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Theoretical interpretation of fetch limited wind-driven sea observations

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Abstract.
We show that the results of major fetch limited field studies of wind-generated surface gravity waves on deep water can be explained in the framework of simple analytical model. The spectra measured in these experiments are described by self-similar solutions of "conservative" Hasselmann equation that includes only advective and nonlinear interaction terms. Interaction with the wind and dissipation due to the wave breaking indirectly defines parameters of the self-similar solutions.

1 Introduction

Theoretical interpretation of experimentally observed wind-driven sea energy spectra is a fundamental problem of physical oceanography, and of the physics of nonlinear waves in generally. At the moment oceanographers accumulated an enormous amount of experimental data, both in field and laboratory, on the wind generated waves.

Among field studies of wind-generated surface gravity waves on deep water, the case of fetch limited wave growth is of special importance. This case is characterized by a constant (in space and time) offshore wind at the normal angle to the straight coastline. Under these idealized conditions, the wave field depends only on the fetch x that is defined as a distance from the shoreline. Highly interesting data sets of field observations are available, for example, from the following studies:

1. Measurements in Nakata Bay (Mitsuyasu et al., 1971);
2. The JONSWAP studies (Hasselmann et al., 1973);
3. Measurements in the Bothnian sea (Kahma, 1981);
4. Measurements in Lake Ontario (Donelan et al., 1985);
5. Measurements off the Nova Scotia coast (Dobson et al., 1989);
6. Measurements in Lake St. Clair, Canada (Donelan et al., 1992).

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The results of these experiments were analyzed by many authors and are summarized by Young (1999).

Theoretical explanation of field and laboratory experiments has to include a clarification of basic physical processes that take place in the boundary layer over the sea surface. This layer is turbulent. The turbulence causes forced oscillations of sea surface; these oscillations can be considered as "seeds" of future wind-driven waves (Phillips , 1957). What is more important, the boundary layer is unstable if the wind velocity $u>u_{\rm crit}$, where

$$u_{crit} = \sqrt{2}(g\sigma)^{1/4} \simeq 0.23\,\mathrm{m/s} \tag{1}$$

Here g is the gravity acceleration, σ is a surface tension. That means that the range of unstable wave numbers is very broad

$$k_{\rm min} < k < k_{\rm max} \tag{2}$$

Here $k_{\rm min}=g/u^2$ is a "standard" wave number associated with the wind velocity u, while

$$k_{\rm max} \simeq \frac{u^2}{\sigma} \simeq k_{\rm crit}\left(\frac{u}{u_{\rm crit}}\right)^2, \tag{3}$$

where $k_{\rm crit}=\sqrt{g/\sigma}$ is the wave number that separates the capillary and gravity waves. For $u\gg u_{\rm crit}$ we have $k_{\rm max}\gg k_{\rm crit}$, and the real upper limit of unstable wave numbers is defined by competing of wind-induced instability and viscous damping of capillary waves. Other physical effects: shear flow, viscosity of air, drop formation, and turbulence are also important in this spectral range.

The linear instability leads to exponential growth in time and space of wave amplitudes. This growth has to be arrested by some nonlinear process; determination of this nonlinear saturation process is the most critical point of the theory. One fact is clear: in a typical situation there is a white-capping on the crest of each major wave. Starting from this observation, Phillips (1958) suggested that the linear instability induced by the wind is arrested by strong nonlinear effects, which lead to formation of the Stokes waves with amplitudes close to limiting. However, this elegant conjecture was not supported by experimental data. The Phillips' theory presumes that the overall wave steepness $\mu=ka$ should be close to its limiting value $\mu_{\text{crit}}\simeq 0.41$; here k is a characteristic wave number and a is a characteristic wave amplitude. Meanwhile, a typical experimental value is $\mu\simeq 0.1$ that is essentially less than μ_{crit}. The overall steepness μ is a measure of the wave nonlinearity. All experimental data show that the average nonlinearity is small. On the other hand, due to broadness of the instability region, the number of degrees of freedom, involved in the process of wave generation, is very large, 10^8 or so, and the wave ensemble has to be described statistically. This fact was realized by Hasselmann (1962, 1963), who described the kinetic equation for average squared wave amplitude, known as Hasselmann's equation. In his derivation, Hasselmann used a systematic expansion in powers of the small parameter μ.

Derivation of the Hasselmann's equation was a great success of the theory. However, this equation couldn't explain the white-capping, which is a universal phenomenon, at least at moderate wind velocity $u\geq 6$ m/s. This fact leaded many oceanographers to a conclusion that in a real situation the weak and strong nonlinearities do coexist. Moreover, to the conclusion that the four-wave resonant processes described by the Hasselmann's equation and the strongly nonlinear white-cap dissipation are equally important for dynamics of wind-driven spectra. In the most clear way this idea was expressed in a well-known article by Phillips (1985). In mathematical terms it can be formulated as follow: the Hasselmann's equation in the presence of wind input and white-cap dissipation can be written as

$$\frac{\partial N}{\partial t} + \nabla\omega_{\boldsymbol{k}} \nabla_{\boldsymbol{x}} N = S_{nl} + S_{in} + S_{ds} \quad (4)$$

Here S_{nl} is the nonlinear interaction term derived by Hasselmann, S_{in} is the wind input term, and S_{ds} is the dissipation term due to white-capping. According to Phillips, all three terms have compatible influence in determination of spectral peak dynamics. This is a rather pessimistic view on the theory capacities, because there is only a little hope to find a reliable, well justified expression for S_{ds}.

In this paper we formulate an alternative view on the problem. We show that more accurate estimates of S_{nl} demonstrate that this term is more important than S_{in} and S_{ds}. That means that on the average, the weakly-nonlinear interactions dominate over the wind input and over the strongly nonlinear interaction; that displays itself as white-capping. The weak and the strong nonlinear interaction coexist; however the white-capping is important in the high-frequency range only. In the area of spectral peak, the nonlinear four-wave interaction is the most important process.

