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Scaling statistics in a critical, nonlinear physical model of tropical oceanic rainfall

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Abstract. Over the last two decades, concepts of scale invariance have come to the fore in both modeling and data analysis in hydrological precipitation research. With the advent of the use of the multiplicative random cascade model, these concepts have become increasingly more important. However, unifying this statistical view of the phenomenon with the physics of rainfall has proven to be a rather nontrivial task. In this paper, we present a simple model, developed entirely from qualitative physical arguments, without invoking any statistical assumptions, to represent tropical atmospheric convection over the ocean. The model is analyzed numerically. It shows that the data from the model rainfall look very spiky, as if generated from a random field model. They look qualitatively similar to real rainfall data sets from Global Atmospheric Research Program (GARP) Atlantic Tropical Experiment (GATE).

A critical point is found in a model parameter corresponding to the Convective Inhibition (CIN), at which rainfall changes abruptly from non-zero to a uniform zero value over the entire domain. Near the critical value of this parameter, the model rainfall field exhibits multifractal scaling determined from a fractional wetted area analysis and a moment scaling analysis. It therefore must exhibit long-range spatial correlations at this point, a situation qualitatively similar to that shown by multiplicative random cascade models and GATE rainfall data sets analyzed previously (Over and Gupta, 1994; Over, 1995). However, the scaling exponents associated with the model data are different from those estimated with real data. This comparison identifies a new theoretical framework for testing diverse physical hypotheses governing rainfall based in empirically observed scaling statistics.

1 Introduction

Over the last two decades, a large body of research has focused on developing statistical rainfall models that are scale dependent (e.g. Waymire et al., 1984; Eagleson and Qinliang, 1985; Gupta and Waymire, 1987; Kavvas et al., 1987; Phelan and Goodall, 1990; Cox and Isham, 1988), and scale invariant (e.g. Lovejoy, 1981, 1982; Lovejoy and Schertzer, 1990; Gupta and Waymire, 1990; Over and Gupta, 1994; Perica and Foufoula-Georgiou, 1996; Foufoula-Georgiou, 1998; Jotithyangkoon et al., 2000); and on determining the extent to which they exhibit certain features of data. However, stochastic models models have faced the difficult problem of determining a relationship between model parameters and the underlying physical processes governing rainfall. Therefore, it is impossible to know how physical parmeters affect a given statistical measure, and how changes in physical processes affect these statistics. Since scaling statistics tend to represent extreme rainfall variability, it is important from a hydrological perspective to have a physical understanding of the statistical nature of rainfall, and a theoretical basis for understanding why rainfall patterns occur the way they do. However, little progress has been made to explicitly link observed scaling statistics in space-time rainfall with physical processes governing rainfall. A great need to make progress on this fundamental problem has existed for a long time (Gupta and Waymire, 1993). In this paper, we develop a phenomenological, dynamical model of tropical oceanic convective rainfall, without any statistical assumptions, in order to investigate how empirically observed statistical scaling in oceanic rainfall is linked to the underlying dynamics.

The stochastic theory of point random fields was first applied to modeling space-time rainfall by LeCam (1961). This line of research was developed substantially through the 1980 s and 1990 s by others (e.g. Waymire et al., 1984; Eagleson and Qinliang, 1985; Gupta and Waymire, 1987; Kavvas et al., 1987; Phelan and Goodall, 1990; Cox and Isham, 1988). It was successful in representing certain features of rainfall statistics based in its geometrical structures, such as clustering in space and time. Some of these models also showed that the Taylor hypothesis of fluid turbulence in the correlation structure holds for a limited time, as observed by Zawadzki (1973). However, in these point process models, a

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parameter was required for representing the geometry at every scale considered. These parameters were also difficult to estimate from data.

In parallel to the above studies, a number of scale-independent model studies were conducted in an attempt to reproduce the observed scaling statistics of rainfall (e.g. Lovejoy, 1981, 1982; Lovejoy and Schertzer, 1990; Gupta and Waymire, 1990; Over and Gupta, 1994; Perica and Foufoula-Georgiou, 1996; Foufoula-Georgiou, 1998; Jotithyangkoon et al., 2000). These 'scaling models', in contrast to the scale-dependent models, generally required only a few parameters to relate the statistics of rainfall fields among different scales. This situation allowed scaling models such as random cascades to be more parsimonious than point random field models. However, applications of spatial random cascade theory to real data have required two sets of generalizations, both of which greatly complicate the underlying mathematical structures. The first is the extension of cascade models from only space to space-time rainfall (Over and Gupta, 1996; Marsan. et al., 1996). This generalization requires cascade generators to be time evolving, which is difficult because it is unclear how to infer the time evolution of cascade generators from data. The second generalization has required scale-based modification of cascade generators. For example, Menabde et al. (1997) introduced a parameter to gradually smooth generators while going down the scales, a generalization that came about due to difficulties with spectral predictions by independent and identically distributed (i.i.d.) cascade generators. Likewise, Jotithyangkoon et al. (2000) found that cascade generators changed with spatial scale in a space-time stochastic model of land-based rainfall. Moreover, they treated spatial rainfall variability independently of its temporal variability.

Stochastic rainfall research has centered upon statistical variability in rainfall intensity in the two-dimensional horizontal plane parallel to Earth's surface. The vertical dimension, where most of the physics of rainfall generally takes place, has not been included in these models. By contrast, the meteorological literature contains many models of rainfall based in physical processes including convection (e.g. Arakawa and Schubert, 1974; Emanuel, 1991; Arakawa and Cheng, 1993; Betts and Miller, 1993; Raymond and Emanuel, 1993). This literature has focused on incorporating microphysics and motions in the vertical dimension. The development of much of this research has occurred primarily to reconcile the small-scale processes associated with convection to the typical grid scales of the order of $10-100$ km in side length in numerical weather and climate models. Thus, meteorological research has been directed towards developing parameterizations to upscale convective effects, rather than towards investigating variability directly, though some efforts have been made recently to downscale rainfall to subgrid scales in dynamical weather forecast models (for an overview, see Foufoula-Georgiou, 1998).

There have been only a few attempts to model rainfall scaling statistics from a physical perspective. For example, Nagel and Raschke (1992), concerned with modeling the fractal properties of cloud cover, developed a simple percolation model. With it they were able to predict the perimeter-area exponent measured for clouds by Lovejoy (1982), as well as the Korcak parameter measured by Lawford (1996). Pelletier (1997) adapted the Khardar-Parisi-Zhang (KPZ) model of depositional growth to model cloud shapes, and found good agreement with both the size-area distribution exponent and the area-perimeter exponent. Recently, Peters et al. (2002a) and Peters and Christensen (2002b) have postulated that the scale-invariance observed in rainfall is analogous to that found in a variety of nonequilibrium processes in nature, such as Earthquakes and avalanches. However, only Peters et al. (2002a) and Peters and Christensen (2002b) have managed to establish a direct link between statistics and physical processes, since the other studies were not directly developed from the physics specific to rainfall.

To address this long-standing open problem, we construct a simple physical model of tropical convective rainfall over the ocean. Our primary objective is to understand how scaling statistics arise under simple but non-trivial parameterizations of convective dynamics. As a first step towards this goal, we construct a model independent of statistical assumptions and based qualitatively on the physics of convective rainfall. Our idealized model couples the horizontal with the vertical through a parameterization of the primary physical process that sustains triggered convection over the tropical oceans, but does not model the mechanisms by which tropical convection is first initiated, e.g. differential surface heating. In this connection, we specify a localized field of storm outflow from a prior rainfall event as initial condition. Thus, our model has been developed specifically to investigate the physical origins of empirically observed scaling statistics in precipitation fields associated with sustained tropical oceanic convection (Over and Gupta, 1994, 1996; Pavlopoulos and Gupta, 2003).

