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Asymmetric multifractal model for solar wind intermittent turbulence

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Abstract. We consider nonuniform energy transfer rate for solar wind turbulence depending on the solar cycle activity. To achieve this purpose we determine the generalized dimensions and singularity spectra for the experimental data of the solar wind measured in situ by Advanced Composition Explorer spacecraft during solar maximum (2001) and minimum (2006) at 1 AU. By determining the asymmetric singularity spectra we confirm the multifractal nature of different states of the solar wind. Moreover, for explanation of this asymmetry we propose a generalization of the usual so-called p-model, which involves eddies of different sizes for the turbulent cascade. Naturally, this generalization takes into account two different scaling parameters for sizes of eddies and one probability measure parameter, describing how the energy is transferred to smaller eddies. We show that the proposed model properly describes multifractality of the solar wind plasma.

1 Introduction

The solar wind is a an example of turbulent and intermittent astrophysical plasma (Burlaga, 1991a, 1992a,b; Marsch, 1991; Carbone, 1993; Marsch and Liu, 1993; Marsch and Tu, 1997; Sorriso-Valvo et al., 2001; Biskamp, 2003; Bruno et al., 2003). For this highly nonlinear system the energy at a given scale is not evenly distributed in space and we can observe how fluctuating parameters affected by intermittency alternate between burst of activity and quiescence. Therefore, based on Richardson's cascade and Kolmogorov's ideas (Kolmogorov, 1941, 1962), followed by Kraichnan (1965), several classes of models have been developed to describe nonuniform distribution of energy in the turbulent flow (Lesieur, 1990; Borgas, 1992; Goldstein and Roberts, 1999), in particular, β-model (Frisch et al., 1978) and random β-model (Benzi et al., 1984). Moreover, due to multifractal models, e.g., p-model (Meneveau and Sreenivasan, 1987), She-Leveque model (She and Leveque, 1994), we can look inside complex nature of intermittent turbulence (Mandelbrot, 1989). Using generalized dimensions and singularity spectra allow us a better description of energy turbulence cascade and the degree of multifractality in the solar wind plasma (Meneveau and Sreenivasan, 1991). It is well known that multifractal nature of solar wind has been observed in the inner heliosphere (Marsch et al., 1996; Macek, 1998, 2002, 2003, 2006, 2007; Macek et al., 2005; Macek and Szczepaniak, 2008a), and in the outer heliosphere (Burlaga, 1991a,b,c, 2004; Burlaga et al., 1993, 2003), also at various phases of the solar cycle (Burlaga, 2001; Burlaga et al., 1993) and various heliographic latitudes (Horbury and Balogh, 1997). However, the multifractal singularity spectrum obtained for the solar wind data has an asymmetric shape and shows a substantial departure from the standard p-model (Burlaga, 1993; Macek, 2007; Macek and Szczepaniak, 2008a). The nature of this departure is still unexplained. Therefore, the main aim of this work is modeling and explanation of this asymmetry. This paper is organized as follows. In Sect. 2 we introduced data collection taken and methods used for analysis. Generalization of the p-model is considered in Sect. 3. Sections 4 and 5 present results and conclusions of our investigations.

2 Data and methods

Using Helios 2 data (Schwenn, 1990) we have demonstrated that intermittent pulses are stronger for asymmetric scaling and a much better agreement with the data is obtained, especially for the negative index of the generalized dimensions (Macek and Szczepaniak, 2008a). In this paper we consider

