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Carl Friedrich Gauss – *General Theory of Terrestrial Magnetism* – a revised translation of the German text

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Abstract. This is a translation of the *Allgemeine Theorie des Erdmagnetismus* published by Carl Friedrich Gauss in 1839 in the *Resultate aus den Beobachtungen des Magnetischen Vereins im Jahre 1838*. The current translation is based on an earlier translation by Elizabeth Juliana Sabine published in 1841. This earlier translation has been revised, corrected, and extended. Numerous biographical comments on the scientists named in the original text have been added as well as further information on the observational material used by Carl Friedrich Gauss. An attempt is made to provide a readable text to a wider scientific community, a text laying the foundation of today's understanding of planetary magnetic fields.

Introductory comments

Carl Friedrich Gauss was named *Princeps Mathematicorum*, the prince of mathematics, already during his lifetime. It would have been appropriate to call him *Princeps Magneticorum* as well because of his seminal work on terrestrial magnetism, the *Allgemeine Theorie des Erdmagnetismus*[1], a translation of which is presented here. The work provided all the necessary tools, both experimental and theoretical, to study the Earth's magnetic field in great depth. It should be noted here that the concept of a field was not used by Gauss. When he spoke about what is nowadays called the magnetic field, he meant the phenomenon of magnetism. This is also reflected in the title of his work. The *Theory* is mainly concerned with a mathematical description of the terrestrial magnetic field. It does not, however, provide any answer to the question of what physical process is generating this field in the interior of the Earth. Though the *Theory* is incomplete in this sense, it is still a fascinating study, a masterpiece of the human mind. The *Theory* introduces a spherical harmonic analysis of the terrestrial magnetic field for the first time. And it describes the possibility of separating the field measured at the surface of the Earth into its contributions of internal and external origin. This method, the Gauss separation algorithm, is still in use today when studying the Earth's magnetic field (e.g., Olsen et al., 2010). However, with modern satellite observations of the magnetic field becoming available for the Earth and other planets, the prime condition for the applicability of the Gauss algorithm, the local electric current-free condition, breaks down. Electric currents of ionospheric or magnetospheric origin (e.g., Baumjohann et al., 2010) inhibit the use of a scalar magnetic potential to describe the magnetic field. Especially for the analysis of planetary magnetic fields, new and modified separation techniques are required, and generalizations of Gauss' algorithm are necessary (e.g., Backus, 1986; Pulkkinen et al., 2003; Mayer and Maier, 2006; Glassmeier et al., 2010; Johnson et al., 2012).

Carl Friedrich Gauss was born in the city of Braunschweig on 30 April 1777 as the son of a street butcher and a maidservant. Already in his early years he proved to be an extremely talented mathematician and scientist. He received his doctorate in 1799 in absentia from the former University of Helmstedt, a town located some 30 km east of Braunschweig. His dissertation provides proof of a fundamental theorem of algebra stating that at least one complex root can be found for every non-constant single variable polynomial with complex

[1]Translators' footnote (footnotes by the translators are indicated with a capital T to discriminate them from the original footnotes of Gauss): a digital version of the original book in German is available from Google Books.

coefficients. Other equally important works followed and made Carl Friedrich Gauss the prince of mathematics. Based on the method of least squares, which Gauss already developed in 1795, he calculated the orbit of the asteroid Ceres in November 1801. This allowed Franz Xaver von Zach (1754–1832) to redetect Ceres on 7 December 1801. This correct prediction of Ceres' orbit made Gauss famous in the international astronomical community as well. As a side note, protoplanet Ceres is the target of the NASA Dawn mission and will be encountered in February 2015 (Russell and Raymond, 2011).

Charles William Ferdinand, Duke of Brunswick-Wolfenbüttel (1735–1806), promised to build an astronomical observatory for Gauss. This plan could not be realized as the Duke was mortally wounded in the Battle of Jena and Auerstedt on 14 October 1806. He died on 10 November 1806 in Ottensen, at that time a small village in the Kingdom of Denmark. Carl Friedrich Gauss therefore accepted an offer from the King of Hanover in 1807 to join the famous University of Göttingen, where he became a professor of astronomy and the director of the newly erected astronomical observatory. In Göttingen Gauss became more engaged in questions related to the terrestrial magnetic field. As Gauss stated in a letter to his friend Wilhelm Olbers (1781–1862), he was interested in the terrestrial magnetic field as early as 1803. This interest was greatly stimulated after meeting Baron Alexander von Humboldt (1769–1859) and Wilhelm Weber (1804–1891) in Berlin in 1828. After 1831, his major collaborator was Wilhelm Weber. Inspired by Alexander von Humboldt, Gauss and Weber realized that magnetic field measurements needed to be done simultaneously and globally with standardized instruments. This research program led to the foundation of the Göttinger Magnetischer Verein in 1836, an organization without much formal structure, only devoted to organizing magnetic field measurements throughout the world. The results of this organization have been published in six volumes as the *Resultate aus den Beobachtungen des Magnetischen Vereins*. The issue for 1838 contains the seminal work *Allgemeine Theorie des Erdmagnetismus*, a revised translation of which is presented here. It is in the *General Theory of Terrestrial Magnetism* where Gauss introduced the concept of the spherical harmonic analysis, applied this new tool to magnetic field measurements, and also introduced a method on how to separate the magnetic field measured at the surface of the Earth into its internal and external contributions.

In the introduction of the *Theory*, Gauss wrote a most interesting remark: "But science, though also equally supporting economic interests, should not be restricted to this, but equal emphasis is required for all of its aspects" (Gauss, 1839). This statement very nicely illustrates Gauss' attitude on the dialectic relation between science and economy. Pure science cannot prosper by itself. Economy, on the other hand, cannot do without scientific achievements as current technological evolution demonstrates. Carl Friedrich Gauss was ingenious in handling both aspects. Land surveying needs of the King of Hanover triggered his interest in theoretical geodesy. Considering problems of the widows' pension system of the University of Göttingen made him one of the founders of insurance actuarial mathematics.

The importance of the *Theory* motivated this current revised English translation. The only other translation known to us is that provided by Elizabeth Juliana Sabine, revised by John Herschel[2] (Gauss, 1841). This original translation was published by Richard Taylor in the *Scientific Memoirs Selected from the Transactions of Foreign Academies and Learned Societies and from Foreign Journals* in London in 1841. Our translation starts with Elizabeth Sabine's version. We significantly revised it, corrected mistakes, and added corrections later published by Carl Friedrich Gauss in a supplement in the same issue of the *Resultate*. In some instances we have indicated important misunderstandings of the text by Mrs. Sabine. We have reverted to Gauss' paragraph structure, contrary to the more recent style used by Sabine. In most cases we have shown Gauss' equations in the original format. Furthermore, we have added information on scientists and collaborators mentioned by Gauss in his treatment. To discriminate the original footnotes of Gauss from those we have added, ours are indicated by a capital letter T. And where possible, we give proper references for the numerous observational data used by Carl Friedrich Gauss.

A note on Elizabeth Juliana Sabine is appropriate here. She was born in 1807 as the daughter of William Leeves from Tortington in Sussex. In 1826, at the age of 19, she married the physicist and later President of the Royal Society Edward Sabine[3] (Brück, 2009). She was a linguistically highly talented and intellectually well-grounded individual. The Irish physicist and astronomer William Rowan Hamilton expressed this in his letter to James William Barlow[4] dated 2 September 1848: "I have known Sabine for many years, and his wife Mrs. Sabine is another old friend of mine. She is rather a learned lady, and has translated many foreign, especially German, papers for Taylor's Memoirs, having no children to occupy her otherwise; and I remember that with her husband she attended a course of lectures that I gave at Trinity College Dublin" (Graves, 1885; Brück, 2009). And Heinrich Wilhelm Dove[5], in his memorial speech on Alexander von Humboldt (Dove, 1869), even judges: "The book has

[2]T: John Herschel (1792–1871), English astronomer; detected that the Magellanic cloud consists of individual stars.

[3]T: Edward Sabine (1788–1883), Irish astronomer and one of the leading magneticians of his time; initiated in Britain the *Magnetic Crusade* (Cawood, 1979), discovered the relation between sunspots and geomagnetic field disturbances (Sawyer Hogg, 1948).

[4]T: James William Barlow (1826–1913), reverend at Trinity College in Dublin, was the son of William Barlow and Catherine Barlow-Disney, the love of William Hamilton's life.

[5]T: Heinrich Wilhelm Dove (1803–1879), German physicist and meteorologist. The mentioned book is the *Cosmos* by Alexander von Humboldt.

been translated into 7 languages, the best one being the translation into English by the most versatile scholar I ever met in England, Herschel excluded, the wife of General Sabine."