Numerical space-time solutions of our phenomenological, dynamical model look like sample paths from a random field, despite the model's lack of built-in statistical assumptions. Therefore, we analyze the model output using spatial moment and fractional wetted area (FWA) analyses, treating the output as though it were stochastic. Scaling features under these analyses were observed by Gupta and Waymire (1993) and Over and Gupta (1994)in the Global Atmospheric Research Program (GARP) Atlantic Tropical Experiment rainfall data sets (GATE; Hudlow and Patterson, 1979). Our model can be formally, but not physically, compared to other models that have been investigated in statistical mechanics and nonlinear chaotic dynamical systems, where the chaotic nature of dynamical output is analysed using stochastic methods; see Beck and Schlogl (1993) for an excellent introductory reference on this topic.

Through these analyses, we find that a dynamical parameter equal to the velocity scale represented by the convective inhibition (CIN) behaves like a critical parameter. This means that just below the critical value of the parameter, the rainfall changes abruptly from non-zero to zero over the entire domain. Near the critical value, the model rainfall field exhibits multifractal scaling and represents long-range spatial correlations, but far from it, model statistics do not show scaling over the entire domain. This feature is similar to that found in nonlinear dynamical and statistical-mechanical systems exhibiting second-order phase transitions (Beck and Schlogl, 1993, ch. 21). Turcotte and Rundle (2002) give other examples of critical phenomena that have been investigated in physical, biological, and social sciences.

A very important implication of this link between scaling statistics and rainfall physics lies in the concept that statistical parameters can be predicted by a dynamical model and compared to empirical values obtained from rainfall data. We illustrate this key idea by comparing two sets of empirical parameters that were computed by Over and Gupta (1994) in GATE rainfall data sets using FWA analysis, and tested against a random cascade model. This comparison identifies a new theoretical framework for testing diverse physical hypotheses and assumptions governing rainfall based in empirically observed scaling statistics.

In Sect. 2, we review some literature on scaling as it is related to the FWA and moment analysis in convective rainfall. In Sect. 3, we present details of our physical model. In Sect. 4, we describe and present results of our statistical scaling analysis from model-generated rainfall fields and compare them to empirical parameters for GATE rainfall data sets. In Sect. 5 we summarize our main findings and discuss some next steps in this exciting new direction for rainfall research.

2 Moment scaling and the Fractional Wetted Area (FWA) analyses

The literature contains numerous examples of scaling analyses associated with the space-time statistics of rainfall. In particular, some of these papers have focused on the analysis of oceanic rainfall through the well-known GATE (Over and Gupta, 1994) and the Tropical Ocean Global Atmosphere Coupled Ocean Atmosphere Response Experiment (TOGA-COARE, Webster and Lukas, 1992) data sets (Pavlopoulos and Gupta, 2003). These analyses have shown that measurements of statistical variability at different spatial scales can be understood in terms of concepts that arise in analyzing multifractal random fields (see e.g. Holly and Waymire, 1992; Gupta and Waymire, 1993). Scaling analyses, in conjunction with random field models like the random cascades, have allowed researchers to characterize empirical statistical scaling features of space-time oceanic rainfall over the tropics. For instance, results from the well-known moment analyses of multifractal rainfall fields, of which the FWA analysis is a special case, have been combined with analytical work on cascade models to determine several persistent statistical scaling features of space-time rainfall (D. Schertzer and Lovejoy, 1987; Gupta and Waymire, 1993; Menabde et al., 1997).

We will briefly introduce some definitions and concepts from the spatial moment analysis of multifractal random fields that have been used by Gupta and Waymire (1993) and Over and Gupta (1994) to analyze GATE rainfall, and for testing a random cascade model on this data set. We will use these concepts in Sect. 4 to analyze our model output. The moment analysis is performed on a radar or a model scan by determining the rain intensities at a variety of pixel scales Δ_n, which in turn are obtained by a "coarse graining" or a "spatial averaging" procedure over the grid. These analyses have been strongly influenced by the desire to empirically test and interpret theoretical results that were developed in the mathematical theory of random cascades (see, e.g. Holly and Waymire, 1992). Although the reader is not required to be familiar with the theory of random cascades to understand the analyses in this paper, we will mention a few important concepts from random cascades to make our definitions and presentation understandable. For this purpose, we will closely follow the definitions and notations used by Over and Gupta (1994).

We define first the spatial moment at scale Δ_n (Over and Gupta, 1994, Eq. (3.6))

$$M_n(q) \equiv \sum_i R^q(\Delta_n^i) \qquad n = 1, 2, 3, \ldots, \tag{1}$$

in which $R(\Delta_n^i)$ is the value of the rain field in the i-th pixel at scale Δ_n^i. Note that the sum is not divided by the total number of pixels, and the units of $R(\Delta_n^i)$ are volume/time, rather than length/time. These two features are introduced to match scaling of spatial moments with theoretical results from random cascades as explained below. To understand this connection, two concepts are needed. First is the branching number b, which determines the number of pixels into which a cascade generator disaggregates mass from one scale to the next scale. The second is a random generator W that is used to redistribute mass across successive scales in constructing a random cascade, though the specific form of W is not important for our purposes. The side length of Δ_n^i at level n represents the spatial scale L_n. If we fix $b = 4$, and $d = 2$ as the spatial dimension, a dimensionless scale parameter can be defined by $\lambda_n \equiv \frac{L_n}{L_0} = b^{-n/d}$ $n = 1, 2, \ldots$, in which L_0 is the length of the entire domain.

A fundamental mathematical quantity in the random cascade theory is the so-called Mandelbrot-Kahane-Peyriere (MKP) function, used to compute various quantities of theoretical significance (Holly and Waymire, 1992). It is defined by

$$\chi_b(q) = \log_b E\left[W^q\right] - (q - 1). \tag{2}$$

The scaling exponents for the scaling of spatial moments in Eq. (1) are defined as

$$\tau(q) \equiv \lim_{\lambda_n \to 0} \frac{\log M_n(q)}{-\log \lambda_n}, \tag{3}$$

representing the slope of a line on a log-log plot of moment versus scale for each q. The theoretical significance of moments as defined in Eq. (1) comes from the fact that the scaling function can be equated with the MKP function as (Over and Gupta, 1994, Thm 3.1)

$$\tau(q) = d \cdot \chi_b(q) \tag{4}$$

This equation shows that the MKP function can be directly estimated from data. For more information on the uses of the MKP function, see Holly and Waymire (1992).

A key feature of random cascades is that they have long-range spatial correlations. This means that the correlation length scale is infinite; see Over (1995) for a careful derivation of this result. What this means is that the existence of scaling relations in the spatial moments of a random field, as depicted by Eq. (2), may be taken as evidence of infinite correlation length scale. This important issue has been discussed in much greater detail by Gupta and Waymire (1993) and Over and Gupta (1994), and applied to the analysis of GATE rainfall using individual scans and in testing a random cascade model. We will use the moment scaling analysis of our dynamical model data in Sect. 4 to the infer presence or absence of an infinite correlation length scale in our model dynamics.

