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Generation of zonal flows by Rossby waves in the atmosphere

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Abstract. A novel mechanism for the short-scale Rossby waves interacting with long-scale zonal flows in the Earth's atmosphere is studied. The model is based on the parametric excitation of convective cells by finite amplitude Rossby waves. We use a set of coupled equations describing the nonlinear interaction of Rossby waves and zonal flows which admits the excitation of zonal flows. The generation of such flows is due to the Reynolds stresses of the finite amplitude Rossby waves. It is found that the wave vector of the fastest growing mode is perpendicular to that of the pump Rossby wave. We calculate the maximum instability growth rate and deduce the optimal spatial dimensions of the zonal flows as well as their azimuthal propagation speed. A comparison with previous results is made. The present theory can be used for the interpretation of existing observations of Rossby type waves in the Earth's atmosphere.

1 Introduction

Generation of the zonal flows by Rossby waves is often observed in the Earth's atmosphere and in laboratory experiments (e.g. Petviashvili and Pokhotelov, 1992 and references therein). Zonal flows are a common feature of many planetary systems and are defined as zero-frequency modes propagating along the parallels. They vary on time scales slower than those corresponding to the Rossby waves. Zonal flows have for a long time been observed in experimental and numerical simulations of rotating neutral fluids, and have been invoked to explain the striped atmospheres of giant planets (Busse, 1994; Aubert et al., 2001, 2002; Cristensen, 2001). Regions of alternating azimuthal velocity have been found on Saturn and Jupiter by the Voyager spacecraft (Ingersoll, 1990). The zonal flows may play a crucial role in the evolution of the vortex structures that regulate the anomalous transport in planetary atmospheres (e.g. Kamenets et al., 1996). They can arise in the Earth's atmosphere which frequently displays azimuthal streams.

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In reality the planetary atmospheres can support both Rossby waves and zonal flows and they thus constitute a dynamical system which exhibits complex nonlinear interactions. Rossby waves are finite frequency dispersive waves propagating along the parallels in the azimuthal direction transverse to the beta plane where the inhomogeneities of the equilibrium atmosphere scale and the planetary rotation frequency play an important role. Since in the Earth's atmosphere the Rossby radius r_0 is large ($\simeq$2000 km) and thus only a few times smaller then the radius of a planet, we focus our attention on perturbations of the atmosphere with spatial horizontal dimensions $\lambda_\perp=2\pi/k_\perp$ comparable or smaller than r_0. In this approximation, the Rossby waves are described by the reduced dispersion relation

$$\omega_{\boldsymbol{k}} = k_x u_*/(1 + k_\perp^2 r_0^2), \tag{1}$$

where $\omega_{\boldsymbol{k}}$ is the wave frequency, $\boldsymbol{k}$ is the wave vector, $k_\perp=(k_x^2+k_y^2)^{1/2}$, $k_{x(y)}$ is the $x(y)-$component of the wave vector, $r_0=(gH_0)^{1/2}/f$ is the Rossby radius, g is the gravitational acceleration, f is the Coriolis frequency which depends on y, $u_*=g\partial_y(H_0/f)$ is the Rossby speed, and H_0 stands for the atmospheric reduced height. For the sake of convenience we consider a shallow rotating atmosphere, assuming that the wave motions are localized in the vicinity of a given latitude $\lambda=\lambda_0$. Similar to Kaladze et al. (2003) we introduce a two-dimensional local Cartesian system of coordinates $(x,\ y)$ with longitudinal, $x=\varphi R\cos\lambda_0$, and latitudinal, $y=(\lambda-\lambda_0)R$, coordinates. In this coordinate system the x-axis is directed from the west to the east and the y-axis points from the south to the north. Furthermore, φ is the latitude and R is distance from the Earth's center.