Elizabeth Juliana Sabine was known personally to both Carl Friedrich Gauss and Alexander von Humboldt. For example, on 11 October 1839 she participated in a breakfast meeting in Humboldt's home in Berlin, jointly with the German astronomer Johann Franz Encke[6], the Irish physicist Humphrey Lloyd[7], and Mrs. Sabine's husband Edward. Three days later the Sabines left Berlin, bound for Göttingen, where they participated in the Little Magnetic Congress organized by Gauss (Biermann et al., 1983). In a letter to Christian Ludwig Gerling[8] dated 30 September 1839, Gauss wrote (Schaefer, 1927): "By the way, Kupffer[9] will come here again mid-October to participate in a kind of Magnetic Congress, to which also Sabine from London, Lloyd from Dublin and Steinheil[10] from Munich will come; actually I should also mention Mrs. Sabine, accompanying her husband, as by her and not by him my General Theory of the Terrestrial Magnetic Field has been translated into English language." This clarification nicely expresses his respect for Mrs. Sabine. Later, in 1848 it was Alexander von Humboldt expressing his respect by sending a Kosmos Medal via the Prussian Ambassador in London to Mrs. Sabine, thereby honoring this woman's truly outstanding contribution to science (Humboldt, 1869). This Kosmos Medal was minted in 1847 by the Prussian Mint to honor Alexander von Humboldt on the occasion of the publication of the second volume of his *Cosmos*, an English translation of which was first provided by Elizabeth Sabine.

Elizabeth Sabine's rendering of Gauss' *Allgemeine Theorie des Erdmagnetismus* into English was the first work that she would tackle as a translator. The brief, parenthetical, mention that she was given immediately below the title of the article – *[Translated by Mrs. Sabine, and revised by Sir John Herschel, Bart.]* – offered her more public recognition than can be found in some of her later translations. The English edition of Humboldt's *Cosmos* (Humboldt, 1849a) does not state directly that this work was translated by Elizabeth Sabine, but rather informs readers that the translation was done *under the superintendence* of Edward Sabine, her husband. Only in an Editor's Preface to the first edition of the English translation do we find a note stating that Elizabeth Juliana Sabine was the translator (Brück, 2009). Mary Brück comments on this in the following way (Brück, 2009): "This was an example of a practice that may have been more widespread than can be discovered, of female family members helping their scientific menfolk anonymously behind the scenes. A glaring example of this transference of attribution from wife to husband is that of the Sabine translations of the works of Alexander von Humboldt."

Elizabeth Sabine's translation of the Berlin physicist Heinrich Wilhelm Dove's *Die Verbreitung der Wärme auf der Oberfläche der Erde* (Dove, 1852), published in 1853 as *The Distribution of Heat over the Surface of the Globe*, was another example of her near-"invisibility" as a translator. Here too, her role in its translation, and, essentially, in the work's international success – Dove was awarded the Royal Society's Copley Medal in 1853 – was downplayed and she received the barest of mention in her husband's preface. Likewise Elizabeth Sabine's translation of extracts from the work of the French mathematician and astronomer Franҫois Arago, published as a compilation of individual pieces in the *Meteorological Essays: With an Introduction by Alexander von Humboldt* (Arago, 1855), gave no acknowledgement of her intellectual and linguistic contribution. The marked exception to all these scientific translations in which Elizabeth Sabine's translatorial voice was apparently "stifled" either by her husband or by her publisher is her English rendering of Humboldt's *Ansichten der Natur*, which appeared with Longman in London in 1849 as *The Aspects of Nature* (Humboldt, 1849b). Here it was not "Lieut. Col. Sabine" who took the limelight as the editor/translator. Rather, the work was formally presented as "Translated by Mrs. Sabine", with a note by the translator, in this instance giving her the prominence she deserved. The edition published by Lea and Blanchard in Philadelphia in the same year (Humboldt, 1849c) was even published with a "Note of the translator", Elizabeth Sabine.

Within the context of mid-19th-century scientific translation in Britain, Elizabeth Sabine's near "invisibility" on the title pages of her translations of Gauss, Humboldt and others was certainly not out of the ordinary. Scholars of translation studies are only now beginning to reveal and research the contribution made by women to the translation of scientific prose in this period. But it is interesting to note that some women were already successfully making a name for themselves as published scientific writers and translators in this period, not the least Mary Somerville, who produced an English version of Pierre Simon Laplace's *Traité de mécanique celeste* (1798–1825) as *The Mechanism of the Heavens* in 1831. Humboldt's oeuvre also attracted other translators besides the Sabines: Thomasina Ross, Helen Maria Williams and Elise C. Otté also produced English versions of his work for a 19th-century British public (some in fierce competition

[6]T: Johann Franz Encke (1791–1865), German astronomer; known for his discovery of the Encke division in the Kronian ring system. He also detected the famous comet 2P/Encke.

[7]T: Humphrey Lloyd (1800–1881), British scientist; known as the inventor of Lloyd's mirror.

[8]T: Christian Ludwig Gerling (1788–1864), German mathematician and astronomer; he was a pupil of Gauss in Göttingen and professor of mathematics in Marburg.

[9]T: Adolph Theodor Kupffer (1799–1865), Baltic physicist; he was a pupil of Gauss in Göttingen, later a professor of physics in Kazan and St. Petersburg. Inspired by Alexander von Humboldt, he conducted magnetic measurements on Mount Elbrus in the Caucasus mountain range and found that the magnetic force varied with height, as suggested by von Humboldt.

[10]T: Carl August von Steinheil (1801–1870), German scientist and entrepreneur.

with Elizabeth Sabine) but were far more visible and vocal in their translations (Martin, 2011).

The *Theory* represents a most influential contribution to our understanding of planetary magnetic fields. The first translation by Elizabeth Sabine was very important in spreading out Gauss' new approach within the English-speaking scientific community. Edward Sabine himself, for example, did not read German. And the *Theory* became important for the later systematic planning of new observations and observational campaigns (e.g., Sabine, 1840). Gauss' *Theory* also helped to resolve several issues of discussion. For example, the introduction of a magnetic potential and its spherical harmonic expansion is a much more powerful tool to describe the terrestrial magnetic field than using a complex distribution of magnets as proposed by Christopher Hansteen. Furthermore, the *Theory* settled an old issue between Gauss and Humboldt. Gauss claimed that only the horizontal force was needed to determine the whole field. The *Theory* finally demonstrates that Gauss was right. Of course, this is correct only if the external field contribution can be omitted.

The mathematical skills needed to understand and use the *Theory* are demanding. A deeper understanding of the associated Legendre polynomials is required, and the new concept of the *potential* requires mathematical power of imagination. In a letter to C. G. J. Jacobi[11] in Berlin, dated 29 April 1839, Alexander von Humboldt notes (citation from Biermann, 1977): "You surely understand that I only have a weak enjoyment from such a treatment, [and I only] understand a little, that is I guess the way the problem is tackled." And he adds: "It is not a shame, that I do not understand more."

The first other magnetician who applied the *Theory* to actual observations was Heinrich Jacob Reinhold Petersen, a German physicist and high-school teacher. He was born in 1815 in Heide in Holstein, and died in 1890 in Kiel. In a series of three contributions, he provided a detailed comparison of the results of the *Theory* with observations made by Georg Adolf Erman during his journey around the Earth (Erman, 1841; Petersen, 1842a, b, c). Petersen calculated the magnetic field components, as we would say now, for 39 observatories using the series expansion coefficients determined by Carl Friedrich Gauss. The deviation between observed and calculated values of the horizontal intensity is of the order of one percent, a remarkable result taking into account that Gauss used only a fourth-order expansion. In later studies Georg Adolf Erman and Heinrich Petersen used the observations of Erman (1841) to determine their own set of Gauss coefficients for a comparison between observed and calculated data (Erman and Petersen, 1872; Petersen, 1873; Erman and Petersen, 1874). Intensive use of Gauss' new mathematical description was also made by Georg von Neumayer (1826–1909) in collaboration with Heinrich Petersen. According to Schröder et al. (2010) they "carried out a new determination of the 24 Gaussian constants of the spherical functions in order to fit them to the actual magnetic field of the Earth." These results are unpublished, but are discussed and presented in Neumayer (1891). Further extension of the *Theory* was presented by Schmidt (1889), taking into account the flattening of the Earth, for example. Gauss' *Theory* had become the most accepted method for studies of the geomagnetic field at the time. With this revised translation and the additional comments and information, we hope to display the logic of Gauss' thinking.