That fact means that the space-time dynamics of the spectral peak can be properly described by the "conservative" Hasselmann's equation

$$\frac{\partial N}{\partial t} + \nabla\omega_{\boldsymbol{k}} \nabla_{\boldsymbol{x}} N = S_{nl} \quad (5)$$

The limited fetch study are described by the stationary conservative Hasselmann's equation

$$\nabla\omega_{\boldsymbol{k}} \nabla_{\boldsymbol{x}} N = S_{nl} \quad (6)$$

In both stationary and non-stationary cases the "rare faces" of the spectra are described by the stationary uniform kinetic equation

$$S_{nl} = 0 \quad (7)$$

Conservative models (Eqs. 5–7) are much simpler than the full nonconservative Eq. (4), which includes rather indefinite term S_{ds}. These models are available for analytic treatment. The simplest stationary uniform Eq. (7) has a family of the Kolmogorov-type solutions

$$N(k,\theta) = \frac{P^{1/3}}{k^4} F\left(\frac{Q\,\omega_k}{P}, \frac{M\,\omega_k}{kP}, \theta\right) \quad (8)$$

Here P, Q, and M are fluxes of energy, wave action, and momentum. Equations (5) and (6) have rich families of self-similar solutions. All these solutions have short-wave Kolmogorov asymptotics (Eq. 8).

In this article we concentrate on self-similar solutions of the stationary conservative Hasselmann's Eq. (6). This equation has a two-parameter family of similarity solutions. We show that these solutions fit very well for description of spectra observed in major fetch-limited experiments. We demonstrate that a proper choice of parameters in the similarity solution makes possible to reach not only qualitative but also quantitative agreement with the field experiments. This statement may appear counter to physical intuition because the surface waves are caused by wind and are accompanied with a substantial energy dissipation due to wave breaking. This controversy easily resolves if one notice that conservative kinetic Eqs. (5)–(6) are incomplete from the mathematical view-point. Their solutions are not defined uniquely by determination of initial data only. To define a solution one should specify also the flux of wave action $Q(\boldsymbol{r},t)$ coming from very high wave numbers. This flux is a result of complicated interplay between the income of wave action from the wind and its dissipation due to white-capping in the high-frequency region. For a self-similar solution of Eq. (6), Q is a powerlike function on fetch x. The exponent and the pre-exponent of Q defines parameters of a similarity solution.

Some authors instead of term "Hasselmann's equation" use "Boltzmann equation", while comparing the kinetic equation for squared amplitudes of surface gravity waves with the kinetic equation for distribution function of a weakly non-ideal gas. We should stress that this comparison cannot be extended too far. In the kinetic gas theory, the Boltzmann equation is complete, in a sense that its solutions are defined uniquely by determination of initial data. The Hasselmann's equation can be better compared with a heat transport equation in a finite domain; solutions of this equation depend essentially on boundary conditions. For Hasselmann's equation, the external flux Q plays the role of boundary condition.

The "conservative" model formulated in the present article is the simplest model that admits an analytical treatment. More accurate model should include into consideration the terms S_{in} and S_{ds}; these terms are considered to be small and can be treated as perturabation. In this model, the stationary Eq. (7) has to be replaced by more accurate equation

$$S_{nl} + S_{in} + S_{ds} = 0 \tag{9}$$

This equation still has the Kolmogorov-type solution (Eq. 8), however the fluxes P, Q, and M are now "slow" functions on frequency. The details of this study will be published in another article.

Let us emphasize that the theoretical model, presented in this article, is based on "first principles" and does not include any adjustable parameters. Nevertheless, the model yields results which not only are qualitative correct but are in a good quantitative agreement with field observations.

2 Basic equations

Let $\eta(\boldsymbol{r}, t)$, $\boldsymbol{r}=(x, y)$ be the surface elevation field, $\eta(k, t)$ its Fourier transform, and $I_k=I_{-k}=<|\eta_k|^2>$ the spectral density as a function of wave number. The wave field is described by the wave action spectral density N_k:

$$I_k = \frac{\omega_k}{2}(N_k + N_{-k}). \tag{10}$$

Here $\omega_k=\sqrt{gk}$ is the dispersion relation, where g is the vertical acceleration due to gravity.

The action density $N_k(\boldsymbol{r}, t)$ satisfies the Hasselmann kinetic equation,

$$\frac{\partial N}{\partial t} + \frac{\partial \omega}{\partial \boldsymbol{k}}\frac{\partial N}{\partial \boldsymbol{r}} = S_{nl} + S_F, \tag{11}$$

where

$$\begin{aligned} S_{nl} &= \pi g^2 \int |T_{kk_1k_2k_3}|^2 \\ &\times [N_2N_3)(N_1 + N_0) - N_0N_1(N_2 + N_3)] \\ &\delta(k + k_1 - k_2 - k_3)\delta(\omega_0 + \omega_1 - \omega_2 - \omega_3)dk_1\, dk_2\, dk_3 \end{aligned} \tag{12}$$

is a nonlinear interaction term and $T_{kk_1k_2k_3}$ is a coupling coefficient. The exact expression for T can be found in (Webb, 1978; Zakharov, 1999). Of great importance is the fact that $T_{kk_1k_2k_3}$ is a homogenous function of the third order:

$$T_{\zeta k, \zeta k_1, \zeta k_2, \zeta k_3} = \zeta^3\, T_{kk_1k_2k_3}. \tag{13}$$

The term $S_F=S_{in}+S_{ds}$ represents a source function which includes the wind input S_{in} and the breaking wave induced dissipation S_{ds}.

