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Frustration and disorder in granular media and tectonic blocks: implications for earthquake complexity

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Abstract. We present exploratory analogies and speculations on the mechanisms underlying the organization of faulting and earthquakes in the earth crust. The mechanical properties of the brittle lithosphere at scales of the order or larger than a few kilometers are proposed to be analogous to those of non-cohesive granular media, since both systems present stress amplitudes controlled by gravity, and shear band (faulting) localization is determined by a type of friction Mohr-Coulomb rupture criterion. Here, we explore the implications of this correspondence with respect to the origin of tectonic and earthquake complexity, on the basis of the existing experimental data on granular media available in the mechanical litterature. An important observation is that motions and deformations of non-cohesive granular media are characterized by important fluctuations both in time (sudden breaks, avalanches, which are analogous to earthquakes) and space (strain localizations, yield surfaces forming sometimes complex patterns). This is in apparent contradiction with the conventional wisdom in mechanics, based on the standard tendency to homogenize, which has led to dismiss fluctuations as experimental noise. On the basis of a second analogy with spinglasses and neural networks, based on the existence of block and grain packing disorder and block rotation "frustration", we suggest that these fluctuations observed both at large scales and at the block scale constitute an intrinsic signature of the mechanics of granular media. The space-time complexity observed in faulting and earthquake phenomenology is thus proposed to result from the special properties of the mechanics of granular media, dominated by the "frustration" of the kinematic deformations of its constitutive blocks.

1 Introduction

The origin of space-time complexity of earthquakes and of tectonic deformations is a subject of large interest and debate, for it has been realized that this complexity is not solely a complication but might provide an Ariane's thread to the unravelling of the fundamental underlying processes. Notwithstanding its importance, progress in this domain is hindered by the still rather limited present state of basic knowledge on the rheology of the crust, the role of inhomogeneities and of pre-existing structures, i.e. the coupling between successive tectonic phases of deformations, the influence of water, of the layered structure of the lithosphere and the coupling between different rheological layers, etc. While waiting for more precise geological, petrographic, rheological and geophysical constraints, a natural alternative has been to develop theoretical and numerical models, based on simplified descriptions and assumptions. Here, we take a similar view, i.e. we attempt to gain some insight in this question by studying a simpler problem. However, in contrast to theoretical/numerical modelling, we use the knowledge accumulated in the study of a model material, namely non-cohesive granular media. The interest in this problem is that, while keeping some of the complexity of the natural geological problem, it can be the object of well-controlled tests and experiments in the laboratory.

The use of non-cohesive sand, for instance, and more generally of granular media as laboratory models of tectonic deformations has a long history (Mandl, 1988). We refer to Mandl (1988), Sornette et al (1990), Davy et al (1990), Sornette (1990), Davy et al (1992) and Sornette et al (1993) for the justification and discussion of the analogy between the mechanics of non-cohesive granular media and tectonic deformations. In these previous works, the granular medium was used as a model of the phenomenon of shear localization occurring in the crust at large scales. The studies were concerned with the formation of faults and other geological structures, starting from underformed materials, or in the presence of well-defined large scale preexisting structures. Here, we want to push to the extreme the analogy between the brittle earth crust and granular media by examining the consequences on the modes of deformations of the microscopic grain structure. Instead of starting from the point of view in which a given granular medium, which is seen homogeneous at large scale, represents the large scale structure of a tectonic system, we focus on the granular or block nature of

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the granular system. The granularity is proposed to model the characteristic highly fragmented structure of the crust, modelled as a complex system of blocks of varying sizes, separated by faults along which the blocks can rotate and slide. This point of view is thus drastically different from previous ones: here, the preexisting structures (blocks) are considered as fondamental and our purpose is to examine their consequences in the nature of the crust deformations. In other words, the present paper does not discuss the origin of the fragmented block structure but rather analyzes the nature of deformations of such "structural" (fragmented block) steady state. At this stage, we should emphasize that the statistical properties of granular media and those of the fragmented crust could be very different and the proposed analogy be somewhat ill-defined. In particular, here we are not taking into account the hierarchical or fractal structure of the blocks.

The main characteristics that will be examined are concerned with the "fluctuations" around an average of the response to some external force. We first review (section 2) the available experimental facts that betray the existence of important fluctuations in the deformation of granular media. In section 3, we develop an analogy between the mechanics of granular media and spinglasses. The mathematical formulation of the spinglass problem and its various versions constitutes the paradigm of the physics of "complex random systems" which we would like to apply here to the mechanics of granular media and of tectonic deformations and earthquakes.

2 Evidence of fluctuations in the mechanics of granular media

2.1 Fluctuations at the macroscopic scale

The mechanics of granular media is established on well-defined experiments carried on standard devices, such as Casagrande boxes (for shear tests), triaxial apparatus (for compressive tests), etc. These various experimental set-ups are complementary in that they provide different references for macroscopic deformation modes, i.e. systems in which the deformation is dominated by "transform", "normal" or "inverse" faulting mode. In these experiments, a continuously increasing deformation is usually applied and various macroscopic quantities (such as the porosity, its derivative with respect to the deformation (dilatancy), the shear stress, the overload, the isotropic stress, etc.) are measured as a function of the deformation.

In these experiments, fluctuations come under two categories:

1. Within a given experiment, the measured quantity (say the reduced uniaxial applied stress q/p in a triaxial test) does not behave continuously as a function of the deformation $\epsilon_1 = \delta h/h$, where h is the height of the triaxial sample, but presents a "noisy" aspect around an averaged continuous variation. It is custumary to consider these fluctuations as an "experimental uncertainty", attributed to experimental errors, limited resolution of the measuring apparatus, external vibrations, etc, in a word as a "noise". In the sequel, we will name it "internal fluctuations".
2. The second type of fluctuations is often observed when repeating the same experiment, using the same experimental set-up, the same granular medium prepared under apparently identical conditions defining equivalent samples (up to the control of the same experimentalist), the same measuring apparatus with the same experimental procedure. It often occurs that a perfect reproducibility is not found and that various physical characteristics are sample dependent. The usual reaction to such observations is to attribute the absence of reproducibility (which will be called "sample to sample" fluctuations) to experimental errors during the preparation of sample or in the measuring process.