The FWA is defined in terms of the zeroth moment $M_n(0)$ by

$$f(\lambda_n) = b^{-n} M_n(0) \ , \tag{5}$$

in which the total number of pixels is given by b^n. As shown in Over and Gupta (1994, Eq. (3.15)), $f(\lambda_n)$ shows log-log linear behaviour with respect to spatial scales λ_n with a slope s that is related to $\tau(0)$ by $-s + 2 = \tau(0)$; again, for details, see Over and Gupta (1994, Eqs. (3.14) and (3.16)).

For an important class of cascade model, the ‘beta’ cascade, Gupta and Waymire (1993) showed that it was possible to compute moments analytically in terms of the model parameter p, the probability that the beta-cascade generator W takes value 0. For spatial moments $M_n(q)$ of arbitrary order q, Over and Gupta used this result to show analytically that $\tau(q)$ was linear in q for a beta cascade. Since $\tau(q)$ was approximately linear for GATE phase I data, though nonlinear and convex for GATE phase II, they argued that a beta cascade was an appropriate model to represent spatial variability of both zero and non-zero rainfall in GATE phase I data. They suggested introducing scale-dependent generators for potentially addressing the deviations from linearity observed in GATE phase II data.

Over and Gupta also inferred the existence of a further scaling relationship between the FWA exponent s and the mean rainfall $\bar{R}$ from the GATE data,

$$\left(\frac{\bar{R}}{R_{\max}}\right)^k = \left(1 - \frac{p}{.75}\right) \tag{6}$$

in which

$$p = 1 - 2^{-s}.$$

Determination of the exponent k and the intercept $R_{\max}$, therefore, is the ultimate goal of the FWA analysis, assuming (6) holds. Over and Gupta (1994) compared results of a FWA analysis of a beta cascade to that computed for the tropical oceanic rainfall data taken in GATE phase I and GATE phase II; and to extratropical land-based rainfall data taken over Elbow, Saskatchewan. They used an appropriate value of p corresponding to $\tau(0)$ in all three cases. In each they found excellent qualitative agreement, and reasonable quantitative agreement since the exponent k and logarithmic intercept $R_{\max}$ were close in value to each other for the two ocean data sets. This very powerful result has since been suggested for use in the disaggregation of General Circulation Model rainfall data by Jotithyangkoon et al. (2000), who also observed it in data taken over land.

We will conduct these moment and FWA scaling analyses in Sect. 4 on output from our dynamical model to test how well they compare to empirically estimated values. A deviation between these two provides a rigorous theoretical framework to test various physical assumptions that are used in our model. This is one of the key new ideas in rainfall research that is introduced and illustrated in this paper.

3 Model description

The phenomenological, dynamical model we develop includes 2 horizontal dimensions and is deterministic. The phenomenology we use operates through triggering from a gust front produced by outflow from local convective rainfall. It is thought to be a principle mechanism behind sustained convection in thunderstorms over land (Emanuel, 1994, pp. 231–235). Naturally, this mechanism is also very important over the tropical oceans (Mapes, 2000). The downdraft is colder than its surroundings, and is also denser than the warm ambient air, since an ideal gas at a fixed pressure P has density $\rho \propto 1/T$. Both cold and warm air masses exist in a state of conditional stability in the region below the Level of Free Convection (LFC), which is the height at which the moist air becomes positively buoyant.

The outflow, or cold pool, provides trigger conditions for packets of incident warm, moist air to rise above the LFC, and thus, to reach the Level of Neutral Buoyancy (LNB), the height at which the moist air is at dynamical equilibrium with its surroundings. In order to reach the LNB, a packet of air must first be pushed above the LFC (Fig. 1). The amount of energy required for a such a successful trigger is commonly known as the Convective Inhibition (CIN; Mapes, 2000). The amount of potential energy stored in the packet for its ascent is known as the Convective Available Potential Energy (CAPE), and is approximately equal to the kinetic energy of a packet once it reaches the LFC; similarly, the potential energy for a downdraft is known as downdraft CAPE, or dCAPE.

As our intention here is primarily to understand how the scaling statistics in convective rainfall arises from dynamical arguments, and since our model represents a first step towards this goal, we are interested in the minimal model produced from physical arguments that will exhibit such behaviour. In this connection, we have made several significant

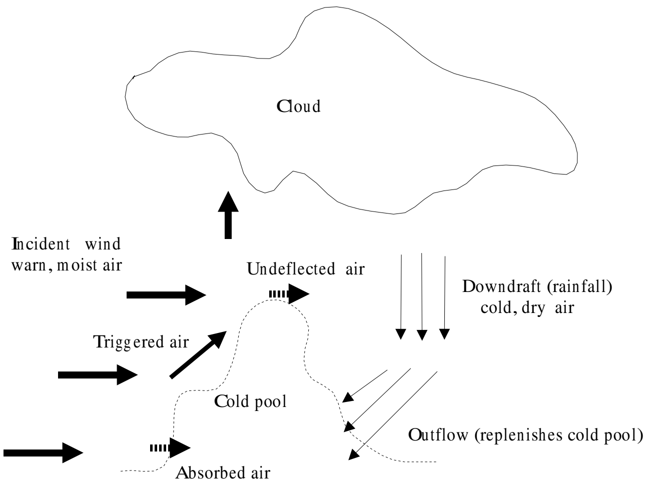

Fig. 1. Schematic of phenomenology used in the model. Warm, moist incident wind strikes the much denser cold, dry pool, and is either triggered above the LFC or not, depending upon the local gradient of the cold pool. Steep gradients present a barrier for the incident wind to puncture, and low gradients allow the incident wind to pass by without gaining upward lift. Downdrafts from the air which is convected replenish the cold pool.

simplifications to the physical models generally used to represent the above physical processes. The first simplification involves the parameterization of the vertical dimension by a timescale, which represents the amount of time required for a packet of air to rise, condense and fall out. The second is the absence of microphysics in our model. The third simplification is the replacement of the commonly-used, nonlinear, two-fluid density current model of surface outflow with a linear diffusion. Details of these simplifications are given below.

We specify a constant velocity $\boldsymbol{U}$ for the horizontal movement of warm air parallel to the surface. As already mentioned, the LNB is specified by a timescale scheme, such that when clouds have developed past a certain threshold they rain out, replenishing the cold pool groundward with more cold air from the outflow. The LFC is considered implicitly, and both the CIN and CAPE are represented phenomenologically as parameter values. dCAPE is a function of the output (rain) field. Justification for a constant CAPE across the domain can be found in Arakawa and Cheng (1993). The model requires as initial condition the existence of a randomized cold pool over the lattice.

The model follows the movement of air masses, considering rainfall to be a passive tracer for reasons of simplicity. The mass of warm air at the surface is not conserved, but rather is treated as a source for the cold pool through the rainfall field. However, the mass of air in cold pools is conserved, with a rainfall source and a sink due to heating. In the meteorological literature, the movement of cold pools is typically represented by a nonlinear fluid model of density currents (Xu et al., 1996; Xue et al., 1997; Emanuel, 1994). This model depicts a fluid mass moving coherently as a spreading packet along the surface, generally with a preferred direction and a velocity $\boldsymbol{v}$ due to surface winds, and a nonlinear front at the leading edge. This model is somewhat complex for our purposes at this stage of development, however, requiring the conservation of mass for both fluids, as well as introducing additional complexity through nonlinearity. Since a diffusion with advection captures the qualitative behaviour of this field in a more linear fashion, we choose to update a depth of cold air $h = h(\boldsymbol{x}, t)$ by

$$h(\boldsymbol{x}, t+\delta t) = h(\boldsymbol{x}, t) + \delta t \, [a\tilde{\nabla}^2 h(\boldsymbol{x}, t) - \boldsymbol{v} \cdot \tilde{\nabla} h(\boldsymbol{x}, t)] \quad (7)$$

with a the diffusion constant and $\boldsymbol{v}$ the velocity of cold pool propagation. Here $\tilde{\partial}_t$, $\tilde{\nabla}^2$ and $\tilde{\nabla}$ denote discretized versions of their respective operators. In addition, we account for heating of the pool by reducing the depth at each timestep by introducing a simple uniform factor $(1 - F)$ such that

$$h(\boldsymbol{x}, t+\delta t) = h(\boldsymbol{x}, t) \times (1 - F), \quad (8)$$

in which $F < 1$ represents the extent of solar heating at the surface. The simplicity of the diffusion model assumption comes at the expense of an altered behaviour at the leading edge of the cold pool and the forced independence of physical parameters that would normally be functionally dependent. We will address this issue in our conclusion.