After these introductory remarks we now proceed to present Carl Friedrich Gauss' *Allgemeine Theorie des Erdmagnetismus*.

I. *General Theory of Terrestrial Magnetism*[12]

The restless zeal, with which in recent times, the direction and intensity of the magnetic force of the Earth everywhere on its surface is examined, is truly admirable the more the purely scientific interest becomes visible. As important as complete knowledge of the lines of declination for navigation is, seafarers' interest does not reach further. They would almost not be interested in any further knowledge. But science, though also equally supporting economic interests, should not be restricted to this. Equal emphasis is required for all aspects of this science[13].

It has been customary to represent the results of magnetic observations by three systems of lines. They are called isogonic, isoclinal, and isodynamic lines. With time these lines undergo considerable changes both in position and in configuration. They are correct only for the epoch in which they were taken. Halley's[14] Chart of Declination for 1700 is very different from that of Barlow[15] for 1833. Hansteen's[16] Dip

[11]Carl Gustav Jacob Jacobi (1804–1851), German mathematician; known for his contribution to the Hamilton–Jacobi formalism.

[12]T: The Latin number refers to the first article in the *Resultate* for 1838.

[13]T: Elizabeth Sabine translated the German word *Elemente* with the word *magnetic elements*. This is not a suitable translation in this context. We interpret the original German half sentence "*sondern fordert für Alle Elemente ihrer Forschung gleiche Anstrengung*" such that Carl Friedrich Gauss expressed the importance of pure science here very clearly. That is, all aspects of science, economic interests, pure curiosity, philosophical insight, etc. are of equal importance. While reading the German word *Elemente,* Elizabeth Sabine was obviously immediately thinking in terms of the magnetic elements, a technical term later used very often in the following text. The magnetic elements denote the three components of the vector magnetic field.

[14]T: Edmund Halley (1656–1741), Astronomer Royal.

[15]T: Peter Barlow (1776–1862), English mathematician and physicist; most famous for his *Barlow's Tables*, listing squares, square roots, cubes, cube roots, and the reciprocals of the integer numbers up to 10 000. Peter Barlow is not a related to James William Barlow.

[16]T: Christopher Hansteen (1784–1873), Norwegian astronomer and physicist; a pioneer in Earth magnetic field measurements.

Chart for 1780 already differs greatly from the present isoclinal lines. Attempts to represent the intensity are too recent to infer similar changes with time. But without doubt such changes with time will occur. In all of these maps there are regions that are either blank or where observations were sparse or not trustworthy. But there is hope for rapid progress towards global coverage, in spite of inaccessibility of portions of the Earth's surface.

From a higher scientific perspective, even a complete representation of the phenomena is not the final objective that is sought. This is analogous to an astronomer who has observed the apparent path of a comet in the heavens. Until the complicated phenomena have been broadened into a general theory, we have only building blocks, not an edifice. To the astronomer, after the celestial body has disappeared from his view, his main work starts. Using the law of gravitation, he calculates the elements of the true path enabling predictions of its future course. Likewise the physicist[17] is challenged to investigate the fundamental processes causing the magnetic phenomena of the Earth and to explain its strength. A physicist needs to describe the available observations in terms of these fundamental processes, and he has to predict the phenomena in regions where observations are not possible. It is important to keep this higher goal constantly in mind and trying to pave the way for it, although a lack of a complete data set merely allows a distant approach to this goal at present.

It is not my intention here to point out earlier fruitless attempts to understand these phenomena, trying to guess right the magnetic riddle without any physical foundation. A physical basis can only be attributed to those attempts treating the Earth as a true magnet first, the action of which can be calculated based on distance. But past attempts have had this common fault: instead of first examining what properties (either simple or complex) this great magnet must have to satisfy the phenomena, certain simple descriptions are often assumed. Then the topic becomes whether the phenomena meet or do not meet the assumed description rather than discussing whether the description also supports insight into the physics of the phenomena. Here, the study of the Earth's magnetic field is a repetition of what has been done in early astronomy and natural sciences according to historical reports.

The simplest hypothesis that one may make assumes a very small magnet at the center of the Earth, or more accurately (it is not likely that anyone believes in the actual existence of such a magnet) one supposes magnetism to be distributed inside the Earth in such a way that the collective action at and beyond the Earth's surface is equivalent to the action of an imaginary infinitely small magnet, just as gravitation being caused by a homogeneous sphere is equivalent to that of a sphere of equal mass condensed in its central point. In this case, the magnetic poles are the two points where the projected axis of the little central magnet intersects the Earth's surface, where the magnetic needle is vertical, and the intensity is also greatest. The great circle midway between these two poles is called the magnetic equator where the dip angle is = 0 and the intensity is half that at the poles. Between the magnetic equator and either pole, the dip angle and the magnetic intensity depend on the distance from the equator (the magnetic latitude). The tangent of the dip angle is equal to twice the tangent of the magnetic latitude. Finally, the direction of the horizontal needle must everywhere coincide with the direction of a great circle drawn through the northern magnetic pole. With all the necessary consequences of this hypothesis, it is only in crude approximation to nature. In reality the line of no dip is not a great circle, but a line of double curvature. Equal intensities do not correspond to equal dip angles. The directions of the horizontal needle do not all converge to a single point, etc. A superficial examination is sufficient to convince oneself that the hypothesis needs to be rejected. Nevertheless, one of the above assumptions is still used as an approximation in deducing the line of no dip from dip observations at small values made at some distance from it.

A similar hypothesis originates from Tobias Mayer[18] about 80 yr ago, but with a modification. Instead of supporting the infinitely small magnet at the center of the Earth, he placed it at about one seventh of the Earth's radius from the center. Probably to simplify the calculations, he also kept the wholly arbitrary assumption that the plane perpendicular to the axis of the magnet passes through the center of the Earth. In this manner, by comparing observed variations and dips at a very small number of places, he found them agreeing very well with his calculations. However, a more extended comparison would have shown that this hypothesis did not give an improved representation of the dip and declination compared to the first one. Intensity measurements had not been made at that time.

Hansteen went a step further, trying to fit the model of two infinitely small magnets of unequal strength and location to the phenomena. The decisive test of a hypothesis must always be the comparison of its results with that of observations. Hansteen compared his model with observations at 48 different locations. There were only 14 places[19] where the intensity was known, and only 6 places where all 3 elements were measured. In these comparisons we still find in the dip differences of up to 13 degrees between calculation and observation[20].

[17]T: Elizabeth Sabine translated the German word *Physiker* into *magnetician*. We do not support this translation as studying the terrestrial magnetic field was and is of far more widespread interest to physical science than just a matter of a specialized group of magneticians.

[18]T: Tobias Mayer (1723–1762), self-taught astronomer and physicist, most famous for his lunar tables.

[19]T: Here we corrected the text following Gauss' addendum with corrections and additions. Mrs. Sabine did not include this correction in her translation.

[20]In the declination there is in one instance a difference of 29 degrees. Of course, the error of the calculation should not be given

If one feels that such large differences are not acceptable as requirements for a satisfactory theory, one cannot avoid drawing the conclusion that the magnetic conditions of the Earth are not such as to admit a representation by means of a concentration in either one or two infinitely small magnets. It is not denied that with a greater number of such fictitious magnets, a sufficient agreement might be ultimately attainable, but how far such a mode of solving the problem might be advisable is quite a different question. Indeed, even in case of two magnets, the calculations are extremely laborious, and with an increased number they would probably present insurmountable difficulties. It will be best to abandon entirely this kind of modeling, which reminds one involuntarily of the attempts to explain the planetary motions by continued accumulation of epicycles.

In the present work it is my purpose to develop the general theory of terrestrial magnetism independent of any hypotheses of the distribution of the magnetic fluids in the Earth's body, and I shall present the first obtained results from my method. As imperfect as these results must be, they will give an idea of what we can hope for in the future when trustworthy and complete observations from all parts of the Earth are available, supporting and improving the theory further.

1.

The force orienting a magnetic needle, suspended at its center by gravity, in a certain direction is called the Earth magnetic force, provided the cause of the force is entirely located in the body of the Earth itself. Here it is assumed that the needle is free from all extraneous influences such as another artificial magnet, or the conductor of a galvanic current. It may indeed be questioned whether the causes of regular or irregular hourly changes of the force under discussion may be assumed to be external relative to the Earth. With the increased attention paid by natural scientists[21] to these phenomena, one may also hope that much future information becomes available on the causes of these short-term variations. However, it should be mentioned that these changes are relatively small. Thus, there must be a much more powerful and constantly acting principal force. We assume the source of this principal force is within the Earth itself[22]. A consequence that follows from this train of thought is that the basic observations on which the study of the principal force is based should be separated from the anomalous changes. This can only be done by using mean values of the magnetic forces. These will be derived from numerous and continued observations. Until we have such distilled results taken from a great number of stations distributed over the whole surface of the globe, the best that one could hope for is an approximation. In this case, there would still remain differences of the order of the size of these anomalies.