By definition, the variance of the surface elevation is given by

$$\sigma^2 = \int \omega_k\, N_k\, dk, \tag{14}$$

and the mean frequency is

$$\bar{\omega} = \frac{1}{\sigma^2}\int \omega_k^2\, N_k\, dk. \tag{15}$$

The overall squared wave steepness is characterized by an integral quantity

$$\mu^2 = \hat{\alpha} = \sigma^2\, \bar{k}^2. \tag{16}$$

Let U_a be the wind speed at a reference height a. We can now introduce the characteristic angular frequency and wave number related to the given wind speed U_a by

$$\omega_0 = g/U_a, \quad k_0 = g/U_a^2 \tag{17}$$

Most authors agree that S_{in} can be presented in the form

$$S_{in} = m\, F(\xi)\, \omega\, N(k), \tag{18}$$

where $\xi=\omega\ \cos\theta/\omega_0$ and

$$m = 0.1 \sim 0.3\ \frac{\rho_a}{\rho_\omega} \simeq 10^{-4}. \tag{19}$$

There is no agreement about the exact form of the function $F(\xi)$. According to (Donelan and Pierson-jr., 1987),

$$F(\xi) = \begin{cases} (\xi - 1)^2 & \xi > 1 \\ 0 & \xi < 1. \end{cases} \qquad m = 0.194\,\frac{\rho_a}{\rho_\omega} \tag{20}$$

According to Hsiao and Shemdin (1983),

$$F(\xi) = \begin{cases} (0.85\xi - 1)^2 & 0.85\xi > 1 \\ 0 & 0.85\xi < 1. \end{cases}, \quad m = 0.12\,\frac{\rho_a}{\rho_\omega} \tag{21}$$

Tolman and Chalikov (1996) proposed a more complicated form of $F(\xi)$. In any case, all such models have $F(\xi)\simeq 1$ for $\xi\sim 0$.

One should take into account that the very concept of an universal expression for S_{in}, depending only on the wind speed U_a, can be easily criticized. The wind input S_{in} is defined by the state of the turbulent air boundary layer. That state depends on many factors; one of them is the difference of water and air temperature. If water is warmer than air (unstable atmosphere), the wind input can be several times higher than in the opposite stable case (see Kahma and Calkoen, 1992). Another important factor is the gustiness of the wind; it is defined by statistical properties of the air boundary layer in mesoscale. Cavaliery and Burgens (1992) showed an enhanced growth rate of ocean waves with the increasing of gustiness. In principle, the growth rate could depend on more fine statistical characteristics of the boundary layer. Thus the scattering and variability of experimental data in fetch-limited studies can be explained not only by an inevitably limited accuracy of measurements, limited reproducibility of experimental conditions such as wind velocity, fetch and duration, but also by a variability of the state of boundary layer.

The dissipation term S_{ds} is studied much less than S_{in}. Hasselmann (1974, see also Komen et al., 1984) proposed the following form:

$$S_{ds} = q \left(\frac{\hat{\alpha}}{\alpha_{pm}}\right)^2 \left(\frac{\omega}{\bar{\omega}}\right)^n \omega N, \quad q = 3.33 \cdot 10^{-5}, \tag{22}$$

where α_{pm}=4.57×10^{-3}.

This relationship is widely used in WAM and SWAN wave prediction models. In most cases, it is assumed that n=2. In our opinion, this choice overestimates S_{ds}, especially in the spectral range $\omega_0<\omega<3\simeq4\,\omega_0$. There is no obvious physical reason for existence of essential dissipation of long waves with frequency near the spectral peak, if the waves are not "too young". Such dissipation has to display itself by creation of large-scale turbulence in the water boundary layer and by formation of vortices with characteristic size of order of the spectral peak wave length $L\simeq g/\omega_0^2$. However, the eddies of such scale have not been observed. All the turbulence produced by wave-breaking is concentrated in the thin layer near the surface (see, for instance Terray et al., 1996). The standard argument in support of dissipation in the spectral peak frequency area is the existence of white caps on wave crests. White caps appear at relatively weak wind ($U_a\sim5-6$ m/s). At $U_a\simeq10$ m/s they are common.

However, the mechanism of white-capping is still unclear. The white caps appear at a moderate wave steepness $\mu\simeq0.07$; this value is much less than the maximum steepness $\mu_{\max}\simeq0.4$ that is typical for breaking Stokes waves. In a real situation, even for young waves the steepness is moderate ($\mu<0.2$) at any small fetches. Typically, $\mu\sim0.1$ or less for the developed sea. The Stokes wave of such steepness is very smooth and close to a linear monochromatic wave.

Apparently the white capping in the real sea has little in common with breaking of strongly nonlinear, almost critical Stokes waves. One can suggest that the white-capping happens due to interaction of high frequency power-like spectral tails with long smooth waves. The spectral tails with Zakharov-Filonenko asymptotic $I_k\simeq k^{-7/2}$ means the formation of fractal structure on the surface. It was shown (Newell and Zakharov, 1992) that at a small wind speed $U_a<(\sigma g)^{1/4}\simeq6$ m/s the surface tension keeps the sea smooth. For higher wind speeds the wave breaking is inevitable. There is no reason to think that the wave-breaking is uniform in space. One can suggest that this process is concentrated mostly near the crests of long waves. This is a very interesting question that deserves to be studied carefully.

It is assumed that in the fetch-limited experiments $\partial N/\partial t$=0, and so Eq. (11) takes the form

$$\frac{\partial\omega}{\partial\boldsymbol{k}}\frac{\partial N}{\partial\boldsymbol{r}} = S_{nl} + S_F \tag{23}$$

We can estimate different terms in Eq. (23). If the sea is not "full developed", the advective term can be estimated as follow

$$\frac{\partial\omega}{\partial k}\frac{\partial N}{\partial r} \simeq \frac{\omega}{k}\frac{N}{L} \tag{24}$$

Here L is the fetch. S_F is positive at least in some spectral domain behind the spectral peak; otherwise the waves could not be exited at all. Certainly, $S_F \le S_{in}$. S_F can be estimated as follow

$$S_{in} < m\,\omega \left(\frac{\omega}{\omega_0}\right)^2 N, \quad m \simeq 10^{-4} \tag{25}$$

If $\omega\gg\omega_0$, then $S_{in}<m\omega N$ at $\omega\simeq\omega_0$.

