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# General calibration methodology for a combined Horton-SCS infiltration scheme in flash flood modeling

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**Abstract.** Flood forecasting undergoes a constant evolution, becoming more and more demanding about the models used for hydrologic simulations. The advantages of developing distributed or semi-distributed models have currently been made clear. Now the importance of using continuous distributed modeling emerges. A proper schematization of the infiltration process is vital to these types of models. Many popular infiltration schemes, reliable and easy to implement, are too simplistic for the development of continuous hydrologic models. On the other hand, the unavailability of detailed and descriptive information on soil properties often limits the implementation of complete infiltration schemes. In this work, a combination between the Soil Conservation Service Curve Number method (SCS-CN) and a method derived from Horton equation is proposed in order to overcome the inherent limits of the two schemes. The SCS-CN method is easily applicable on large areas, but has structural limitations. The Horton-like methods present parameters that, though measurable to a point, are difficult to achieve a reliable estimate at catchment scale. The objective of this work is to overcome these limits by proposing a calibration procedure which maintains the large applicability of the SCS-CN method as well as the continuous description of the infiltration process given by the Horton's equation suitably modified. The estimation of the parameters of the *modified* Horton method is carried out using a formal analogy with the SCS-CN method under specific conditions. Some applications, at catchment scale within a distributed model, are presented.

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## 1 Introduction

When dealing with flash flood forecasting, there is always the need to combine different requirements that, in many cases, force tradeoffs on the completeness of the processes schematization used in models. A model that is to be implemented in a flash flood forecasting chain is required to be reliable and portable, because, due to their limited sizes, the majority of the catchments affected by this particular hazard are un-gauged. Catchments affected by this particular hazard are ungauged, so that a large ensemble of model runs is usually needed to deliver a usable forecast product (Siccardi et al., 2005; Franz et al., 2005). From this, the need for simple schematization has led to the diffuse utilization of hydrological models working at event scale, using simplified schemes for rainfall abstraction. Although event models have been successfully employed in this field, nowadays their value, which is based on their simplicity and robustness, is more and more confined to off-line applications such as risk assessment, hydraulic design, etc. Real-time forecasting problems now call for continuous, spatially distributed models, because of their ability in estimating initial conditions (i.e., soil moisture distribution) in combination with distributed forcing (e.g., rainfall, temperature, radiation) provided by real-time measurements (e.g. meteorological radars, satellites) and meteorological models (Reed and Zhang, 2007). The common belief for flash floods is that the the influence on peak discharge of initial moisture conditions is poor, flaws when the performance of a forecasting chain is evaluated during a long period and not on the basis of a few case studies that produced extreme discharge values. The added value of a forecasting chain resides in its ability to dealing with ambiguous cases, when false or missed alarms can occur. In these conditions, a good representation of the initial state, together with a proper infiltration scheme, can substantially improve the forecasting chain skill, enabling the model to discriminate events that produce different runoff volumes

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starting from similar volumes and intensities of rainfall.

The need to develop continuous and distributed modeling frameworks has generated many infiltration models able to run continuously in time starting from schematization originally intended for event modeling. Many of these efforts considers the modifications of the SCS-CN schematization (e.g., Mishra and Singh, 2003), which is one of the most widely used because of its applicability and reliability (e.g., Ponce and Hawkins, 1996). This method was developed within the empirical framework, giving it world-wide credibility, and asks for a deep understanding of the mathematical consistency of the formula when a modification or addition is proposed. Recently, it has been proven that it is impossible to introduce a continuous schematization of the soil moisture accounting without rewriting the method consistently (e.g., Michel et al., 2005). In doing this, it is the opinion of the authors that one of the major added values of the method, which resides in the tremendous amount of experimental work done since it was developed, tends to be lost. It is hardly acceptable to modify the structure of an empirically driven method without going through an extensive calibration which takes into account the proposed modifications. It would then be more straight forward to use an infiltration model with less empirical implications, conceptually suitable to run in a continuous framework.

Many choices are possible and several applications are present in the literature that could be of help in such a choice. Given the operational focus of this work, a suitable choice is a method which is not over-parameterized and does not require the acquisition of soil or land use information with details normally not available outside experimental catchments. Simple schematization of this type, based on physically interpretable parameters, start from the Horton equation (e.g., Bauer, 1974; Aron, 1990; Diskin and Nazimov, 1994). The limit of these models is usually the difficulty of setting parameter values based on physical characteristics of the catchment which are representative for basin scale simulations. This often forces the hydrologist to consider them as only parameters to be calibrated on available rainfall/discharge time series, thus limiting the portability of the model itself.

The aim of this paper is to formalize a simple conceptual schematization based on a modification of the Horton's infiltration equation and propose a calibration methodology of its parameters on the basis of some formal analogies with the SCS-CN method. The final result is a general relation between SCS-CN and *modified* parameters of Horton's methods that increases the portability of the latter exploiting the proved applicability and reliability of the SCS-CN method.

The proposed methodology leads to an infiltration method which behaves as the SCS-CN method in the range of events where the latter ought to perform at its best with no additional calibration, and requires little calibration for events whose intensity and duration is out of this range.

The calibration is performed by implementing the two infiltration schemes in the same semi-distributed hydrological model (Giannoni et al., 2000, 2003, 2005). The capability of the calibrated *modified* Horton's infiltration model to correctly represent the rainfall-runoff separation and the portability of the results, are discussed, presenting some case studies.