We note that the dispersion relation (1) resembles the drift wave dispersion relation (e.g. Hasegawa and Mima, 1978; Hasegawa et al., 1979) for a nonuniform magnetoplasma where the atmospheric reduced height, the Rossby speed and the Rossby radius are replaced by the drift wave potential, the electron diamagnetic drift velocity and the ion gyroradius at the electron temperature, respectively. The Coriolis force in fluids is similarly replaced by the ion Lorentz force. However, electrostatic drift waves are pseudo-three-dimensional contrary to flute-like Rossby waves. Both Rossby and drift waves are obtained in the geostrophic or drift approximations where the characteristic wave frequency is much smaller than the Coriolis or the ion gyrofrequency. Clearly, there are similarities between Rossby and drift waves, despite the fact that the two waves appear under different physical conditions. The dynamics of nonlinear Rossby and drift waves is governed by the Charney (1948) and Hasegawa-Mima (1978) equations, in which the nonlinearity comes from the fluid advection and the nonlinear ion polarization drifts, respectively.

Recently, there has been renewed interest in examining the nonlinear coupling between coherent and incoherent drift waves and zonal flows (or convective cells) in nonuniform magnetoplasmas (e.g. Smolyakov et al., 2000; Manfredi et al., 2001; Shukla and Stenflo, 2002). It has been found that pseudo-three dimensional drift waves strongly couple with zonal flows whose dynamics is governed by the drift wave stresses driven Navier-Stokes equation. Thus, the latter is nonlinearly coupled with the Hasegawa-Mima equation in the drift wave-zonal flow theory. On the other it has been shown (Shukla and Stenflo, 2003) that the zonal flow generation in fluids can be considered within a simple model of Rossby-wave turbulence described by the two-dimensional Charney equation (Charney, 1948; Pedlosky, 1987). In the short wavelength limit, viz. $k_\perp r_0 \gg 1$, the latter is

$$r_0^2 \nabla_\perp^2 \partial_t h - u_* \partial_x h + f r_0^4 \{h, \nabla_\perp^2 h\} = 0. \tag{2}$$

In this model the atmosphere is treated as an incompressible shallow water fluid of depth $H = H_0(1+h)$, where H_0 is the unperturbed depth which may depend weakly on the coordinate y and h stands for the dimensionless wave amplitude. Furthermore, $\{A, B\} \equiv (\partial_x A)\partial_y B - (\partial_y A)\partial_x B$ denotes the Poisson bracket.

Since the zonal flow varies on a much longer time scale than the small-scale Rossby waves one can use a multiple scale expansion, assuming that there is a sufficient spectral gap separating the large- and small-scale motions. Following the standard procedure to describe the evolution of the coupled system (Rossby waves plus zonal flows), we decompose the perturbation of the atmospheric dimensionless depth h into its low- and high-frequency parts, that is $h = \hat{h} + \tilde{h}$, where $\hat{h}(y, t)$ refers to the large-scale zonal flow and $\tilde{h}(\boldsymbol{r}, t)$ to the short-scale Rossby wave. Averaging Eq. (2) over the fast short scales, we obtain the evolution equation for the mean flow

$$\nabla_\perp^2 \partial_t \hat{h} = -f r_0^2 \overline{\{\tilde{h}, \nabla_\perp^2 \tilde{h}\}}, \tag{3}$$

where the bar denotes the averaging process.

In Eq. (3) the term on the right-hand side describes the Reynolds stresses induced by the short-scale Rossby waves. The nonlinear coupling of these waves with the zonal flow is governed by

$$r_0^2 \nabla_\perp^2 \partial_t \tilde{h} - u_* \partial_x \tilde{h} = -f r_0^4 (\{\tilde{h}, \nabla_\perp^2 \hat{h}\} + \{\hat{h}, \nabla_\perp^2 \tilde{h}\}). \tag{4}$$

The Rossby waves are considered as a superposition of the pump wave and two sidebands, that is $\tilde{h} = h_0 + \tilde{h}_+ + \tilde{h}_-$, where for the pump wave we have $h_0 = h_{\boldsymbol{k}} \exp(i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega_{\boldsymbol{k}} t)) + h_{\boldsymbol{k}}^* \exp(-i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega_{\boldsymbol{k}} t))$ with the frequency $\omega_{\boldsymbol{k}}$ given by Eq. (1).