If n=2, Eq. (22) can be rewritten as follow

$$S_{ds} \simeq 1.58\,\mu^4 \left(\frac{\omega}{\bar{\omega}}\right)^2 \omega N \tag{26}$$

One can see that in this case S_{ds} has exactly the same dependance on frequency as S_{in} if $\omega\gg\omega_0$. For $\omega\sim\omega_0$, S_{ds} dominates over S_{in}.

There is no any theoretical justification for such strong assumptions. As for experimental data, we will show that they can be explained without using of any particular empirical expressions for S_{ds}, which inevitably includes indefinite tunable parameters. Instead of using such forms of S_{ds}, we assume that S_{ds} at $\omega\sim\omega_0$ is negligibly small and grows with ω faster than S_{in}. For instance, Eq. (22) can be used if we put n=4 and assume that q is less in two orders of magnitude.

3 Estimate for S_{nl}

The central point of our theory is a predominance of S_{nl} over S_F. To be sure that this is correct, we show that this is a rather sophisticated problem. Suppose that the spectrum has a peak at $k_p\simeq\omega p^2/g$ and the value of N at the spectral peak is N_p. From Eq. (52) one can obtain a "naive" estimate

$$S_{nl} \simeq \omega_p N_p \frac{|T(k_p,k_p,k_p,k_p|^2}{\omega_p^2} k_p^4 N_p^2 \tag{27}$$

or

$$S_{nl} \simeq \omega_p N_p \left(\frac{\sigma^2\omega_p^4}{g^2}\right)^2 \tag{28}$$

In fact, estimates Eq. (27, Eq. 28) are vulnerable for critics. S_{nl} is the intergal operator, and it looks more reasonable to use an averaged value of frequency instead of its local value in the spectra. If the average frequency $\bar{\omega}$ is used instead of the peak frequency, Eq. (28) reads

$$S_{nl} \simeq \bar{\omega} N_p \left(\frac{\sigma^2\overline{\omega}^4}{g^2}\right)^2 \tag{29}$$

Estimates Eq. (28, Eq. 29) differ by the factor $\left(\bar{\omega}/\omega_p\right)^9$. This is not a small factor. Suppose that the spectrum has Zakharov-Filonenko asymptotics $\varepsilon_\omega\simeq\omega^{-4}$ and has no "peakeness". Thus,

$$\varepsilon_\omega \simeq c_1 P^{1/3}\theta(\omega-\omega_p)\,\omega^{-4} \tag{30}$$

In this case

$$\frac{\bar{\omega}}{\omega_p} = \frac{3}{2}, \quad \left(\frac{\bar{\omega}}{\omega_p}\right)^9 \simeq 38.4 \tag{31}$$

For the Zakharov-Zaslavskii asymptotics $\epsilon_\omega \simeq \omega^{-11/3}$, we have

$$\varepsilon_\omega \simeq c_2\, Q^{1/3}\, \theta(\omega-\omega_p)\, \omega^{-11/3}, \tag{32}$$

$$\frac{\bar\omega}{\omega_p}=\frac{8}{5}, \quad \left(\frac{\bar\omega}{\omega_p}\right)^9 \simeq 68.7 \tag{33}$$

A dramatic difference between Eqs. (28) and (29) stems from the fact that the coupling coefficient $|T_{kk_1k_2k_3}|^2$ is a very fast growing function of frequencies. Indeed,

$$W_{\omega\omega_1\omega_2\omega_3}=|T_{kk_1k_2k_3}|^2 \tag{34}$$

is a homogenous function of order 12

$$W_{\epsilon\omega,\epsilon\omega_1,\epsilon\omega_2,\epsilon\omega_4}=\epsilon^{12}\, W_{\omega\omega_1\omega_2\omega_3} \tag{35}$$

As far as S_{nl} is a nonlocal operator, integration in Eq. (11) involves into process the integration of the more short waves that the waves that are close to the spectral peak. This effect of "blurring" leads to essential enhancing of S_{nl}. Actually models (Eqs. 30 and 32) are too simple to describe real spectra. In reality the spectra have an essential "peakeness", such that the most part of energy is concentrated in a narrow spectral range $\Delta_\omega \ll \omega_p$ near the spectral peak. The peakeness decreases the effect of "blurring" but not supresses it completely. Our numerical calculation (Badulin et al., 2002) shows that even in the presence of peakeness

$$\frac{\bar\omega}{\omega_p}\simeq 1.3, \quad \left(\frac{\bar\omega}{\omega_p}\right)^9 \simeq 10.4 \tag{36}$$

and estimate Eq. (29) differs from estimate Eq. (28) in order of magnitude.

The peakeness is itself a factor for enhancing of S_{nl}. Near the spectral peak one can put $\boldsymbol{k}=\boldsymbol{k_p}+\delta\boldsymbol{k}$ and expand ω_k in powers of δ_k

$$\omega_k=\omega_p+\boldsymbol{v}\,\delta\boldsymbol{k}+\frac{1}{2}\,T_{\alpha\beta}\,\delta k_\alpha\,\delta k_\beta+\cdots \tag{37}$$

$$T_{\alpha\beta}=\frac{\partial^2}{\partial k_\alpha\,\partial k_\beta}\,\omega(k)\Big|_{k=k_p}, \quad \boldsymbol{v}=\frac{\partial\omega}{\partial\boldsymbol{k}}\Big|_{k=k_p} \tag{38}$$