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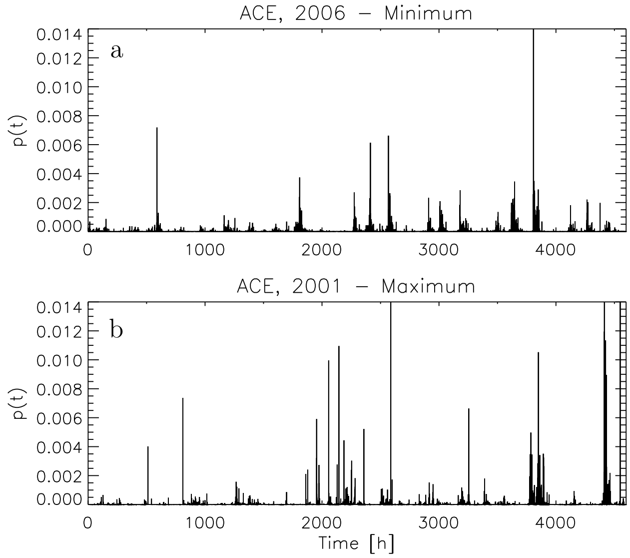

Fig. 1. Energy transfer rate for solar wind turbulence as a multifractal measure for **(a)** solar minimum (2006) and **(b)** solar maximum (2001), correspondingly.

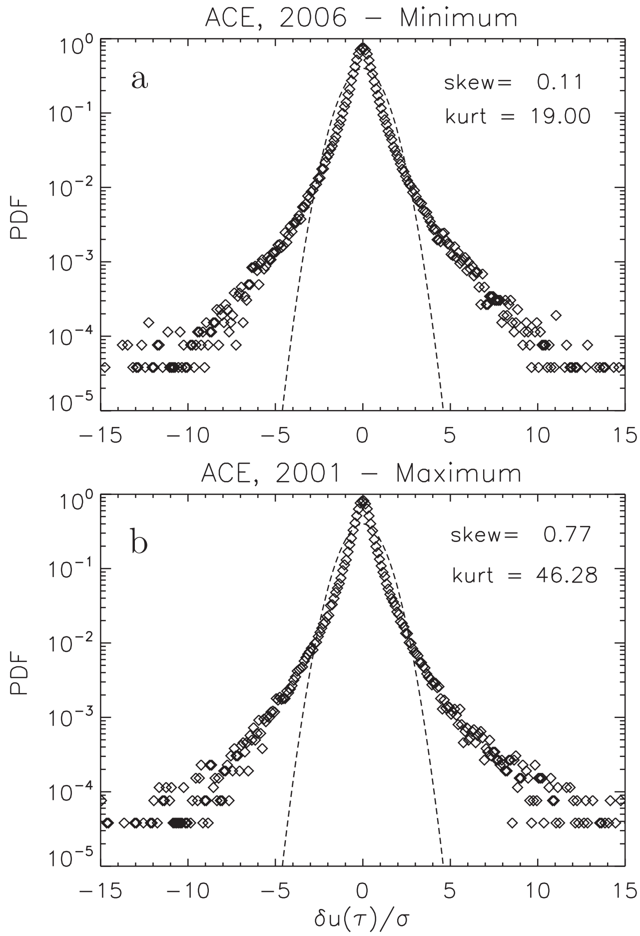

Fig. 2. Probability density functions of fluctuations of the solar wind radial velocity for **(a)** solar minimum (2006) and **(b)** solar maximum (2001), correspondingly, τ=64 s, as compared with the normal distribution (dashed lines).

changes of the multifractality of the energy transfer rate in the solar wind turbulence with the solar cycle activity. For this purpose we use two-years samples (2001 and 2006) of the velocity parameter measured in situ by Advanced Composition Explorer (ACE). These intervals are representative for a broad range of the solar wind conditions, in particular, we take into account both slow and fast wind streams and changes during the solar activity cycle. Our data of the resolution of 64 s have been obtained at about 1 AU in GSE system, near Lagrangian point ($L1$). For these data under study we apply multifractal formalism, which is one of the most adequate method for describing local scaling properties of energy transfer rate in nonhomogeneous turbulence.

