} \qquad (5)$$

and

$$\nabla\times\boldsymbol{B}=\frac{4\pi}{c}\boldsymbol{j}+\frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t}, \qquad (6)$$

where the current $\boldsymbol{j}$ is

$$\boldsymbol{j}=e(n_{\mathrm{i}}\boldsymbol{v}_{\mathrm{i}}-n_{\mathrm{e}}\boldsymbol{v}_{\mathrm{e}}). \qquad (7)$$

In Eqs. (1)–(7), the indices "i" and "e" denote the ions and electrons, respectively, $j=$i,e, e and m_{i} are the ion charge and mass ($-e$ is the electron charge, $m_{\mathrm{e}}=0$), $\boldsymbol{v}_j$ is the hydrodynamic velocity, n_j is the number density, p_j is the thermal pressure, γ is the adiabatic constant, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the wave and induced nonlinear electric and magnetic fields, $\boldsymbol{E}_0$ is the background electric field, $\boldsymbol{B}_0$ is the geomagnetic field, $\boldsymbol{g}$ is the gravitational force, c is the speed of light in vacuum, and $d/dt\equiv\partial/\partial t+\boldsymbol{v}_{\mathrm{i}}\cdot\nabla$. For generality, we take into account the displacement current in Eq. (6).

3 Diamagnetic field

The first question we would like to discuss is the expression for the quasi-stationary nonlinear magnetic field induced by the electromagnetic wave. This field can be found from the time-averaged Eqs. (5) or (6). Let us consider, for example, the averaged Eq. (6)

$$\nabla\times\langle\boldsymbol{B}_2\rangle=\frac{4\pi}{c}\langle\boldsymbol{j}_2\rangle, \qquad (8)$$

where $\langle \boldsymbol{B}_2 \rangle$ is the induced magnetic field (magnetic moment), $\langle \boldsymbol{j}_2 \rangle$ is the quasi-stationary nonlinear current, the angle brackets $\langle ... \rangle$ denote the time-averaging, and the index 2 denotes the nonlinear values. The well-known expression for the induced magnetic field in a cold plasma is the following (Pitayevsky, 1960):

$$\langle \boldsymbol{B}_{21} \rangle = \frac{1}{4} \frac{\partial \varepsilon_{ik}}{\partial \boldsymbol{B}_0} E^*_{10i} E_{10k}, \tag{9}$$

where $\boldsymbol{E}_{10}$ is the complex amplitude of the wave electric field, ε_{ik} is the tensor of the dielectric permeability, and the sign $*$ denotes the complex conjugate. This expression has been used, in particular, in Guglielmi et al. (1993, 1995). However, we will show below that Eq. (9) describes only one part of $\langle \boldsymbol{B}_2 \rangle$.

We see from Eq. (7) that the nonlinear current $\langle \boldsymbol{j}_2 \rangle$ can be written in the form

$$\langle \boldsymbol{j}_2 \rangle = \langle \boldsymbol{j}_{21} \rangle + \langle \boldsymbol{j}_{22} \rangle,$$

where

$$\langle \boldsymbol{j}_{21} \rangle = e \left(\langle n_{\mathrm{i}1} \boldsymbol{v}_{\mathrm{i}1} \rangle - \langle n_{\mathrm{e}1} \boldsymbol{v}_{\mathrm{e}1} \rangle \right),$$

$$\langle \boldsymbol{j}_{22} \rangle = e n_0 \left(\langle \boldsymbol{v}_{\mathrm{i}2} \rangle - \langle \boldsymbol{v}_{\mathrm{e}2} \rangle \right).$$

Here n_{j1} and $\boldsymbol{v}_{j1}$ are linear disturbances of the density and velocity, $\left\langle \boldsymbol{v}_{j2} \right\rangle$ is the nonlinear time-averaged velocity, n_0 is the background number density (we assume that the background velocities of species are equal to zero). We find, at first, the current $\langle \boldsymbol{j}_{21} \rangle$. Let the background magnetic field $\boldsymbol{B}_0$ be directed along the z-axis and the electromagnetic wave field depend mainly on the z-coordinate and only weakly, say, on the y-coordinate. We here discuss a general physical problem which is independent on the specific choice of the coordinate system. It can be some local system in the magnetosphere or laboratory. For simplicity, we only have chosen the y-dependence of the wave amplitude. This does not exclude a possible x-dependence and has no influence on the result obtained below. We further consider equations of motion (1) and (2) and continuity Eq. (3) in the linear approximation over the wave amplitude. We find n_{j1}, $v_{j1\mathrm{x}}$, and, consequently, $\left\langle n_{j1} v_{j1\mathrm{x}} \right\rangle$ (the index 1 denotes the linear values). We consider the wave which travels along the background magnetic field and has the circular polarization. Then, we obtain for the wave with a given frequency ω (here and below $\omega > 0$)