There are four-wave resonant conditions

$$\omega_k+\omega_{k_1}=\omega_{k_2}+\omega_{k_3}, \quad \boldsymbol{k}+\boldsymbol{k_1}=\boldsymbol{k_2}+\boldsymbol{k_3} \tag{39}$$

which have to be replaced by

$$\frac{T_{\alpha\beta}}{2}\Big[\delta k_\alpha\,\delta k_\beta+\delta k_{1\alpha}\,\delta k_{2\beta} -\delta k_{2\alpha}\,\delta k_{2\beta}-\delta k_{3\alpha}\,\delta k_{3\beta}\Big]=0 \tag{40}$$

$$\delta\boldsymbol{k}+\delta\boldsymbol{k_1}=\delta\boldsymbol{k_2}+\delta\boldsymbol{k_3} \tag{41}$$

It is clear that in presence of peakeness, estimate Eq. (29) has to be replaced by the following estimate

$$S_{nl}\simeq\overline{\omega}\,N_p\left(\frac{\sigma^2\,\overline{\omega}^4}{g^2}\right)^2\left(\frac{\omega_p}{\Delta\omega}\right)^2 \tag{42}$$

In a typical situation, $\Delta\omega/\omega_p \sim 1/3$ and the factor $(\omega_p/\Delta\omega)^2$ adds to S_{nl} another order of magnitude.

Summarizing the results of our consideration, one can offer for S_{nl} in the area of spectral peak the following estimate, expressed in terms of σ^2 and ω_p:

$$S_{nl}\simeq\lambda\,\omega_p\,N_p\left(\frac{\sigma^2\,\omega_p^4}{g^2}\right)^2 \tag{43}$$

Here λ is the "enhancing" factor, which accumulates the effects of "peakeness" and "blurring". Due to corraboration of two effects, λ is rather large,

$$50<\lambda<100 \tag{44}$$

This estimate is supported by numerical simulations (Badulin et al., 2002; Pushkarev et al., 2003). In other words, our study shows that the "naive" estimate Eq. (28) diminishes the real value of S_{nl} almost by two orders of magnitude.

S_{nl} is the integral operator consisting of four terms of different signs. In the area of spectral peak these terms don't compensate each other; they are compensated by the advective term. However on the "rare face" of the spectrum at $\omega>\omega_p$, the advective term is too weak to compensate S_{nl}. There is only one possibility to reach balance in this spectral range: the different terms in S_{nl} should compensate each other. In this area Hasselmann's Eq. (4) is reduced to the simplest Eq. (7). At the same time, the separate terms in S_{nl} can be estimated as follow:

$$S_{nl}\simeq\lambda\,\omega\,N\left(\frac{\varepsilon_\omega\,\omega^5}{g^2}\right)^2 \tag{45}$$

The fastest decrease of ε_ω compatible with weak-turbulent theory is the Zakharov-Filonenko spectrum $\varepsilon_\omega \simeq \omega^{-4}$. Even in this case the particular terms in S_{nl} grow with ω as vigorously as ω^3. That means that if S_{nl} surpasses S_{in} in the area of spectral peak, this is also true for the whole "rare face" area.

The fast increasing with frequency of particular terms in S_{nl} is a manifistation of the fact that the weakly-nonlinear approximation together with the Hasselmann's equation is applicable only in the finite range of frequencies, approximately for $\omega<5\omega_p$. In the high frequency region the nonlinearity becomes strong, the white-capping occur, and weak-turbulent Kolmogorov spectra transit to the Phillips spectra $\varepsilon_\omega \simeq \alpha/\omega^5$ (Kitaigorodskii and Lumley, 1983; Kitaigorodskii et al., 1983; Newell and Zakharov, 1992). We will not discuss this very interesting subject the present article.

4 Self-similar solution

Let us introduce dimensionless variables

$$x=\chi/k_0, \quad \boldsymbol{k}=k_0\,\boldsymbol{\kappa}, \quad \omega=\omega_0\,\Omega, \quad \Omega=\sqrt{\kappa},$$

yielding

$$N(k)=\frac{1}{\omega_0\,k_0^4}\,n(\boldsymbol{\kappa}). \tag{46}$$

The non-dimensional surface height variance and the non-dimensional average frequency are:

$$\epsilon = k_0^2\,\sigma^2 = \frac{g^2}{U_0^4}\,\sigma^2, \tag{47}$$

$$\nu = \frac{1}{2\pi}\,\frac{\bar{\omega}}{\omega_0}. \tag{48}$$

Both ϵ and ν can be expressed in terms of $n(\boldsymbol{\kappa})$. Apparently,

$$\epsilon = \int \sqrt{\kappa}\, n(\boldsymbol{\kappa})\, d\boldsymbol{\kappa}, \tag{49}$$

$$\nu = \frac{1}{2\pi}\,\frac{\int |\kappa|\, n(\kappa)\, d\boldsymbol{\kappa}}{\int \sqrt{|\kappa|}\, n(\kappa)\, d\kappa}. \tag{50}$$

Under limited-fetch conditions, $n(\boldsymbol{\kappa}, \chi)$ is governed by the kinetic equation

$$\frac{\cos\theta}{2\Omega}\,\frac{\partial n}{\partial\chi} = \tilde{S}_{nl} + \tilde{S}_F, \tag{51}$$

where

$$\begin{aligned}\tilde{S}_{nl} &= \pi \int |T_{\kappa\kappa_1\kappa_2\kappa_3}|^2\, \delta(\boldsymbol{\kappa} + \boldsymbol{\kappa_1} - \boldsymbol{\kappa_2} - \boldsymbol{\kappa_3})\\ &\quad \times \delta(\Omega + \Omega_1 - \Omega_2 - \Omega_3) \times \\ &\quad \times [(n_0 + n_1)n_2 n_3 - n_0 n_1 (n_2 + n_3)]\, d\kappa_1\, d\kappa_2\, d\kappa_3,\\ \tilde{S}_F &\simeq m\,\Omega\, F(\Omega)\, n(\kappa, \chi).\end{aligned} \tag{52}$$

In Eq. (52) the parameter m is a small one (see Eq. 19, $m{\simeq}10^{-4}$). In a first approximation, we can set μ=0, and obtain the "conservative" kinetic equation:

$$\frac{\cos\theta}{2\Omega}\,\frac{\partial n}{\partial\chi} = \tilde{S}_{nl}. \tag{53}$$

This governing equation is the main focus of our analytical effort. It contains a family of self-similar solutions. In polar coordinates κ, θ on $\boldsymbol{\kappa}$ plane, these solutions can be presented as:

$$n(\kappa, \theta, \chi) = a\,\chi^\alpha\, P_\beta(b\,\chi^\beta\,\kappa, \theta). \tag{54}$$

Here a, b, α, β are constants.