It is our purpose here to revisit published and unpublished experiments to show that there exist strong indications that these two types of fluctuations ("internal fluctuations" and "sample to sample fluctuations") are genuine intrinsic properties of granular media. Of course, this does not mean that non-reproducibility and experimental uncertainty cannot appear in addition as a result of careless experimental procedures. We address the case of "well-done" and "well-controlled" experiments!

2.1.1 The triaxial test

Notwithstanding the almost complete absence of a clear statement of this fact, there are strong reasons to believe from the literature that two strictly identical triaxial tests do not lead to the same final state ("sample to sample fluctuations"). For instance, it is well-known that a bulging bifurcation (Vardourakis, 1983) may occur, and that very tiny changes on boundary conditions make successive tests non perfectly "reproducible" (Helttler and Vardourakis, 1984). One also observes variations of the yield surface inclination when a single yield surface is observed, or fluctuations of the stress-strain characteristics when homogeneous tests are achieved. It is even possible to quantify the sample-to-sample fluctuations of the results in some special well-defined cases, for instance, when the state of the granular material is supposed to be perfectly defined such as in the case of the so-called critical state (Frossard, 1979; Kolymbas and Wu, 1990). According to the common wisdom of soil mechanics specialists, the "sample-to-sample fluctuations" of the measured friction angle in this "critical state" is at least 2 degrees. To account for the observed randomness in the mechanics of strain localization, soil mechanics specialists have introduced various remedies in the standard theoretical framework (yield design theory (Salençon, 1990) or continuum bifurcation formulation (Adamard, 1903; Thomas, 1961; Hill, 1962; Varlourakis, 1979; Sulem and Varlourakis, 1990)), for instance probabilistic failure criteria (Alpa and Gambarotta, 1990), plastic deformation constitutive laws with a history dependence.

The evaluation of the "internal fluctuations" is much harder since most of the many experimental triaxial test curves which are reported in the literature are smoothed prior to publication. Only a few partial data are reported with some "fluctuations" or slope discontinuities. Some relevant examples may be found in Helttler and Vardourakis (1984), Frossard (1979), Kolymbas and Wu (1990), Allersma (1987) and Konrad et al. (1991). Fluctuations appear particularly on the behavior of the deviatoric stress (Helttler and Vardourakis, 1984; Frossard, 1979; Kolymbas and Wu, 1990; Allersma, 1987) and on the dilatancy (Helttler and Vardourakis, 1984; Frossard, 1979; Kolymbas and Wu, 1990) (see in particular fig. 21 of Helttler and Vardourakis, 1984). It is also interesting to mention the data on polystyrol (Frossard, 1979) which exhibit a very large and unexplained "internal fluctuations".

A very important experimental observation is the following: in general, much larger "internal fluctuations" are found in 2-D experiments (i.e. with biaxial cell test on rods (Greco, 1991)) than with grains in 3-D apparatus. This strengthens the idea that these fluctuations are genuine, because it is a general fact that 2-D systems are much more sensitive to fluctuations due to the larger mode density at low wavenumbers.

2.1.2 Other classical soil mechanics testing devices

These two kinds of fluctuations are also found when testing 3-D granular materials or 2-D packing of rods, using a Casagrande box (Greco, 1991; Rowe, 1969), i.e. in a shear experiment in rotation. There are also found in penetration (Allersma, 1987; Konrad et al., 1991) and footing experiments or during the uplift of an anchorage (Tran Vo Nhiem, 1972a; Tran Vo Nhiem and Giroud, 1972b). Similar fluctuations appear in bulldozing (Bagnold, 1954, 1966). In the same spirit, it is well known that the stress in a silo at rest fluctuates from point to point (Blight, 1986).

Important geometrical fluctuations are also observed in the formation of localization bands. Since Coulomb (1773), it is well known that, as the deformation increases, a granular sample generally produces a yield surface where strains are localized. This occurs even when the mechanical test is highly symmetric such as in the triaxial test. This yield surface is understood as the consequence of a mechanical instability or equivalently a bifurcation. To account for this behavior, two standard approaches have been developed. The first one makes use of the yield design theory (Salençon, 1990) and evaluates the shape of the most probable yield surface. The second one uses a continuum theory together with a formalism of bifurcations (Adamard, 1903; Thomas, 1961; Hill, 1962; Varlourakis, 1979; Sulem and Varlourakis, 1990) associated to a stress-strain mechanical law (Schofield and Wroth, 1968; Collins, 1990; Rowe, 1962). This second approach may be combined with finite element techniques (Schofield and Wroth, 1968; Collins, 1990; Rowe, 1962) based on the use of a incremental mechanical law determined directly from experimental triaxial test data. These theoretical approaches predict a single well-defined major crack. This is true at least when the sample symmetry is low, since in some cases, when the symmetry of boundary conditions is larger, different failure surfaces are equally possible at the beginning of deformation (Mandel, 1966; Desrues and Mokni, 1991); in this case, these different failures compete and several may be observed simultaneously at small deformation, but an increase of deformation leads eventually to the selection of one of them (Desrues and Mokni, 1991).

Experimentally, the situation is not as simple and, in many cases, the selection of one leading localization band occurs only in the very late stage at very large deformations. In the mean time, the different bands have time to grow and have been found to form complex fractal patterns of widly varying lengths and positions (Sornette et al, 1990; Davy et al, 1990; Sornette, 1990; Davy et al, 1992; Sornette et al, 1993). Also, notable differences between two fractal localization band patterns obtained under apparently the same conditions have been observed (Davy et al, 1990), confirming again the ubiquitous presence of sample to sample fluctuations. These fluctuations come from the fact that the major crack is not produced at once but results from the growth and fusion of many smaller precursors largely spread in the sample volume (Lockner et al, 1991).