$$\langle n_{\mathrm{i}1} v_{\mathrm{i}1\mathrm{x}} \rangle = -\frac{e^2 n_0}{4 m_{\mathrm{i}}^2} \frac{\sigma}{\omega (\omega_{\mathrm{i}} - \sigma \omega)^2} \frac{\partial \left\langle \boldsymbol{E}_1^2 \right\rangle}{\partial y}, \tag{10}$$

where $\omega_{\mathrm{i}} = e B_0 / m_{\mathrm{i}} c$ is the ion cyclotron frequency, $\boldsymbol{E}_1$ is the wave electric field, $\sigma = \pm 1$ marks the left- ($\sigma = +1$, Alfvén ion cyclotron waves) or right- ($\sigma = -1$, magnetosonic waves) polarization. The expression for $\langle n_{\mathrm{e}1} v_{\mathrm{e}1\mathrm{x}} \rangle$ can be obtained from Eq. (10) with $m_{\mathrm{i}} \to 0$. Calculating the nonlinear current $\langle j_{21\mathrm{x}} \rangle$ and substituting it into Eq. (8), we find expression for $\langle B_{21\mathrm{z}} \rangle$

$$\langle B_{21\mathrm{z}} \rangle = -\frac{1}{4} \frac{\omega_{\mathrm{pi}}^2 (2\omega_{\mathrm{i}} - \sigma \omega)}{\omega_{\mathrm{i}} (\omega_{\mathrm{i}} - \sigma \omega)^2} \frac{\left\langle \boldsymbol{E}_1^2 \right\rangle}{B_0}, \tag{11}$$

where $\omega_{\mathrm{pi}} = \left(4\pi n_0 e^2 / m_{\mathrm{i}} \right)^{1/2}$ is the ion plasma frequency. It is easy to verify that expression (11) follows from Eq. (9) for the wave under consideration. Thus, Pitayevsky's formula (9) takes only into account the current $\langle \boldsymbol{j}_{21} \rangle$.

Now we calculate the nonlinear current $\langle \boldsymbol{j}_{22} \rangle$. By considering equations of motion for ions and electrons in the second approximation over the wave amplitude, we obtain

$$\langle \boldsymbol{v}_{\mathrm{i}2} \rangle - \langle \boldsymbol{v}_{\mathrm{e}2} \rangle = -\frac{1}{\omega_{\mathrm{i}} B_0} \langle \boldsymbol{A}_2 \rangle \times \boldsymbol{B}_0, \tag{12}$$

where

$$\boldsymbol{A}_2 = \boldsymbol{v}_{\mathrm{i}1} \cdot \nabla \boldsymbol{v}_{\mathrm{i}1} - \frac{e}{m_{\mathrm{i}} c} (\boldsymbol{v}_{\mathrm{i}1} - \boldsymbol{v}_{\mathrm{e}1}) \times \boldsymbol{B}_1.$$

Here $\boldsymbol{B}_1$ is the wave magnetic field. In our case, we must find the value $\left\langle A_{2y} \right\rangle$. Carrying out the necessary calculations and taking into account Eq. (5), we find

$$\left\langle A_{2y} \right\rangle = \frac{e^2}{4 m_{\mathrm{i}}^2} \frac{\sigma \omega}{\omega_{\mathrm{i}} (\omega_{\mathrm{i}} - \sigma \omega)^2} \frac{\partial \left\langle \boldsymbol{E}_1^2 \right\rangle}{\partial y}. \tag{13}$$

Substituting Eq. (13) into Eq. (12), we find the current $\langle j_{22\mathrm{x}} \rangle$. Using Eq. (8), we obtain

$$\langle B_{22\mathrm{z}} \rangle = -\frac{1}{4} \frac{\omega_{\mathrm{pi}}^2 \sigma \omega}{\omega_{\mathrm{i}} (\omega_{\mathrm{i}} - \sigma \omega)^2} \frac{\left\langle \boldsymbol{E}_1^2 \right\rangle}{B_0}. \tag{14}$$

