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Complex-network description of seismicity*

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Abstract. The seismic data taken in California and Japan are mapped to growing random networks. It is shown in the undirected network picture that these earthquake networks are scale-free and small-work networks with the power-law connectivity distributions, the large values of the clustering coefficient, and the small values of the average path length. It is demonstrated how the present network approach reveals complexity of seismicity in a novel manner.

1 Introduction

Seismicity is governed by yet unknown dynamics of the earth crust as a complex system. Although seismology has a long tradition, only a few universal laws have been discovered. The celebrated examples are the Omori law (Omori, 1894) for the temporal pattern of aftershocks and the Gutenberg-Richter law (Gutenberg and Richter, 1954) for the scaling relation between frequency and magnitude. Although there are some discussions about the theoretical bases of these laws, it seems fair to say that essentially they still remain empirical. The situation shows how understanding physics of earthquakes is far from maturity. And, there may be much to be explored even at the empirical level. This in turn suggests a possibility that approach from the viewpoint of science of complexity may shed new light on seismicity.

In the recent investigations (Abe and Suzuki, 2003, 2005a), we have analyzed the spatio-temporal properties of seismicity from the viewpoint of nonextensive statistical mechanics (Abe and Okamoto, 2001). Nonextensive statistical mechanics is constructed based on the Tsallis entropy (Tsallis, 1988) and generalizes Boltzmann-Gibbs statistical mechanics in order to treat complex systems. We have found that both the spatial distance and time interval between two successive earthquakes are well described by the q-exponential distributions, which are characteristics of nonextensive statistical mechanics and maximize the Tsallis entropy under appropriate constraints. The fact that two successive earthquakes obey such definite statistical laws implies that successive events are indivisibly correlated, no matter how large their spatial distance is. In fact, there is a report (Steeples and Steeples, 1996), which shows that an earthquake can be induced by a foregoing earthquake more than 1000 km away. This means that the seismic correlation length may be enormously large, indicating a strong similarity to phase transition phenomena and making it inappropriate to put spatial windows in analysis of seismicity, in general. Thus, we are naturally led to a conclusion that the earth crust always stays in a critical state.

In this article, we discuss a novel method of describing complexity of seismicity, which has recently been introduced in the literature (Abe and Suzuki, 2004b, c). This method uses the concept of complex networks (Albert and Barabási, 2002; Dorogovtsev and Mendes, 2003), in particular, scale-free networks (Albert and Barabási, 2002) and small-world networks (Watts and Strogatz, 1998). We define the mapping of the seismic data to a growing random graph and then study its topological properties. Such a network representation ideally realize the above-mentioned fact that two successive events are indivisibly correlated, irrespectively of their spatial distance.

The article is organized as follows. In Sect. 2, the method of constructing earthquake networks is explained. In Sect. 3, the results are presented for the connectivity distributions (or, the degree distributions) of the earthquake networks, which manifest the fact that the networks are scale free. In Sect. 4, an aspect of the earthquake networks as the small-world networks is discussed employing the data in both California and Japan. We calculate the values of the clustering coefficient, and then perform full analysis of the average path length over the whole data, which improves and expands the previous

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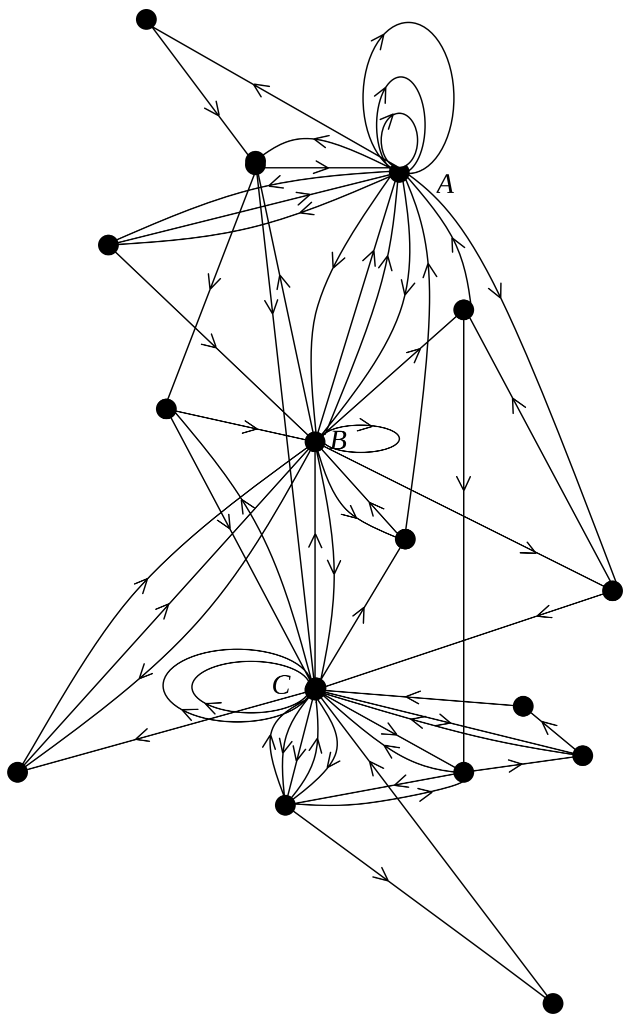