Parallel to the observations made on triaxial tests, experiments of 2-D footings and 2-D upliftings (Tran Vo Nhiem, 1972a; Tran Vo Nhiem and Giroud, 1972b) or using a 2-D Casagrande device (Greco, 1991) exhibit fluctuations larger than those obtained in 3-D devices. This confirms that the enhancement of fluctuations in 2-D devices compared to 3-D ones is a general phenomenon.

Important fluctuations are also found in the problem of slope instability at the free surface of granular-material, notably in the statistics of sand avalanches (Evesque and Rajchenbach, 1988; Jaeger et al., 1989; Held et al., 1990; Evesque, 1991). For instance, one finds a typical mean avalanche size $< \delta\theta > \simeq 2^\circ$ which is the difference between the inclinations of the free surface before and after a typical avalanche; the width of the distribution of $\delta\theta$ is also about 2°. These quantities seem to be quite independent of the pile size (Evesque and Rajchenbach, 1988) (for piles larger than 40 grains) and of gravity (Jaeger et al., 1989) in the range $1 - 100g$ ($1g$ is the acceleration of gravity). Note that these fluctuations of the avalanche size correspond also to the fluctuations of the maximum angle of repose θ_m, so that these fluctuations can be viewed as the "reproducibility" with which one may define the macroscopic friction coefficient $\Phi = \theta_m$. This result confirms the evaluation of the "sample to sample fluctuations" of a triaxial experiment, since the angle of the localization band is also controlled by the same macroscopic friction coefficient. A few series of sand-avalanche experiments have also been performed on 2-D packings (Evesque, 1991). Their results confirm that fluctuations of the avalanche size are much larger in 2-D than in 3-D.

2.1.3 Summary

Summarizing the above experimental observations, macroscopic "internal fluctuations" and "sample to sample fluctuations" of mechanical tests measuring intensive quantities such as the deviatoric stress level, the porosity, as well as other mechanical variables appear to be genuine intrinsic phenomena, which are present even at a macroscopic level. It is not clear how the amplitude of these fluctuations depend on the sample size. However, it does not seem that these fluctuations average out when taking large volume limits, contrary to the usual expectation of standard statistical physics and thermodynamics (central limit theorem). An important observation is that "sample to sample fluctuations" are always larger than "internal fluctuations". Furthermore, the size of these fluctuations are enhanced in 2-D experiments compared to those observed in 3-D. This implies that two tectonic plates with similar preexisting structures and boundary conditions may develop largely different fault patterns as a result of the "sample to sample" fluctuations phenomenon.

2.2 Fluctuations at the microscopic scale: Microscopic stress localization and restructuration

Dantu has been the first to use photoelasticity to demonstrate that stress is a wildly varying quantity within a loaded granular material (Dantu, 1968, 1967; LCPC, 1980; Dantu, 1957). (Photoelasticity enables to visualize the paths taken by the stress field (Nisida, 1986; Dantu, 1957; LCPC, 1980)). It has been found that, at low global load, only a small fraction of the grains carry a significant stress. This fraction of grains builds up a loose complex disordered connected system of stress paths (Allersma, 1987; Konrad et al., 1991; Dantu, 1968, 1967; LCPC, 1980; Dantu, 1957). Due to the vault effect, many grains are thus screened and carry a small or no stress at all. Increasing the load on the sample, this first leads, for very weak load increase, to deform the material in a continuous fashion with a continuous evolution of the local stress paths. But very soon, one observes (Dantu, 1967) many "bifurcations" corresponding to a complete and sudden redistribution of the geometry of the local stress paths. As the load increases more, the system of stress path densify progressively while exhibiting large non-local reorganizations. The visualization of the fluctuations of the stress paths within granular samples allows one to get a vivid illustration at the microscopic grain scale of the "internal fluctuations" reviewed above at the macroscopic level.

More quantitatively, different works have brought useful informations on the distribution of the forces at the contact between two grains. In Cambou (1979), an experiment on brittle grains loaded by an increasing force has shown that the first grain rupture occurs for a load ten times smaller than the one estimated for identical contact forces between all pairs of grains: this result suggests that the ratio between the maximum intergranular force and its average is of the order of 10. Contact forces have been estimated in small 2D Schneebeli (parallel cylinders) models made of 40 cylinders, by measuring the friction force between cylinders in solid contact and using a linear friction law. The ratio maximum/average for the contact force is about 2 in this small system. In Gourves and Mezghani (1988) and Delyon et al. (1990), the ratio maximum/average for the contact force in a 3D sand grain system containing 2000 grains is found about 9. This value around 10 is often observed and seems to be the asymptotic value for large systems. These results show that large deviations of the forces of contacts exist around the average, implying the existence of important "internal fluctuations".

Within the picture where a tectonic plate can be modelled as an ensemble of blocks (certainly of varying sizes, but this is not necessary in our discussion), these observations can be directly translated to draw useful conclusions on the nature of the mechanics of the lithosphere. In particular, these observations suggest a high degree of stress heterogeneity in the crust, in a large part due to screening and vault effects, which are ubiquitous in granular systems. This has important consequences for the understanding of the coupling between earthquakes in space and time. Indeed, this coupling is predicted to be highly heterogeneous along preferential stress paths connecting particular blocks, these special stress paths being very sensitive to the particular block structure and applied stress pattern.

2.3 Summary of the keys results

Fluctuations have been found experimentally in two guises: 1) "internal" fluctuations and 2) "sample to sample" fluctuations. They are observed both at the macroscopic and microscopic level. They concern both spatial geometrical features as well as mechanical quantities. In other words, these fluctuations occur both in space (geometry) and time (variation of various mechanical quantities with the load), these two aspects being intimately related.

3 Analogy between granular media and spinglasses

It is our purpose now to outline an analogy between the mechanics of granular media (and consequently the mechanics of tectonic deformations and earthquakes) and the physics of complex random systems, whose paradigms are spinglasses and neural networks. We are presenting more a "metaphoric" approach than a precise model as we believe that if,the classic spin glass models we review below are useful for the sake of our arguments, the derivation of appropriate and specific "spinglasses" should be called for a quantitative development of our ideas.

