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The influence of convective current generator on the global current

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Abstract. The mathematical generalization of classical model of the global circuit with taking into account the convective current generator, working in the planetary boundary layer was considered. Convective current generator may be interpreted as generator, in which the electromotive force is generated by processes, of the turbulent transport of electrical charge. It is shown that the average potential of ionosphere is defined not only by the thunderstorm current generators, working at the present moment, but by the convective current generator also. The influence of the convective processes in the boundary layer on the electrical parameters of the atmosphere is not only local, but has global character as well. The numerical estimations, made for the case of the convective-unstable boundary layer demonstrate that the increase of the average potential of ionosphere may be of the order of 10% to 40%.

1 Introduction

The elucidation of the nature of the lower atmosphere electric field and its temporal variation is one of the most important problems of atmospheric electricity. The conception, according to which the main sources supporting the electric field in lower atmosphere is being thunderstorms, is adopted at present. One of the first model of electric field of atmosphere (or the global circuit) was the model of the spherical condenser (Wilson, 1925). The most developed mathematical model is at present the quasistationary model of the global circuit (Hays and Roble, 1979) in which the dipole model of quasistationary thunderstorm is used as electrical current generator supporting the electric field of the lower atmosphere. This model is a computer realization of Wilson's model. The orography of the Earth's surface, the anisotropy of the conductivity in the top layer of the atmosphere are taken into account by this model. The time variations of the electric field, for example UT diurnal variation, are caused by the world thunderstorm temporal changing in the limits of thunderstorm theory of the atmospheric electric fields origin. For maintaining the equilibrium of the global current circuit Hays-Roble's model requires large values of charges (>100 C) in this model of thunderstorm which is necessary for the creation, equal to 0.5 A–1.0 A and flowing upward from thunderstorm to the ionosphere (Selezneva, 1984). Because of this one has to find the additional generators of the electric current working in the lower atmosphere. One of such generators may be the convective current generator working in the planetary boundary layer. Indeed the appearance of this layer leads to the appearance of the convective electric current for the case of the unstable boundary layer or the turbulent electric current if the boundary layer has the neutral stratification. Appearance of this currents leads to the changing of the global circuit parameters and it is possible to lower the requirements to thunderstorm generators in the stationary model (the process of current or additional electric field generation were discussed also by Pulinets et al., 1998). On the other hand, the variations of thunderstorm activity do not always correspond in phase to the variations of the electric field strength (Morozov, 1981).

It was pointed out by Imyanitov and Kolokolov (1984) that the processes of convective and turbulent transfer of electric charges in the atmosphere averaged for the globe can lead to the stable universal time variations of the electric field. Therefore, in the present paper the generalization of the quasistationary model of the global circuit is considered which includes the thunderstorm generators and the convective current generator working in the planetary boundary layer.

2 The mathematical model

Let us consider the problem of the definition of the electrical potential of the atmosphere φ in the region between the boundary layer and the ionosphere. The thunderstorms will be modeled as the dipole current generator (Holser and Saxon, 1952; Hays and Roble, 1979). The convective electrical current will be neglected in this region. Then the equation

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for electrical potential φ in the stationary case may be expressed in the following form, in CGSE system:

$$\operatorname{div}(\lambda \operatorname{grad}\varphi) = -4\pi\lambda \sum_i q_i \delta(\boldsymbol{r} - \boldsymbol{r}_i) \,, \tag{1}$$

where λ is conductivity of the atmosphere, q_i, $\boldsymbol{r}_i$ are charge and radius-vector of the I-th thunderstorm sources, $\delta(\boldsymbol{r}-\boldsymbol{r}_i)$ – Dirac's function.

The boundary conditions for φ are

$$\begin{aligned} &\left(\frac{\partial\varphi}{\partial \boldsymbol{n}} - K\varphi\right)\Big|_{S=S_1} = 0 \,, \\ &K = \left|\int_0^H E'(z)\, dz\right|^{-1} , \\ &\frac{1}{4\pi}\oint_{S_2} \varphi d\Omega = \varphi_\infty \,, \end{aligned} \tag{2}$$

where S_1 – high limit surface of boundary layer, H – height of the boundary layer, $\boldsymbol{n}$ – is normal vector of boundary layer surface, S_2 is surface lying in top layer of atmosphere, φ_∞ – average potential of ionosphere, $E'(z)$ is nondimensional electric field strength in the boundary layer which is determined below.