For ascent of incident moist air, we consider a coupling

$$v_a = \boldsymbol{U} \cdot \nabla h \quad (9)$$

between the incident wind $\boldsymbol{U}$ and the gradient of the cold pool, ∇h. v_a can be expressed as

$$v_a = U|\nabla h| \cos(\theta_h) \quad (10)$$

where $\theta_h = \theta(\nabla h)$ is defined as the angle between $\boldsymbol{U}$ and ∇h and $U = |\boldsymbol{U}|$ is a numerical constant representing the strength of the incident warm wind. We can express θ_h in terms of ∇h if we define a unit vector $\hat{x}$ in the direction of $\boldsymbol{U}$. Then

$$\hat{x} \cdot \nabla h = dh/dx = \tan\theta_h, \quad (11)$$

and $\cos\theta_h = [1 + (\nabla h)^2]^{-1/2}$. Hence,

$$\begin{aligned} v_a &= U|\nabla h| \cos(\theta_h) = U|\nabla h|[1 + (\nabla h)^2]^{-1/2} \\ &= U[1 + (\nabla h)^{-2}]^{-1/2}, \end{aligned} \quad (12)$$

which is monotonic and increasing in ∇h.

Low values of the dimensionless quantity v_a/U suggest the incident wind is nearly tangent to the gradient, in which case there will be little upward component to motion. For convection, therefore, v_a should be above some threshold value b_1. b_1 has units of velocity and is clearly a function of the CIN, since CIN is a measure of the energy required for a parcel to be triggered. High values of v_a/U, on the other hand, imply that θ_h is large. An angle that is too great will allow the wind to penetrate the cold pool rather than ascend, so that for ascent v_a should also be below some threshold value b_2. b_2 is a function of the density difference between the cold pool and the incident wind, since high densities differences will prevent penetration of the cold pool. b_2 is required to be non-zero for the phenomenology of the model to make sense.

Thus, in our deterministic model, we use the Heaviside function $\Theta(\cdot)$ to update cloudiness $C(\boldsymbol{x}, t)$, defined here as the quantity of moist air aloft, by

$$C(\boldsymbol{x}, t + \delta t) = C(\boldsymbol{x}, t) + \delta t \; c\Theta(\kappa), \tag{13}$$

where c is a constant value for packets convected and

$$\kappa(\boldsymbol{x}, t) = (\boldsymbol{U} \cdot \tilde{\nabla} h - b_1)(b_2 - \boldsymbol{U} \cdot \tilde{\nabla} h) \tag{14}$$

is the phenomenological triggering energy.

Specifying downdrafts from convected clouds requires the construction of a criterion for fallout. The simplest method would be to introduce a constant time scale τ, after which convected water vapor would return to Earth. However, cloudy regions of higher mass should rain more quickly than those of lower mass. We can therefore spread the timescale locally by introducing a mean time of ascent and fallout,

$$\bar{t} = \frac{1}{N(\boldsymbol{x}, t)} \sum_{i=1, N(\boldsymbol{x}, t)} w_i t_i \Theta(\kappa(t_i)) \; . \tag{15}$$

in which $t_i = t_i(\boldsymbol{x})$ is the time associated with a convection event i, i.e. i such that $\theta(\kappa(t_i)) = 1$. $N(\boldsymbol{x})$ is the number of such events since the initial development of convection at $\boldsymbol{x}$. w_i are weights we take to be 1 for simplicity, though we note here that it is possible to represent more complex microphysics with different weights. Now, if $t - \tau > \bar{t}$, we take rainfall to be

$$R(\boldsymbol{x}, t) = rC(\boldsymbol{x}, t) \; , \tag{16}$$

in which r is a constant fraction of cloud that falls as rain; we further update cloudiness by

$$C(\boldsymbol{x}, t + \delta t) = C(\boldsymbol{x}, t) - \delta t \; R(\boldsymbol{x}, t) \; ; \tag{17}$$

and we add to the cold pool the cold air from above,

$$h(\boldsymbol{x}, t + \delta t) = h(\boldsymbol{x}, t) + R(\boldsymbol{x}, t) \; . \tag{18}$$

The full model equations can now be written

$$\begin{aligned} \tilde{\partial}_t h &= a\tilde{\nabla}^2 h - \boldsymbol{v} \cdot \tilde{\nabla} h - Fh + R \\ \tilde{\partial}_t C &= c\Theta(\kappa) - R \\ \kappa &= (\boldsymbol{U} \cdot \tilde{\nabla} h - b_1)(b_2 - \boldsymbol{U} \cdot \tilde{\nabla} h) \\ R &= rC\Theta(t - \tau - \bar{t}) \\ \bar{t} &= \frac{1}{N(\boldsymbol{x}, t)} \sum_{i=1, N(\boldsymbol{x}, t)} w_i t_i \Theta(\kappa(t_i)). \end{aligned} \tag{19}$$

The model is initialized with a randomly generated cold pool in the center of the model grid, and all other fields are initialized uniformly to 0. The computational dynamics are extremely simple, evolving with a first-order forward scheme. The boundary conditions are taken to be periodic for simplicity.

3.1 Model parameters

All of the parameters used in this model can be interpreted physically. As noted above, the replacement of the density current dynamics with a linear diffusion has simplified the dynamical behaviour of the model at the expense of decoupling parameters that would otherwise have been functionally related. For instance, in a density current model the local difference in velocity between the movement of the cold pool and the incident wind $|\boldsymbol{U} - \boldsymbol{v}|$ is proportional to the square root of the depth of the cold pool h, and the density difference between the air masses (Emanuel, 1994, p. 231). If we had used a density current model, therefore, these quantities would have been represented by spatially dependent fields rather than parameters.

Parameters used in the model include b_1, which as discussed above must be proportional to $\sqrt{CIN}$; and b_2, which represents a penetration speed for the cold pool, another function of the density difference between the incident wind and the outflow. The incident velocity, $\boldsymbol{U}$, provides a measure of the component of the incident wind along the gradient of the cold pool, $v_a = \boldsymbol{U} \cdot \nabla h$, and from this we can determine the constant of proportionality between CIN and $\frac{1}{2}b_1^2$. Since, from Eq. (10), $v_a = U|\nabla h| \cos(\theta_h)$, and since $|\nabla h| = \tan(\theta_h)$ from Eq. (11), we can write $v_a = U \sin(\theta_h)$. This represents the vertical component of the deflected wind, or the velocity of the updraft deflected from the cold pool. b_1 therefore represents an escape velocity for this updraft, and thus, $CIN = \frac{1}{2}b_1^2$.