## 2 The Soil Conservation Service Curve Number (SCS-CN) method and its limitations

The SCS-CN method was developed in the 1954 and documented in the United States Department of Agriculture (1954). Since then, this method has been widely used and modified (McCuen, 1982; Mishra and Singh, 2003). The method is simple, useful for ungauged watersheds, and accounts for most runoff producing watershed characteristics as soil type, land use, surface condition, and antecedent moisture condition. It is founded on the hypothesis that the ratio between potential runoff volume ($P-I_a$), where $I_a$ is the initial abstraction, and runoff volume $R$ equals the ratio between potential maximum retention $S$ and infiltration volume $F$:

$$\frac{(P-I_a)}{R}=\frac{S}{F} \tag{1}$$

The combination between this hypothesis and a simplified mass balance equation gives:

$$R=\frac{(P-I_a)^2}{R+S-I_a} \quad \text{for} \quad P>Ia \tag{2}$$

The potential maximum retention of the soil is determined by selecting a Curve Number parameter (CN):

$$S=25.4\left(\frac{1000}{\mathrm{CN}}-10\right)\ [\mathrm{mm}] \tag{3}$$

The Curve Number is a function of the soils type, land cover and the so called *Antecedent Moisture Condition* (AMC). Three levels of AMC are considered: AMC-I dry soil (but not to the wilting point), AMC-II average case and AMC-III saturated soil. The initial abstraction term $I_a$ combines the short term losses – surface storage, interception and infiltration prior to runoff – and it is linearly related to the potential maximum retention $S$. The results of empirical studies give $I_a$=0.2$S$. Hydrologists have shown that the SCS-CN formulation is nominally consistent with both infiltration-excess (Hjelmfelt, 1980a,b) and saturation-excess or Varied Source Area – VSA – hydrology (Steenhuis et al., 1995). The traditional SCS-CN method is most commonly used after the assumption that infiltration excess is the primary runoff mechanism. The strength of the method results from an exhaustive field investigation carried out in several USA watersheds and it is empirically corrected to match the outputs of the watersheds used for this study. The SCS-CN method is, in fact, empirically calibrated in order to satisfy, for the calibration events, at catchment scale, Eq. (1) (United States Department of Agriculture, 1954).

The impressive sample size used in the calibration gives reliability and, as proved by different authors (Singh et al., 2002), good method application when used to evaluate the cumulative runoff on durations comparable to the ones used for its calibration, (i.e., 24 h, in most cases the curve number was developed using daily rainfall-runoff records corresponding to the maximal annual flow derived from gauged watersheds (United States Department of Agriculture, 1954; Mishra and Singh, 2003). Recent applications have also shown the possibility of using the SCS-CN method in a distributed way and on time scales finer than the event scale, so that each cell of a distributed model is described by a CN value and the runoff/infiltration separation is performed at cell scale over a time step (Grove et al., 1998; Moglen, 2000). This is not surprising, due to the general scalability of the continuity equation which is at the base of the method, although a careful recalibration would be needed (Grove et al., 1998; Michel et al., 2005). The SCS-CN method does not consider the infiltration to lower soil layers, therefore, setting the soil residual infiltration capacity to zero. Generally, this assumption is physically unacceptable, but holds when daily or shorter rainfall annual maximum precipitation events are studied and and limited sizes of catchments are of a concern. From this, comes the necessity of re-initializing the moisture conditions of the watershed for long rainfall events. The revision of the antecedent moisture condition varies considerably with the morphology and climatology of the site where the hydrologic model is applied. For an example, in Northwestern Apennines the antecedent moisture conditions should be revised mostly after about 36–40 h, this comes from the decade-long experience of the authors in applying this method operationally in in the area. So the SCS-CN method, in its classical configuration, is neither suitable for continuous simulations nor to represent long events with complex rainfall temporal distribution. Another intrinsic limitation of the method is that it refers to cumulated rainfall and not to instantaneous intensity inputs. This, for equal volumes, leads to a constant runoff coefficient independent from the temporal distribution of rainfall. This characteristic, though the method has been used successfully to reproduce flood events, fails theoretically in capturing complex temporal distribution of runoff when infiltration-excess is the dominant runoff-producing mechanism. With the increasing knowledge of rainfall spatial and temporal distribution provided by modern sensors, this limitation becomes more and more evident.

Initial soil moisture conditions are crucial in determining the performance of a hydrologic model in an operational flood-forecasting chain. In the SCS-CN method the AMC condition is discretized by only three different values. On one hand, this simplicity enhances the applicability of the method, but, on the other hand, it does not allow for an accurate representation of the catchment initial state, hampering the performance of the model. All these issues limit the applicability of the SCS-CN method for operational flood forecasting and leads us to move towards another scheme for the infiltration process simulation.

## 3 The *modified* Horton's equation

Horton (1933) proposed an exponential decay equation to describe the variation in time of the infiltration capacity of the soil during a rainfall event as:

$$f(t-t_0) = f_1 + (f_0 - f_1)e^{-k(t-t_0)} \tag{4}$$

$f(t-t_0)$=infiltration rate at time $t-t_0$ from the beginning of the rainfall event $[LT^{-1}]$,

$f_0$=initial infiltration rate at $t_0$ $[LT^{-1}]$,

$f_1$=infiltration rate for $(t-t_0)\rightarrow\infty$ $[LT^{-1}]$,

$t_0$=beginning time of the rainfall event $[T]$,

$k$= exponential decay coefficient $[T^{-1}]$.

The main restrictions to the application, at catchment scale, of the Horton's equation in its original form are: the difficulty of taking into account rainfall with intensities lower than $f_0$, the impossibility to describe the effect of (even short) dry periods inside the rainfall event, and the difficulty of obtaining reliable estimates for its parameters, namely $f_0$, $f_1$ and $k$. Several authors suggested modifications of the Horton's equation to estimate infiltration for intermittent storm events including rainfall intensities lower than $f_0$. Bauer (1974) accounted for dry periods through the recovery of soil infiltration capacity by coupling the Horton's equation with a drainage equation into lower soil layers. Following Bauer's formulation, Aron (1990) proposed a modification of the Horton's equation that makes the infiltration rate a function of cumulative antecedent infiltration estimating the water storage capacity of the soil from the potential maximum retention $S$ given by the SCS-CN. A similar approach is proposed in the SWAT model (Arnold et al., 1993). Aron (1990) smoothed the SCS-CN infiltration process by linearly filtering the infiltration increments. He computed the cumulative infiltration $F$ according to SCS-CN and defined the potential infiltration rate, $f$, proportional to the available soil moisture storage capacity, given by $S-F$.