For estimating K the following equations describing the stationary one-dimensional structure of electric field in the boundary layer are used:

$$\begin{aligned} &\overline{\rho' v_z'} + \bar{\lambda}(z)\bar{E}(z) = j_0 \,, \\ &\frac{d\bar{E}}{dz} = 4\pi\bar{\rho} \,, \\ &-\frac{d}{dz} D_T(z) \frac{d\bar{\lambda}}{dz} = 2ebq - \frac{\bar{\alpha}}{2eb}\bar{\lambda}^2 - \gamma\bar{\lambda} \,, \end{aligned} \tag{3}$$

where $\bar{\rho}$ is average density of electrical charge in the boundary layer, $D_T(z)$ – the coefficient of the turbulent diffusion, $\bar{\lambda}$ is average conductivity, $\bar{E}$ is average vertical component of the electric field strength, b is mobility of small ions, $\bar{\alpha}$ $(b_+ = |b_-|)$ – is recombination coefficient, q is intensity of ionization, ρ' are fluctuations of the density of electrical charge, v_z' – fluctuations of vertical hydrodynamical velocity, e is charge of electron, j_0 is density of electrical current, γ is the electrical density of aerosols.

The system of Eqs. (3) has been obtained by the presentation of quantities $f = \bar{f} + f'$ and averaging the initial equations in horizontal plane: $\bar{f} = \frac{1}{S}\int f\, dS$ (Monin and Yaglom, 1965; Willet, 1979).

The boundary conditions for the system of Eqs. (3) may be expressed in the form

$$\begin{aligned} &\bar{\lambda}(z = z_0) = 0, \qquad \bar{\lambda}(z = H) = 2eb\sqrt{\frac{q}{\alpha}} \,, \\ &\frac{d\bar{E}}{dz}\Big|_{z=z_0} = 0 \,, \qquad E(z = H) = \frac{j_0}{\lambda(H)} \,, \end{aligned} \tag{4}$$

where z_0 is roughness scale of the Earth's surface ($z_0 \approx 2.5\times10^{-3}$ m) (Monin and Yaglom, 1965; Willet, 1979), in this case is $E'(z) = \frac{\bar{E}(z)}{\bar{E}(z=H)}$.

For finding the solution of Eq. (1) with the boundary conditions (2) φ may be expressed in the form:

$$\varphi = \sum_i q_i G_i + \psi \tag{5}$$

where G_i is Green's function of the i-th source which is solution of the boundary problem:

$$\begin{aligned} &\operatorname{div}(\lambda \operatorname{grad} G_i) = -4\pi\lambda \sum_i q_i \delta(\boldsymbol{r} - \boldsymbol{r}_i) \,, \\ &G_i|_{S=S_1} = G_i|_{S=S_2} = 0 \end{aligned} \tag{6}$$

Function ψ is found from the solution of the following equation:

$$\begin{aligned} &\operatorname{div}(\lambda \operatorname{grad}\psi) = 0 \\ &\left(\frac{\partial\psi}{\partial \boldsymbol{n}}\right) + \sum_i q_i \frac{\partial G_i}{\partial \boldsymbol{n}} - K\psi\Big|_{S=S_1} = 0 \end{aligned} \tag{7}$$

Thus, the problem of the estimation of electric potential in the region between surfaces S_1 and S_2 is reduced to the problem of estimating function G_i and ψ. To estimate these functions the conductivity $\lambda(r)$ in the Earth atmosphere should be prescribed. It is estimated from the solution of the third equation of system (3) in the boundary layer. We use the following presentation for in the area between surfaces S_1 and S_2 (Hays and Roble, 1979).

$$\lambda(r, \theta, \phi) = \lambda(r_1, \theta, \phi)\, e^{\alpha(r-r_1)} \,, \tag{8}$$

where (r, θ, ϕ) are spherical coordinates beginning in the center of the Earth, r_1 is distance from the center of the Earth to the top of the boundary layer, $\alpha = 0.2\,\text{km}^{-1}$.

The following presentation of Green's function may used taking into account that correction defined by deflexion from plane case and correction defined by latitude changing λ (r, θ, ϕ) are small (Frenkel, 1949):

$$G_i = e^{-\frac{\alpha}{2}(r-r_i)} \left(\frac{e^{-\frac{\alpha}{2}\rho_i}}{\rho_i} - \frac{e^{-\frac{\alpha}{2}\rho_i'}}{\rho_i'}\right) , \tag{9}$$

where $\rho_i = |\boldsymbol{r} - \boldsymbol{r}_i|$, ρ_i' is distance from the point of observation to the mirror charge situated symmetrically relative to surface S_1 which may be considered as plane in the area of working of thunderstorm sources.