The parameter c is the quantity, measured in meters, of warm moist air convected during a convection event. Since such an event lasts for a timestep, we can determine the velocity of updrafted air, c/dt, and therefore specify the CAPE via

$$CAPE = \frac{1}{2}(c/dt)^2. \tag{20}$$

Similarly, the parameter r, representing an inverse time scale for rain fallout, can be related to the dCAPE with

$$dCAPE(\boldsymbol{x}) = \frac{1}{2}(rC(\boldsymbol{x})/dt)^2. \tag{21}$$

The fact that CAPE is a model parameter and dCAPE is a field is an artifact of the model. This situation could be changed by making c a function of position. Justification for a constant CAPE across the region can be found in the derivation of the Arakawa-Schubert parameterization (Arakawa and Cheng, 1993; Emanuel, 1994). The parameter F is a heating frequency, and is the inverse of the timescale for heating of the cold pool. τ is simply a timescale for turnover of convective storms in the tropics. a, the diffusion constant, was set somewhat arbitrarily for stability in the model. Values of all parameters used are listed in Table 1.

4 Analysis

Although our model itself is a space-time deterministic model, the output appears to be stochastic in nature and may

Table 1. Parameter values used

Parameter	interpretation	value
dt	timestep	6 s
dx	spatial res.	200 m
v_x	x comp. cold pool vel.	10 m/s
v_y	y comp. cold pool vel.	10 m/s
a	diffusion constant	400 m^2/s
U_x	x comp. incident wind vel.	10 m/s
U_y	y comp. incident wind vel.	10 m/s
b_2	penetration vel.	5 m/s
τ	vertical time scale for convection	30 min
c	velocity scale for convection	84 m/s $\rightarrow$ CAPE=50 J
r	frequency scale for rainout	1 h^{-1}
F	frequency scale for heating	2 h^{-2}

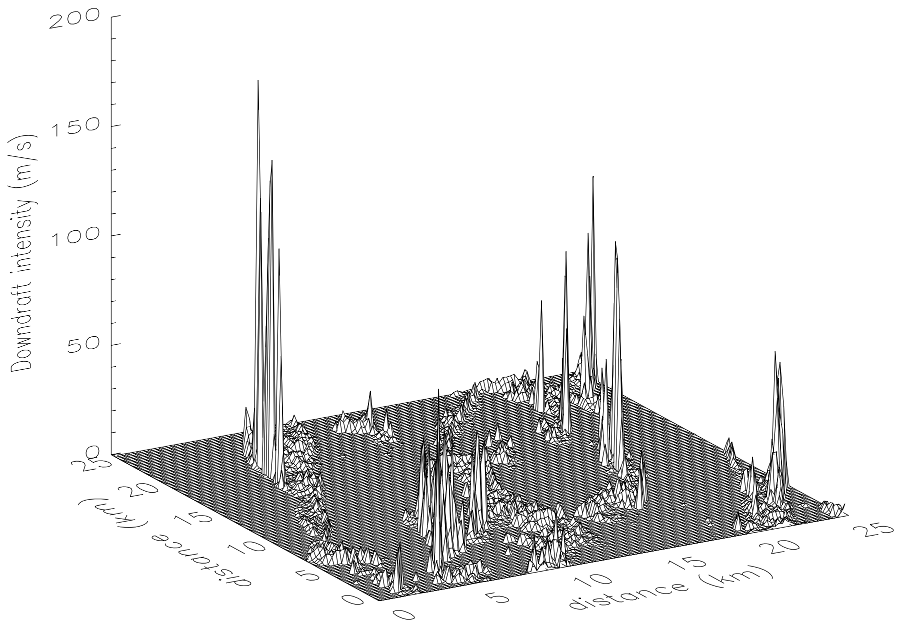

Fig. 2. Sample of model output with $b_1 = 2.50$ m/s, far from critical. For clarity we reproduce here a subset of the domain, since at this parameter value rain covered a large portion of the lattice.

well be chaotic. Figure 2 shows output from the model for the parameter $b_1 = 2.5$ m/s. It looks qualitatively quite similar to the real data shown in Fig. 3 in that both the data sets exhibit a wide range of variability in rainfall intensity. A fairly minor difference between output for $b_1 = 2.5$ m/s and real data is a slightly enhanced tendency for the model output to organize into bands. This tendency increases as b_1 approaches 3.42, which is the critical value of b_1 as discussed below. Just below this critical value, the rainfall occurs only in one narrow, time-dependent band that stretches completely across the lattice and wraps around due to lattice periodicity (Fig. 4). Output is still highly variable at criticality, but is not geometrically similar in any qualitative way to real tropical oceanic rainfall, since it all occurs in this band.

Naturally, a quantitative assessment of model output is necessary. The variability inherent in our model has led us to view the output from this model as a random field by constructing a distribution from the deterministic data. This is a well-known approach in the field of nonlinear dynamics (Tabor, 1989; Beck and Schlogl, 1993). It has been applied in a great number of cases ranging from extremely simple models such as the tent map to models capable of reasonable comparison to real data (Birnir et al., 2001). In this approach, it is common to vary a single parameter in the model to estimate the effect of that parameter on statistical output. In this paper, that parameter will be $b_1 = \sqrt{2\,CIN}$.

We analyze the data generated by our dynamical model for the power law multiscaling found in the GATE data set under the FWA analysis, as well as the higher-order spatial moment scaling analysis presented by Gupta and Waymire (1993) and

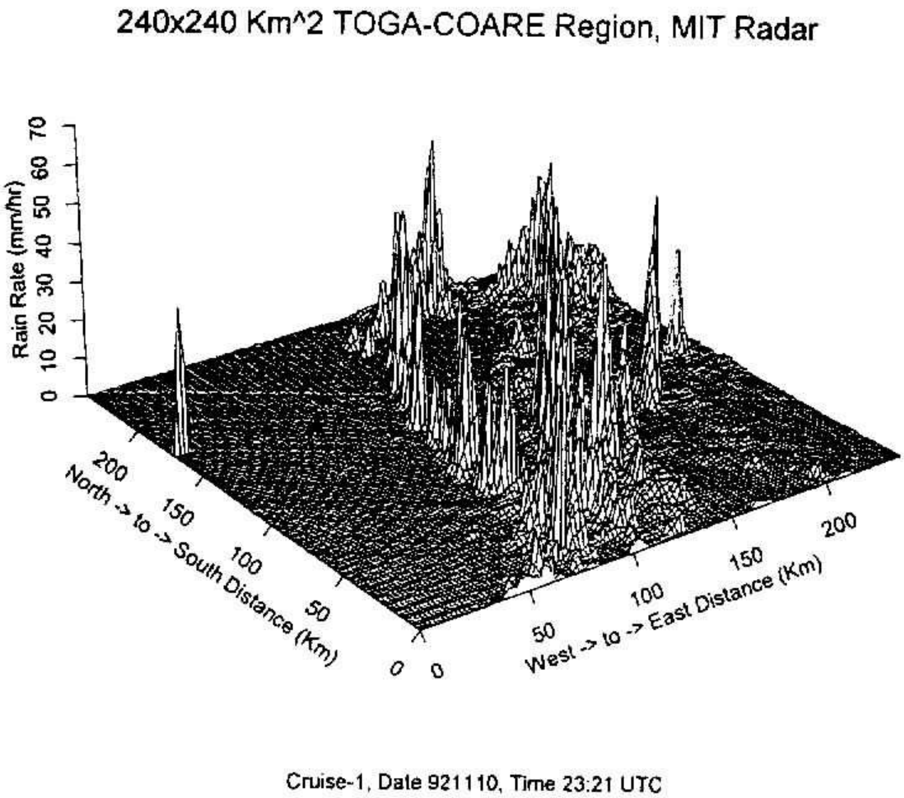

Fig. 3. Sample of data from TOGA-COARE reprinted from PG02.