2.

The foundation of our studies is the assumption that the terrestrial magnetic force is due to the collective action of all the magnetized parts within the Earth. We imagine that magnetization is due to a separation of the magnetic fluids. Based on this assumption the action of these magnetic fluids (repulsion between similar particles, attraction between dissimilar particles, and force decreasing with the square of the distance) is a well-established physical fact. No change in the results would be caused by changing this mode of representation to that of Ampère, which assumes magnetism being due to galvanic currents within the minutest particles of bodies. Nor would there be a difference if the terrestrial magnetism were due to a mixed origin, such as having both magnetic fluids and galvanic currents within the Earth. As is generally known each galvanic current may be substituted by a distribution of magnetic fluids at the surface bounded by the current and causing precisely the same force at each point of external space as the galvanic current.

3.

As in *Intensitas vis magneticae* etc.[23] we take as a positive unit for the measurement of the Earth's magnetic fluids that quantity of north polarity fluid that at a unit distance exerts a force on the same amount of north polarity fluid, which is equivalent to the unit force. When we speak of the magnetic force observed at any point of space as the result of another magnetic fluid, we also have in mind that this force is exerted at this point on a unit of positive magnetic fluid. Therefore the magnetic fluid μ concentrated at a point exerts at the distance ρ the magnetic force $\mu/\rho\rho$. The force can be either one of repulsion or attraction along the straight line ρ, depending on whether μ is positive or negative. By a, b, and c we denote the coordinates of μ in relation to three axes crossing each other under right angles and by x, y, and z the coordinates of that point where the force is exerted. Thus, $\rho = \sqrt{((x-a)^2 + (y-b)^2 + (z-c)^2)}$. Resolving

by the number of degrees of declination, but by the true angular difference between the calculated and observed directions, which in the case in question is 11 1/2 degrees.

[21]T: Mrs. Sabine neglected the German word *Naturforscher* in her translation.

[22]T: The possibility of external sources of the terrestrial magnetic field is excluded here. But later, in Chapters 36–40 Gauss relaxes this assumption. He extends his mathematical description of the field allowing external sources, and provides a means to separate the effects of both internal and external contributions at the Earth's surface. He a posteriori finds that external contributions are small.

[23]T: The *Intensitas Vis Magneticae Terrestris Ad Mensuram Absolutam Revocata*, that is *The Intensity of the Earth's Magnetic Force Reduced to Absolute Measurement*, was presented by Gauss in 1832 to the Königliche Gesellschaft der Wissenschaften zu Göttingen and published in 1833. In this treatise Gauss demonstrated that absolute magnetic field measurements can be obtained from the measurement of mass, length, and time (Gauss, 1833).

the force components along the coordinate axes, the components are

$$\frac{\mu(x-a)}{\rho^3}, \quad \frac{\mu(y-b)}{\rho^3}, \quad \frac{\mu(z-c)}{\rho^3},$$

which, as is easily seen, are the partial differential coefficients of $-\mu/\rho$ relatively to x, y, and z.

If besides the source μ, there are also other point-source magnetic fluids, μ', μ'', μ''', etc., where the distances from the origin are ρ', ρ'', ρ''', etc., then the components of the whole resulting magnetic force, parallel to the coordinate axes, are equal to the partial differential coefficients of

$$-\left(\frac{\mu}{\rho}+\frac{\mu'}{\rho'}+\frac{\mu''}{\rho''}+\frac{\mu'''}{\rho'''}+\text{etc.}\right)$$

with respect to x, y, and z.

4.

From this one can easily deduce the magnetic force exerted at each point in space by the Earth, independent of the distribution of magnetic fluids within it. Imagine the whole volume of the Earth containing free point-source magnetic fluids, that is, containing separated magnetic fluids, subdivided into infinitely small elements. The free amount of magnetic fluid in each element is designated by $\mathrm{d}\mu$, where the south polarity fluid is negative. The distance of the element with $\mathrm{d}\mu$ to an undetermined point of space with rectangular coordinates x, y, and z is ρ. Finally, let V be the aggregate of these $\mathrm{d}\mu/\rho$ with inverse sign, taken over all magnetic particles of the Earth, and then one has

$$V=-\int\frac{\mathrm{d}\mu}{\rho}.$$

Thus V has a determinate value at each point in space, or stated another way, it is a function of x, y, z, or any other set of three dependent parameters, whereby we may define points in space. The magnetic force Ψ in every point of space and the components of Ψ parallel to each of the coordinate axes, ξ, η, ζ, can be found by the formulas

$$\xi=\frac{\mathrm{d}V}{\mathrm{d}x}, \quad \eta=\frac{\mathrm{d}V}{\mathrm{d}y}, \quad \zeta=\frac{\mathrm{d}V}{\mathrm{d}z}, \quad \Psi=\sqrt{\xi\xi+\eta\eta+\zeta\zeta}.$$

5.

In a next step some general propositions that are independent of the form of V will be derived. They are remarkable by their simplicity and elegance. The complete differential of V is

$$\begin{aligned}\mathrm{d}V &= \frac{\mathrm{d}V}{\mathrm{d}x}\cdot\mathrm{d}x+\frac{\mathrm{d}V}{\mathrm{d}y}\cdot\mathrm{d}y+\frac{\mathrm{d}V}{\mathrm{d}z}\cdot\mathrm{d}z\\ &= \xi\,\mathrm{d}x+\eta\,\mathrm{d}y+\zeta\,\mathrm{d}z.\end{aligned}$$

If one denotes by $\mathrm{d}s$ the distance between two points with values V and $V+\mathrm{d}V$, and by θ the angle that the direction of the magnetic force Ψ makes with $\mathrm{d}s$, one derives

$$\mathrm{d}V=\Psi\cos\theta\cdot\mathrm{d}s,$$

because ξ/Ψ, η/Ψ, ζ/Ψ are the cosines of the angles that Ψ makes with the coordinate axes. On the other hand $\mathrm{d}x/\mathrm{d}s$, $\mathrm{d}y/\mathrm{d}s$, $\mathrm{d}z/\mathrm{d}s$ are the cosines of the angles between $\mathrm{d}s$ and the same axes. Thus $\mathrm{d}V/\mathrm{d}s$ is equal to the force in the direction of $\mathrm{d}s$. The same follows from the equation $\mathrm{d}V/\mathrm{d}x=\xi$, remembering that the coordinate axes may be chosen arbitrarily.

6.

If two points in space, P^0 and P', are connected by an arbitrary line for which $\mathrm{d}s$ is an indeterminate element, and if θ is the angle between $\mathrm{d}s$ and the direction of the magnetic force and Ψ its intensity, one has

$$\int\Psi\cos\theta\cdot\mathrm{d}s=V'-V^0,$$

if one carries out the integration along the whole line, and designates by V^0, V', etc. the values of V at the endpoints.

The following corollaries of this fruitful proposition deserve special notice[24].

I. The integral $\int\Psi\cos\theta\,\mathrm{d}s$ is independent of the path chosen between P^0 to P'.

II. The integral $\int\Psi\cos\theta\,\mathrm{d}s$ along any closed loop is always = 0.

III. Along a closed loop part of the values θ must be greater than and another part must be less than 90°, provided θ is not = 90° throughout.

7.

The surface in which all points of space have a value = V^0 divides those points of space where V is greater than V^0 from those where V is less than the V^0 value[25]. From the proposition in Chapter 5 it is easily seen that the magnetic force at each point on this surface has a direction perpendicular to this surface and is directed towards the side where the higher values of V are found. Let $\mathrm{d}s$ be an infinitesimal line perpendicular to the surface, and $V^0+\mathrm{d}V^0$ be the value of V at

[24] T: These corollaries are actually different formulations of the classical Stokes' theorem. Already in 1813 Gauss presented a special version of this theorem (Gauss, 1813). Concerning a detailed historical note on the Stokes' theorem see Katz (1979)

[25] If the function V were allowed to have an arbitrary form, then in some cases maximum or minimum values of V might correspond to isolated points or lines, around which only greater or only lesser values might be found. Or the topology might correspond to a surface where on both sides greater or lesser values are found. Due to the nature of the function V, these cases are not possible. Because this is not directly relevant to our present discussion, a full discussion of this topic will be reserved for a later occasion.

its endpoints. Then the intensity of the magnetic force will be given by $= \mathrm{d}V^0/\mathrm{d}s$. The set of points with $V = V^0 + \mathrm{d}V^0$ forms a second surface, infinitesimally close to the first one. At the different points in the intervening space between the surfaces, the intensity of the magnetic force is in the inverse ratio of the distance between the surfaces. Let V be altered by infinitesimally small but equal steps. A system of surfaces will be produced, dividing space into infinitely thin layers. The inverse ratio of the thickness of the layers to the intensity of the magnetic force holds not only for different points on the same layer, but also for different layers.