There are several techniques to evaluate the multifractality and to obtain the generalized dimensions (Hentschel and Procaccia, 1983) or multifractal spectra (Halsey et al., 1986). Some methods are based on the calculations of the scaling exponents of structure functions (Anselmet et al., 1984), and are related to the generalized dimensions D_q (Frisch, 1995; Tsang et al., 2005). It is also possible to obtain the multifractal spectrum directly from data (Chhabra and Jensen, 1989). Here, we construct the transfer rate of the energy flux as a multifractal measure and consider its scaling properties. Namely, to each ith eddy of size l at nth cascade step (i=1, ...N=2^n) we associate a probability measure

$$p_i(l) = \frac{\varepsilon_i(l)}{\sum_{i=1}^{N} \varepsilon_i(l)} \tag{1}$$

where $\varepsilon_i(l) \sim |u(x+l)-u(x)|^3/l$ (Marsch et al., 1996). In Fig. 1 we show the multifractal measure obtained using $N=2^n$, with n=18, data points for (a) solar minimum (2006) and (b) solar maximum (2001), correspondingly. One can notice that intermittent pulses are somewhat stronger for data at solar maximum. This results in fatter tails of the probability distribution functions as shown in Fig. 2, for solar maximum and minimum with large deviations from the normal distribution (dashed lines).

In the next step we identify the inertial range, $\eta \ll l \ll L$, where η is the dissipation scale and L is the size of the whole system. Calculation of this range is essential, because it provides information as to whether turbulence is fully developed and the energy cascade is actually present (Sorriso-Valvo et al., 2007). This may indicate that in fact we can have a fully developed turbulence during solar maximum. One can therefore expect that in this case the distribution of the energy between cascading eddies is more inhomogeneous, and consequently the behaviour of intermittent pulses are stronger for solar maximum, Fig. 1. We identify the inertial range

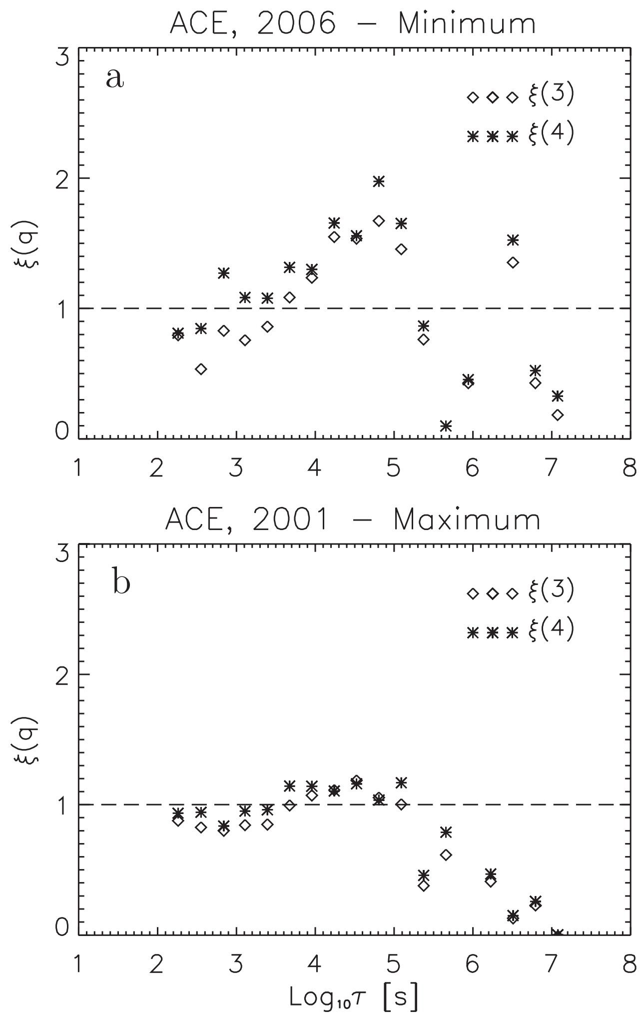

Fig. 3. The measured third and fourth orders scaling exponents $\xi(3)$ and $\xi(4)$ as indicators of the inertial range (Carbone, 1994).

by considering the scale dependence of the usual third and fourth orders caling exponents $\xi(3)$ and $\xi(4)$ (Carbone, 1994; Horbury et al., 1997; Horbury and Balogh, 1997). The results based on the experimental values are presented in Fig. 3. We see that the scaling range is much clearer and wider for solar maximum.