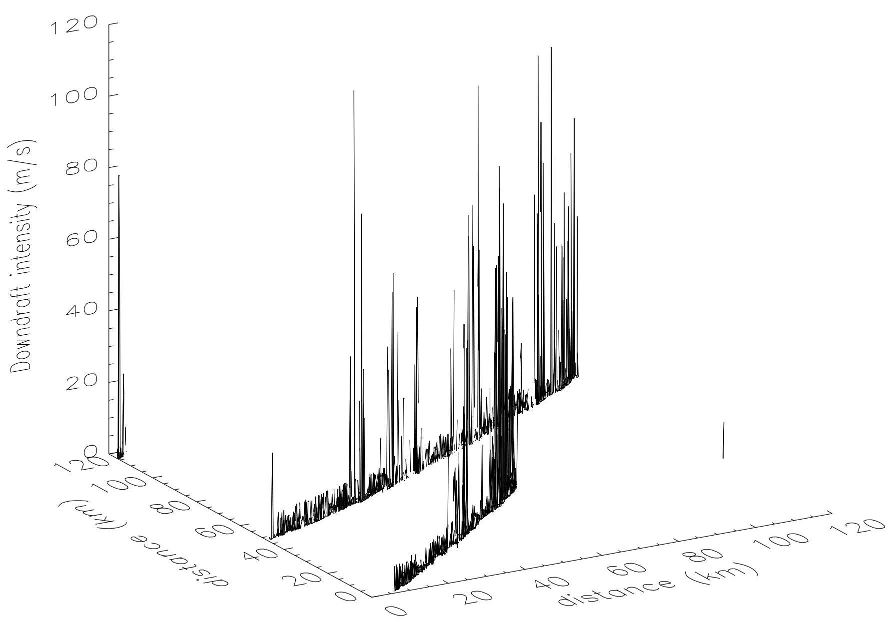

Fig. 4. Sample of model output with $b_1 = 3.41$ m/s, near critical.

Over and Gupta (1994), and reviewed in Sect. 2. For this analysis, we compare model results to those obtained from data in the original paper. Over and Gupta (1994) used GATE data on a square lattice 100 sites per side at 4 km resolution. We take our data on a 600-site grid at 0.2 km resolution, so that our grid is roughly a quarter the size of the domain analyzed by Over and Gupta (1994). Factors of 600 corresponding to observed spatial scales are contained in the set $\ell_{600} = \{1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 25, 30, 40, 50, 60, 75, 100, 120, 150, 200, 300, 600\}$, giving us 24 potential scales to analyze over more than 2.5 orders of magnitude. Our analysis does not involve dimensional quantities, so it is not nec-

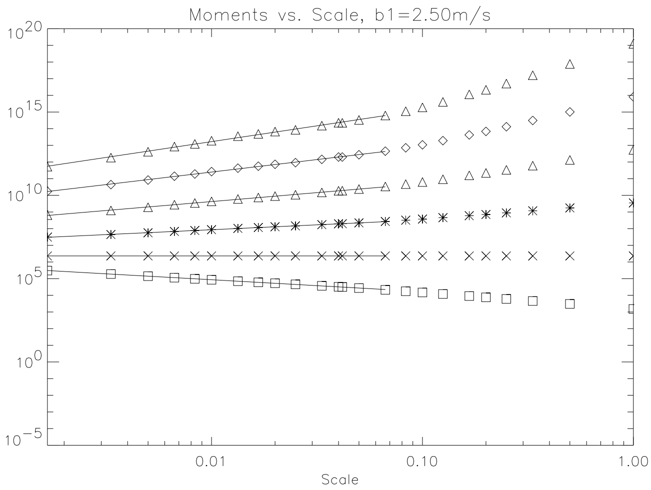

Fig. 5. Scaling of moments for the 1000th recorded scan with $b_1 = 2.5$ m/s, far from critical. Moments are plotted in half-order increments; the smallest moment plotted corresponds to $q = 0.5$ and the largest to $q = 3.0$. These moments clearly scale over only a very limited range of spatial scales.

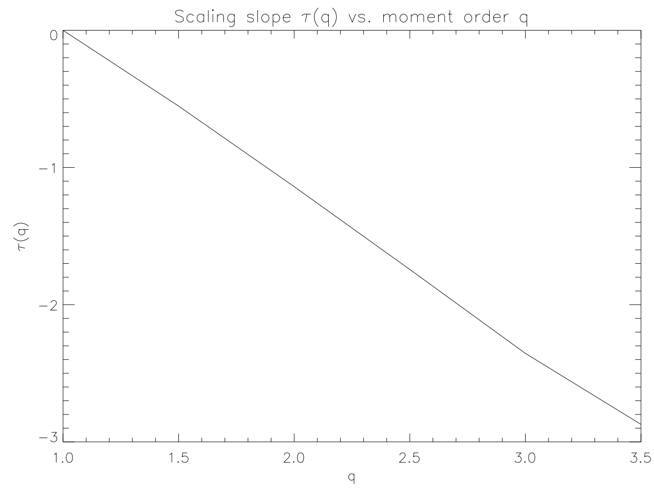

Fig. 7. Exponent $\tau(q)$ for moment scaling plotted as a function of q for the 3000th recorded scan with $b_1 = 3.41$ m/s, near critical.

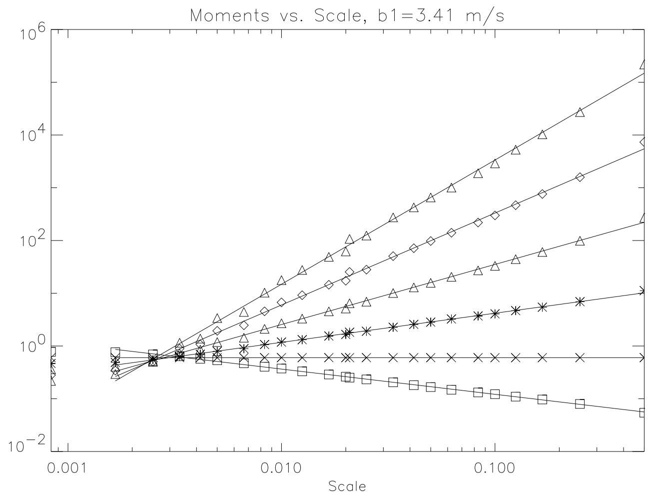

Fig. 6. Scaling of moments for the 1000th recorded scan with $b_1 = 3.41$ m/s, near critical. Moments are plotted in half-order increments; the smallest moment plotted corresponds to $q = 0.5$ and the largest to $q = 3.0$. These moments, from the model at close to critical, exhibit scaling over more than two orders of magnitude.

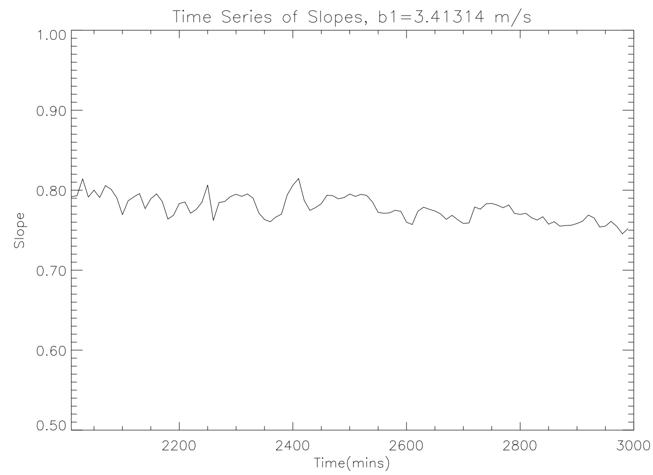

Fig. 8. Time series of slopes for FWA plots for the case $b_1 = 3.41$ m/s, near critical.

essary to transform our model output from units of downdraft intensity to units of mass.