Substituting Eq. (54) into Eq. (53) one finds

$$\alpha = 5\,\beta - 1/2, \quad a = b^5,$$

and ultimately,

$$n(\kappa, \theta, \chi) = b^5\,\chi^{5\beta - 1/2}\, P_\beta(b\,\chi^\beta\,\kappa,\ \theta). \tag{55}$$

In Eq. (55), β and b are constants that are unknown yet, and $P_\beta(z, \theta)$ is a function of two variables with $z{=}b\chi^\beta\kappa$. Let us emphasize that this function is independent on b; it satisfies the following integro-differential equation:

$$\begin{aligned}&\frac{\cos\theta}{2\sqrt{z}}\left[(5\beta{-}1/2)P_\beta{+}\beta z P_z\right]{=}\pi \int |T_{z,z_1,z_2,z_3,\theta,\theta_1,\theta_2,\theta_3}|^2\\ &\times \delta(z\cos\theta + z_1\cos\theta_1 - z_2\cos\theta_2 - z_3\cos\theta_3)\\ &\times \delta(z\sin\theta + z_1\sin\theta_1 - z_2\sin\theta_2 - z_3\sin\theta_3)\\ &\times \delta(\sqrt{z} + \sqrt{z}_1 - \sqrt{z}_2 - \sqrt{z}_3)\\ &\times \Big[P_\beta(z_1,\theta_1)\, P_\beta(z_2,\theta_2)\, P_\beta(z_3,\theta_3) + \\ &\quad +P_\beta(z,\theta)\, P_\beta(z_2,\theta_2)\, P_\beta(z_3,\theta_3)\\ &\quad -P_\beta(z,\theta)\, P_\beta(z_1,\theta_1)\, P_\beta(z_2,\theta_2)\\ &\quad -P_\beta(z,\theta)\, P_\beta(z_1,\theta_1)\, P_\beta(z_3,\theta_3)\Big]\\ &\quad z_1\, z_2\, z_3\, dz_1\, dz_2,\, dz_3\, d\theta_1\, d\theta_2\, d\theta_3.\end{aligned} \tag{56}$$

This is the equation that has to be solved numerically.

Let us denote:

$$A_\beta = \int \sqrt{z}\, P_\beta(z, \theta)\, z\, dz\, d\theta, \tag{57}$$

$$B_\beta = \int z\, P_\beta(z, \theta)\, z\, dz\, d\theta. \tag{58}$$

Then, we find

$$\begin{aligned}\epsilon &= b^{5/2}\,\chi^{\frac{5\beta-1}{2}}\, A_\beta,\\ \nu &= \frac{1}{2\pi}\, b^{-1/2}\,\chi^{-\beta/2}\,\frac{B_\beta}{A_\beta}.\end{aligned} \tag{59}$$

These equations can be written as:

$$\begin{aligned}\epsilon &= u\,\chi^p,\\ \nu &= v\,\chi^{-q}\ ,\end{aligned} \tag{60}$$

where

$$q = \frac{2p+1}{10}, \qquad \beta = \frac{2p+1}{5}, \tag{61}$$

$$v = \frac{1}{2\pi}\,\bar{u}^{1/5}\, C_\beta, \qquad C_\beta = \frac{B_\beta}{A_\beta^{1/5}}. \tag{62}$$

After integrating the Eq. (51) over $\boldsymbol{\kappa}$ we obtain the balance equation

$$\frac{1}{2}\int \frac{\cos\theta}{\Omega}\,\frac{\partial n}{\partial\chi}\, d\boldsymbol{\kappa} = \int \tilde{S}_F\, d\kappa = Q(\chi). \tag{63}$$

Here Q is the total input of wave action; a net result of wind forcing and breaking wave dissipation. Substituting the self-similar solution Eq. (55) into Eq. (53), we find

$$Q \simeq \chi^{\frac{7\beta-3}{2}}\, b^4 \tag{64}$$

Therefore, the solution (Eq. 55) implies the presence of the wave action source Q at high wave numbers. For $\beta{=}\beta_{\rm crit}{=}3/7$, the net input Q=const, and the intensity of the source does not depend on the fetch. For $\beta{>}\beta_{\rm crit}$, the input Q grows with an increasing fetch. For this special self-similar solution

$$n_{\rm crit}(k, \theta, \chi) = b\,\chi^{23/14}\, P_{3/7}\,(b\,\chi^{3/7}\,\kappa, \theta), \tag{65}$$

$$p = \frac{4}{7} = 0.57,$$

$$q = \frac{3}{14} = 0.21,$$

Solution (Eq. 65) was studied by Zakharov and Zaslavskii (1983 1983) and Glazman (1994). For $\beta{>}3/7$ the input Q

grows on increasing fetch. Note that according Eq. (65) the total wave action is

$$N = \int \kappa\, n(\kappa)\, d\kappa\, d\theta \simeq b^3\, \chi^{3\beta-1/2}. \tag{66}$$

Self-similar solution (Eq. 54) describes swell if Q=0, N=const. In this case β=1/6 and Eq. (54) takes the form

$$n(\kappa,\theta,\chi) = b^5\, \chi^{2/3}\, P(b\chi^{1/6}\kappa,\theta). \tag{67}$$