Next, we analyse the log-log plots $[\sum_i^N p_i^q(l)]^{\frac{1}{q-1}}$ versus l for different steps (n) of the cascade. The slopes of this curves correspond to the generalized dimensions, D_q (Meneveau and Sreenivasan, 1991). The multifractal measure $\mu{=}\varepsilon/\langle\varepsilon_L\rangle$ on the unit interval for several steps of the construction of the generalized p-model is presented in Fig. 4. As usual the generalized dimensions are defined by

$$D_q = \lim_{l\to 0} \frac{1}{q-1} \frac{\log \sum_{i=1}^N p_i^q(l)}{\log l} \tag{2}$$

To obtain multifractal spectra we use the methods described

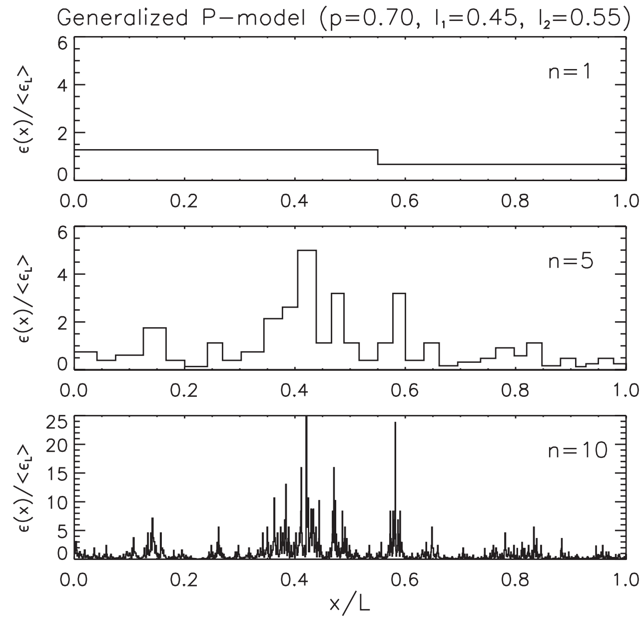

Fig. 4. The multifractal measure $\mu{=}\varepsilon/\langle\varepsilon_L\rangle$ on the unit interval for **(a)** first, **(b)** fifth and **(c)** tenth step of the construction of the generalized p-model.

by Chhabra and Jensen (1989). Finally, we verify the multifractal spectra $f(\alpha)$ (Halsey et al., 1986; Stanley and Meakin, 1988) obtained from D_q using Legendre transform (Ott, 1993; Macek and Szczepaniak, 2008b[1]).

$$\alpha = \frac{d}{dq}[(q-1)D_q] \qquad f(\alpha) = q\alpha - (q-1)D_q \tag{3}$$

3 Asymmetric model

A generalized two-scale Cantor set, which is a combination of asymmetric and weighted Cantor set, is a theoretical ground for the cascade model (Halsey et al., 1986; Ott, 1993). In general, at each step of the cascade construction we use two different scales l_1 and l_2 for the segment generated at each level, and two different, in general, weights, p and $1-p$. For $l_1{=}l_2{=}\frac{1}{2}$ one recovers the standard p-model (Meneveau and Sreenivasan, 1987) resulting in a symmetric shape of the multifractal singularity spectrum function. Direct relation between q and D_q for the proposed model is obtained from the following transcendental equation:

$$p^q l_1^{(1-q)D_q} + (1-p)^q l_2^{(1-q)D_q} = 1 \tag{4}$$

[1]Macek, W. M. and Szczepaniak, A.: Asymmetric multifractal scaling of solar wind turbulence, J. Geophys. Res., submitted, 2008b.