Our line of reasoning is based on three essential assumptions:

1. the existence of "intrinsic" and "sample to sample" fluctuations, both in the mechanics of granular media and in spinglasses,

2. the existence of an analog variational principle which governs the state of equilibrium of both systems, based on similar ingredients, disorder and frustration;

3. the local tendency of two grains (or blocks) which are in contact to deform by counter-rotation in order to minimize the dissipation work of friction, which is analogous to an antiferromagnetic coupling (the spin being here the rotation vector).

3.1 Brief discussion on spinglasses, neural networks and complex systems (Binder and Young, 1986; Mezard et al., 1987)

The term "spinglass" has been introduced initially to describe a given specific composite systems of diluted randomly placed Fe atoms among Au atoms. Each Fe atom carries a spin (intrinsic magnetic moment) which interacts with the other Fe atoms through an interaction which is an oscillatory function of the distance between the pair of Fe atoms (so-called RKKY interaction). Since Fe atoms are positioned at random, their pair interactions take arbitrary but fixed (quenched) signs, a positive coupling coefficient favoring the ferromagnetic alignment of the spins while the negative sign favors anti-ferromagnetic ordering. The set of random coupling coefficients introduces "frustration", is characterized by disorder, produces novel bizarre behaviors (long time relaxations, breakdown of ergodicity, etc.) and important fluctuations.

The spinglass problem is often formalized as follows (see Maddox, 1994) for a discussion of the relevance of spin models to real complex systems). Consider a network of nodes i and of links ij which bind the nodes i and j together. Attribute to each node i a variable (called a spin) S_i usually taken discrete and equal to ± 1 (it can also be continuous between say -1 and $+1$). Attribute to each link an algebraic number J_{ij} which characterizes the strength of the interaction between the two spins S_i and S_j along the link ij. The simplest interaction energy H_{ij} is usually taken of the form

$$H_{ij} = -J_{ij} S_i S_j \tag{1}$$

If $J_{ij} > 0$, a stable state, for which the energy is the lowest, corresponds to two spins having the same sign (they are aligned). This corresponds to the so-called ferromagnetic ordering. If all J_{ij} are positive, the stable state corresponds to two equivalent configurations in which all spins are of the same sign either all $+$ or all $-$ (ferromagnetic state).

If $J_{ij} < 0$, the energy is the lowest (stable state) when the two spin have opposite sign (they are anti-aligned). This corresponds to the so-called antiferromagnetic ordering. If all J_{ij} are negative, the stable state corresponds to the antiferromagnetic phase characterized by the fact that spins alternate in sign from one site to the next one. Note that on square lattice, this is the solution which minimizes the energy. However, in a triangular lattice, this is not so as can be seen by following the perimeter of a triangle, say clockwise. Starting from a spin $+$ at one node, the neighbor must be $-$ to minimize the interaction energy with $J_{ij} < 0$. Its neighbor is then $+$. But this spin is also the neighbor of the first spin which is also $+$. As a result, the energy of interaction of these two spins is not minimum. This situation has been coined "frustration" (Toulouse, 1977) since at least one interaction must be frustrated on each plaquette. The existence of frustration leads to the existence of several degenerate state of equivalent minimum energy. More generally, frustration arises in a system whose interactions compete or conflict in such a way that not all constraints on the system can be simultaneously satisfied.

If in addition, the J_{ij} couplings are distributed at random in the system, frustration will appear whatever the topology of the underlying lattice. To verify this point, it suffices to follow a closed loop and compute the product $\prod J_{ij}$ of all J_{ij} of the links which constitute this closed contour. A negative product corresponds to frustration on this loop (Toulouse, 1977). The problem defined by the hamiltonian (1) with random algebraic coupling coefficient J_{ij} is called a spinglass.

Summarizing drastically this rapidly developing field (Binder and Young, 1986; Mezard et al., 1987), let us recall briefly the key results which bear on the proposed correspondence with the mechanics of granular media and tectonics. It has thus been discovered that these two ingredients, disorder and frustration, lead to the existence of, not a single well-defined stable minimal state, but rather to a very large number of disordered minimum states of equivalent energies. The number of these minimum states usually increases exponentially with the number of degrees of freedom (spins). The energy landscape is extremely complicated with a hierarchy of barriers of increasing sizes separating the minimum states. In other words, in order to go from one minimum state to another, an energy barrier must be passed. It has been discovered that the set of minimum states can be ordered by defining a "distance" in the space of states, the distance between two states increasing with the height of the energy barrier which needs to be passed to go from one state to the other. With this distance, the space of minimum states has been found to be "ultrametric", i.e. it can be viewed as a hierarchy of states of increasing distances, arranged according to a hierarchical tree. This hierarchical structure is responsible for instance for the anomalous time dependence of thermodynamical properties of spinglasses (Souletie et al., 1987).

These results are obtained for a fixed (or "quenched") set of coupling coefficients J_{ij}. However, it is interesting and richer to think of systems whose coupling coefficients J_{ij} can (slowly) evolve in response to the spin configuration. The paradigm of this problem is the so-called "neural network" problem. Each neuron is either in a firing $+$ state or is quiet $-$ and is interacting with other neurons with dendrites connections J_{ij} which can be amplifying ($J_{ij} > 0$) or inhibitory ($J_{ij} < 0$). These connections may evolve as a function of sollicitations imposed on the neural network and its response. Many different models can be defined which differ in the way the J_{ij} evolve as a response to inputs in the state variables S_i. It is thus believed that neural systems of animals may function somewhat like spinglasses with self-organizing learning rules (Becker and Hinton, 1992) which amount to make evolve the coupling coeffients. As a result, the neural network is capable to learn and evolve in response to stimulii.