8.

We will now take into consideration the values of V on the surface of the Earth.

At a point P on the Earth's surface, let Ψ be the intensity, PM the direction of the total magnetic force, ω the intensity, and PN the direction of the force, projected onto the horizontal plane. Or assume PN as the direction of the magnetic meridian such that the south pole of the magnetic needle points towards the direction of the north pole. The angle i is the angle between PM and PN or the dip angle; θ and t are the angles formed by the element ds of a line on the surface of the Earth and the directions PM and PN, respectively. Lastly, V and $V + \mathrm{d}V$ correspond to the starting and endpoints of ds. We have consequently

$$\cos\theta = \cos i \cos t\,,\ \omega = \Psi \cos i.$$

And the equation in Chapter 5 becomes

$$\mathrm{d}V = \omega\ \cos t \cdot \mathrm{d}s.$$

Therefore, if the two points P^0 and P' on the Earth's surface at which V has the values V^0 and V', respectively, are connected by a line traced on the surface of the Earth and if ds is an indeterminate element on this line, then

$$\int \omega \cos t\, \mathrm{d}s = V' - V^0,$$

if the integration is extended along the whole line. It is obvious that three corollaries hold, similar to those in Chapter 6, namely:

I. The integral $\int \omega \cos t \cdot \mathrm{d}s$ is constant and independent of the path of integration along the surface of the Earth from P^0 to P'.

II. The integral $\int \omega \cos t \cdot \mathrm{d}s$ taken along a closed loop on the surface of the Earth is always = 0.

III. On such a closed loop, either $t = 90°$ throughout, or one part of the values of t are acute and another part is obtuse.

9.

Propositions I and II of the foregoing chapter (which are only two different ways of saying the same thing) may be tested by observation, at least approximately. Let the points P^0, P', P''... P^0 be a polygon on the surface of the Earth, the sides of which are the shortest lines that can be drawn between their respective endpoints. These lines are therefore portions of great circles, assuming that the Earth is treated as a sphere. Let ω^0, ω', ω'', etc. be the intensities of the horizontal magnetic force at the points P^0, P', P'', etc. Furthermore, let δ^0, δ', δ'', etc. be the declinations using the standard convention for the latter values, west of north as positive, east of north as negative. Lastly, let (01) be the azimuth of the line P^0P' at P^0, by convention measured from south to west. In like manner let (10) be the azimuth of the same line taken backwards at P', and so on.

It should be noted that t changes continuously in each of the sides of the polygon, but discontinuously at the corners, exhibiting two different values here; for example, at P' t has the value $(10) + \delta$ if P' is the endpoint of the line P^0P'. And it has the value $180° + (12) + \delta'$ at P' if P' is the endpoint of $P'P''$.

For the integral $\int \omega \cos t\, \mathrm{d}s$, extended through P^0P', one can use the approximation

$$\frac{1}{2}(\ \omega^0\ \cos t^0 + \omega' \cos t') \cdot P^0P',$$

where t^0 and t' denote the values of t at P^0 as the starting point and at P' as the endpoint of P^0P'. This approximation is the best that one can do because we have the values of ω and t only at the endpoints P^0, P'. The shorter the line, the greater the confidence. The given expression is, in our notation,

$$= \frac{1}{2}(\ \omega' \cos((10) + \delta') - \omega^0 \cos((01) + \delta^0)) \cdot P^0P'.$$

In a similar manner, the approximate value of the integral, extended through $P'P''$, is

$$= \frac{1}{2}(\omega'' \cos((21) + \delta'') - \omega' \cos((12) + \delta')) \cdot P'P''$$

and so on throughout the whole polygon.

Therefore, for a triangle our proposition gives the approximatively correct equation

$$\begin{aligned} &\omega^0\ (P^0P' \cos((01) + \delta^0) &&- \quad P^0P'' \cos((02) + \delta^0)) \\ +\ &\omega'\ (P'P'' \cos((12) + \delta') &&- \quad P^0P' \cos((10) + \delta')) \\ +\ &\omega''\ (P^0P'' \cos((20) + \delta'') &&- \quad P'P'' \cos((21) + \delta'')) \\ &= 0. \end{aligned}$$

It is obvious that in this equation the units of intensities and distances are arbitrary[26].

[26]T: This chapter provides a most remarkable application of what was later called the Stokes' theorem, that is, the theorem relating

10.

As an example, we will apply the formula to the magnetic elements of[27]

Göttingen	$\delta^0 = 18°38'$	$i^0 = 67°56'$	$\Psi^0 = 1.357$
Mailand	$\delta' = 18\ 33$	$i' = 63\ 49$	$\Psi' = 1.294$
Paris	$\delta'' = 22\ 04$	$i'' = 67\ 24$	$\Psi'' = 1.348$

from which it follows that[28]

$$\omega^0 = 0.50980$$
$$\omega' = 0.57094$$
$$\omega'' = 0.51804$$

With the geographical positions below as a basis

Göttingen	51°32′ latitude	9°58′ longitude from Greenwich
Mailand	45 28	9 09
Paris	48 52	2 21

and performing the calculation for a spherical surface only, one finds

$$\left.\begin{array}{lll}(01) & = & 5°\ 11'\ 31'' \\ (10) & = & 184\ 35\ 35\end{array}\right\} \quad P^0P' = 6°\ 5'\ 20''$$

$$\left.\begin{array}{lll}(12) & = & 128\ 47\ 31 \\ (21) & = & 303\ 48\ 01\end{array}\right\} \quad P'P'' = 5\ 44\ 06$$

$$\left.\begin{array}{lll}(20) & = & 238\ 20\ 20 \\ (02) & = & 64\ 10\ 12\end{array}\right\} \quad P^0P'' = 5\ 32\ 04$$

Substituting these values in our equation, and those given above for δ^0, δ', δ'', we have

$$0 = 17556\,\omega^0 + 2774\,\omega' - 20377\,\omega'',$$

or

$$\omega'' = 0.86158\,\omega^0 + 0.13613\,\omega'.$$

From the observed horizontal intensities at Göttingen and Milan, we deduce that one at Paris to be $\omega'' = 0.51696$, agreeing almost exactly with the measured value of 0.51804[29].

By the way, it is easily seen that if we permit ourselves to take their sines instead of the distances P^0, P', etc., then the above formula can be expressed immediately by the geographical longitudes and latitudes of any particular location.

11.

The line on the Earth's surface where in all points V has the same value V^0 in general divides those parts of the surface where V is greater than V^0 from those where it is less. The direction of the horizontal magnetic force in each point on this line is obviously perpendicular to it and is directed towards the side where the values of V are greater. If ds is an infinitely small line in this direction and if $V^0 + \mathrm{d}V^0$ is the value of V at the other end of this line, then $\mathrm{d}V^0/\mathrm{d}s$ is the intensity of the horizontal magnetic force at this place. The series of points corresponding to the value of $V = V^0 + \mathrm{d}V^0$ forms a second line situated infinitesimally close to the first. It demarks on the entire surface of the Earth a *zone*, within which the values of V are between V^0 and $V^0 + \mathrm{d}V^0$, and where the horizontal intensity is in an inverse ratio to the width of the zone. By making V vary by infinitesimally small but equal steps from the lowest value on the surface of the Earth to the highest, the whole surface of the globe becomes divided into an infinite number of infinitesimally narrow zones. The direction of the horizontal magnetic force is everywhere perpendicular to the dividing lines and inversely related to the width of the zone at the place in question. The two extreme values of

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the surface integral, or flux of the curl of a vector field $\boldsymbol{B}$ through a given two-dimensional surface in the Euclidean space to the line integral of the vector field along the boundary of this surface. The practical application presented here is based on the assumption that the terrestrial magnetic field at the Earth surface is a curl-free field. This assumption is well justified as any atmospheric electric current density can be neglected, although that was not known in Gauss' time. In space, however, this assumption is not justified. And any deviation from the proposition discussed by Gauss can be used to estimate the electric current density through the surface defined by three observational points. Dunlop et al. (2002) present a more detailed, practical, modern application, using magnetic field measurements made on board the four CLUSTER spacecraft.