The change in the zonal flow amplitude is given by $\hat{h} = h_{\boldsymbol{q}} \exp(i(\boldsymbol{q}\cdot\boldsymbol{r} - \Omega t)) + h_{\boldsymbol{q}}^* \exp(-i(\boldsymbol{q}\cdot\boldsymbol{r} - \Omega t))$, where $\boldsymbol{q} = q\ \hat{\boldsymbol{y}}$ and $\hat{\boldsymbol{y}}$ is the unit vector along the latitude. For the Rossby side-bands we have $\tilde{h}_\pm = h_{\boldsymbol{k}_\pm} \exp(i(\boldsymbol{k}_\pm\cdot\boldsymbol{r} - \omega_{\boldsymbol{k}_\pm} t)) + h_{\boldsymbol{k}_\pm}^* \exp(-i(\boldsymbol{k}_\pm\cdot\boldsymbol{r} - \omega_{\boldsymbol{k}_\pm} t))$, where $\omega_{\boldsymbol{k}_\pm} = \omega_{\boldsymbol{k}} \pm \Omega$ and $\boldsymbol{k}_\pm = \boldsymbol{k} \pm \boldsymbol{q}$ are the frequencies and wave vectors of the Rossby sidebands.

Substituting h_0, $\hat{h}$ and $\tilde{h}_\pm$ into Eq. (3), we obtain

$$\Omega \hat{h}_{\boldsymbol{q}} = -i f r_0^2 \frac{(\boldsymbol{k}\times\boldsymbol{q})_z}{q^2} \cdot \left((\boldsymbol{k}_+^2 - \boldsymbol{k}^2) h_{\boldsymbol{k}_+} h_{\boldsymbol{k}}^* - (\boldsymbol{k}_-^2 - \boldsymbol{k}^2) h_{\boldsymbol{k}_-}^* h_{\boldsymbol{k}} \right). \tag{5}$$

where the expressions for Fourier amplitudes $h_{\boldsymbol{k}_+}$ and $h_{\boldsymbol{k}_-}^*$, found from Eq. (4), are

$$h_{\boldsymbol{k}_+} = -i \frac{f r_0^2 (\boldsymbol{k}\times\boldsymbol{q})_z}{\Omega + \delta\omega_+} \frac{k^2 - q^2}{k_+^2} \hat{h}_{\boldsymbol{q}} h_{\boldsymbol{k}}, \tag{6}$$

$$h_{\boldsymbol{k}_-}^* = i \frac{f r_0^2 (\boldsymbol{k}\times\boldsymbol{q})_z}{\Omega - \delta\omega_-} \frac{k^2 - q^2}{k_-^2} \hat{h}_{\boldsymbol{q}} h_{\boldsymbol{k}}^*, \tag{7}$$

where $\delta\omega_\pm \equiv \omega_{\boldsymbol{k}} - k_x u_* / r_0^2 \boldsymbol{k}_\pm^2$. Substituting Eqs. (6) and (7) into Eq. (5) we have (cf. Shukla and Stenflo, 2003)

$$\Omega = -f^2 \frac{(\boldsymbol{k}\times\boldsymbol{q})_z^2 (k^2 - q^2) r_0^4 |h_0|^2}{q^2} \cdot \left(\frac{(\boldsymbol{k}_+^2 - \boldsymbol{k}^2)}{\boldsymbol{k}_+^2} \frac{1}{\Omega + \delta\omega_+} + \frac{(\boldsymbol{k}_-^2 - \boldsymbol{k}^2)}{\boldsymbol{k}_-^2} \frac{1}{\Omega - \delta\omega_-} \right). \tag{8}$$