3.2 Analogy between the mechanics of granular media, spinglasses and neural networks.

In order to simplify the problem, let us consider a granular medium as a disordered packing of rigid spheres. Then, dissipation comes from solid friction between the spheres. This friction depends on the connectivity, the normal and tangential forces applied at each grain-grain contact, and the relative grain-grain motion. In order to link the mechanics of granular media to the spinglass problem, we make use of the formalism of virtual work, corresponding to the work associated to an admissible virtual deformation. This theory allows one to determine the limit of stability of an initially stable configuration and the different routes that the system may choose to follow when reaching the stability-instability threshold. The method of the virtual work consists in considering admissible grain motions δu and compute the corresponding energy dissipation:

$$\begin{aligned} -dW &= \frac{1}{2}\sum_{n,m} kf(n,m) \mid \delta u(n,m) \mid \\ &\quad - \sum_{n} kF(n) \mid \delta u(n,ext) \mid \end{aligned} \qquad (2)$$

The first sum in the r.h.s. is over the pairs of grains in contact with each other and the second sum is over the grains which are in contact with a boundary. k is the solid friction coefficient between the grains and between the grains and the boundaries. $f(n,m)$ is the symmetric normal force (from the action/reaction theorem) applied on grain n by grain m (or reversely on grain m by grain n). $\delta u(n,m)$ is any possible (virtual or not) displacement of the point of contact $n-m$. $F(n)$ is the external normal force applied to the grain n. $\delta u(n,ext)$ is any possible displacement of the application point of $F(n)$. The symbol $\mid . \mid$ takes the absolute value of the quantity inside the vertical bars.

Assuming for the time being that grains in contact remain in contact during an infinitesimal deformation, $\delta u(n,m)$ can be decomposed into the sum of three sets of rotations. Indeed, the grains being rigid, they can only rotate: two sets of rotations ($\delta\Omega_n$ and $\delta\Omega_m$) around the two centers of the grains describe the rotation of each grain independently of each other. Restricting to two dimensional grains (disks), $\delta\Omega_n$ and $\delta\Omega_m$ are the analog of spin variables. Each rotation can indeed be positive (anticlockwise) or negative (clockwise). The third set of rotations $\delta\Omega_{nm}$ gives the relative displacement of the two grains. Denoting $r(n,m)$ the vector joigning the center of n to the contact point with grain m, one can write

$$\begin{aligned} \delta u(n,m) &= \delta\Omega_n \times r(n,m) - \delta\Omega_m \times r(m,n) \\ &\quad + \delta\Omega_{nm} \times [r(n,m) - r(m,n)] \end{aligned} \qquad (3)$$

We can then draw a loose analogy with spinglasses by noting that the rotation variables $\delta\Omega_n$ are similar to (continuous) spins. Neglecting the relative displacement of the two grains, we note that the virtual work is minimum when $\delta\Omega_n \times r(n,m) - \delta\Omega_m \times r(m,n)$ vanishes. For grains of equal size, this implies $\delta\Omega_n = -\delta\Omega_m$. We thus obtain an "antiferromagnetic" coupling between neighboring grains, which is nothing but the standard condition of rotation without sliding, which indeed corresponds to a vanishing friction dissipation. "Frustration" appears due to the disordered topology of the grains. It is also present, as in spin problems, for most compact grain topologies (disordered or not), one of the simplest topology being the triangular lattice in two dimensions. From this analogy, one can expect most of the qualitative properties of spinglasses to characterize granular media.

Let us now formulate more precisely the analogy we have in mind: let us consider the set of forces $f(n,m)$ between all pairs of grains within the system, for a given configuration of the grains (given position and rotation variables), under the action of a set of boundary conditions and forces supposed, to simplify, to be parametrized by a single parameter F (think of F for instance as being the magnitude of the stress applied at the boundary). For a given configuration of grains, a given value of the external parameter F determines the set of boundary conditions and thus the set of forces $f(n,m)$ between all pairs of grains. There will be a critical value F^*, such that the set of forces becomes $f^*(n,m)$ and satisfies the condition that the virtual work dW given by eq.(2) becomes zero. This condition corresponds to the threshold for instability, i.e. the onset of grain motion. It is then well-known that the set of motions in this instable regime is convex, i.e. any linear superposition of displacements in the unstable regime is itself unstable.

This reasoning holds for a given initial grain configuration. Of course, we can consider many different initial configurations of grains and ask the same question for each of them, namely, determine the value $F^{*\prime}$ such that the set of forces between all pairs of grains is $f^{*\prime}(n,m)$ and satisfies the condition $dW = 0$ for the onset of instability. In other words, each initial grain configuration will determine its own critical value F^* for the onset of unstable motion.

We are now interested in characterizing the space of grain configurations, with respect to this instability of the onset of motion, or in other words with respect to the critical value of the boundary condition parameter F^*. We thus view a given macroscopic state of the granular medium as resulting from the response of a given grain configuration which is selected during the preparation of the sample among many others possible. In the language of thermodynamics, this corresponds to view the macrostate as resulting from one of the many possible microstates which can be explored by the system. The difference with standard thermal thermodynamics is that there are no thermal fluctuations here which allow the system to sample all the permissible microstate configurations. The sand system is thus much more sensitive to fluctuations from sample to sample. This is precisely the main difference between an annealed ferromagnetic or antiferromagnetic system, without disorder and coupled to a thermal bath, and a spinglass system with quenched disorder at a low temperature so that thermal fluctuations can be neglected. More precisely, we conjecture that the dependence of the critical force F^* as a function of initial microstate grain configurations (that we shall call the critical force "landscape") exhibits a multivalley hierarchical structure, reminiscent of that of the energy landscape for the spinglass problem.

Concretely, this means that some grain configurations will have a low value of F^* and will be separated from other grain configurations with similar low values of F^* by grain configurations with much larger values of F^*. We conjecture that this picture should remain roughly self-similar at all levels of F^*, thus defining the multi-valley structure characteristic of a spinglass or more generally what is believed to characterize generally a frustrated random system. In order to make the analogy more precise, we need to introduce a distance in the multi-dimensional space (so-called "phase" space) in which a point represents a given grain configuration, i.e. a distance between two different grain configurations: a natural choice is simply the square root of the sum over all discernable grains of their squared difference in position and orientation. Then, if the spinglass analogy has any sense, the distance between two different grain configuration should be an increasing function of the difference $F^{*\prime} - F^*$ of their respective instability thresholds. This picture, if correct, has important implications for our understanding of fluctuations in granular media. However, this formulation is still too restrictive to capture the deformation processes.