The modification of Horton's equations, proposed here, starts from the original formulation of (Bauer, 1974) and (Diskin and Nazimov, 1994). It accounts for: (i) initial soil moisture conditions linking them to initial infiltration capacity $f_0$; (ii) intermittent and low-intensity rainfall (namely lower than $f_0$). The terms are properly managed through a mass balance equation applied to the "root zone" (*RZ*). *RZ*

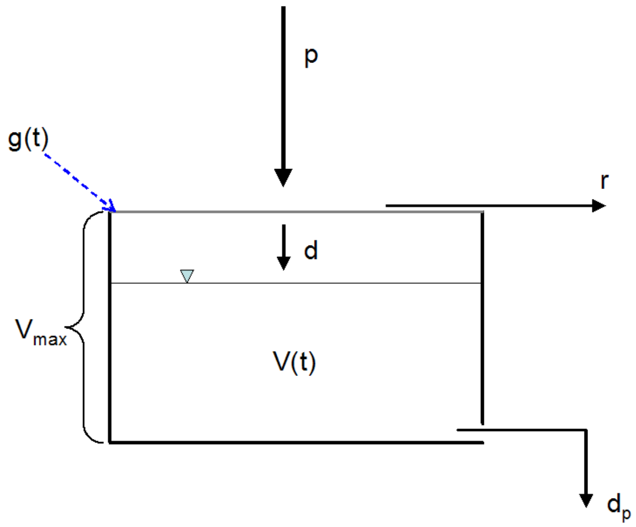


**Fig. 1.** A scheme of the modified Horton method for the parameters calibration; $g(t)$ is the filter, $p$ is the rainfall, $r$ is the runoff, $d$ is the infiltration and $d_p$ is the percolation to lower strata.

soil layer is modelled as a linear reservoir of total capacity $V_{\max}$. The state of the reservoir is quantified by the state variable $V(t-t_0)$ $(0<V(t-t_0)<V_{\max})$. It represents the water volume stored in the reservoir. The input to the system is regulated using a time variant filter $g(t-t_0)$. Percolation to deeper soil layers is introduced as a time variant output $d_p(t-t_0)$. Both $g(t-t_0)$ and $d_p(t-t_0)$ are linearly dependent from $V(t-t_0)$. The filter $g(t-t_0)$ that represents the infiltration capacity at any time is constrained to assume the value $f_0$ for dry soil condition and the value $f_1$ for saturated soil condition (Diskin and Nazimov, 1994).

Figure 1 shows the structure of the model. The initial time step $t_0$ can assume any value. For the sake of simplicity, from now on, we put $t_0$=0. The dynamic mass-balance equation used is then:

$$\frac{dV(t)}{dt} = g(t) - d_p(t) \tag{5}$$

The input and output terms are modeled as follows:

$$g(t) = \begin{cases} \text{if } p(t) \le f_0 - (f_0 - f_1)\,\frac{V(t)}{V_{\max}} : \\ r(t) \\ \text{if } p(t) > f_0 - (f_0 - f_1)\,\frac{V(t)}{V_{\max}} : \\ f_0 - (f_0 - f_1)\,\frac{V(t)}{V_{\max}} \end{cases} \tag{6a}$$

$$d_p(t) = f_1 \frac{V(t)}{V_{\max}} \tag{6b}$$

where $d_p(t)$ is the percolation rate to deep layers $[LT^{-1}]$, $p(t)$ is the rainfall rate $[LT^{-1}]$ and $f_1$ is the percolation rate at $t\to\infty$ $[LT^{-1}]$. The mass balance equation takes then the form:

$$\frac{dV(t)}{dt} = \begin{cases} \text{if } p(t) \le g(t) : \\ p(t) - d_p(t) = p(t) - f_1 \frac{V(t)}{V_{\max}} \\ \text{if } p(t) > g(t) : \\ g(t) - d_p(t) = f_0 \left(1 - \frac{V(t)}{V_{\max}}\right) \end{cases} \tag{7}$$

Equation (7) can be applied to numerical models assuming that $p(t)$ can be considered a constant $p_i$ on time intervals $i\cdot\Delta t{:}(i+1)\cdot\Delta t$, $i\in N$[1]. Integrating it over $\Delta t$ gives:

$$V(t_{i+1}) = \begin{cases} \text{if } p_i \le g_i : \\ \frac{p_i V_{\max}}{f_1} + e^{-\frac{f_1}{V_{\max}}\cdot\Delta t}\left[V(t_i) - \frac{p_i V_{\max}}{f_1}\right] \\ \text{if } p_i > g_i : \\ V_{\max}\left[1 - e^{-\frac{f_0}{V_{\max}}\Delta t}\right] + V(t_i)e^{-\frac{f_0}{V_{\max}}\Delta t} \end{cases} \tag{8}$$

where $p_i$ and $g_i$ are the values of $p(t)$ and $g(t)$ assumed constant inside the time interval $i\cdot\Delta t{:}(i+1)\cdot\Delta t$. The tuning of the 3 model parameters, $f_0$, $V_{\max}$, $f_1$ is described in the next section.

## 4 Combining SCS-CN and *modified* Horton's equation: a methodology for parameters calibration

One of the most appealing characteristics of the SCS-CN method and the reason for its wide application in catchment modeling, is the simplicity in estimating its parameters. Thus, it is not surprising that many authors exploited this potential by linking calibration procedures of other infiltration schemes to SCS-CN parameters (e.g., Risse et al., 1995). This has been done especially for Horton's equation-based methods. The easiest way to link Horton's equation and SCS-CN is to assume analytical equality between their outputs under the same rainfall and soil moisture conditions. (e.g., Bras, 1990; Mishra and Singh, 2003). The assumptions that are needed to equate the two models lead the final formulation of the Horton's equation to fall back in many of the limitations of the SCS-CN method (e.g., Pitt et al., 1999). The equations yield a null percolation rate which is not physically appealing. Moreover, in order to obtain an analytical expression of $f_0$, it is necessary to introduce strong hypotheses which hamper the use of the method in a continuous-like

[1]Usually $\Delta t$ is the step on which rainfall is measured by raingauges or radar (typically ranges from 10 min to 1 h).

framework (see Appendix A for the analytical details). It is necessary to change perspective in order to overcome such limitations.