The dispersion relation (8) is in general too cumbersome for analysis, and it can thus, only be solved numerically. In order to simplify it we consider the case of most interest $q \ll k$, when the typical scales of the zonal flows are much larger then the scales of the Rossby waves. In this limiting case we have the following expansions

$$\delta\omega_\pm \simeq \mp \boldsymbol{q}\cdot\boldsymbol{v}_g - \frac{q^2 v_g'}{2}, \tag{9}$$

$$\frac{(\boldsymbol{k}_+^2 - \boldsymbol{k}^2)}{\boldsymbol{k}_+^2} \frac{1}{\Omega + \delta\omega_+} + \frac{(\boldsymbol{k}_-^2 - \boldsymbol{k}^2)}{\boldsymbol{k}_-^2} \frac{1}{\Omega - \delta\omega_-} \simeq -\frac{q^2 \Omega v_g'}{\omega_{\boldsymbol{k}}((\Omega - \boldsymbol{q}\cdot\boldsymbol{v}_g)^2 - (v_g' q^2/2)^2)}, \tag{10}$$

where

$$v_g' \equiv \frac{\partial v_g}{\partial k_y} = \frac{\partial^2 \omega_{\boldsymbol{k}}}{\partial k_y^2} = -\omega_{\boldsymbol{k}} \frac{2(k_x^2 - 3k_y^2)}{k^4}. \tag{11}$$

Here $v_g \equiv \partial\omega_{\boldsymbol{k}}/\partial k_y = -2k_y\omega_{\boldsymbol{k}}/k^2$ is the latitudinal (y-component) pump Rossby group velocity. We note that v_g' can change sign when $k_x = \pm 3^{\frac{1}{2}} k_y$. This occurs on the Rossby wave caustics. Substituting Eqs. (9)–(11) into Eq. (8), we obtain the dispersion relation

$$\Omega_\pm \simeq \boldsymbol{q} \cdot \boldsymbol{v}_g \pm i \cdot \left(-v_g' \frac{(\boldsymbol{k} \times \boldsymbol{q})_z^2 f^2 r_0^4 k^2 |h_0|^2}{\omega_{\boldsymbol{k}}} - \left(\frac{v_g' q^2}{2} \right)^2 \right)^{1/2} . \quad (12)$$

If $v_g' \omega_{\boldsymbol{k}}^{-1} < 0$, then the upper sign in Eq. (12) yields an instability. This condition is similar to the Lighthill criterion for modulation instability in nonlinear optics (Lighthill, 1965).

According to Eqs. (11) and (12) for a given k_x ($k_x > 0$) the instability condition applies to Rossby pump waves with wave vectors localized in the cone $k_x / 3^{\frac{1}{2}} > k_y > -k_x / 3^{\frac{1}{2}}$. The maximum growth is attained at the axis of the cone when $k_y = 0$. In this case the mode is purely growing with the growth rate

$$\gamma = -i \Omega_+ = \left(2 f^2 q^2 k^2 r_0^4 |h_0|^2 - \frac{q^4}{k^4} \omega_{\boldsymbol{k}}^2 \right)^{1/2} . \quad (13)$$

Expression (13) describes the initial (linear) stage of zonal flow growth due to the parametric instability of the short-scale Rossby waves.

It follows from Eq. (13) that the wavenumbers q of the growing modes are localized in the range

$$0 < \left(\frac{q}{k} \right)_{\max}^2 < 2 \left(\frac{f}{\omega_{\boldsymbol{k}}} \right)^2 (k r_0)^4 \, |h_0|^2 \quad (14)$$

For fixed k, the fastest wave growth is attained at the value of $(q/k)^2$ given by

$$\left(\frac{q}{k} \right)_{\max}^2 = \left(\frac{f}{\omega_{\boldsymbol{k}}} \right)^2 (k r_0)^4 \, |h_0|^2 \quad (15)$$

This maximum growth rate is thus

$$\gamma_{\max} = \frac{f^2}{k \, |u_*|} (k r_0)^6 \, |h_0|^2 , \quad (16)$$

which shows that $\gamma_{\max}$ increases as k^5 in the short wavelength limit ($k r_0 \gg 1$). Physically, this instability is a manifestation of an inverse cascade. It shows that the spectral energy of the short-scale Rossby wave turbulence is transferred into the long scales of the zonal flows, i.e. the Rossby wave energy is converted into the energy of slow zonal motions. The transport of energy from short- to large scales is in two-dimensional turbulence theory consistent with conservation of the total energy and enstrophy (Charney, 1948).