Indeed, once a deformation is triggered, grains move and the set of contacts between grains is rearranged until one reaches again a static state of mechanical equilibrium under the action of external boundary conditions $F \geq F^*$. This new evolutionary problem then enters the general framework of neural networks and self-organizing systems, which are a direct extension of spinglasses in the sense that bond interaction couplings are now allowed to become a function of the grain response to their previous values. Then, the global picture we propose is the following: starting from any grain configuration, we have for very small deformation the multivalley spinglass structure, which at larger deformation progressively organizes itself in response to the various sollicitations.

3.3 Implications of the spinglass analogy

3.3.1 Fluctuations and spinglass hierarchical structure

The existence of "sample to sample" fluctuations in physical measurements is a well established fact in spinglass physics. From the proposed analogy, it is thus natural to observe it in the mechanics of granular media. We thus suggest to rationalize the experimental observations of microscopic and macroscopic stress fluctuations during the deformation of a packing of grains as stemming from a hierarchical structure of the landscape of the threshold function F^* in the space of grain configurations. A similar discussion applies to the "intrinsic" fluctuations.

3.3.2 System size dependence of the fluctuations

A prediction of this analogy is that these fluctuations should not decay or decay very slowly with the sample size. This seems to be born out for instance in the measurements of the fluctuations of the macroscopic friction coefficient (or equivalently of the angle of repose of a sandpile).

3.3.3 Sensitive dependence to small sollicitations

Another consequence of the proposed analogy is the evolution of the state of deformation of the grains within the granular medium and correspondingly the stress field, under evolving boundary conditions or external forces. Indeed, the hierarchical structure of the state space of spinglasses implies that a small change of a local field or of a coupling coefficient may result in a drastic reorganization of the spin configuration (Bray and Moore, 1987; Zhang, 1987). Similarly, we thus expect that a small change of boundary condition and applied stress may result in large readjustment in the grain configurations and consequently stress field. This prediction rationalizes the observation of Dantu and others made by photoelasticimetry (Allersma, 1987; Konrad et al., 1991; Dantu, 1968, 1967; LCPC, 1980; Dantu, 1957). This large susceptibility stems from the delicate hierarchical nature of the organization of the grain or block configurations. This could provide an explaination for the often observed triggering of earthquakes by relatively small stress sollicitations induced by mining or dam filling (Gupta and Rastogi, 1976; Gupta, 1985; Guyoton et al., 1992).

3.3.4 Aging and long memory effects

The dynamics of the block motion and earthquake activity should present the typical features of multi-state systems. One of these features is the existence of aging in spinglasses or in glasses (Bouchaud et al., 1994). In the tectonic context, this implies that the recovery of the stress field as a function of time after a large event should depend on the size of the event and should proceed rather slowly in a self-similar way over long time scales. This may provide a new mechanism to explain the time and space correlation between large earthquakes. Another feature is the competition between different fault structures, i.e. different metastable states: for a long time, earthquakes can be trapped on a given fault domain, and then shift rather abruptly to another domain (Miltenberger et al., 1993; Sornette et al., 1994).

3.3.5 Nonlinearity in the mechanics and elastic wave propagation

The disordered and "frustrated" block structure of the crust implies that very large nonlinear elastic responses should be present for relatively small strains, much smaller that those necessary to enter the nonlinear regime of homogeneous elastic rocks, with amplitudes far greater than for homogeneous systems. This phenomenon has been documented recently in the litterature (Travers et al., 1986; Roux, 1991), with the discovery of a very large nonlinearity (force proportional to strain to the power 3-4) resulting from an amplification of the Hertz effect by the occurrence of new grain-grain contacts upon increasing the applied force. This phenomenon could be at the origin of the recent observations of nonlinear elastic wave reponse in rock (Johnson and McCall, 1994; McCall, 1994), attributed to structural discontinuities such as microcracks and grain boundaries. In the model proposed here, the structural discontinuities allow for the sensitive formation and/or disappearance of stress path as the amplitude of the applied stress wave varies. This modification of the topology of stress paths could be a major cause of the observed nonlinearity. This large nonlinear response may be responsible for significant spectral alterations of a seismic wave at amplitudes and distances currently considered to be within the linear elastic regime. However, only the upper brittle crust should exhibit such a large nonlinear effect intimately associated to its fragmented block structure. Therefore, only bulk skimming waves and surface waves should be sensitive to this phenomenon as they can propagate long enough in the nonlinear superficial domain. The existence of large nonlinear effects at low wave amplitudes is further confirmed by recent sound propagation measurements on model granular media (Liu and Nagel, 1992, 1994), also resulting from a sensitive alteration of stress paths as the sound amplitude increases.

3.3.6 Multiple scattering of elastic waves

It has been shown that, due to the fragile nature of the contacts between grains and the resulting high sensitivity to the exact position of each of the grains, a thermal expansion of a single grain of about 3000Å can produce a change as large as 25% in the total transmission of the sound in a model granular medium, even though this expansion is four to five orders of magnitude smaller than the wavelength of the sound or the size of the grain (half a centimeter) (Liu, 1994). These phenomena have been rationalized (Feng and Sornette, 1993) using the physical picture that sound waves are multiply scattered at grain-grain contacts and are thus in the diffusive regime, close to Anderson localization. This phenomenon could be involved in the formation of the Coda of seismic signals. Multiple wave scattering (at frequencies above a few hertz in the crust) is well-known to lead to about 100% relative fluctuations in the wave intensity (the so-called "speckle" phenomenon). This phenomenon is even stronger if the block or granular structure favors a strong enhancement of the multiple scattering, called "weak localization", leading eventually to the trapping of the wave by the disorder, the so-called "Anderson localization" phenomenon. One could speculate whether some site effects observed in strong earthquakes could be the result of the local trapping of seismic waves by the Anderson localization phenomenon.