We ran our model for values of b_1 in the set $\{2.0, 2.5, 3.0, 3.41\}$. There appears to be a critical value $b_1^* \approx 3.42$ of the parameter b_1 above which rainfall is only a transient phenomenon . For the full details of this analysis, refer to Nordstrom (2002). Here we will provide one example significantly below the critical point ($b_1 = 2.5$ m/s) and one example just below it ($b_1 = 3.41$ m/s). From the form of Eq. (14), two important dimensionless quantities $b' = b_1/b_2$ and $U' = U/b_2$ can be constructed. As b' approaches 1, the range of gradients ∇h that may be selected by U' narrows, until at some critical value $b'^* < 1$, determined by U', that range becomes negligible. Therefore, the critical value $b_1^* = b_1^*(b_2, U)$. While it is clearly the ratio b' that is important to the model rather than the actual physical quantity $b_1 = \sqrt{2\,CIN}$, in subsequent discussion we will take it to be implicit that we are changing the ratio b' but retain our dimensional parameter notation.

Results of regressing spatial moments $M_\lambda(q)$ against λ for scans of $b_1 = 2.5$ m/s and $b_1 = 3.41$ m/s are presented in Figs. 5 and 6. The $\tau(q)$ function for these moments corresponding to $b_1 = 3.41$ m/s is plotted in Fig. 7. As discussed in Sect. 2, $\tau(q)$ is the principal statistic used to relate cascade models to real data. It is nonlinear and convex for our model output and compares well to those published in Over

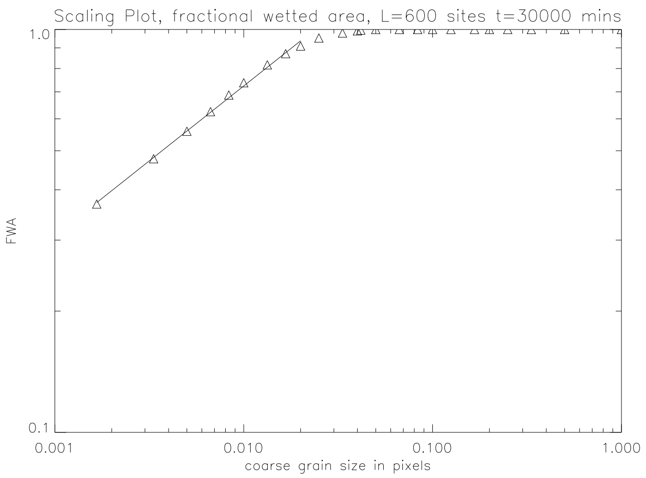

Fig. 9. Sample scaling plot for $b_1 = 2.50$ m/s, far from critical. Scaling occurs only in a transient fashion over scales.

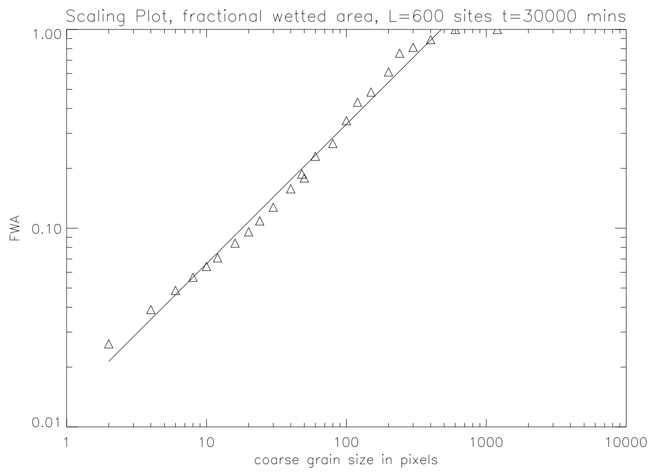

Fig. 10. Sample scaling plot for $b_1 = 3.41$ m/s, near critical. Scaling is persistent over a large range of scales, spanning more than two orders of magnitude.

and Gupta (1994) Fig. 8 for GATE phase II, which was found by Over and Gupta (1994) to match a lower cascade parameter p. Our $\tau(q)$, with an average slope of -1.12, also compares quantitatively better to GATE phase II (average slope for $q \in [0, 4]$ was -1.23) than to GATE phase I (slope -1.52), which was linear.

For the FWA analysis defined by Eq. (5), we compute a time series of slope s for plots of $\log_2 f(\lambda_n)$ vs. $\log_2 \lambda_n$. Since in each case we run the model for 30 000 timesteps (50 h), and we record every 10th scan, we have 3000 scans to consider, all of which integrate to a stationary state. In order to be assured of a quasi-stationary state, we begin our analysis towards the end of the integration at the 2000th recorded scan. Time series plots of these slopes are included in Fig. 8, from which it is clear that the slope of the curves is almost constant and therefore quasi-stationarity is maintained.

Sample scaling plots at the 3000th recorded scan for both values of b_1 are included in Figs. 9 and 10. It is customary to require 2 or more orders of magnitude of varying spatial scales for analysis in order to confirm scaling. Scaling is to be expected if the system really is at critical, since in such cases the correlation length generally diverges. A long correlation length demonstrates a long-range coherent behaviour and therefore large clusters (Fig. 4) . Since a clustered field shows less change under aggregation transformations than an uncorrelated one does, we expect to see power law behaviour under such a transformation. For $b_1 = 2.5$ m/s, the scaling occurs over a very limited range of scales, but close to the critical point with $b_1 = 3.41$ m/s, it occurs for more than two orders of magnitude. These features are similar to those found in nonlinear dynamical and statistical-mechanical systems exhibiting second-order phase transitions (Beck and Schlogl, 1993, ch. 21).

To compare to Over and Gupta, we plot $\ln \bar{R}$ vs $\ln(1-(1-$ $-2^{-s})/.75)$ to fit the function (6). The linear fit in this case is very highly significant, and the slope of the line is ≈ 0.078. The slopes calculated by Over and Gupta were $k = .242$, and $k = .252$ for GATE phase I data and phase II data, respectively. Therefore, the slope determined from our model with this parameter set is thus, about $\frac{1}{3}$ the empirical value. The intercept of the line determines the value of $R_{\max}$, effectively the value $\bar{R}$ takes when the slope $s \to \infty$, which in this case was $R_{\max} = 716.082$. By contrast, the values found $R_{\max}$ in the GATE data set were 7.52 and 7.06. These values are quantitatively very different from the values calculated for our model, which produced a maximum rain value two orders of magnitude too large. $R_{\max}$ is very sensitive to changes in the slope k, however, and if we perform the same calculation using the intercept calculated from model output with a hypothetical slope of $k = 0.25$ (to match that found in the GATE data), we find an $R_{\max} \approx 7.78$, which is in extremely good agreement with the values found in GATE. This indicates that our model run with this set of parameters produces a reasonably good estimate of the intercept, but not for the slope. These results are summarized in Table 2.

A clear discrepancy between the model prediction of the slope k and the empirically observed value shows that certain aspects of our model requires modification, a situation less than surprising given its simplicity. However, the existence of realistic looking output (Fig. 2) and the evidence of nontrivial scaling suggests a new theoretical framework for testing different physical assumptions and hypotheses underlying rainfall generation.