As follows from our analysis, the small-scale Rossby waves in a shallow rotating atmosphere are unstable with respect to the long wavelength perturbations. These perturbations are accompanied by the excitation of long wavelength modes with the velocity $v_{zf} = f r_0^2 q \, |h_0|$, i.e. zonal flows. Replacing q by its optimum value from Eq. (15) we obtain

$$v_{zf} = \frac{(f r_0)^2}{|u_*|} (k r_0)^4 \, |h_0|^2 . \quad (17)$$

The peculiar feature of this instability is that it appears solely for Rossby waves that are localized in the cone bounded by the caustics for which $v_g' = 0$. This can lead to the formation of a so-called caustic shadow in the spectrum of the Rossby waves.

For typical parameters of the Earth's atmosphere $f \approx 1.6 \times 10^{-4}$ s^{-1}, $r_0 \approx 2 \times 10^6$ m, $h_0 \simeq 10^{-2}$, $k r_0 \simeq 5 - 10$ and $|u_*| \simeq 3 \times 10^2$ m/s we obtain $\gamma_{\max} \simeq (2 \times 10^{-5} - 8 \times 10^{-4})$ s^{-1}, $\lambda_{\max} = 2\pi / q_{\max} \simeq (110 - 1700)$ km and $v_{zf} = (22 - 350)$ m/s. These estimates are consistent with existing observations. Thus, it is possible that the parametric instability of Rossby waves is responsible for the generation of mean flows in the atmospheres of rotating planets.

The parametric instability investigated in the present paper is generic to a wide class of similar instabilities in space and laboratory plasmas. On one hand it can play a role in the Earth's ionosphere where it results in the formation of a turbulent Alfvén boundary layer (Pokhotelov et al., 2003). On the other hand, the parametric instability can give rise to the generation of shear flows in laboratory plasmas where it may substantially influence the plasma drift turbulence suppressing the transport coefficients (e.g. Smolyakov et al., 2000; Manfredi et al., 2001). We note that the expression for the instability growth rate obtained by Smolyakov et al. (2000) and Manfredi et al. (2001) is similar to that of Eq. (13) except for the second term in the square root. In the mathematical formalism (the wave equation in the eikonal approximation based on the Hasegawa-Mima model (Hasegawa and Mima, 1978; Hasegawa et al., 1979) of Smolyakov et al. (2000) this additional term corresponds to wave dispersion spreading which was unfortunately neglected in both these papers. However, our analysis presented above shows that by taking this additional effect into consideration we can treat the case of main interest, that is, we can consider the fastest growing mode. This makes it possible to obtain the expression for the maximum instability growth rate and to define the optimal parameters of the zonal flows.

In the present study we have demonstrated how zonal flows in a shallow rotating atmosphere can be excited by finite amplitude Rossby waves. The driving mechanism of this instability is due to the Reynolds stresses which are inevitably inherent for finite amplitude short scale Rossby waves. Hence, our investigation provides an essential nonlinear mechanism for the transfer of spectral energy from short scale Rossby waves to long scale enhanced zonal flows in the Earth's atmosphere. We note that finite amplitude Rossby waves can be nonlinearly coupled to other atmospheric modes, e.g. to large amplitude long wavelength inertial waves (Falkovich, 1992; Pokhotelov et al., 1995). These authors considered a scenario when the inertial sidebands interact with the inertial pump wave and produce a low-frequency advection force, which becomes a driver for Rossby type perturbations.

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