[27]T: The unit for the magnetic intensity used here is the *Humboldt unit*, which is based on comparing oscillation times of a particular compass needle at an observation point and a reference station. Alexander von Humboldt used Micuipampa (Peru) as his standard station (Chapman and Bartels, 1951). This unit is also known as the *German unit of absolute intensity* (Petersen, 1873). A proper conversion factor to the SI system is 3.49412×10^4 nT (Chapman and Bartels, 1951). The magnetic intensity at Göttingen at the time of Gauss was 47 415 nT. See also Chapter 31 of the *Theory*. It should also be noted that Gauss used the comma as a decimal marker. We use the point as the decimal marker in this translation. Furthermore, here and in the following tables we did not convert the names of the towns and villages into English notation, but reproduce the German names as used by Gauss.

[28]T: Here and in the following we use a dot to mark the radix point and thin space as a group-of-three separator.

[29]T: Here Gauss applied an observational test demonstrating that the terrestrial magnetic field at the surface of the Earth is a curl-free vector field and can be represented by a scalar potential. If the deduced value of the field at Paris did not agree with that one observed, then $\nabla \times \boldsymbol{B} \neq 0$ would follow. This would imply that significant electric currents would flow in the atmosphere. He earlier used a similar way of experimentally testing a mathematical theorem when measuring the angles of a triangle formed by three hills in the Göttingen region, namely Brocken, Hoher Hagen, and Inselsberg. He tried to learn whether the surface of the Earth was hyperbolic, elliptic, or flat by comparing the sum of interior angles with π.

V correspond to two points, enclosed by the zones, at which the horizontal force becomes = 0, and where therefore the whole magnetic force can only be vertical. These two points are termed the magnetic poles of the Earth.

The lines dividing the zones are nothing but the intersections of the surfaces considered in the seventh Chapter with the surface of the Earth, while at the poles they are merely in contact with it.

12.

The form of the system of lines described in the preceding section is actually of the simplest type, allowing for many exceptions if taking into account every possible distribution of magnetism in the Earth. However, we shall not go into great detail here. We merely add a few remarks on exceptions, as due to the *true* magnetic condition of the Earth, the form of the system of lines on its surface corresponds almost to that one described already. At least there are certainly no large-scale exceptions, although there probably may be local ones.

Some physicists[30] have considered models where the Earth has two north and two south magnetic poles. However, it seems that most essential conditions are not satisfied, and a *precise* definition of what one terms a magnetic pole was not given. We intend to use this term for every point on the Earth's surface where the horizontal intensity is zero. Of course, here the dip angle is = 90°. We also include the singular case when the total intensity is = 0, if it exists. If one intends to call magnetic poles those places where the total intensity is a maximum (i.e., greater than anywhere in the surrounding vicinity), that would be quite different from the above definition. There is not necessarily any connection between these latter points and the former, neither with respect to their location nor their number. And it would confuse the situation if they were given the same name.

If we ignore the real state of the Earth and consider the general case, there might exist two poles of the same polarity. But it does not appear to have been noticed that if, for example, two north poles exist, a third point between them is required, which is likewise a magnetic pole. It is neither a north nor a south pole, but if one prefers to say, it has the properties of both.

To clarify this subject nothing is more useful than our system of lines.

If the function V has a maximum value V^* at a point of the Earth's surface P^*, that is, there are only smaller values all around P^*, then a series of stepwise decreasing values will correspond to a system of rings. Each of these will enclose all the preceding ones and the point P^*. The direction of the horizontal magnetic force, or that of the north pole of the magnetic needle, will be *inwards*[31]. This is the characteristic signature of a magnetic north pole[32]. It is clear that the rings may be made so small, or the corresponding values of the function V may differ so little from V^* that any other point may be excluded.

We will designate by S the space included by all the points on the surface of the Earth for which the value of V is greater than a given value W. It is clear that S may either be one connected surface or consists of several detached areas. On the bounding line or the bounding lines that separate S from other parts where V is less than W, one has $V = W$. By increasing or decreasing W, we enlarge or contract the area S.

Now let us assume P^{**} is a second point that has similar properties to P^* so that at it V may also have a maximum value V^{**}. Following the previous discussion, one can attribute to the quantity W a value less than V^* and deviating so little from this that P^{**} may fall outside that part of S where P^* lies. Now, assuming that V^{**} is not less than V^* (which is allowed), but greater than W, then P^{**} will necessarily also be part of S. Thus, P^{**} and P^* will both lie inside S, but in separated regions of S.

On the other hand, one can assume W to be so small that P^* and P^{**} will both be situated in one connected part of S. This holds as if choosing W small enough, S can be made to cover the entire surface of the Earth.

If W is made to pass stepwise through all the values between the first and the second, there must be a final value $= V^{***}$, for which both P^* and P^{**} are still located in separate parts of S, which join in the case of further decreasing W.

If this junction occurs at a point P^{***}, the bounding line on which $V = V^{***}$ will have the shape of the number 8, with its crossing at that point. It is easily seen that the horizontal intensity must be zero there. In fact, the crossing either does or does not take place under a measurable angle. In the first

[30] T: This time Elizabeth Sabine translated the German word *Physiker* into *philosopher*, suitable in English in 1849, but not now.

[31] These infinitesimally small rings are not necessarily circular, but generally speaking oval in shape, so that the normal direction of the magnetic needle in reference to them only coincides with the direction towards P^* at four points in each ring. Large errors may therefore occur if one simply assumes that the intersection of the projections of two compass directions at considerable distances is P^*.

[32] We here follow the definition established by Captain James Ross, although properly speaking it is a south pole in as much as the Earth is considered as a magnet. T: Additional translators' note: In physics the point from which the lines of magnetic induction diverge is defined as the magnetic north pole; the point toward which the lines converge is the magnetic south pole. The geomagnetic north pole, however, is that point on the surface of the Earth towards which the lines of magnetic induction converge and to which the magnetic south pole of a compass needle points. Conversely, the geomagnetic south pole is that point from which the lines of magnetic induction diverge and to which the magnetic north pole of the compass needle points. According to Carl Friedrich Gauss, we owe this confusion to James Clark Ross (1800–1862), the English seafarer and surveyor. Ross and his companions discovered the geomagnetic north pole near the Boothia Peninsula.

case, if the horizontal force is not = 0, it must be directed at the normal to two different tangents, which is absurd. In the second case, where the two halves of the number 8 touch each other at P^{***} or would have the same tangent, the force normal to this tangent could only be directed towards the interior of one half surface of the number 8. This is contradictory, as the value of V increases towards both sides. Therefore, P^{***} is a true magnetic pole by our definition, but it must be considered as a south pole by regarding the points nearest to it inside the two loops of the number 8. It is a north pole when considering the points that lie outside. Figure 1 may serve to illustrate this form of system of lines.

If the junction takes place at two different points, the previous discussion will be true for both points. One may easily note that an insular space would be formed inside the space enclosing P^{*} and P^{**}. This space would gradually contract as W decreases. It will eventually be resolved as a true south pole.

The situation is similar when the junction takes place at three or more singular points. But if it occurs along a whole line, then the horizontal force must disappear at all points along that line.

By the way, it is evident that the assumption of two south poles necessitates the existence of a third pole point, which would be neither a south pole nor a north pole. It would be both south and north at the same time.

13.

From what has been derived in the previous section, one can easily understand the peculiar matters of several possible exceptions to the simplest type of our system of lines. The whole of the points to which a certain value of V corresponds may be a line consisting of several parts, of which each returns back into itself but at the same time are distinct. It may be a line that crosses itself. Finally, it might be such a line to which on both sides areas are attached where V is entirely greater or less than on the line.

We may generally state that on the Earth there are no major deviations from the simplest type. But local deviations may certainly exist at places where magnetic masses are located close to the surface with vanishing effect at large distances, but dominating the terrestrial magnetic force locally and surpassing and masking the Earth's magnetic force. In the simplest case the system of lines in such a local area may take the form presented in the second figure.

14.