4 Concluding remarks

In this paper, we have proposed a tentative analogy between the mechanics of granular media and of tectonic deformations, on one hand, and the physics of spinglasses and neural networks, on the other hand. This analogy is based essentially on the recognition of the key role of "frustration" and disorder in both fields. The analogy is largely supported at the qualitative level by the large set of experimental observations of "intrinsic" and "sample to sample" fluctuations in laboratory experiments of mechanical deformations of granular media. From our point of view, the suggested analogy possesses the following main interests:

-its connects the mechanics of granular media and of tectonic block deformations to a large class of other problems, thereby allowing the use of reasoning and methods already known in these other fields;

-it gives confidence in the existence of fluctuations as intrinsic to the mechanics of granular media and tectonic blocks deformations. This then demands a deeper reexamination of previous experimental results and suggests many other experiments which can focus to unravel these fluctuations and their significance.

-it may be used to develop "toy" models of granular media and tectonic block structures, in the same spirit as for the spin model for ferromagnetism. "Toy" models are not made to describe all details of the systems, but to study a much simplified model which encapture some of the key feature of reality, and which can be solved or at least are opened to some useful analysis.

The main difference between spinglasses, on one hand, and granular media and tectonic blocks, on the other hand, is the fact that the disorder is given a priori in a spinglass, whereas it can be self-constructed in the case of granular media. However, it has been shown (Golay, 1982; Bernasconi, 1987; Bouchaud and Mezard, 1994) that frustration alone (i.e. in absence of quenched disorder) may be sufficient to create a complex multistate situation very similar to the situation occurring in genuine quenched spinglasses. Thus, the origin of the quenched disorder, be it intrinsic or stemming from a quench of the initially free degrees of freedom in a situation where frustration dominates, does not matter for the existence of the fundamental multi-state nature of the granular media and tectonic blocks at the origin of the "intrinsic" and "sample to sample" fluctuations.

An additional insight in this problem is provided by a recent model of self-organization of faulting and earthquake space-time structures (Cowie et al., 1993; Miltenberger et al., 1993; Sornette et al., 1994). Basically, this is a model of a simple tectonic plate made of blocks interacting by elastic forces and sliding with respect to each other above some threshold. It describes the spontaneous formation of complex multifractal fault patterns organized by the accumulation of earthquakes. We have found in this model that the complexity may emerge both in presence and in absence of quenched randomness (in the block elasticity or sliding thresholds), due to the possible quenching of the initial configuration disorder, thus giving another indication of the plausibility of the proposed analogy.

In the future, we intend to explore further at a more quantitative level the consequences of the analogy which has been proposed here between the mechanics of deformations of a tectonic plate modelled as an ensemble of blocks, the physics of granular media, and that of spinglasses.

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References

Mandl G., Mechanics of tectonic faulting, *Elsevier, Amsterdam*, 1988

Vardourakis I., *Acta Mechanica, 49*, 57-79, 1983.

Hettler A. and Varlourakis I., em Géotechnique,*34*, 183-98, 1984.

Frossard A., *Géotechnique, 29*, 341-50, 1979.

Kolymbas D. and Wu W., *Powder Technology, 60*, 99-109, 1990.

Salençon J., *Europ. J. Mech., A9*,477-499, 1990.

Adamard J., Leçons sur la Propagation des Ondes et les Equations de l'Hydromécanique, *Paris* 1903.

Thomas T. Y., Plastic Flow and Fracture in Solids, *Academic Press*, New York)1961.

Hill R., *J. Mech. Phys. Solids, 10*, 1, 1962.

Varlourakis I., *Acta Mechanica, 32*, 35-54, 1979.

Sulem J. and Varlourakis I., *Acta Mechanica, 83*, 195-212, 1990.

Alpa G. and Gambarotta L., *J. Mech. Phys. Solids, 38*, 491-503, 1990.

Allersma H.G.B., Optical Analysis of Stress and Strain in Photoelastic particle Assemblies, *Thesis*, Delft University of Technology, The Netherlands, Delft, 1987.

Konrad J.M., Flavigny E. and Meghachou M., *Revue Française de Géotechnique, 54*, 53-63, 1991.

GRECO-Géomatériaux, *Passage micro-macro, AUSSOIS, France*, Nov. 1991.

Rowe P.W., Géotechnique, *19*, 75-86, 1969.

Tran Vo Nhiem, *Bulletin de liaison des Laboratoires des Ponts et Chaussées (special issue 'Comportement des sols avant la rupture')*, 127-137, June 1972.

Tran Vo Nhiem and Giroud J.-P., *Bulletin de liaison des Laboratoires des Ponts et Chaussées, (special issue 'Comportement des sols avant la rupture')*, 138-145, June 1972.

Bagnold R.A., *Proc. Roy. Soc. Lond. Ser., A225*, 49, 1954, and *ibid, A295*, 219, 1966.

Blight G.E., *Géotechnique, 36*, 33-46, 1986.

de Coulomb C., Mémoires de Mathématiques et de Physique présentés à l'Académie Royale des Sciences par Divers savants et lues dans les Assemblées, *L'Imprimerie Royale, Paris*, 343 1773.

Schofield A.N. and Wroth P.C., Critical State of Soil Mechanics, *Mac Graw Hill, London*, 1968.

Collins I.F., *J. Mech. Phys. Solid, 38*, 1-25, 1990.

Rowe P.W., *Proc. Roy. Soc. Lond., A269*, 500-527, 1962.

Mandel J., *J. Mech. Phys. Solids, 14*, 303-308, 1966.

Desrues J., and Mokni M., *X ECSMFE, Florence*, May 1991.

Sornette A., Davy P. and Sornette D., Growth of fractal fault patterns, *Phys.Rev.Lett., 65*, 2266-2269, 1990.