SCS-CN parameters were calibrated using daily rainfall-runoff records corresponding to maximum annual flows derived from gauged watersheds in the USA for which information on their soils, land cover, and hydrologic conditions were available (United States Department of Agriculture, 1954). Because of the empirical nature of the method, the CN values would need, if not a complete “re-calibration”, at least an adjustment on historical events to match the observed discharge in the catchments which might belong to different morphology contexts. This problem is often overlooked by hydrologists who rely on literature values. Although the authors feel that this is a key problem in the application of the SCS-CN method, it is out of the scope of the present work to discuss it in detail. It is only necessary to notice that the CN values used in the presented test-case have been carefully tested for the study area (Liguria Region, see below) on the basis of the maximum annual flows recorded there, following techniques similar to the original SCS-CN work (Boni et al., 2007). Their work proved that CN values give a reliable description of the hydrological behavior of the soils in the region when the class of events that causes the maximum annual flows is of concern. It would, therefore, be reasonable to force the *modified* Horton's method to give the same results for the same class of events. In order to properly select the events to be used for extensive calibration of the *modified* Horton's model parameters, it is necessary to identify unambiguously the characteristics of the intense events in the region. Specifically the characteristics influencing, at most, the runoff production: the total duration, the range of rainfall intensities spanned and their temporal distribution.

According to the SCS-CN method, only for this calibration step, the final infiltration rate $f_1$ is set to null. This limit is justified by the fact that percolation cumulates on these specific kind of events and short durations is negligible if compared to runoff and infiltration; this hypothesis, not feasible for real catchments, will be relaxed further on in the paper. Following Aron (1990), the maximum soil capacity $V_{\max}$ is set equally to the potential maximum retention $S$ in average conditions (AMC-II):

Figure 2 illustrates the scheme adopted for this step of the calibration. Under the formulated assumptions, $f_0$ is the only parameter of the *modified* Horton method which needs calibration.

Referring to the analysis of historical series and METEOSAT satellite images (Deidda et al., 1999; G.N.D.C.I., 1992; Boni et al., 2007) a typical duration of the intense events in Liguria region is about 12 h. As already mentioned, because of the extensive calibration on regional statistics of maximum flows, the SCS-CN method gives good results for such durations. Such limited temporal extension justifies also the assumption of negligible cumulate losses and, therefore, of null residual infiltration rate. Therefore, the total duration

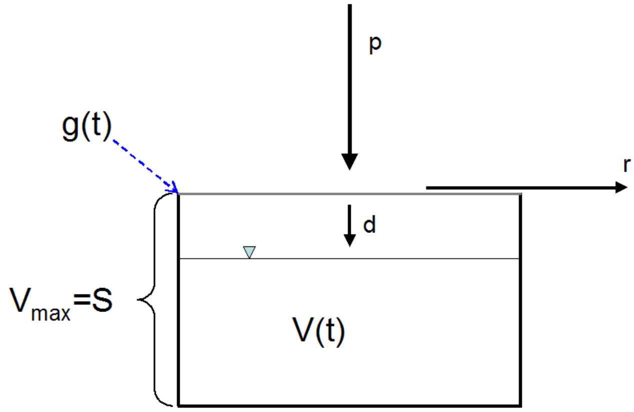


**Fig. 2.** Scheme of the modified Horton method for the parameters calibration; $g(t)$ is the filter, $p$ is the rainfall, $r$ is the runoff, $d$ is the infiltration.

of the design events $N$ has been set equal to 12 h, and the cumulated rainfall $P$ have been fixed by the annual maximum values recorded from 125 stations in Liguria Region from 1951 to 2001.

The fact that the CN method gives identical results in terms of final cumulated runoff regardless of the hyetograph shape leads to an ill-posed mathematical problem. Further assumptions are therefore needed in order to define a proper distribution of the rainfall intensities $I_{nt}$. Several different shapes are possible (linear increasing, Chicago Hyetograph, etc.) which give slightly different values of $f_0$. In order to avoid an arbitrary choice, we have searched for the more appropriate event's shape by comparing the runoff produced by the two methods not only on the total duration $N$ of the event, but also on smaller durations. The shape of the hyetograph that optimizes such a comparison has been sought by minimizing the following objective function:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(r_{H_i} - r_{\mathrm{CN}_i})^2}{N}} \tag{9}$$

where $N$ is the duration of the event in hours, $r_{H_i}$ and $r_{\mathrm{CN}_i}$ are the runoffs obtained by the *modified* Horton method and the SCS-CN method, respectively[2]. The procedure has been repeated for each CN. The unknowns of the minimization problem are: the duration $N$, the intensity $I_{nt}$ of the $N-1$ showers, the cumulated rainfall $P$ and the initial infiltration

[2]RMSE has been adopted to quantify the error in terms of the unit of the variable. It is know that, like variance, mean squared error has the disadvantage of weighting outliers. This is a result of the squaring of each term, which effectively weights large errors more heavily than small ones. However using mean absolute errors (MAE) leads to similar results indicating that significant outliers are not generated within the comparison.