Fig. 1. A schematic description of the earthquake network. The vertices represent the cells, in which earthquakes occurred, and the edges replace the patterns of complex event-event correlation. A, B, and C are the cells containing main shocks and have the large values of connectivity, playing role of hubs.

analysis using the method of random sampling only for the data in California (Abe and Suzuki, 2004c). Section 5 is devoted to concluding remarks.

2 Mapping seismic data to a growing random graph

Seismic data basically consists of the series of a set of values of occurrence time, hypocenter (or, focus), and magnitude of each earthquake. It can therefore be seen as a field-theoretical system, in which magnitude as a field strength is defined on discrete spacetime points. Unlike ordinary field theories, however, both the field strength and spacetime points are inherently probabilistic in the case of seismicity. A basic idea here is to represent seismic data by a growing random graph.

Our proposal for constructing an earthquake network is quite simple. A geographical region under consideration is divided into a lot of small cubic cells. A cell is regarded as a vertex if earthquakes with any values of magnitude occurred therein. Two successive events define an edge between two vertices. If two successive events occur in the same cell, they form a loop. This procedure enables us to map the seismic data to a growing random graph. This graph, referred to as the earthquake network, represents dynamical information of seismicity in a peculiar manner.

Several comments on this construction are in order. First of all, it contains a single parameter: the cell size, which is a scale of coarse graining. Once the cell size is fixed, the earthquake network is unambiguously defined. However, since there exist no a priori operational rule to determine the cell size, it is of importance to examine how the properties of the earthquake network depend on this parameter. Secondly, it should be noticed that edges and loops efficiently represent correlation between successive earthquakes, the crucial importance of which is emphasized in the preceding section. Thirdly, the earthquake network is a directed graph in its nature. Directedness does not bring any difficulties to statistical analysis of connectivity (degree, the number of edges attached to the vertex under consideration) since, by construction, in-degree and out-degree (Pastor-Satorras and Vespignani, 2004) are identical for each vertex. We shall not distinguish in-degree and out-degree from each other in the analysis of the connectivity distributions. However, directedness becomes essential when the path length (i.e., the number of edges) between a pair of connected vertices, i.e., the degree of separation between the pair, is considered (Abe and Suzuki, 2005b). In the directed network picture, the path length corresponds to natural time of the newer vertex measured from the older one. Recent investigations (Varotsos et al., 2002; Abe and Suzuki, 2004a; Tirnakli and Abe, 2004; Abe et al., 2005) show how the natural time representation plays a prominent role in analyzing complex time series. Finally, directedness has to be ignored in the small-world picture, and the path length in this case should be defined as the smallest value among the possible numbers of edges connecting the pair of vertices. Also, loops have to be removed and multiple edges are replaced by single edges. That is, an undirected simple graph has to be considered in the small-world picture.

3 Scale-free nature of the earthquake networks

An earthquake network thus constructed is schematically depicted in Fig. 1. There, one sees that there are a few special vertices, labeled by A, B, and C, which have large values of connectivity. Such vertices are termed “hubs”. A striking feature we discovered from data analysis is that aftershocks