For determination of ψ we present it in the form of expansion in spherical harmonics functions (Morse and Feshbah, 1960):

$$\psi(r, \theta, \phi) = \sum_{n, m \le n} A_{n,m} R_n(r)\, Y_{n,m}(\theta, \phi), \; n = 1, 2, 3 \ldots \tag{10}$$

where $A_{n,m}$ – constant, $R_n(r)$ – radial part of the function $\psi(r, \theta, \phi)$.

Substituting Eq. (10) in Eq. (7), we get equation for function $R_n(r)$:

$$\frac{d^2 R_n(r)}{dr^2} + \left(\frac{2}{r} + \alpha\right)\frac{dR_n(r)}{dr} - \frac{n(n+1)R_n(r)}{r^2} = 0 \,. \tag{11}$$

Solving Eq. (1) we get the following asymptotic expressions for $R_n(r)$:

$$\begin{aligned} &R_0(r) = B + \int_r^\infty \frac{e^{-\alpha y}}{y^2} dy \approx B + \frac{e^{-\alpha r}}{\alpha r^2} \\ &R_n(r) \approx \frac{e^{-\alpha r}}{\alpha r^2} \,, \quad n > 0 \end{aligned} \tag{12}$$

where B is constant.

Using the second boundary condition (2) and taking into account that spherical harmonic functions are orthogonal, we obtain that the average value of ψ on sphere is equal to $A_{0,0}R_0(r)$ we get:

$$\varphi_\infty = BA_{0,0} \quad (13)$$

Substituting Eqs. (12) and (13) in Eq. (10), ψ is obtained from:

$$\begin{aligned}\psi &= \varphi_\infty + \frac{e^{-\alpha r}}{\alpha r^2}\ \nu(\theta,\phi)\ , \\ \nu(\theta,\phi) &= \sum_{n,m\le n} A_{n,m}Y_{n,m}(\theta,\phi)\end{aligned} \quad (14)$$

Using the boundary condition (7) for ψ on the surface S_1, $\nu(\theta,\phi)$ is obtained from:

$$\nu(\theta,\phi) = \frac{e^{\alpha r_1} r_1^2}{\left(1+\frac{K}{\alpha}\right)}\left[\left(\frac{\partial}{\partial r}\sum_i q_i G_i\right)_{r=r_1} - K\varphi_\infty\right] \quad (15)$$

When deriving Eq. (15) the differentiation with respect to the normal direction of surface S_1 is replaced by the differentiation with respect to the radius direction, since the angle of the slope of surface S_1 to horizontal plane caused by variation of the weight of boundary layer is equal to $\frac{\Delta H}{R}$, where ΔH is variations of the height of boundary layer, R – radius of the Earth and is small at $\Delta H \sim 1$ km.

Substituting Eqs. (14) and (15) in Eq. (5) we obtain the expression for potential φ in the spherical layer between S_1 and S_2:

$$\begin{aligned}\varphi(\boldsymbol{r}) &= \sum_i q_i G_i + \varphi_\infty\left(1 - \frac{K}{K+\alpha}e^{-\alpha(r-r_1)}\right) \\ &\quad + \frac{e^{-\alpha(r-r_1)}}{K+\alpha}\left(\frac{\partial}{\partial r}\sum_i q_i G_i\right)_{r=r_1}\end{aligned} \quad (16)$$

Using stationary condition: $\oint\limits_{S_1} \lambda \frac{\partial\varphi}{\partial \boldsymbol{n}} dS = 0$ for estimation of φ_∞ we obtain φ_∞ from:

$$\varphi_\infty = -\frac{1}{4\pi\alpha}\frac{\sum_i q_i \oint\limits_{S_1}\left(\frac{\partial G_i}{\partial r}\right)_{r=r_1}\frac{K\lambda}{K+\alpha}d\Omega}{\frac{1}{4\pi\alpha}\oint\limits_{S_1}\frac{K\lambda}{K+\alpha}d\Omega} \quad (17)$$

Since every function G_i is not equal to zero in small region of the space, K may be constant in this region and taken to be $K_i = K(\theta_i,\phi_i)$. Then, for the numerator of Eq. (17) we get:

$$\begin{aligned}&\frac{1}{4\pi}\sum_i q_i \oint\limits_{S_1}\left(\frac{\partial G_i}{\partial r}\right)_{r=r_1}\frac{K\lambda}{K+\alpha}d\Omega \\ &\quad = \sum_i q_i\left(\frac{\partial\, g_{i0,0}}{\partial r}\right)_{r=r_1}\frac{K_i\lambda_i}{K_i+\alpha}\ ,\end{aligned} \quad (18)$$

where $g_{i0,0}$ is zero coefficient of expansion of function G_i in spherical harmonic functions equal to the average value of G_i of the sphere.