This term is not taken into account by Eq. (9). Thus, the total induced quasi-stationary nonlinear magnetic field is the sum of Eqs. (11) and (14) and equals to

$$\langle B_{2\mathrm{z}} \rangle = \langle B_{21\mathrm{z}} \rangle + \langle B_{22\mathrm{z}} \rangle = -\frac{1}{2} \frac{\omega_{\mathrm{pi}}^2}{(\omega_{\mathrm{i}} - \sigma \omega)^2} \frac{\left\langle \boldsymbol{E}_1^2 \right\rangle}{B_0}. \tag{15}$$

We note that when $\omega \to \omega_{\mathrm{i}}$ ($\sigma > 0$) expressions (11) and (14) are equal to each other. Thus, Pitaevsky's formula gives in this case a result that is a factor of 2 smaller than the one of Eq. (15).

The magnetic field $\langle B_{2\mathrm{z}} \rangle$ is negative, i.e. is a diamagnetic one. The nonlinear current $\langle \boldsymbol{j}_{22} \rangle$ must be also taken into account for other kinds of waves. The expression (15) can be shown to be suitable when $L_{y(x)} \ll L_z$ and $\tau c_A \gg L_{y(x)}$, where $L_{y(x)}$ and L_z are typical inhomogeneity lengths of the wave amplitude across and along the background magnetic field, τ is the typical time of the wave amplitude change, and $c_A = B_0 / (4\pi \rho_0)^{1/2}$ is the Alfvén velocity, $\rho_0 = m_{\mathrm{i}} n_0$. When these inequalities are not satisfied, the magnetic field $\langle B_{2\mathrm{z}} \rangle$ in a cold plasma is determined by some differential equation (see Nekrasov and Feygin, 2006). We note that the kinetic expression for the induced magnetic moment in the thermal plasma has been derived in Nekrasov and Petviashvili (1979).

In the Appendix, we show that the magnetic field $\langle B_{21z}\rangle$ is produced by the circular microcurrents across the background magnetic field generated by waves under consideration. However, this conclusion has a general physical sense and can be applied to other kinds of waves.

4 Ponderomotive force

In this section, we will find the ponderomotive force along the background magnetic field $\boldsymbol{B}_0$. Let us add Eqs. (1) and (2) and take into account Eqs. (6) and (7). Having in mind that $n_\mathrm{i} \simeq n_\mathrm{e} = n$ for waves under consideration, we obtain

$$m_\mathrm{i} n\left(\frac{\partial \boldsymbol{v}_\mathrm{i}}{\partial t}+\boldsymbol{v}_\mathrm{i}\cdot\nabla\boldsymbol{v}_\mathrm{i}\right) = -\nabla p + m_\mathrm{i} n\boldsymbol{g} + \frac{1}{4\pi}(\nabla\times\boldsymbol{B}) \times(\boldsymbol{B}_0+\boldsymbol{B}) - \frac{1}{4\pi c}\frac{\partial \boldsymbol{E}}{\partial t}\times(\boldsymbol{B}_0+\boldsymbol{B}), \tag{16}$$

where $p = p_\mathrm{i} + p_\mathrm{e}$.

We are interested in a slow nonoscillatory motion of plasma along the magnetic field $\boldsymbol{B}_0$. It is obvious that such a motion is possible under the action of the nonlinear force in Eq. (16). Projecting this equation on the z-direction and averaging over fast oscillations, we obtain

$$\rho_0\frac{\partial\langle v_{\mathrm{i}2z}\rangle}{\partial t} + \frac{\partial\langle p_2\rangle}{\partial z} - m_\mathrm{i} g_z\langle n_{\mathrm{i}2}\rangle = -\frac{1}{8\pi}\frac{\partial\left\langle \boldsymbol{B}_1^2\right\rangle}{\partial z} - \frac{1}{4\pi c}\left\langle\left(\frac{\partial\boldsymbol{E}_1}{\partial t}\times\boldsymbol{B}_1\right)_z\right\rangle + \frac{1}{c}\left\langle(\boldsymbol{j}_1\times\boldsymbol{B}_0)_z\right\rangle. \tag{17}$$

In Eq. (17), the equilibrium condition $g_z = \partial p_0/\rho_0\partial z$ and equalities $v_{\mathrm{i}1z} = B_{1z} \simeq 0$ satisfied in our case have been taken into account. The last term on the right-hand side of Eq. (17) is connected with the curvature of the magnetic field lines (see below). The right-hand side of Eq. (17) is the longitudinal ponderomotive force F_pz.