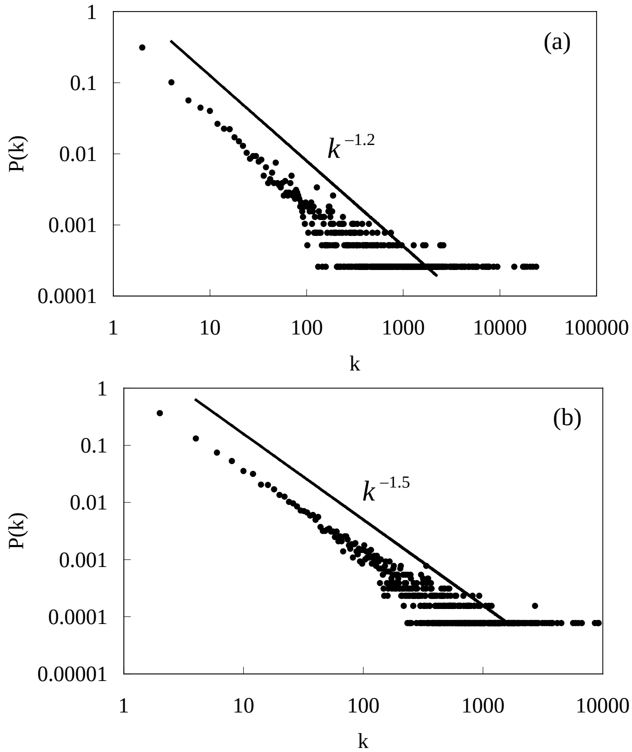

Fig. 2. The log-log plots of the connectivity distributions of the earthquake networks in California for the cell sizes **(a)** 10 km×10 km×10 km, and **(b)** 5 km×5 km×5 km. The total numbers of vertices are (a) N=3869, and (b) N=12 913, respectively.

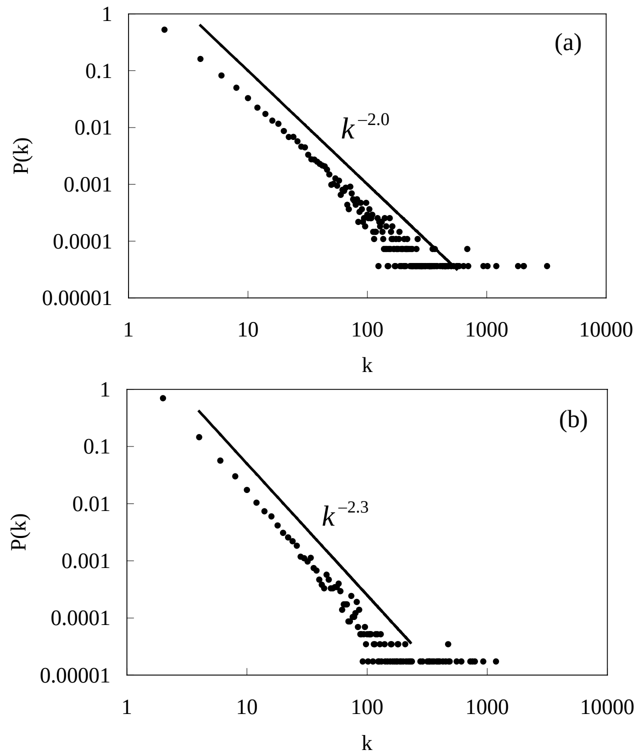

Fig. 3. The log-log plots of the connectivity distributions of the earthquake networks in Japan for the cell sizes **(a)** 10 km×10 km×10 km, and **(b)** 5 km×5 km×5 km. The total numbers of vertices are (a) N=27 599, and (b) N=57 768, respectively.

associated with a main shock tend to return to the locus of the main shock, geographically, making the vertex of the main shock a hub. This has an analogy with the preferential attachment rule for a growing network (Albert and Barabási, 2002; Dorogovtsev and Mendes, 2003; Pastor-Satorras and Vespignani, 2004). That is, a newly created vertex is connected to the ith vertex with connectivity k_i with probability

$$\Pi\left(k_i\right)=\frac{k_i}{\sum_j k_j}. \tag{1}$$

This rule is known to generate a scale-free network characterized by the power-law connectivity distribution:

$$P\left(k\right)\sim k^{-\gamma}, \tag{2}$$

where γ is a positive exponent.

A scale-free network is in contrast to the Erdös-Rényi classical random graph (Erdös and Rényi, 1959; Bollobás, 2001; Dorogovtsev and Mendes, 2003), the connectivity distribution of which is Poissonian.

The observation of the above-mentioned feature of aftershocks may lead to the reasoning that the earthquake network is a scale-free network. Below, we shall see that this is indeed the case.

We have constructed the earthquake networks from the seismic data taken in California and Japan. The data sources are i) the Southern California Earthquake Data Center (http://www.data.scec.org/), and ii) the Japan University Network Earthquake Catalog (http://kea.eri.u-tokyo.ac.jp/CATALOG/junec/monthly.html). The time intervals are i) between 00:25:8.58 on 1 January 1984 and 22:21:52.09 on 31 December 2003, and ii) between 01:14:57.63 on 1 January 1993 and 20:54:38.95 on 31 December 1998. The regions covered are i) 29°06.00′ N–38°59.76′ N latitude and 113°06.00′ W–122°55.59′ W longitude with the maximal depth 175.99 km, and ii) 25.730° N–47.831° N latitude and 126.433° E–148.000° E longitude with the maximal depth 599.9 km. The total numbers of events are i) 367 613, and ii) 123 390. The data in California contains no threshold for magnitude, but we exclude artificial “quarry blasts” from the data. On the other hand, the data in Japan contains only the events with magnitude larger than 2.