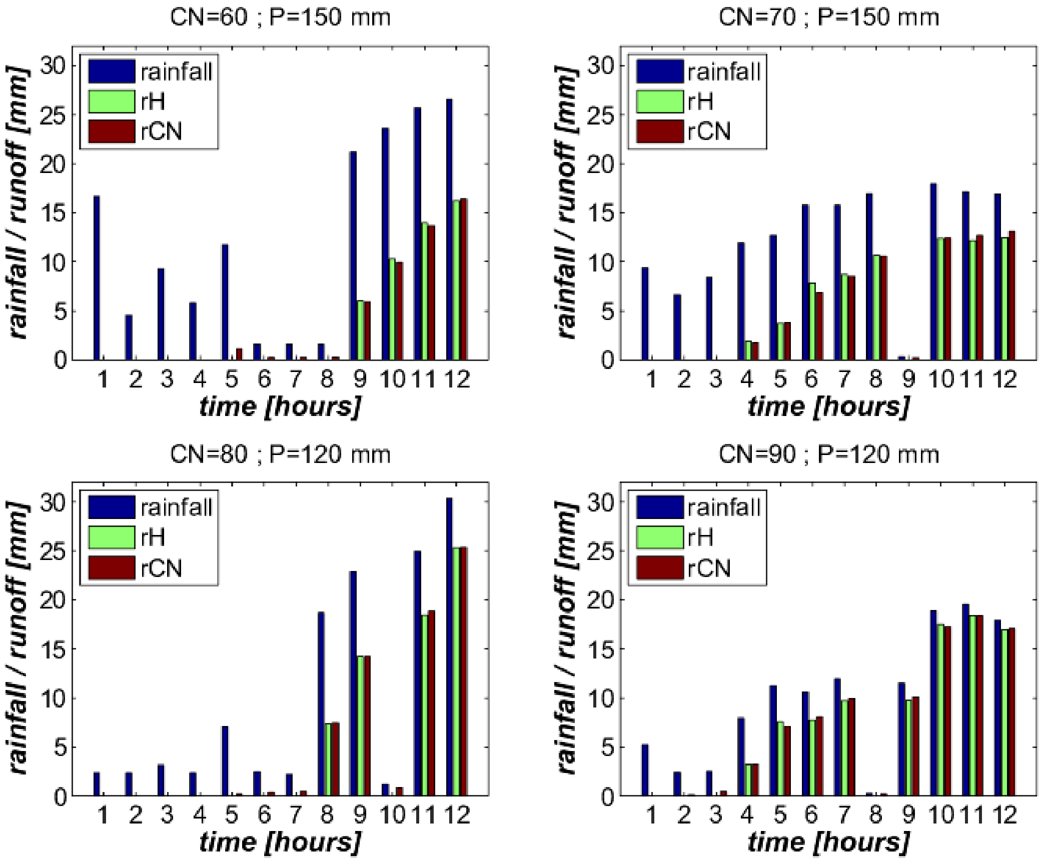


**Fig. 3.** Examples of hyetographs obtained by searching rainfall events that minimize the differences between the runoff obtained by the *modified* Horton method and the SCS-CN method. $V_{max}$ is set equal to S(AMC-II).

rate $f_0$. In total, there are $N+2$ unknowns. The system of equations is constituted by the $N+2$ derivatives of Eq. (9). However, we have reduced the unknowns by fixing the values of cumulated rainfall $P$ and of the total duration of the event $N$.

The variables of function 9 are hence the $N-1$ intensities of the showers, $I_{nt}$, and the initial infiltration rate, $f_0$. The minimum research procedure is based on the Conjugate Gradient Method and starts from a random distribution of hourly showers. The procedure is repeated starting from different initial distributions of the single showers. By looking at the hyetographs obtained by the minimization of the Eq. (9) (see Fig. 3) one notices in the first part of the event the values of hourly showers are apparently random because the cumulated rainfall is less than the initial abstraction $I_a$ in the SCS-CN method, and the runoff is null. When the cumulated rainfall reaches $I_a$ the runoff production begins and the showers increase with a rather linear trend. The shape of the rainfall event that minimizes the runoff differences between the two methods has been identified as a linearly increasing triangular event (Fig. 4).

Once the cumulated rainfall exceeds $I_a$, the SCS-CN method always produces runoff, while the runoff produced by the *modified* Horton method is a function of the rainfall intensity (if $p(t)<g(t)$ $r(t)=0$). The shape obtained by the described procedure ensures the greater runoff values (in each single step) in the SCS-CN method and the contemporary presence of runoff in the *modified* Horton method.

The linearly increasing triangular event has been used to create a conversion table from CN values to $f_0$ values for the region, as the average on all the minimizations over the entire set of values of $P$ described above. The results are summarized in Table 1 and in Fig. 5.

The variability of $f_0$ values, see Table 1 and Fig. 5, is significant for low CN. It can be explained considering that for low values of CN the quantity $I_a$ is comparable with the total cumulated rainfall values $P$, this leads, in the proposed numerical procedure, to a great variation of potential runoff $P-I_a$. Moreover, the SCS-CN method, as stated above, always produces runoff when the cumulated rainfall exceeds the values $I_a$, regardless of the intensity of rainfall (not physical but reasonable, considering that the method has

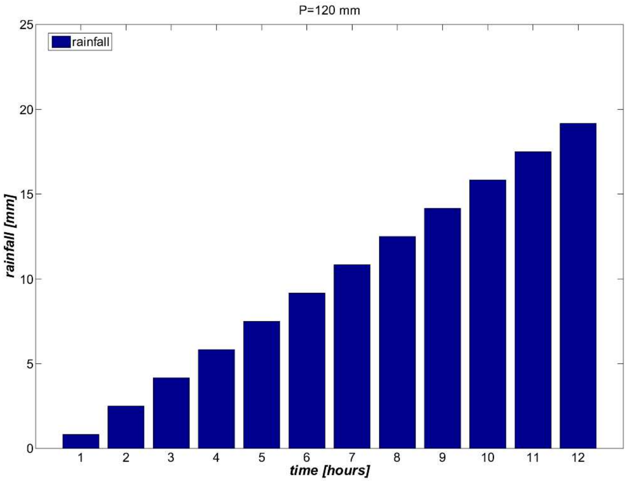


**Fig. 4.** An example of hyetograph used in the assessment of the initial infiltration capacity $f_0$.

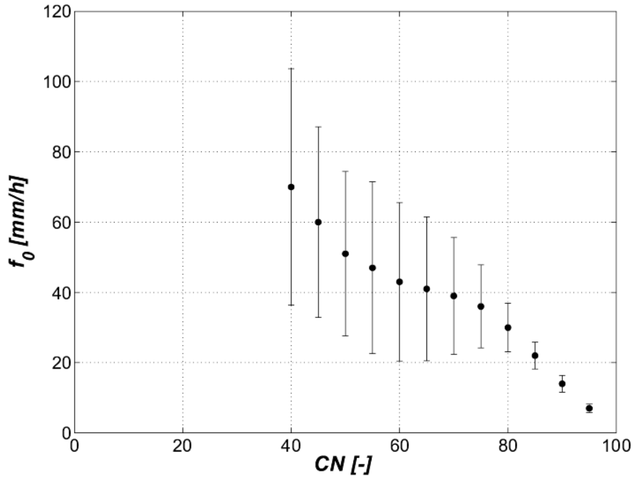


**Fig. 5.** Trend of the initial infiltration rate $f_0$ as a function of the CN(AMC-II). Standard deviation error bars are plotted.

been originally conceived for catchment scale application). The *modified* Horton method, during the calibration process, adapts the parameter $f_0$ trying to reproduce the effects on the runoff estimation caused by the described issues.

With this calibration procedure, from the information of CN maps, the spatial distribution of *modified* Horton method's parameters have been obtained.