The peak of the self-similar solution is posed ar $z\simeq 1$. The rare face of the similarity solution is a Kolmogorov type solution. In the primitive variables, it is described by Eq. (32), where Q is given by Eq. (64), while the fluxes of energy and momentum are

$$P(\Omega,\,\theta) \simeq b^{7/2}\, \chi^{\frac{5\beta-3}{2}}$$
$$M(\kappa,\,\theta) \simeq b^3\, \chi^{\frac{3}{2}(\beta-1)} \tag{68}$$

Ar very large κ, solution (27) becomes isotropic. Asymptotically,

$$n(\kappa) \simeq \frac{c_1\, Q^{1/3}}{\kappa^{23/6}} \tag{69}$$

This is Zakharov-Zaslavskii spectrum that describes the inverse cascade (Zakharov, 1966; Zakharov and Zaslavskii, 1982). In the case of swell, Q=0 and the self-similar solution has another asymptotics

$$n(\kappa) = \frac{c_0\, P^{1/3}}{\kappa^4}, \tag{70}$$

where $P\simeq\chi^{1/6}$. This is the Zakharov-Filonenko spectrum of direct energy cascade.

Study of self-similar solutions shows that the conservative kinetic Eq. (53) is incomplete. It must be accomplished by the boundary condition

$$n(\kappa) \to \frac{c_1\, Q^{1/3}}{\kappa^{23/6}} \tag{71}$$

that defines the flux of wave action $Q(\boldsymbol{r},t)$ coming from high wave numbers. In a general case, $Q(\boldsymbol{r},t)$ is an arbitrary function of coordinates and time. Asymptotics (Eq. 62) plays a role of a boundary condition for the conservative kinetic equation.

5 Comparison with experiment

We shall now compare the similarity solution (Eq. 55) with the field observations under limited-fetch conditions. Let us first notice that an elementary analysis of observed spectra shows their self-similar behavior. The similarity is implicit in the fact that the spectra can be expressed by a universal form that involves a finite number of parameters.

The frequency spectrum can be introduced as follows:

$$F(f) = k\,\omega_k\, \frac{dk}{df} \int_0^{2\pi} N(k,\theta)\, d\theta, \quad k = (2\pi\, f)^2/g. \tag{72}$$

Table 1. First group of experiments.

Study (3 pt)	$\epsilon(\chi)$	$\nu(\chi)$
Nakata Bay (Mitsuyasu et al., 1971) (3 pt)	$2.89\times10^{-7}\,\chi$	$3.12\,\chi^{-0.33}$
JONSWAP (Hasselmann et al., 1973) (3 pt)	$1.6\times10^{-7}\,\chi$	$3.5\,\chi^{-0.33}$
Bothnian Sea (Kahma, 1981) (3 pt)	$2.6\times10^{-7}\,\chi$	$3.18\,\chi^{-0.33}$
Lake St. Clair (Donelan et al., 1992) (3 pt)	$1.7\times10^{-7}\,\chi$	$3.6\,\chi^{-0.33}$

For example, the results of the JONSWAP experiment are summarized by the spectral form (Hasselmann et al., 1973):

$$F(f) = \frac{\alpha\, g^2}{2\pi^4 f^5} \exp\left[-\frac{5}{4}\left(\frac{f}{f_p}\right)^{-4}\right] \gamma^{\exp\left[-\frac{(f-f_p)^2}{2\sigma^2 f_p^2}\right]} \tag{73}$$

At moderate values of the non-dimensional fetch $\chi<10^5$, parameters γ and σ are approximately constant

$$\gamma \simeq 3.3, \quad \sigma \simeq 0.08,$$

while α and f_p are powers of χ. Form Eq. (73) is explicitly self-similar. Actually, the f^{-5} asymptotic of Eq. (73) represents the regime described by the Phillips spectrum (Phillips, 1958). A more accurate analysis of the JONSWAP spectra (Battjes et al., 1987) shows that a f^{-4} behavior is more relevant. Donelan et al. (1985) also found the f^{-4} asymptotic for their spectra. This is exactly what the theory predicts, but a discussion of this issue is outside the scope of this article.

In accordance with the main assumption of self-similarity, the fetch dependence of ϵ and ν is described by powerlike functions of χ. To assure that the spectra are described by the similar solution (Eq. 55), one has to compare these functions with the theoretically predicted dependencies (Eq. 61). The results of the major fetch limited experiments can be summarized in two tables:

Expression (61) makes it possible to find q if p is give. For the first group of experiments $p=1$, and one obtains:

$$q = 0.3, \quad \beta = 0.6$$

For the Lake Ontario experiment:

$$p = 0.76, \quad q = 0.25, \quad \beta = 0.5$$

For the North Atlantic study:

$$p = 0.75, \quad q = 0.25, \quad \beta = 0.5$$

Table 2. Second group of experiments.

Study (3 pt)	$\epsilon(\chi)$	$\nu(\chi)$
Lake Ontario (Donelan et al., 1985) (3 pt)	$8.415\times10^{-7}\,\chi^{0.76}$	$1.85\,\chi^{-0.23}$
North Atlantic (Dobson et al., 1989) (3 pt)	$12.7\times10^{-7}\,\chi^{0.75}$	$1.7\,\chi^{-0.24}$

Table 3. Comparison with theory.