It is followed from Eq. (5) that, for example, for the traveling wave $\sim\cos\left(\int^z k_z dz - \omega t\right)$ we have the relation

$$\left\langle\boldsymbol{B}_1^2\right\rangle = N^2\left\langle\boldsymbol{E}_1^2\right\rangle, \tag{18}$$

where $N^2 = k_z^2c^2/\omega^2$ is the refractive index. Here, we do not consider the time-dependence of the wave amplitude, supposing that $\partial\left\langle\boldsymbol{E}_1^2\right\rangle/\partial z \gg (k_z/\omega)\partial\left\langle\boldsymbol{E}_1^2\right\rangle/\partial t$ (Washimi and Karpman, 1976). Then, the second term on the right-hand side of Eq. (17) can be represented in the form

$$\left\langle\left(\frac{\partial\boldsymbol{E}_1}{\partial t}\times\boldsymbol{B}_1\right)_z\right\rangle = \frac{1}{2}c\frac{\partial\left\langle\boldsymbol{E}_1^2\right\rangle}{\partial z}. \tag{19}$$

To calculate the last term on the right-hand side of Eq. (17), we take into account that the magnetic field $\boldsymbol{B}_0$ can have a curvature. Let $\boldsymbol{B}_0$ have the form

$$\boldsymbol{B}_0 = \left[-\frac{1}{2}\frac{\partial B_0}{\partial z}x, -\frac{1}{2}\frac{\partial B_0}{\partial z}y, B_0(z)\right] \tag{20}$$

in some local system of coordinates. The equation $\nabla\cdot\boldsymbol{B}_0 = 0$ is satisfied. Substituting expression (20) in this term, we obtain

$$\left\langle(\boldsymbol{j}_1\times\boldsymbol{B}_0)_z\right\rangle = \frac{1}{2}\frac{\partial B_0}{\partial z}\left\langle x_{\mathrm{i}1}j_{1y} - y_{\mathrm{i}1}j_{1x}\right\rangle. \tag{21}$$

Accomplishing calculations of expression in the angle brackets of Eq. (21), we find in our case

$$\left\langle x_{\mathrm{i}1}j_{1y} - y_{\mathrm{i}1}j_{1x}\right\rangle = -\frac{e^3 n_0}{m_\mathrm{i}^2\omega_\mathrm{i}}\frac{\left\langle\boldsymbol{E}_1^2\right\rangle}{(\omega_\mathrm{i}-\sigma\omega)^2}. \tag{22}$$

Let us now substitute expressions (18), (19), (21), and (22) into Eq. (17). As a result, we obtain the following equation:

$$\rho_0\frac{\partial\langle v_{\mathrm{i}2z}\rangle}{\partial t} + \frac{\partial\langle p_2\rangle}{\partial z} - m_\mathrm{i} g_z\langle n_{\mathrm{i}2}\rangle = -\frac{1}{8\pi}\frac{\partial}{\partial z}\left(N^2+1\right)\left\langle\boldsymbol{E}_1^2\right\rangle + \frac{1}{4\pi}\langle B_{2z}\rangle\frac{\partial B_0}{\partial z}, \tag{23}$$