**Table 1.** Parameters $f_0$ and $V_{\max}$ of the *modified* Horton method as resulting from the calibration; $c_v$ is the coefficient of variation of $f_0$ and $\sigma_{f_0}$ is the standard deviation of $f_0$.

| $CN_{AMC-II}$ | $f_0$ [mm/h] | $\sigma_{f_0}$ [mm/h] | $cv$ | $V_{\max}$ [mm] |
|---|---|---|---|---|
| 40 | 70 | 33.6 | 0.38 | 375 |
| 45 | 60 | 27.1 | 0.36 | 306 |
| 50 | 51 | 23.4 | 0.38 | 250 |
| 55 | 47 | 24.4 | 0.39 | 205 |
| 60 | 43 | 22.5 | 0.38 | 167 |
| 65 | 41 | 20.5 | 0.36 | 135 |
| 70 | 39 | 16.6 | 0.30 | 107 |
| 75 | 36 | 11.8 | 0.21 | 83 |
| 80 | 30 | 6.9 | 0.12 | 63 |
| 85 | 22 | 3.8 | 0.08 | 44 |
| 90 | 14 | 2.3 | 0.08 | 28 |
| 95 | 7 | 1.2 | 0.07 | 13 |

## 5 Catchment applications

In order to evaluate the *modified* Horton method behavior in hydrological simulations, this infiltration scheme has been implemented in a semi-distributed rainfall-runoff model, DRiFt – Discharge River Forecast – (Giannoni et al., 2005). Several "multi-peaks" events, for which the SCS-CN method usually failed, have been considered. DRiFt with the *modified* Horton infiltration scheme uses the distribution of the soil capacity $V_{\max}$ and the initial infiltration $f_0$ maps as previously determined from the Curve Numbers for the different catchments (see Table 1). The final infiltration rate $f_1$ has been re-introduced using the scheme of Fig. 1 as a percentage of the infiltration rate for dry soil $f_0$:

$$f_1 = c_f f_0 \tag{10}$$

where $c_f$ is a calibration parameter that belongs to interval [0,1] and represents a physically meaningful simplification. In this way, the spatial distribution if $f_1$ is fixed by the distribution of $f_0$ and the parameter $c_f$ is a calibration parameter at catchment scale. Just one event per each catchment (not shown) has been used to provide calibration for $c_f$, chosen in a range where the residual infiltration capacity counts the most. The initial moisture conditions have been consistently chosen with the cumulated rainfall of the five days antecedent the event for both *modified* Horton and SCS-CN methods. Figures 6, 7, 8 and 9 show some validation events that compare the performances of both methods with the observed hydrographs. In Table 2, the values of Nash-Sutcliffe coefficient (NS), root mean square error (RMSE) and mean absolute error (MAE) are reported.

Figure 6 reports the observed hydrograph for the 6–13 November 1997 event at the Bisagno creek closed at Gavette (89 km$^2$) and the simulations obtained with the *modified* Horton method and with the SCS-CN method. When using the SCS-CN method it is not possible to simulate the whole event but it is necessary to break it into three parts and

**Table 2.** Nash-Sutcliffe coefficient (NS), root mean square error (RMSE) and mean absolute error (MAE) for the considered events in the case of use of the *modified* Horton method and in the case of SCS-CN method.

| Date | Basin | Horton Mod. | | | SCS method | | |
|---|---|---|---|---|---|---|---|
| | | NS | RMSE | MAE | NS | RMSE | MAE |
| 6 Nov 1997 | Bisagno | 0.87 | 11.8 | 6.8 | 0.55 | 22.0 | 10.9 |
| 25 Nov 2002 | Bisagno | 0.71 | 29.7 | 7.6 | 0.48 | 34.4 | 10.8 |
| 2 Aug 1965 | Magra | 0.90 | 77.5 | 37.5 | 0.96 | 80.6 | 40.9 |
| 2 Dec 1966 | Magra | 0.91 | 115.8 | 20.2 | 0.85 | 147.8 | 26.0 |

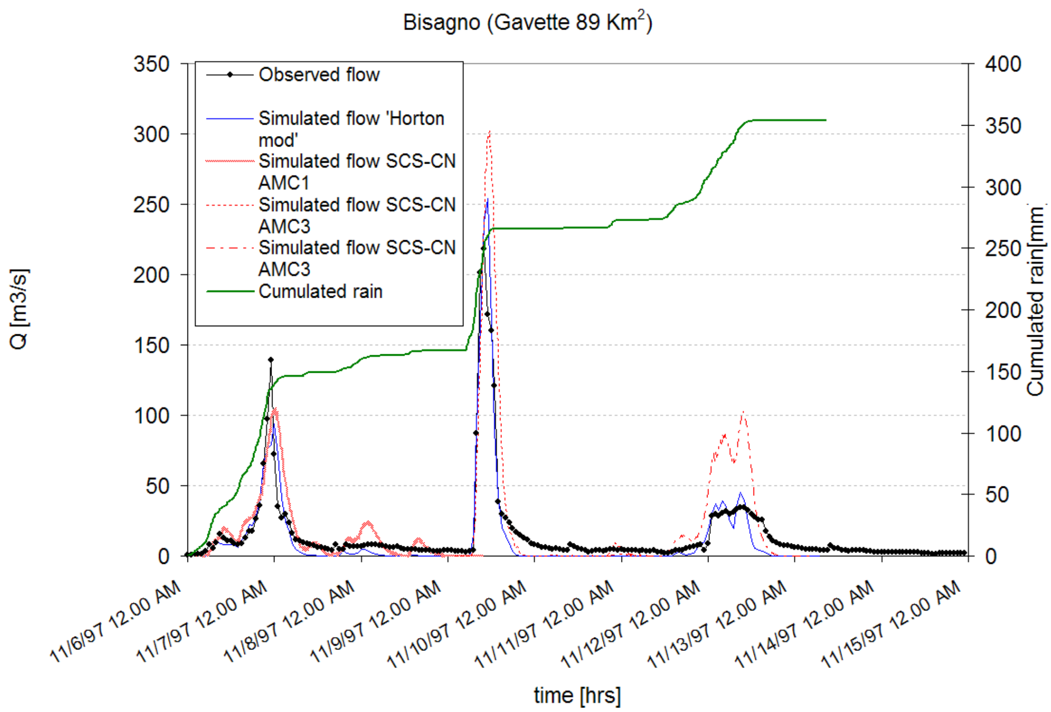


**Fig. 6.** 6–13 November 1997 event, Bisagno river closed at Gavette (89 km$^2$). Comparison between the simulations performed with the DRiFt model using the *modified* Horton method and the SCS-CN method. The event has been broken into three parts for simulation using SCS-CN method.

re-initialize the model[3], while the *modified* Horton method allows for a continuous simulation of the event, improving the performance in reproducing the last peak. Figures 7, 8 and 9 illustrate some events in which the performances of the two model are comparable.