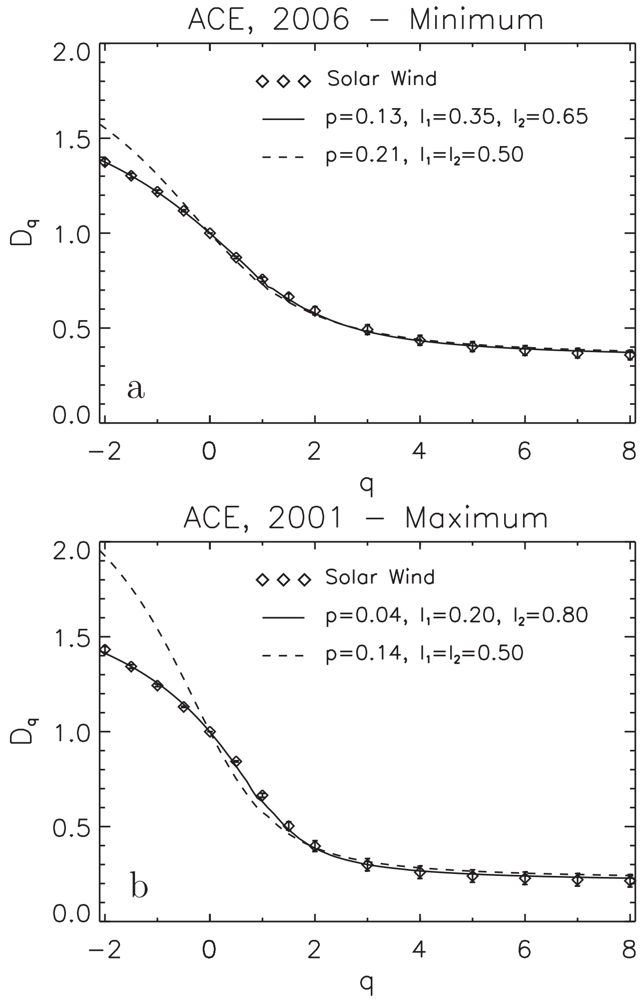

Fig. 5. The generalized dimensions D_q for the energy transfer rate in the solar wind turbulence at **(a)** solar minimum (2006) and **(b)** solar maximum (2001), correspondingly.