Making use of the well known expansion of function $\frac{e^{-\beta\rho}}{\rho}$ by Legandre polynomials (Morse and Feshbah, 1960) for surface S_1, we obtain:

$$\left(\frac{\partial g_{i0,0}}{\partial r}\right)_{r=r_1} = -\frac{1}{r_1^2}$$

Changing in Eq. (17) from summing up by separate sources to summing up by dipoles and taking into account that charge of the positive pole of dipole q_{i+} and charge of the negative pole of dipole q_{i-} are connected with the value of charging current J_i in the dipole model of thunderstorm by equation: $J_i = 4\pi\lambda_{i+}q_{i+} = -4\pi\lambda_{i-}q_{i-}$ (Holser and Saxon, 1952), φ_∞ is obtained from:

$$\begin{aligned}\varphi_\infty &= \frac{\frac{1}{4\pi R^2\alpha}\sum_i J_i\left(\frac{1}{\lambda_{i-}} - \frac{1}{\lambda_{i+}}\right)\frac{K_i\lambda_i}{K_i+\alpha}}{\overline{\frac{K\lambda}{K+\alpha}}} \\ \overline{\frac{K\lambda}{K+\alpha}} &= \frac{1}{4\pi}\oint\limits_{S_1}\frac{K\lambda}{K+\alpha}d\Omega\end{aligned} \quad (19)$$

If $K\to\infty$ is equivalent to solving the problem for the estimation of electrical potential φ with zero boundary condition on the Earth's surface, the Eq. (19) coinsides with the equation for φ_∞ from paper (Hays and Roble, 1979). In "fair weather region" where the thunderstorm sources are absent, the electrical potential φ is obtained from the following expressions from Eq. (16):

$$\varphi = \varphi_\infty\left(1 - \frac{K}{K+\alpha}e^{-\alpha(r-r_1)}\right) \quad (20)$$

The vertical component of the electrical field strength E_r is determined by differentiation of Eq. (20):

$$E_r = -\frac{K\alpha}{K+\alpha}e^{-\alpha(r-r_1)}\varphi_\infty \quad (21)$$

In the regions where boundary layer is absent ($K\to\infty$) we obtain:

$$\begin{aligned}\varphi &= \varphi_\infty\left(1 - e^{-\alpha(r-R)}\right)\ , \\ E_r &= -\alpha\varphi_\infty e^{-\alpha(r-R)}\end{aligned} \quad (22)$$

The obtained results are valid not only in the stationary case but also in the quasistationary case under the conditions: $T \gg \tau_\lambda$ where T is the characteristic time of changing the electrical parameters of convective and the thunderstorm current generators, τ_λ is time of the electrical relaxation. Therefore it follows from the Eqs. (20–22) that if φ_∞ and K change so that the atmosphere has time to relax under this changing, then changes of the electric field strength and electrical potential may be caused not only by change of the globe thunderstorm activity, but also by temporary change of convective current generator determined by the working of some average boundary layer, existing on the Earth's surface.

Let us consider the problem of finding the values of quantity K, characterizing the intensity of the convective current generator. As mentioned above, this quantity is determined from the solution of the Eqs. (3). This system of equations may be supplemented with the equation determining the density of turbulent electrical current $\overline{\rho' v_z'}$. For its estimation one may use either diffuse approximation: $\overline{\rho' v_z'}=-D_T(z)\frac{\partial\bar{\rho}}{\partial z}$ or develop the method of its estimation based on the closure of correlation moments of the third order, as it was done for convective-unstable boundary layer (Willet, 1979). The results of computer simulations considered in this paper have show that for convective-unstable boundary layer with inversion: K=4×10^{-4} m^{-1} for H=10^3 m. All other stratifications of boundary layer give the values of K much higher than the value of K for convective-unstable boundary layer. Let us estimate changes of the average potential of ionosphere φ_∞ using the numerical value of K for convective-unstable boundary layer. First of all let us note that in the case of homogeneous distribution of boundary layer on the Earth's surface (model of spherical Earth): $\xi_i=\frac{K_i\lambda_i}{K_i+\alpha}\Big/\frac{\overline{K\lambda}}{K+\alpha}=1$ and the contribution of the current generator working in the planetary boundary layer potential of the ionosphere is equal to zero. In real case the thickness of the atmospheric boundary layer depends on latitude and longitude therefore $\xi_i\neq 1$. Experimental data about geographical distribution of the planetary boundary layer on the Earth's surface are absent at present, therefore below we shall give some simple estimations of the value of changing φ_∞. Let us assume that $\lambda_I=\lambda$ and in the region of working of thunderstorm's sources: $K_i\to\infty$, $\frac{K_i}{K_i+\alpha}=1$. Then φ_∞ is obtained from