[3]The antecedent moisture conditions for the SCS-CN method are coherently given with the rainfall observed in the five days preceding the event.

## 6 Conclusions

A procedure that maps the CN values onto the initial infiltration rate $f_0$ of a *modified* Horton method is proposed. The procedure allows for an easy calibration of the main parameter of the *modified* Horton method at catchment scale exploiting the advantages offered by the SCS-CN method. The two methods are bounded to give the same performance in terms of cumulated run-off in a specific range of rainfall intensities and durations where the hypothesis of the SCS-CN method do not represent a limitation. A further calibration step is due for $f_1$ on events where the residual infiltration rate is important. The $f_1$ spatial distribution simply shifts the

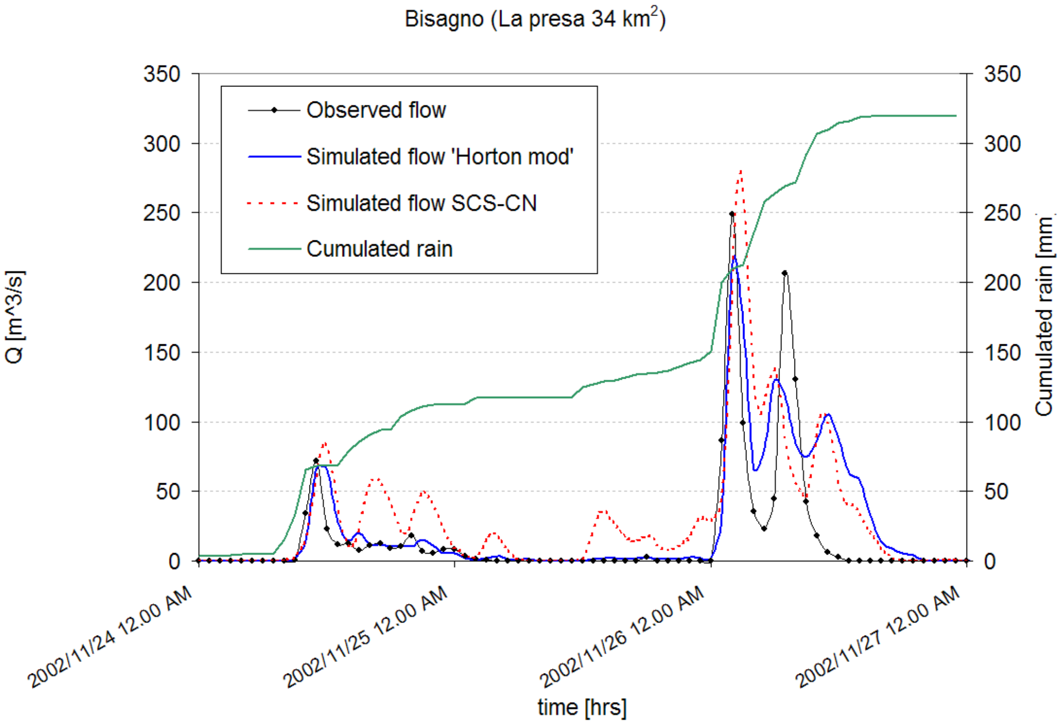


**Fig. 7.** 25–27 November 2002 event, Bisagno river closed at La Presa (34 km$^2$). Comparison between the simulations performed with the DRiFt model using the *modified* Horton method and the SCS-CN method.

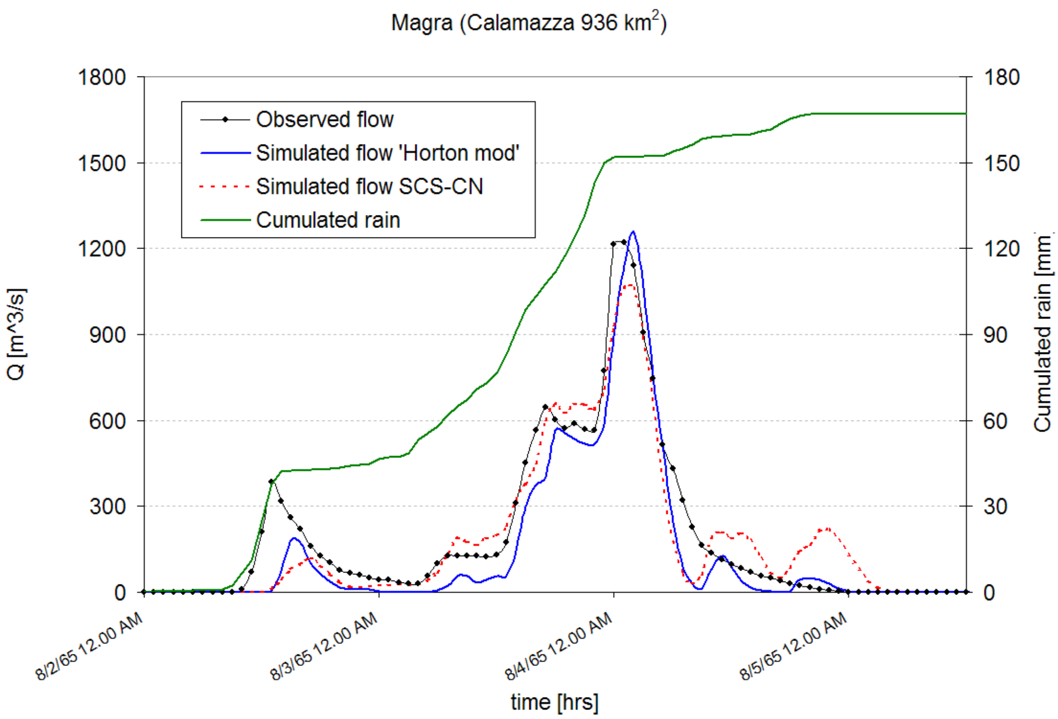


**Fig. 8.** 2–5 August 1965 event, Magra river closed at Calamazza (936 km$^2$). Comparison between the simulations performed with the DRiFt model using the *modified* Horton method and the SCS-CN method.