where $\langle B_{2z}\rangle$ is defined by Eq. (15). This equation is appropriate in the region $y(x) \ll L_{y(x)}$ because we have not taken into account the contribution of $\langle \boldsymbol{j}_2\rangle$ in Eq. (17). The first term of the ponderomotive force F_pz in Eq. (23) is the high-frequency pressure and the second term appears in the magnetic medium embedded in an external inhomogeneous magnetic field. This specific term is the well-known Pitayevsky's force (Pitayevsky, 1960). We note that this force can be written in the vector form as $\boldsymbol{F}_P = (\boldsymbol{M}\cdot\nabla)\boldsymbol{B}_0$, where $\boldsymbol{M} = (1/4\pi)\langle\boldsymbol{B}_2\rangle$. The last form is a general one for a force acting on medium with the magnetic moment $\boldsymbol{M}$ in the inhomogeneous magnetic field $\boldsymbol{B}_0$ (Landau and Lifshits, 1982). In our case, $\boldsymbol{M} = M\boldsymbol{b}_0$, where $\boldsymbol{b}_0$ is the unit vector along $\boldsymbol{B}_0$. Therefore, we have $\boldsymbol{F}_P = M\partial\boldsymbol{B}_0/\partial\boldsymbol{b}_0$ or $F_{P\boldsymbol{b}_0} = \boldsymbol{b}_0\cdot\boldsymbol{F}_P = \boldsymbol{M}\cdot\partial\boldsymbol{B}_0/\partial\boldsymbol{b}_0$. This is a general expression available for the arbitrary curve magnetic field. We see that important is the inhomogeneity of the magnetic field along its direction to have the longitudinal force. In the vicinity of some point, where we calculate the force $\boldsymbol{F}_P$, a local magnetic force line has in a general case the form (20). An inhomogeneity of the magnetic field along only the x- or y-directions does not influence on the value $\boldsymbol{F}_P$. We also note that if we take the inhomogeneous magnetic field with straight field lines, $\boldsymbol{B}_0 = [0, 0, B_0(x,y)]$, then we obtain $\boldsymbol{F}_P = M\nabla B_0(x,y)$.

The expression for F_pz in Eq. (23) can be rewritten through only the gradient of the electric field amplitude. To show this, we note that the wave electric field amplitude in a weakly inhomogeneous medium is proportional to $N^{-1/2}$. Thus, we have that $\left\langle\boldsymbol{E}_1^2\right\rangle \sim N^{-1}$. Using this relation and taking into account Eq. (18), we obtain

$$\frac{\partial}{\partial z}\left\langle\boldsymbol{B}_1^2\right\rangle = \frac{\partial}{\partial z}N^2\left\langle\boldsymbol{E}_1^2\right\rangle = -N^2\frac{\partial}{\partial z}\left\langle\boldsymbol{E}_1^2\right\rangle. \tag{24}$$

We also see that $\langle \boldsymbol{B}_1^2 \rangle \sim N$. Thus, we have $\langle \boldsymbol{B}_1^2 \rangle \langle \boldsymbol{E}_1^2 \rangle = \text{const}$. For low-frequency electromagnetic waves traveling along the background magnetic field, the refractive index N is, as known, equal to

$$N^2 = 1 + \frac{\omega_{\mathrm{pi}}^2}{\omega_{\mathrm{i}}(\omega_{\mathrm{i}} - \sigma\omega)}. \tag{25}$$

For our purpose, we don't need the small thermal corrections to N. As is known (e.g. Hasegawa and Uberoi, 1982; Voitenko and Goossens, 2004), this contribution for Alfvén waves $\omega \ll \omega_{\mathrm{i}}$ and $N^2 \gg 1$ is the following: $\omega = k_z c_A \left(1 + k_\perp^2 \rho_s^2\right)^{1/2}$, where ρ_s is the ion acoustic gyroradius and $k_\perp$ is the wave number across the background magnetic field. In the case $k_\perp^2 \rho_s^2 \ll 1$ which is here considered, this term can be neglected in our calculations for the real value $N^2 \gg 1$. Using Eqs. (15), (24), and (25), we can express the ponderomotive force F_{pz} in the following form:

$$F_{\mathrm{pz}} = \frac{1}{8\pi}\left(N^2 - 1\right)\left(\frac{\partial \langle \boldsymbol{E}_1^2 \rangle}{\partial z} - \frac{\omega_{\mathrm{i}}}{\omega_{\mathrm{i}} - \sigma\omega}\langle \boldsymbol{E}_1^2 \rangle \frac{\partial \ln B_0}{\partial z}\right). \tag{26}$$