$$\begin{aligned}\varphi_\infty &= \varphi_\infty^0\Big/\frac{\bar{K}}{K+\alpha}\,,\\ \varphi_\infty^0 &= \frac{1}{4\pi R^2\alpha}\sum_i J_i\left(\frac{1}{\lambda_{i-}}-\frac{1}{\lambda_{i+}}\right)\end{aligned} \qquad (23)$$

The following values are assumed for the area occupied by convective-unstable boundary layer: S/S_0=0.9; 0.5; 0.1; where S_0 is the area of the Earth's surface, then $\overline{K\big/K+\alpha}$=0.7; 0.84; 0.97 and for ratio $\varphi_\infty/\varphi_\infty^0$ we obtain values: $\varphi_\infty/\varphi_\infty^0$=1.42; 1.20; 1.03.

The estimations show that convective current generator working in the boundary layer of the atmosphere can make contribution to the average potential of the ionosphere which is ∼10% (maximum value is 42%). Undoubtedly the estimations obtained are of preliminary character and will be improved in future.

3 Conclusion

Thus, the generalization of the classical model of global circuit taking into account the convective current generator working in the boundary layer of the atmosphere is obtained. The calculations show that the convective current generator exerts not only the local influence on the distribution of the electrical parameter of the atmosphere, but also determines the values of the average potential of the ionosphere along with thunderstorm current generators. The numerical estimations of changing of potential of ionosphere are given for the case of the convective-unstable boundary layer, assuming that the region of the action of thunderstorm sources and convective current generators are spatially divided. The value of changing is equal to 10–40%. Thus, along with thunderstorm activity, convective processes in the atmosphere make the contribution to the global temporary variation of the electrical parameters of the atmosphere. Further development of this conception is determined by the investigations of the geographical distribution of the planetary boundary layer on the Earth's surface and the investigations of electrical processes in these regions of the atmosphere.

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References

Frenkel, Y. I.: Theory electric phenomena of atmosphere, L.-M., Gostehizdat (in Russian), 155 p., 1949.

Hays, P. B. and Roble, R. G.: A quasi-static model of global atmospheric electricity. I. Lower Atmosphere, J. Geophys. Res., 84(A7), 3291–3305, 1979.

Holser, R. E. and Saxon, D. S.: Distribution of electrical conduction current in the vicinity of thunderstorm, J. Geophys. Res., 52(2), 207–217, 1952.

Imjanitov, I. M. and Kolokolov, V. P.: Investigations of electric field of atmosphere, Trudy MGO (in Russian), 334, 232–250, 1974.

Monin, A. S. and Yaglom, A. I.: Statistical hydromechanics. Part 1. M., Science (in Russian), 639 p., 1965.

Morozov, V. N.: Models of atmospheric circuit, Ser. "Meteorology", 8, VNIIGMI-MCD, Obninsk (in Russian), 56 p., 1981.

Morse F. M. and Feshbah, G.: Methods of theoretical physics. 2. M.-L. (in Russian), 886 p., 1960.

Pulinets, A., Khegai, V. V., Boyarchuk, K. A., and Lomonosov, A. M.: Atmospheric Electric field as a Source of Ionospheric Variability, Physics-Uspekhi (in Russian), 41(5), 515–522, 1998.

Selezneva, A. N.: Influence of thunderstorm generators on atmospheric circuit. Atmospheric Electricidy, Proceeding of 2th symposium of USSR, L., Gidrometeoizdat (in Russian), 17–19, 1984.

Willet, J. C.: Fair-Weather electric charge transfer by convection in an unstable planetary boundary layer, J. Geophys. Res., 84(C2), 703–718, 1979.

Wilson, C. T. R.: Investigations of lightning discharged and electric fields of thunderstorms, Phyl. Trans. Roy. Soc., London, 221, 75–115, 1925.