After this geometrical representation of the horizontal magnetic force, we proceed to develop a method to use for calculational purposes. On the surface of the Earth, V becomes a simple function of two variables: the geographical longitude measured in an eastward direction from an arbitrary first meridian, and the distance from the north pole of the Earth. The former will be designated by λ, the latter, the complement to the geographic latitude, by u. If we consider the Earth as being generated by the rotation of an ellipse with major semi-axis = R, minor semi-axis = $(1-\epsilon)\,R$, and being rotated around the latter, then an element of the meridian is

$$= \frac{(1-\epsilon)^2\, R\cdot \mathrm{d}u}{(1-(2\epsilon-\epsilon\,\epsilon)\cos u^2)^{3/2}},$$

and an element of the parallel is

$$= \frac{R\ \sin u \cdot \mathrm{d}\,\lambda}{\sqrt{(1-(2\epsilon-\epsilon\,\epsilon)\cos\ u^2)}}.$$

Separating the horizontal magnetic force into two parts with X acting along the direction of the geographical meridian, and the other, Y, acting perpendicular to that meridian and assigning X as a positive value if it is directed towards the north, and assuming Y as positive when directed towards the west[33] results into

$$X = -\frac{(1-(\,2\epsilon-\epsilon\,\epsilon)\cos u^2)^{3/2}}{(1-\epsilon)^2}\cdot\frac{\mathrm{d}V}{R\,\mathrm{d}u},$$

$$Y = -\sqrt{(1-(\,2\epsilon-\epsilon\,\epsilon)\cos u^2)}\cdot\frac{\mathrm{d}V}{R\sin u\cdot \mathrm{d}\,\lambda}.$$

The total horizontal force is then

$$= \sqrt{X\,X+Y\,Y},$$

and the tangent of the declination

$$= \frac{Y}{X}.$$

Neglecting the square of the oblateness ϵ, the expressions become

$$X = -(1+(2-3\cos u^2)\,\epsilon)\cdot\frac{\mathrm{d}V}{R\,\mathrm{d}u},$$

$$Y = -(1-\epsilon\cos u^2)\cdot\frac{\mathrm{d}V}{R\sin u\cdot\mathrm{d}\,\lambda},$$

or, if we completely neglect the oblateness

$$X = -\frac{\mathrm{d}V}{R\,\mathrm{d}u}$$

$$Y = -\frac{\mathrm{d}V}{R\sin u\cdot\mathrm{d}\,\lambda}.$$

The data furnished by the observations at this time are much too sparse, and most of them much too inaccurate. It is currently not advisable to take into account the ellipsoidal shape of the Earth. Doing so is not difficult, but would prevent easy calculations without giving any advantages. We will therefore adhere to the last mentioned formula considering the Earth as a sphere with radius = R[34].

[33]T: Note that Gauss counts the Y component positive towards the west, different from current practice.

[34]T: This was done later by, for example, Adolf Schmidt (1860–1944) in his extensions of the mathematical theory of the description of the terrestrial magnetic field (Schmidt, 1889).

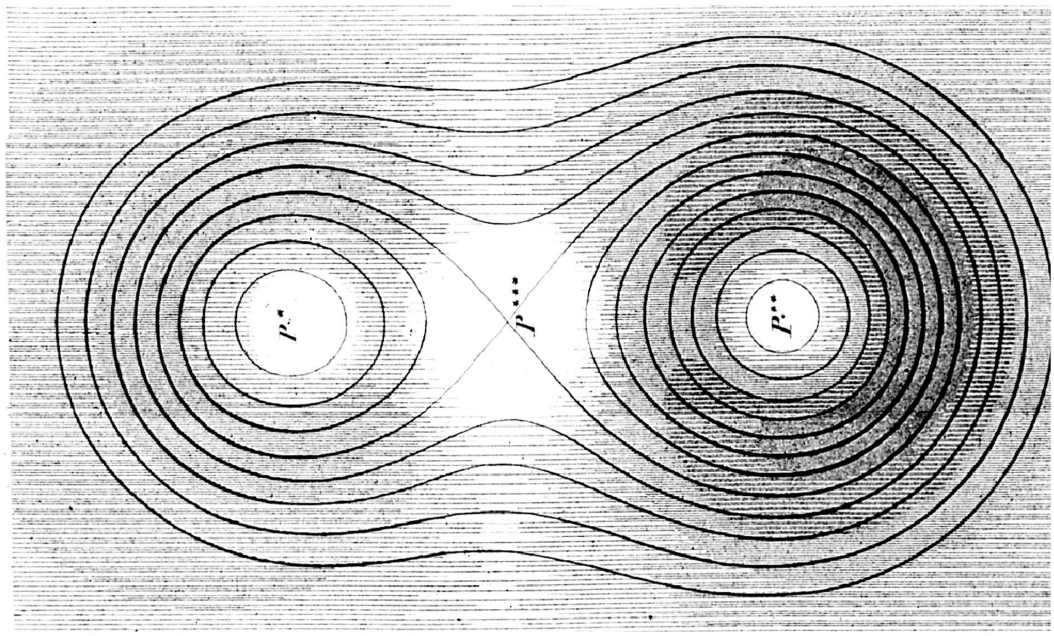

Figure 1. The system of magnetic lines near magnetic poles. Figure 1 is referenced in the original text, but was not included in the article itself. The figure was published in an annex volume *10 Tafeln zu Gauss und Weber, Resultate: Jahrgang 1838*, Weidmannsche Buchhandlung, Leipzig, 1839. Source: Library of the Technische Universität Braunschweig.

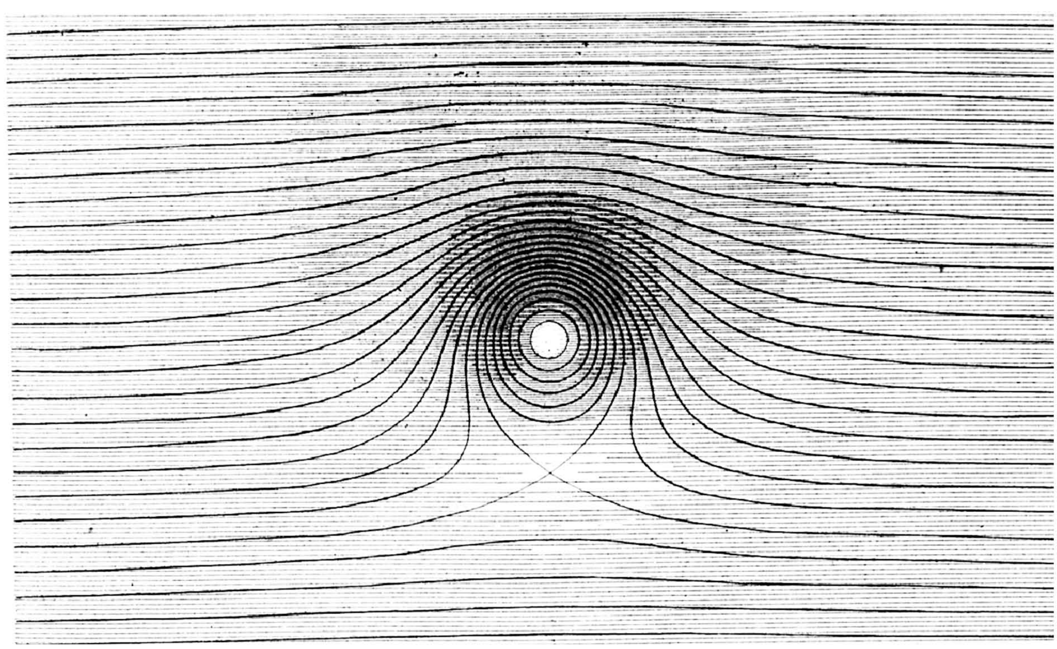

Figure 2. The system of magnetic lines around magnetic anomalies. Figure 2 was referenced in the original text, but was also not included in the article itself. It was published together with Fig. 1 and other tables in the annex *10 Tafeln zu Gauss und Weber, Resultate: Jahrgang 1838*, Weidmannsche Buchhandlung, Leipzig, 1839. Source: Library of the Technische Universität Braunschweig.

15.

If X is expressed by a given function of u and λ, Y can be deduced from it a priori. Define the integral $\int_0^u X\mathrm{d}u = T$ by considering λ as a constant in the integration. It is then clear that if we differentiate likewise with respect to u, $\mathrm{d}(V + R\,T)/\mathrm{d}u = 0$; that is $V + RT$ is a quantity independent of u, or, what is the same thing, constant in all points along the meridian. It must also be absolutely constant because all meridians converge and meet at the poles. Denoting the value of V at the north pole by V^*, then

$$T = \frac{V^* - V}{R},$$

and hence

$$Y = \frac{\mathrm{d}\,T}{\sin u \cdot \mathrm{d}\,\lambda}.$$

This result can also be expressed as

$$Y = \frac{1}{\sin u} \int_0^u \frac{\mathrm{d}X}{\mathrm{d}\,\lambda} \cdot \mathrm{d}\,u.$$

16.