5 Conclusion

We have presented a simple mathematical model of oceanic convective rainfall that was developed entirely from qualitative physical arguments, without invoking any statistical assumptions. In this model, we have made simplifying assumptions in qualitatively incorporating various physical mecha-

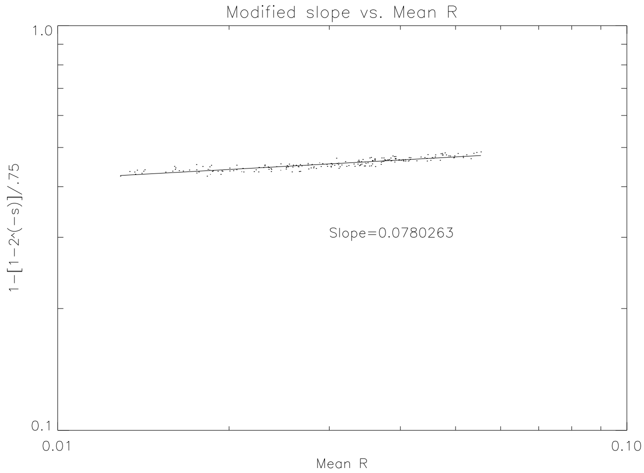

Fig. 11. Plot of $\bar{R}$ vs. $1-(1--2^{-s})/.75$ for $b_1 = 3.41$ m/s, near critical. The slope of the line was ≈ 0.078, about $1/3$ that calculated for data GATE phases I and II. The intercept of this line, however, is very close to that found by Over and Gupta (1994) for GATE data.

Table 2. $R_{\max}$ and k for plots of $\ln(R)$ vs $1-(1-2^{-n/2})/.75$

	$R_{\max}$	k	σ_k
b_1 = 3.41 314 :	716.082	0.078	0.00 291
GATE phase I:	7.52	0.242	N/A
GATE phase II:	7.06	0.252	N/A

nisms that are known to produce sustained convection over tropical oceans. Yet, despite these simplifications and despite a lack of statistical assumptions, it has produced multifractal output qualitatively similar to that found in real rainfall data.

We have found a critical value of a dynamical parameter b_1 such that just below this value the model exhibits scaling statistics. Such statistics have been observed in many studies of convective oceanic rainfall, as well as modelled previously by random cascade models. For values of b_1 away from the critical point, the model does not exhibit scaling over the entire domain. The scaling statistics for our model output have been found in this paper using a scaling analysis of spatial moments with slopes given by a function $\tau(q)$. This function plays a basic role in describing both random and non-random multifractal mass distributions. We have observed that our model produces a slightly convex $\tau(q)$ that compares better with that published for GATE phase II rainfall data set than for GATE phase I rainfall, which was found to be linear. We have scaled the FWAs for various plots in our data at the critical point, and then scaled the slopes against the mean rain $\bar{R}$, as in Over and Gupta (1994). As shown in Table 2, and discussed in Sect. 4, our estimates for the exponent k are off by a factor of 3 and our estimates of $R_{\max}$ are off by two orders of magnitude. However, the intercepts for these plots are in very good agreement with data. From the estimate of probable error in Table 2, it is quite clear that our model predicts an equation like Eq. (6), which has also shown promise of playing an important role in the analysis of land-based rainfall (Jotithyangkoon et al., 2000).

The parameter choices for b_1 at which model output looks similar to real data does not yield scaling statistics, which have been observed in many studies of convective rainfall. Conversely, the parameter choice for which the model output looks least like real rainfall yields scaling statistics quite similar to those found in real data. This unrealistic geometrical behaviour at criticality is due to the establishment of a fixed value for the "parameters" b_1 and b_2 across an entire lattice, a situation related to our use of a highly idealized model for the outflow field. In reality, and in density current models, b_1 and b_2 (and other parameters) would be represented by interdependent fields. In our model, they are fixed such that as b_1 approaches b_2, there is an increasingly narrower range of cold-pool gradients that are selected. Such a narrow gradient range is in turn restricted by the diffusion process to a narrow contiguous area aligned on an equi-gradient surface around the cold pool. This is clear from Fig. 4. The band of rainfall traces the outline of a wide oval shape across the domain, exactly the two-dimensional projection expected for the non-zero values of a directed diffusion process such as that which we have used to represent the cold-pool dynamics. In light of this finding, we conjecture that the processes we have used in the model contain the minimum set of relevant physics necessary to produce scaling statistics in tropical oceanic convective rainfall. However, the dynamical representation needs modifications in order to capture the geometry of observed rainfall. Therefore, we must relax some of the physical assumptions made here. A key example of such an assumption is the use of a diffusion model to represent the cold pool, which should be replaced with the more appropriate but more complex density current model as discussed in Sect. 3. With more physically correct cold-pool dynamics, it seems likely that suitable values of our physically-based model parameters other than b_1 could be found to bring our results into quantitative agreement with data.

A discrepancy between our physical model-derived and empirically observed statistical parameters provides a new theoretical framework for testing diverse physical hypotheses and assumptions governing rainfall. This is a foundational issue in future research that deserves a brief discussion here. It has been conventional to validate physical models by looking at individual realizations of the model output and comparing them against data. However, this kind of sample path approach is more sensitive to errors in forcing and parameterization than a statistical-mechanical approach like that used here. This latter approach is used widely to study other nonlinear chaotic dynamical systems (Beck and Schlogl, 1993). In addition, since our model output exhibits statistical variability, it must be analysed using appropriate statistical methods as a way to test different physical hypotheses. This variability, or 'spikiness' in our model output can also be contrasted with state-of-the-art atmospheric models. Any tendency for spikiness in such models is typically smoothed away in order to prevent infinite energies from occurring due to frequency aliasing and the turbulent cascade. In our model, the extreme variability is a stable feature, and is a direct consequence of the promotion of moderate gradients in our uplift parameterization (14). Therefore, this model, or another another model like it, is ideally suited to the study of empirically observed statistical variability in rainfall fields. Such an approach has recently been made in the study of floods. Gupta (2003) has explained how scaling statistics in floods can be used to test different physical hypotheses covering complex runoff dynamics on channel networks.

Due to the existence of scaling behaviour in our model at criticality, we conjecture that convective rainfall over the tropical ocean is produced by a self-tuning system running near a critical point. This is consistent with the claims of self-organized criticality made for rainfall and other nonequlibrium systems by Peters et al. (2002a); Peters and Christensen (2002b); Nagel and Raschke (1992) and others. Our model itself is not self-organized critical, but must be brought to a critical regime by fixing the value of an external parameter, b_1. This is presumably due to the neglect of some feedback mechanism between our CIN parameter b_1 and the output of the model. Such a feedback is surely plausible, as suggested by the coupling of parameters in the density current model discussed in Sect. 3. This possibility remains to be investigated in future work.

Additionally, there are several other statistical analyses to be performed, including the Zawadzki test of the Taylor hypothesis (Zawadzki, 1973) and the Pavlopoulos and Gupta duration multiscaling analysis (Pavlopoulos and Gupta, 2003). Some of this work was carried out by Nordstrom (2002). However, additional work is required on these analyses, and it is to be published later. As far as we know, this is the first model of tropical oceanic rainfall, developed directly from the physical processes governing convection, which shows significant multifractal statistical scaling as observed empirically. The eventual goal of this modeling effort is to produce scaling statistics directly linked to the more complex physical processes occurring over land. Such a model will be of great practical value in predicting hydrological response in ungauged basins to convective storms.

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