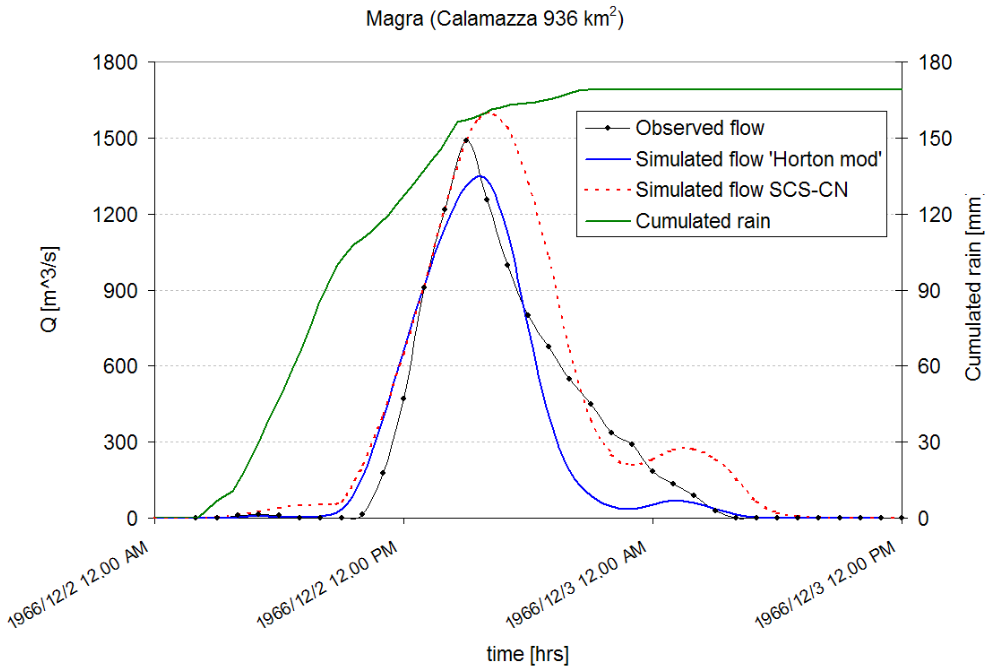


**Fig. 9.** 2–3 December 1966 event, Magra river closed at Calamazza (936 km$^2$). Comparison between the simulations performed with the DRiFt model using the *modified* Horton method and the SCS-CN method.

$f_0$ spatial distribution at catchment scale. The results within a simple distributed hydrological model show how the calibrated Horton method outperforms the SCS-CN method on long, multi-peak events. The structure of *modified* Horton method is suitable for implementation in continuous models without schematization straining and, within the proposed framework, maintains simplicity in the calibration phase of its main parameters.

## Appendix A

### Analytic comparison

The analytical derivation follows.

Infiltration by SCS-CN is given by:

$$F=\frac{(P-I_a)}{(P+S-I_a)}S \tag{A1}$$

Cumulated infiltration by *modified* Horton is given by equations 8. To carry out the analytical comparison we have to make some significant hypotheses due to the different formalization of the two methods.

First of all, the expression of cumulated infiltration $F$ of the SCS-CN method is valid when $P>I_a$ with $I_a$=0.2$V_{\max}$. This portion of rainfall ($I_a$), in the case of *modified* Horton method, is processed by the equations and its analytical expression depends on the temporal distribution of rainfall intensity. It could, or not entirely, infiltrate depending on temporal distribution of $P$.

The same problem affects the initial condition $V(t_{i-1})$, so that it is necessary to make an assumption to integrate the equations from $t_{i-1}$ to $t_i$.

Moreover, we have to bind $\frac{dP}{dt}(t)$ to be always greater than $g(t)$. In this way, an analytical equation of the methods is possible. Equations A1 and A3, assuming $V_{\max}=S$ and starting from $V(t_{i-1})=I_a=0.2V_{\max}$, which assumes $\frac{dP}{dt}(t)<g(t)$ in the period before the cumulates reaches $I_a$, the Eq. A4 is obtained: if $p_i\leq g_i$ :

$$V(t_{i+1})=\frac{p_i V_{\max}}{f_1}+e^{-\frac{f_1}{V_{\max}}\cdot\Delta t}\left[V(t_i)-\frac{p_i V_{\max}}{f_1}\right] \tag{A2}$$

if $p_i>g_i$ :

$$V(t_{i+1})=V_{\max}\left[1-e^{-\frac{f_0}{V_{\max}}\Delta t}\right]+V(t_i)e^{-\frac{f_0}{V_{\max}}\Delta t} \tag{A3}$$

$$V_{\max}\left[1-e^{-\frac{f_0}{V_{\max}}\Delta t}\right]+0.2V_{\max}e^{-\frac{f_0}{V_{\max}}\Delta t}=\frac{(P-0.2V_{\max})}{(P+0.8V_{\max})}V_{\max} \tag{A4}$$

and solving for $f_0$ if $P>0.2V_{\max}$ :

$$f_0=-\frac{V_{\max}}{0.8\Delta t}ln\frac{(V_{\max})}{(P+0.8V_{\max})} \tag{A5}$$

On the other hand, the *modified* Horton method switches between the two equations which describe the evolution of $V(t)$ (Eq. 8) depending on the temporal distribution of $P$, therefore, on the values of the rainfall intensities $\frac{dP}{dt}(t)$. The switch in the equations is difficult to manage analytically and can be tackled numerically as proposed in the paper.

The hypothesis of equating runoff or infiltration respectively operated in the numerical approach and in the mathematical comparison are identical under the assumption $P>0.2V_{\max}$. $\Delta t$ and $P$ in the analytic comparison are the cumulated and the duration on the whole event while in the numerical framework the differences between the two methods are computed on each single step.

As can be easily deduced from Eq. A5, that the value of $f_0$ results are strongly dependent on cumulated rainfall $P$ and on which duration $\triangle t$ it is occurred, independently of its temporal distribution. Knowing this, the Eq. A5 could be used step by step always under the assumption that $\frac{dP}{dt}>g(t)$, but in this case the evolution of $V(t)$ needs to be introduced ($V(t_{i-1})\neq 0$).

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