In Eq. (23), the inhomogeneity of the electric field amplitude can be defined by the inhomogeneity of medium, nonlinearity, finite spectral width of the linear harmonics with the different weight, and wavelength for standing waves. Below, we examine the first case. Considering the WKB-solution for the stationary wave electric field amplitude and substituting expression (25) into Eq. (26), we obtain

$$\begin{aligned} F_{\mathrm{pz}} = & -\frac{\left(N^2 - 1\right)}{N^2} \frac{\langle \boldsymbol{E}_1^2 \rangle}{16\pi} \\ & \left[\left(N^2 - 1\right)\frac{\partial \ln \rho_0}{\partial z} + \frac{\sigma\omega}{\omega_{\mathrm{i}} - \sigma\omega}\left(N^2 - 1\right)\frac{\partial \ln B_0}{\partial z} \right. \\ & \left. + \frac{2\omega_{\mathrm{i}}}{\omega_{\mathrm{i}} - \sigma\omega}\frac{\partial \ln B_0}{\partial z}\right]. \end{aligned} \tag{27}$$

The expression (27) differs from the analogous one obtained by Guglielmi et al. (1995) by the coefficient in the second term in the square brackets. For the case $\langle \boldsymbol{E}_1^2 \rangle \sim B_0/N$ considered in Guglielmi et al. (1993, 1995), we also have a different result. Note that for magnetosonic waves ($\sigma = -1$), the second term in the square brackets in Eq. (27) changes the sign. In the case $N^2 \gg 1$ and $\omega_{\mathrm{i}} \gg \omega$, expression (27) takes the form

$$F_{\mathrm{pz}} = -\frac{N^2 \langle \boldsymbol{E}_1^2 \rangle}{16\pi}\left[\frac{\partial \ln \rho_0}{\partial z} + \left(\frac{\sigma\omega}{\omega_{\mathrm{i}}} + \frac{2}{N^2}\right)\frac{\partial \ln B_0}{\partial z}\right]. \tag{28}$$

For the model $B_0 \sim \rho_0^\gamma$, the last term in square brackets of this expression can be omitted. We see from Eqs. (27) or (28) that for the low-frequency Alfvén and magnetosonic waves the ponderomotive force has a negative sign (the equator position is at $z = 0$). Therefore, it is directed to the equator of the geomagnetic field and can drive the plasma upward.

5 Nonlinear density and pressure evolution

We further consider the equation for the mass density evolution $\langle \rho_2 \rangle = m_{\mathrm{i}} \langle n_2 \rangle$. Applying the operator $\partial/\partial t$ to the corresponding equation derived from Eq. (3) and using Eqs. (17) and (28), we obtain

$$\frac{\partial^2 \langle \rho_2 \rangle}{\partial t^2} - \frac{\partial^2 \langle p_2 \rangle}{\partial z^2} + \frac{\partial \langle \rho_2 \rangle g_z}{\partial z} = \frac{\partial}{\partial z}\left(\frac{\langle \boldsymbol{B}_1^2 \rangle}{16\pi}\frac{\partial \ln \rho_0}{\partial z}\right), \tag{29}$$

where $g_z = 2g_0\psi/L^2$, $\partial/\partial z = (1/R_{\mathrm{E}}L)\partial/\partial\psi$, $\psi < 1$ is the geomagnetic latitude in the vicinity of the equator, $g_0 = 9.8\,\mathrm{m\,s^{-2}}$, L is McIllwain's parameter, and R_{E} is the Earth's radius. It is followed from Eq. (29) that the thermal pressure $\langle p_2 \rangle$ can have a considerable influence on the evolution of density. We see from Eq. (17) that the sign of $\partial \langle p_2 \rangle/\partial z$ coincides with the sign of the ponderomotive force and is negative (see Eq. 28). Therefore, the growth of $\partial \langle p_2 \rangle/\partial z$ will hinder the movement of plasma due to the ponderomotive force and lead to the establishment of a stationary state in which $\langle v_{\mathrm{i2z}} \rangle = 0$. The pressure equation for ions (Eq. 4) can be written by using Eq. (17) in the form

$$\begin{aligned} & \frac{\partial^2 \langle p_{\mathrm{i2}} \rangle}{\partial t^2} - 2c_{\mathrm{si}}^2 \frac{\partial^2 \langle p_{\mathrm{i2}} \rangle}{\partial z^2} + 2\left(c_{\mathrm{si}}^2 \frac{\partial \rho_0}{\rho_0 \partial z} - \frac{\partial p_{\mathrm{i0}}}{\rho_0 \partial z}\right)\frac{\partial \langle p_{\mathrm{i2}} \rangle}{\partial z} \\ & \qquad = -c_{\mathrm{si}}^2 \frac{\partial G_z}{\partial z} + \left(c_{\mathrm{si}}^2 \frac{\partial \rho_0}{\rho_0 \partial z} - \frac{\partial p_{\mathrm{i0}}}{\rho_0 \partial z}\right) G_z, \end{aligned} \tag{30}$$

where $c_{\mathrm{si}}^2 = \gamma p_{\mathrm{i0}}/\rho_0$ and $G_z = g_z \langle \rho_2 \rangle + F_{\mathrm{pz}}$. For simplicity, we assume that $p_{\mathrm{e}} \sim p_{\mathrm{i}}$ that results in $\langle p_{\mathrm{e2}} \rangle \sim \langle p_{\mathrm{i2}} \rangle$.