Study (3 pt)	Experiment	Theory	Optimized
Nakata Bay (3 pt)	$3.12\,\chi^{-0.33}$	$3.23\,C_1\,\chi^{-0.30}$	$3.20\,\chi^{-0.30}$
JONSWAP (3 pt)	$3.5\,\chi^{-0.33}$	$3.64\,C_1\,\chi^{-0.30}$	$3.6\,\chi^{-0.30}$
Bothnian Sea (3 pt)	$3.18\,\chi^{-0.33}$	$3.1\,C_1\,\chi^{-0.30}$	$3.06\,\chi^{-0.30}$
Lake St. Clair (3 pt)	$3.6\,\chi^{-0.33}$	$3.6\,C_1\,\chi^{-0.30}$	$3.56\,\chi^{-0.30}$
Lake Ontario (3 pt)	$1.85\,\chi^{-0.23}$	$2.6\,C_2\,\chi^{-0.25}$	$1.84\,\chi^{-0.25}$
North Atlantic (3 pt)	$1.7\,\chi^{-0.24}$	$2.4\,C_2\,\chi^{-0.25}$	$1.7\,\chi^{-0.25}$

In Eq. (62), coefficients C_β are unknown constants defined by the shapes of the solutions to Eq. (54). Let us denote:

$$C|_{\beta=0.6}=C_1,\quad C|_{\beta=0.5}=C_2$$

Now we can compare experimental and theoretical results for $\nu(\chi)$. These results are summarized in Table 3. Coefficients C_1 and C_2 in the second column of Table 3 are not adjustable parameters. They have definite values to be found by numerical solution of Eq. (25). We plan to determine these values in a later study. At the present time we propose a hypothesis that their values are "optimal", so that the third column in Table 3 is sufficiently close to the first column. Optimization by the least square method gives:

$$C_1=0.99,\quad C_2=0.71$$

Table 3 demonstrates a good agreement between theory and experiment.

6 Discussion

1. The close agreement between the theoretical and experimental results indicates that the Hasselmann kinetic equation is an adequate model describing the evolution of wind-driven surface gravity waves. Moreover, the evolution of the spectral peak at moderate fetches can be faithfully described by the "conservative" kinetic equation where the forcing and dissipation terms are dropped.

   Self-similar solution (Eq. 54) describes downshift of the peak frequency. This is a direct consequence of the "inverse cascade" of energy and wave action. The physical origin of the inverse cascade is the existence of an additional integral of motion – the wave action. The wave action is preserved only in four-wave interactions. One can say that the very fact of the downshifting of the spectral peak indicates a dominant role of four-wave nonlinear interaction.

   This qualitative analysis finds, as we just showed, a reasonable quantitative confirmation.

2. Different groups of experiments give two different values for β: β=0.6 and β=0.5. Both of them are larger than the critical value $\beta=3/7\simeq0.43$. This means that the input of wave action increases with an increasing fetch. The explanation here is rather simple. According to Eqs. (60) and (64), the characteristic steepness $\hat{\alpha}$ decreases as the fetch increases:

   $$\hat{\alpha}\simeq\chi^{\frac{\beta-1}{2}}. \qquad (74)$$

   According to Eq. (22) (see Hasselmann, 1974, and many other models of the breaking wave dissipation), S_{ds} is very sensitive to $\hat{\alpha}$ and it rapidly decreases as $\hat{\alpha}$ decreases. As a result, the white capping is more vigorous for "young" seas. When χ grows, S_{ds} becomes suppressed and the wave action input from wind increases.

3. According to Komen et al. (1995) theoretical prediction, the self-similar solution (Eq. 54) is valid only for not too large fetches. What happens after? It depends on the structure of S_F for long waves. If, as it was estimated in models (Eqs. 5–6), S_F=0 for $\omega<\omega_0$, the downshift continues infinitely long. Asymptotically it is described by the self-similar solution with $\beta=\beta_{\rm crit}=3/7$. Behind the spectral peak, the spectrum has an asymptotic form $F(f)\simeq f^{-11/3}$.

   In the experiments of Donelan et al. (1992) performed on Lake St.Clair, as well as in some earlier experiments by Pierson and Moskowitz (1964), and SMB CERC (1977), the downshift is arrested approximately at $\chi_{\rm crit}\simeq5\times10^4$. It corresponds to a very long fetch, $\chi_{\rm crit}\simeq10^4\,L$, where L is a characteristic wave length. In a typical case $L\simeq100$ m, thus $\chi_{\rm crit}\simeq10^3$km.

According to Pierson and Moskowitz (1964); Young (1999) who summarized these results, stabilization of the downshift (inverse cascade) is going up to the level:

$$\epsilon = 4 \sim 5 \times 10^{-3}, \quad \nu \simeq 0.13$$

Thus, $\bar{\omega} \simeq 0.81\,\omega_0$. In other words, the maximal "wave age", observed in these experiments, does not exceed unit. In this scenario, when $\chi \rightarrow \chi_{\rm crit}$, the self-similarity is violated. γ is not a constant any more. At $\chi \rightarrow \chi_{\rm crit}$, $\gamma \rightarrow 1$, and the spectrum loses its conspicuous peak. This process is called "sea maturing". Not all researchers agree with this concept. According to Glazman (1994), the inverse cascade is not arrested at wave age of order of one, but continue to the spectral area of waves with mean frequency $\omega < \omega_0$. In his experiments near Hawaii island, he observed the wave of age $\omega_0/\omega \geq 3$. Anyway, downshift in this area is more slow than predicts the critical self-similar solution (Eq. 65), and the spectrum tail becomes less steep: $F(f) \simeq f^{-3}$. This question urgently needs more experimental studies.

Nevertheless, it is clear that both the slowdown and the arrest of the inverse cascade ("maturing of the sea"), occur due to dissipation of long waves with phase velocities close to the wind speed or exceeding it. The decrement of this dissipation, β, is very small: $\beta/\omega \simeq \mu \sim 10^{-4}$. A mechanism of this dissipation is still unclear. It could be a combination of wave breaking and friction over a turbulent air. Meanwhile, a scenario of the inverse cascade, the slowdown and arrest is very sensitive to the exact value and the details of this dissipation. It makes the problem of wave prediction extremely difficult from a theoretical viewpoint.

4. The ideas presented in this article can be applied to wind-driven waves on a finite-depth fluid. However, this problem is much more difficult. On a finite-depth the kinetic equation has no self-similar solutions and the solutions for comparison with experiment can be obtained only by the use of a massive numerical simulation. Anyway, the idea of predominance of four-wave nonlinear interaction in the area of spectral peak is still applicable, at least as a first approximation. The next important factor to be taken into consideration is a bottom friction.

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