The connectivity distributions in California and Japan are shown in Figs. 2 and 3, respectively. There, we compare the results with two cell sizes: (a) 10 km×10 km×10 km, and (b) 5 km×5 km×5 km. The minimal case (b) is legitimate from the geophysical viewpoint regarding the typical size of

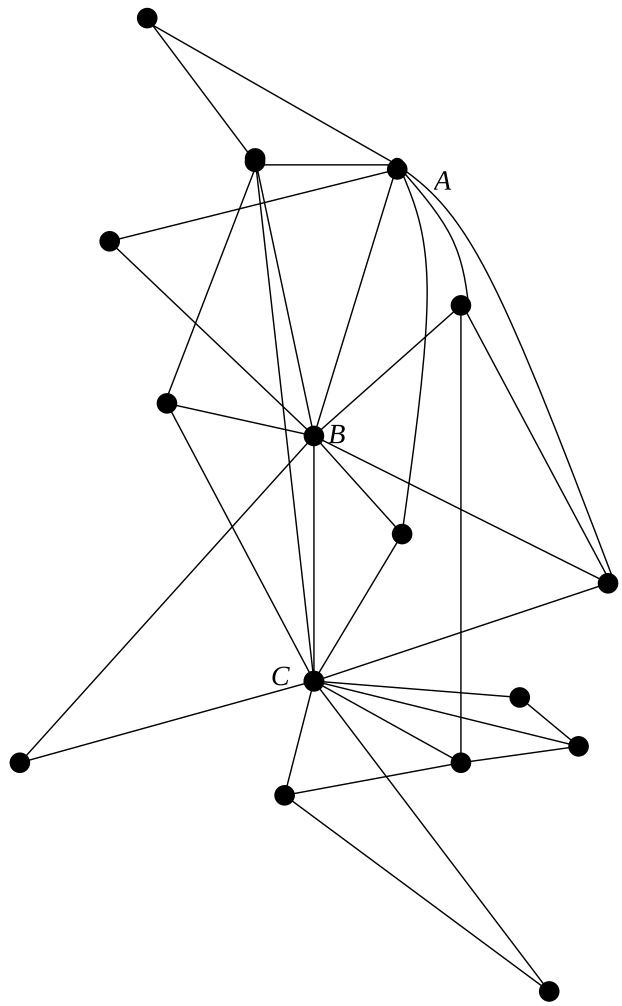

Fig. 4. Reduction of the network in Fig. 1 to the corresponding undirected simple graph.

a fault. From them, we conclude that the earthquake networks are scale-free networks characterized by the connectivity distributions of the form in Eq. (2). As mentioned in Sect. 2, in-degree and out-degree are not distinguished here since they are identical for each vertex. The smaller the cell size is, the larger the exponent, γ, is. This is natural since the number of vertices with large values of connectivity decreases as the cell size becomes smaller. However, the trend remains unchanged for the different cell sizes. We have also examined the effect of threshold for magnitude on the connectivity distributions by using the data in California. As expected, again the trend does not change for the threshold values: M_{th}=0∼3.

The result may physically be interpreted as follows. As already mentioned, aftershocks associated with a main shock tend to be connected to the vertex of the main shock, realizing preferential attachment. On the other hand, the Gutenberg-Richter law states that frequency of earthquakes with large values of seismic moment decays slowly as a power law with respect to the value of moment. This implies that there appear quite a few giant components, and accordingly the network becomes highly inhomogeneous. However, see also the comment on hierarchical organization in Sect. 5.

4 Small-world structure of the earthquake networks

Small-worldness is also an important ingredient of complex networks. To examine the small-world structure of earthquake network, it is essential to notice the following point: loops have to be removed and multiple edges should be replaced by single edges, and then directedness is ignored. This is because, in the small-world picture, we are concerned only with static pattern of whether vertices are connected or not. Consequently, the original earthquake network has to be reduced to an undirected simple graph. In Fig. 4, we show the reduction of the original network in Fig. 1.

Two important characteristics (Watts and Strogatz, 1998) of a small-world network are a large value of the clustering coefficient compared to the Erdös-Rényi classical random graph and a small value of the average path length (i.e., the number of edges connecting two vertices).