From Eq. (30), we can conclude that the typical time for the stationary state to be established is of the order of $\tau \sim H/c_{\mathrm{s}}$, where H is the typical inhomogeneity scale length and $c_{\mathrm{s}} \sim [(T_{\mathrm{e}} + T_{\mathrm{i}})/m_{\mathrm{i}}]^{1/2}$ is the sound velocity. During this time, the plasma displaces by the distance $\triangle z \sim \langle v_{\mathrm{i2z}} \rangle \tau$. The typical velocity $\langle v_{\mathrm{i2z}} \rangle$ is $\langle v_{\mathrm{i2z}} \rangle \sim F_{\mathrm{pz}}\tau/\rho_0$. Thus, we obtain an estimation

$$\frac{\triangle z}{H} \sim \frac{H}{c_s^2 \rho_0} F_{\mathrm{pz}}. \tag{31}$$

Substituting expression (28) for F_{pz} into Eq. (31), we find

$$\frac{\triangle z}{H} \sim \frac{\langle \boldsymbol{B}_1^2 \rangle}{16\pi\gamma p_0}. \tag{32}$$

Depending on the relation $\langle \boldsymbol{B}_1^2 \rangle / p_0$, the value $\triangle z/H$ can be smaller, of the order of or larger than unity.

The amount of plasma per unit of area, which displaces into the region of the equator, is $\triangle\rho \sim \rho_0 \triangle z$. If the ponderomotive force acts from both sides of the equator, then the plasma will be accumulated in this region. The change of the density $\delta\rho$ will be equal to $\delta\rho/\rho_0 \sim \triangle z/l$, where l is the dimension of the region, where plasma is accumulated. We note that the stationary state can be achieved, if $\Delta L/c_A \gtrsim H/c_{\mathrm{s}}$, where ΔL is the length of the wave packet.

If the stationary state is not achieved, the displacement is determined by the duration of the ponderomotive force action.

6 Displacement of plasma under the action of narrow packets

The Pc 1 pulsations are sufficiently narrow wave packets in space, dimension of which is much smaller than the length of magnetic field force lines in the region of their generation (e.g. Gendrin et al., 1971; Arnoldy et al., 2005; Loto'aniu et al., 2005, and references therein). The group velocity of these waves is equal to the Alfvén velocity. The forward front of the wave accelerates the plasma under the action of the ponderomotive force and the backward front decelerates it leading to a finite plasma displacement. If $\triangle L$ is the typical dimension of the packet, then the time of interaction with plasma, through which the packet travels, is $\triangle t \sim \triangle L/c_A$. The typical nonlinear velocity is $\langle v_{\mathrm{i2z}}\rangle \sim F_{\mathrm{pz}}\triangle t/\rho_0$. The plasma displacement is $\triangle z \sim \langle v_{\mathrm{i2z}}\rangle \triangle t$. For the ponderomotive force of the form

$$F_{\mathrm{pz}} = -\frac{1}{8\pi}\frac{\partial\langle \boldsymbol{B}_1^2\rangle}{\partial z}$$

(see Eq. 17, where the contribution of the magnetic moment is neglected), the value $\triangle z$ is estimated as

$$\frac{\triangle z}{\triangle L} \sim \frac{\langle \boldsymbol{B}_1^2\rangle}{B_0^2} \ll 1. \quad (33)$$

If we take the ponderomotive force defined by Eq. (28), then we obtain the same estimation as Eq. (33). Thus, Pc 1 pulsations do not modify the background plasma distribution.