Davy P.,Sornette A. and Sornette D., Some consequences of a proposed fractal nature of continental faulting, *Nature, 348*, 56-58, 1990.

Sornette A., *PhD thesis, Orsay University*, 1990.

Davy P., Sornette A. and Sornette D., Experimental discovery of scaling laws relating fractal dimensions and the length distribution exponent of fault systems, *Geophys.Res.Lett., 19*, 361-364, 1992.

Sornette A., Davy P. and Sornette D., Fault growth in brittle-ductile experiments and the mechanics of continental collisions, *J.Geophys.Res., 98*, 12111-12139, 1993.

Lockner D.A., J.D. Byerlee, V. Kuksenko, A. Ponomarev and A. Sidorin, Quasi-static fault growth and shear fracture energy in granite, *Nature, 350*, 39-42, 1991.

Evesque P. and Rajchenbach J., *C.R. Acad. Sc. (Paris), 307, Série II*, 223, 1988.

Jaeger H.M., Liu C.-H., Nagel S., *Phys. Rev. Lett., 62, 40*, 1989.

Held G.A., Solina D.H., Keane D.T., Haag W.J., Horn P.M. and Grinstein G., *Phys. Rev. Lett., 65, 1120*, 1990.

Evesque P., *Phys. Rev., A43*, 2720-2740,1991.

Dantu P., *Géotechnique, 18*, 50-55, 1968.

Dantu P.,*Annales des Ponts et Chaussées, IV*, 1-10, 1967.

Au coeur des Milieux Granulaires, La Photoélasticité, *L.C.P.C. video tape, L.C.P.C., 58 bd Lebvre, 75732 PARIS Cédex 15, France.*

Dantu P., *Proceeding of the 4th International Conference on Soil Mechanics and Foundations Engineering, London*, 1957.

Nisida M. ed., Photoelasticity, *Proceeding of the International Symposium on Photoelasticity, Tokyo 1986, Springer-Verlag, Tokyo*, 1986.

Cambou B., *Thèse de Doctorat, Université Claude Bernard, Lyon*, 1979.

Gourvès R. and Mezghani F., *Revue Française de Géotechnique, 42*, 23-34, 1988.

Delyon F., Dufresne D. and Lévy Y.E., Physique et génie civile: deux illustrations simples, *Annales des Ponts et Chaussées, 1er-2ème trimestre*, 22-29, 1990.

Binder K. and Young A.P., *Rev. Mod. Phys., 58*, 801-976,1986.

Mézard M., Parisi G. and Virasoro M.A., Spinglass Theory and Beyond, *World Scientist Lecture Notes in Physics, Vol. 9*, 1987.

Maddox J., *Nature, 368*, 493, 1994.

Toulouse G., Theory of the frustration effect in spinglasses, *Commun.Phys.*, 2, 115-119, 1977.

Souletie J., Vannimenus J. and Stora R. eds., Chance and Matter, *Les Houches 1986 Session XLVI, North Holland, Amsterdam*, 1987.

Becker S. and Hinton G.E., Self-organizing neural networks that discovers surfaces in random-dot stereograms, *Nature, 355*, 161-163, 1992.

Bray A.J. and Moore M.A., *Phys. Rev. Lett., 58*, 57-60, 1987.

Zhang Y-C, *Phys. Rev.Lett., 59*, 2125-2128, 1987.

Golay M.J.E., *IEEE, IT-23*, 43, 1977; *IEEE, IT-28*, 543, 1982.

Bernasconi J., *J. Physique France, 48*, 559, 1987.

Bouchaud J.-P. and Mézard M., *Self-induced quenched disorder: a model for the glass transition*, preprint april 1994.

Cowie P., Vanneste C. and Sornette D., Statistical physics model for the spatio-temporal evolution of faults , *J.Geophys.Res., 98*, 21809-21821, 1993;

Miltenberger P., Sornette D. and Vanneste C., Fault self-organization as optimal random paths selected by critical spatio-temporal dynamics of earthquakes, *Phys.Rev.Lett., 71*, 3604-3607, 1993.

Sornette D., Miltenberger P. and Vanneste C., Statistical physics of fault patterns self-organized by repeated earthquakes, *Pageoph (in press), (special volume on Rock Friction, Faulting and Earthquake Mechanics, edited by C.J. Marone and M.L. Blanpied*, 1994.

Bouchaud J.-P., Vincent E. and Hammann J., Towards an experimental determination of the number of metastable states in spinglasses, *J.Phys.I France*, *4*, 139-146, 1994.

Gupta H.K. and Rastogi B.K., Dams and earthquakes, *Elsevier, Amsterdam*, 1976.

Gupta H.K., The present status of reservoir induced seismicity: investigations with a special emphasis on Kyona earthquakes, *Tectonophysics*, *118*, 257-279, 1985.

Guyoton F., Grasso J.-R. and Volant P., Interrelation between induced seismic instabilities and complex geological structures, *Geophys. Res. Lett.*, *19*, 7, 1992.

Travers T. et al., *J.Phys.A*, *19*, L1033, 1986.

Roux S., Effect of disorder on the elastic behavior of piling, *in Physics of Granular Media, Les Houches Series, Nova Science Publishers, New York*, 1991.

Johnson P.A. and McCall K.R., Observation and implications of nonlinear elastic wave response in rock, *Geophys.Res.Lett.*, *21*, 165-168, 1994.

McCall K.R., Theoretical study of nonlinear wave propagation, *J.Geophys.Res.*, *99*, 2591-2600, 1994.

Liu C.-H. and Nagel S.R., Sound in sand, *Phys.Rev.Lett.*, *68*, 2301-2304, 1992.

Liu C.-H. and Nagel S.R., Sound in granular material: disorder and nonlinearity, *in press*, 1994.

Liu C.-H., Spatial patterns of sound propagation in sand, *in press*, 1994.

Feng S. and Sornette D., Can sound be localized in granular media?, *Phys.Lett.*, *A184*, 127, 1993.