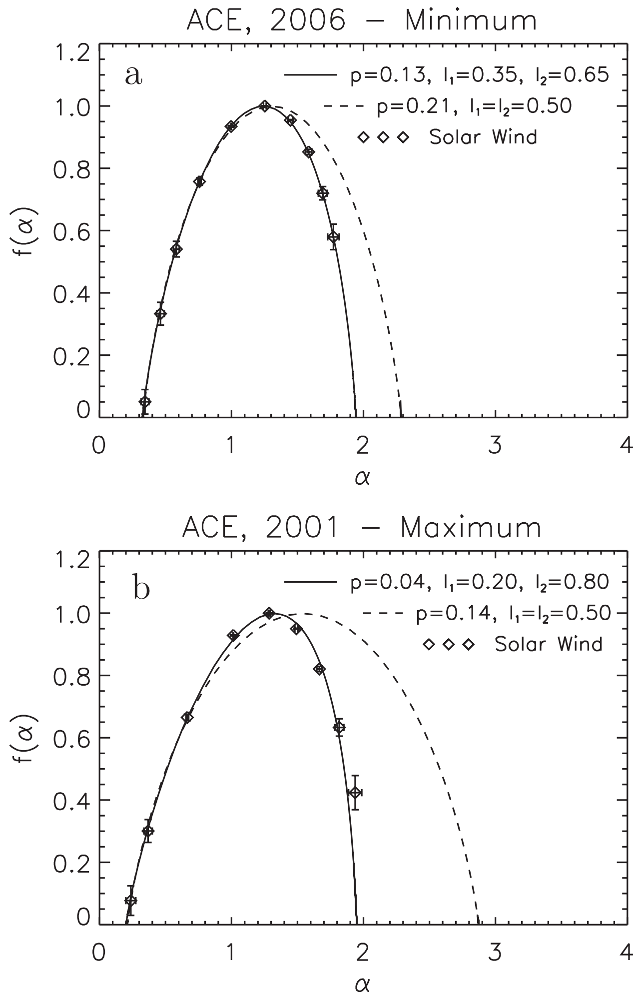

Fig. 6. The multifractal spectrum $f(\alpha)$ for the energy transfer rate in the solar wind turbulence at **(a)** solar minimum (2006) and **(b)** solar maximum (2001), correspondingly.

We also consider the degree of multifractality $\Delta \equiv \alpha_{\max} - \alpha_{\min}$, which is given by Halsey et al. (1986):

$$\Delta = D_{-\infty} - D_{\infty} = \left| \frac{\log(1-p)}{\log l_2} - \frac{\log(p)}{\log l_1} \right| \tag{5}$$

and the degree of asymmetry:

$$A = \frac{\alpha_0 - \alpha_{\min}}{\alpha_{\max} - \alpha_0}, \tag{6}$$

where the singularity spectrum has its maximum $f(\alpha_0)=1$ (Ott, 1993; Macek and Szczepaniak, 2008b[1]).

4 Results

The results for the generalized dimensions D_q as a function of q, calculated from Eq. (2) using the ACE data and compared with those obtained from Eq. (4) for solar wind turbulence at 1 AU during solar minimum (2006) and solar maximum (2001) are presented in Fig. 5a and b, correspondingly (cf. Macek and Szczepaniak, 2008a, Fig. 3). The related singularity spectra $f(\alpha)$ as a function of singularity strength α are depicted in the corresponding Fig. 6a and b (cf. Macek and Szczepaniak, 2008b[1], Fig. 7). In particular, in agreement with other studies, we confirm the universal shape of the multifractal spectrum as noticed, e.g., by Burlaga (2001). Since the Cantor set is sensitive to initial conditions the multifractal spectrum for intermittent turbulence can be naturally related to the Lyapunov spectrum as discussed by Chian et al. (2006).

We have also calculated the degree of multifractality Δ given in Eq. (5), which is equal to 1.75 for solar maximum and 1.62 for solar minimum. Hence we observe that the solar wind is multifractal during the whole solar cycle. It is worth

Table 1. Degree of multifractality Δ and asymmetry A.

	Δ	A
Solar Minimum	1.62	1.30
Solar Maximum	1.75	1.37

noting that the shape of the multifractal singularity spectrum is rather asymmetric, which cannot be explained by the usual p-model, which involves only a one-scale Cantor set. The actual degree of asymmetry A defined in Eq. (6) is of about 1.3 for both solar minimum and maximum, as summarized in Table 1.

5 Conclusions

We have studied the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behaviour of solar wind turbulence in the inner heliosphere. In particular, we have demonstrated that for the model with two different scaling parameters a much better agreement with the real data is obtained, especially for $q<0$. By investigating the ACE data we have shown that as the solar activity increases the solar wind becomes somewhat more multifractal and more asymmetric. Admittedly, it seems that the degree of asymmetry of the singularity spectrum for one-year samples is rather weakly correlated with the phase of the solar activity. The dependence for slow and fast streams is thoroughly studied in another paper by Macek and Szczepaniak (2008b)[1].

Basically, the generalized dimensions for the solar wind are consistent with the generalized p-model for both positive and negative q, but rather with different scaling parameters for sizes of eddies, while the usual p-model can only reproduce the spectrum for $q\geq 0$. Therefore we propose this cascade model describing intermittent energy transfer for analysis of turbulence in various environments.

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