7 Plasma displacement under the action of standing waves

It is followed from Eq. (17) that the standing wave displaces the plasma to its nodes. In this case, the wave amplitude inhomogeneity is defined by its wavelength. If the standing wave would have a node at the equator and an amplitude maximum in the conjugate points of the ionosphere, then the wave could cause a movement of plasma only upward. It would be possible, if a quarter of the wavelength would contain between the Earth and the magnetic equator. In a general case, the wave equation must be solved for an inhomogeneous medium with the boundary conditions.

8 Conclusion

In the present paper, we have considered the ponderomotive action of the low-frequency Alfvén waves traveling along the nonuniform geomagnetic field. The magnetic moment induced by the electromagnetic field has been taken into account. We have shown that Pitayevsky's formula for the magnetic moment does not include the additional magnetic moment induced by the nonlinear current which is connected with the quasi-stationary nonlinear particle velocities. The total magnetic moment has a negative sign, i.e. it is directed against the background magnetic field. This diamagnetic field results, in particular, in the nonlinear shift of the cyclotron frequency (Nekrasov and Petviashvili, 1979).

We have obtained the expression for the ponderomotive force for the quasi-monochromatic wave packets which are nonuniform due to the background inhomogeneity. Our expression differs from the analogous result obtained by Guglielmi et al. (1993, 1995). In our consideration, we also have taken into account the right-polarized magnetosonic waves. We have shown that in this case one term which is proportional to the magnetic field inhomogeneity in the expression for the ponderomotive force changes the sign (see Eq. 27 for $\sigma = -1$).

The equation for the plasma density evolution under the action of the induced thermal pressure and ponderomotive force has been considered. We have shown that the nonlinear thermal pressure decelerates the plasma flux and can lead to the establishment of a stationary state in which the flux is equal to zero. This important effect of the nonlinear thermal pressure for traveling pulsations has not been considered in papers cited above. For this case, we have obtained the estimation for the plasma displacement to the equator (Eq. 32). If the stationary state is not achieved, the displacement is determined by the duration of the ponderomotive force action.

We have also obtained an estimation for the plasma displacement under the action of the narrow wave packets such as Pc 1 pulsations. In this case, the displacement is much smaller than the space dimension of the packet. Thus, Pc 1 pulsations can not modify the background plasma distribution. This conclusion considerably differs from the previous results according to which the Pc 1 pulsations lead to accumulation of plasma in the equatorial region.

The long standing wave having node only at the equator can cause an upward movement of plasma.

Appendix A

We here show that expression (11) can be obtained, using the magnetic moment of the circular current of the charged species. Under the action of the circularly polarized Alfvén wave, the electrons and ions move over circle in the direction of the wave polarization around the magnetic field $\boldsymbol{B}_0$. The current I_j of one particle j is equal to

$$I_j = \sigma\frac{q_j\omega}{2\pi}, \quad (A1)$$

where q_j is the charge of species j. The magnetic moment $\boldsymbol{\mu}_j$ of this current is given by

$$\boldsymbol{\mu}_j = \mu_j \boldsymbol{b}_0 = -\frac{I_j}{c} S_j \boldsymbol{b}_0, \tag{A2}$$

where S_j is the circle area (e.g. Sivukhin, 2002). It can be shown (e.g. Nekrasov and Feygin, 2006) that the radius of the circle a_j is

$$a_j = \frac{|q_j|}{m_j} \frac{1}{\omega |\omega_j - \sigma\omega|} E_{10}, \tag{A3}$$

where E_{10} is the wave amplitude and the brackets || denote an absolute value. Substituting expressions (A1) and (A3) into Eq. (A2), we obtain the magnetic moment of one particle

$$\mu_j = -\frac{q_j^3}{2m_j^2 c} \frac{\sigma}{\omega \left(\omega_j - \sigma\omega\right)^2} E_{10}^2. \tag{A4}$$

The total magnetic moment of the unit of volume is equal to $M = n_0(\mu_{\mathrm{e}} + \mu_{\mathrm{i}})$. Using expression (A4), we find

$$M = -\frac{1}{8\pi} \frac{\omega_{\mathrm{pi}}^2 (2\omega_{\mathrm{i}} - \sigma\omega)}{\omega_{\mathrm{i}} (\omega_{\mathrm{i}} - \sigma\omega)^2} \frac{E_{10}^2}{B_0}. \tag{A5}$$

We see from Eq. (A5) that the value $4\pi M$ coincides with $\langle B_{21z} \rangle$ within a factor 1/2.

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