The clustering coefficient is defined as follows. Let $A=(a_{i\,j})$ be the adjacency matrix of a simple graph. $a_{i\,j}=1$ if the ith and jth vertices are connected by an edge, whereas $a_{i\,j}=0$ if they are not directly connected. $a_{i\,i}=0$ because of the absence of loops. Using the adjacency matrix, the clustering coefficient, C, is given by

$$C = \frac{1}{N} \sum_{i=1}^{N} c_i, \tag{3}$$

$$c_i = \frac{1}{k_i\,(k_i-1)/2}\,(A^3)_{i\,i}, \tag{4}$$

where N is the total number of vertices. This quantity shows tendency of two neighboring vertices of a given vertex being connected to each other. It is known (Watts and Strogatz, 1998; Albert and Barabási, 2002; Dorogovtsev and Mendes, 2003) that the clustering coefficient of a small-world network is much larger than that of the classical random graph of Erdös and Rényi given by

$$C_{\mathrm{random}} = \frac{<k>}{N} \ll 1, \tag{5}$$

where $<k>$ is the average value of connectivity.

We have analyzed the same data as those in the discussion about the connectivity distributions in Sect. 3. In Table 1, we present the results obtained by performing full analysis (in contrast to the previous limited study of the average path length based on the method of random sampling only for the data in California; Abe and Suzuki, 2004c). (However, because of its heavy combinatorial-problem nature, still we

Table 1. The values of the number of vertices, N, the clustering coefficient, C, (compared with those of the classical random graphs, C_{random}) and the average path length, L. Because of the large number of vertices, the average path length of the earthquake network in Japan with the cell size 5 km×5 km×5 km could not be evaluated (see the text).

cell size	10 km×10 km×10 km	5 km×5 km×5 km
California	N=3869 C=0.630 (C_{random}=0.014) L=2.526	N=12 913 C=0.317 (C_{random}=0.003) L=2.905
Japan	N=27 599 C=0.045 ($C_{\text{random}}=0.298\times10^{-3}$) L=3.825	N=57 768 C=0.015 ($C_{\text{random}}=7.111\times10^{-5}$) L= − −−

could not obtain the definite value of the average path length for the data in Japan with the cell size 5 km×5 km×5 km, unfortunately.) From it, one clearly appreciates that the values of the clustering coefficient are much larger than those corresponding to the associated classical random graphs. The values of the average path length are also seen to be very small, between 2 and 4, taking into account the numbers of vertices. Thus, the earthquake networks are in fact small-world networks.

5 Concluding remarks

We have discussed the complex-network approach to seismicity and have shown how such an approach sheds new light on complexity of the phenomenon. We have analyzed the topological properties of the earthquake networks constructed from the seismic data in California and Japan. We have shown that the earthquake networks are scale-free networks characterized by the power-law connectivity distributions and have given a physical interpretation to this result based on network growth with the preferential attachment rule together with the Gutenberg-Richter law. Then, we have studied the small-world structure of the earthquake networks reduced to undirected simple graphs. Improving and generalizing the previous study using only the data in California, we have performed full analyses of the clustering coefficient and the average path length for the data not only in Californian but also in Japan. The values of clustering coefficient are found to be much larger than those of the classical random graphs. In addition, the values of the average path length are found to be very small. Thus, the earthquake networks are scale-free small-world networks.

There may be a number of important issues still to be addressed. Recently, we have studied the period distribution of the directed earthquake network in California (Abe and Suzuki, 2005b), which show after how many events the earthquake returns to the initial vertex. This is of interest in view of earthquake prediction. There, we have found that the period distributions obey a power law, suggesting a fundamental difficulty of predicting the period. Such investigations using data in other regions are of obvious importance. More recently, we have investigated the hierarchical structure as well as the mixing property of earthquake networks (Abe and Suzuki, 2006). We have discovered that the earthquake networks possess hierarchical organizations and assortative mixing. Assortative mixing means that vertices with large values of connectivity tend to be connected to each other. That is, a main shock induces other main shocks. These features are important due to the following reason: the hierarchical organization cannot be realized by a simple combination of network growth and the preferential attachment rule, and, therefore, still there must be more physical mechanisms to be revealed.

The complex-network approach also enables one to examine seismological models from a peculiar viewpoint. Recent works (Peixoto and Prado, 2004, 2006) discuss the complex-network approach to a self-organized-criticality model. It was shown that under certain conditions a scale-free network can in fact be realized by the model.

It is our expectation that the present complex-network approach may lead to deeper understanding of physics of seismicity.

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