The converse of this extraordinary proposition that *if the northward component of the horizontal magnetic force is given for the whole of the surface of the Earth, then the component directed towards the west (or towards the east) can be derived from this* is true only with a certain modification: if Y is expressed by a given function of u and λ, and if U represents the indeterminate integral $\int \sin u \cdot Y \mathrm{d}\,\lambda$ and if u is assumed constant in the integration, then $\mathrm{d}(V + RU)/\mathrm{d}\lambda = 0$, and $V + RU$ becomes a quantity independent of λ, generally speaking a function of u. Thus, also $\mathrm{d}(V + RU)/R\mathrm{d}u = \mathrm{d}U/\mathrm{d}u - X$ is such a function. That is to say the formula $\mathrm{d}U/\mathrm{d}u$ gives an imperfect expression for X, a part of which depends on u only and remains undetermined. This shortcoming may be cured if, besides the expression for Y, one also knows an expression for X for a given meridian or along any line extending from the north pole to the south pole. It is seen that, *if one knows both the component of the horizontal magnetic force in the direction towards the west for the whole of the Earth's surface, and the component in the northward direction for all points along a line from the north pole to the south pole, the latter component follows for the whole of the Earth's surface.*

17.

The above considerations only apply to the horizontal part of the Earth's magnetic force. In order to include the vertical force as well, we must consider the general case. V must be regarded as a function of three variables, describing the position of an arbitrary point O in space. For this we select the distance r from the center of the Earth, the angle u that r makes with the northern part of the Earth's axis, and the angle λ between a plane passing through r and the axis of the Earth and a fixed meridian, counted positive in the eastward direction.

Let the function V be expanded into a series with decreasing powers of r of the following form:

$$V = \frac{R\,R\,P^0}{r} + \frac{R^3 P'}{r\,r} + \frac{R^4 P''}{r^3} + \frac{R^5 P'''}{r^4} + \text{etc.}$$

The coefficients P^0, P', P'', etc. here are functions of u and λ. In order to illustrate on how they are related to the distribution of the magnetic fluid in the Earth, let $\mathrm{d}\mu$ be an element of this, ρ its distance from O. Let r^0, u^0, and λ^0 be the coordinates with respect to $\mathrm{d}\mu$ as r, u, λ are the coordinates with respect to O. Then we have $V = -\int \mathrm{d}\mu/\rho$ being expressed by every $\mathrm{d}\mu$. Further, $\rho = \sqrt{rr - 2rr^0\,(\cos u \cos u^0 + \sin u \sin u^0 \cos(\lambda - \lambda^0)) + r^0 r^0}$, and if one expands $1/\rho$ into the series

$$\frac{1}{\rho} = \frac{1}{r}\,(T^0 + T' \cdot \frac{r^0}{r} + T'' \cdot \frac{r^0 r^0}{rr} + \text{etc.}),$$

one derives[35] $R\,R\,P^0 = -\int T^0 \mathrm{d}\mu$, $R^3\,P' = -\int T'\,r^0\,\mathrm{d}\mu$, $R^4 P'' = -\int T'' r^0\,r^0\,\mathrm{d}\mu$, etc.

As $T^0 = 1$, the fundamental assumptions that we started with, namely equal quantities of positive and of negative fluid in every measurable part of its carrier, thereby also within the *whole* Earth, imply $\int \mathrm{d}\mu = 0$. Thus

$$P^0 = 0,$$

or the first term of our series for V vanishes. One further notices that P' has the form

$$R^3 P' = \alpha \cos u + \beta \sin u \cos \lambda + \gamma \sin u \sin \lambda,$$

where $\alpha = \int \cos u^0 \mathrm{d}\mu$, $\beta = -\int \sin u^0 \cos \lambda^0\,\mathrm{d}\mu$, and $\gamma = -\int \sin u \sin \lambda^0\,\mathrm{d}\mu$. Therefore, according to the explanation on page 13 of *Intensitas Vis Magneticae*, $-\alpha, -\beta$, and $-\gamma$ are the moments of the Earth's magnetism with respect to three rectangular axes. The first one is the axis of the Earth, and the second and the third are the equatorial radii for longitudes 0° and 90°.

The general formulas for all coefficients of the series for $1/\rho$ we can assume to be known. For our purpose it is sufficient to note that with respect to u and λ, the coefficients are rational integral functions of $\cos u$, $\sin u \cos \lambda$, and $\sin u \sin \lambda$. Actually T'' is of the second order, T''' of the third order, etc. The same also holds for the coefficients P'', P''', etc.

The series for $1/\rho$ and for V converge as long as r is not less than R, or rather not less than half the diameter of that sphere including all the magnetic particles of the Earth.

[35] T: The original German version exhibits a misprint here later corrected by Gauss in an addendum (see further down). Elizabeth Sabine already corrected these misprints in her English translation.

18.

The function V, constructed via $-\int \mathrm{d}\mu/\rho$, satisfies the following partial differential equation:

$$0 = \frac{r\mathrm{dd}rV}{\mathrm{d}r^2} + \frac{\mathrm{dd}V}{\mathrm{d}u^2} + \cot u \cdot \frac{\mathrm{d}V}{\mathrm{d}u} + \frac{1}{\sin u^2} \cdot \frac{\mathrm{dd}V}{\mathrm{d}\lambda^2},$$

which is only a transformation of the well-known equation

$$0 = \frac{\mathrm{dd}V}{\mathrm{d}x^2} + \frac{\mathrm{dd}V}{\mathrm{d}y^2} + \frac{\mathrm{dd}V}{\mathrm{d}z^2}$$

with x, y, and z denoting the coordinates of the point O. If one substitutes the expression for V,

$$V = \frac{R^3 P'}{rr} + \frac{R^4 P''}{r^3} + \frac{R^5 P'''}{r^4} + \text{etc.},$$

it is clear that there will likewise be partial differential equations for each coefficient P', P'', P''', etc., for which the general expression is

$$0 = n(n+1)\, P^{(n)} + \frac{\mathrm{dd}P^{(n)}}{\mathrm{d}u^2} + \cot u\, \frac{\mathrm{d}P^{(n)}}{\mathrm{d}u} + \frac{1}{\sin u^2} \cdot \frac{\mathrm{dd}P^{(n)}}{\mathrm{d}\lambda^2}.$$

From this equation and the remark in the preceding section, one derives the general form of P^n. If $P^{n,m}$ describes the following function of u[36],

$$\begin{aligned}(\cos^{n-m}u - \frac{(n-m)\,(n-m-1)}{2\,(2n-1)}\cos^{n-m-2}u \\ + \frac{(n-m)(n-m-1)(n-m-2)(n-m-3)}{2\cdot 4\,(2n-1)\,(2n-3)}\cos^{n-m-4}u \\ -\text{etc.})\ \sin^m u,\end{aligned}$$

then $P^{(n)}$ has the form of a series with $2n+1$ terms:

$$\begin{aligned}P^{(n)} = g^{n,0}\, P^{n,0} &+ (g^{n,1}\cos\lambda + h^{n,1}\sin\lambda)\, P^{n,1} \\ &+ (g^{n,2}\cos 2\lambda + h^{n,2}\sin 2\lambda)\, P^{n,2} \\ &+ \text{etc.} + (g^{n,n}\cos n\lambda + h^{n,n}\sin n\lambda)\, P^{n,n},\end{aligned}$$

where $g^{n,0}, g^{n,1}, h^{n,1}, g^{n,2}$, etc. are numerical coefficients to be specified.

19.

If the magnetic force at point O is decomposed into three orthogonal forces X, Y, and Z, where Z is directed towards the center of the Earth, and X and Y are tangential to a spherical surface concentric with the Earth and passing through O with X being directed northwards in a plane passing through O and the axis of the Earth and Y being directed westwards parallel to the equator of the Earth[37], then

$$X = -\frac{\mathrm{d}V}{r\,\mathrm{d}u},\ Y = -\frac{\mathrm{d}V}{r\sin u\,\mathrm{d}\lambda},\ Z = -\frac{\mathrm{d}V}{\mathrm{d}r},$$

and consequently

$$\begin{aligned}X &= -\frac{R^3}{r^3}\left(\frac{\mathrm{d}P'}{\mathrm{d}u} + \frac{R}{r}\cdot\frac{\mathrm{d}P''}{\mathrm{d}u} + \frac{R\,R}{r\,r}\cdot\frac{\mathrm{d}P'''}{\mathrm{d}u}\ \text{etc.}\right) \\ Y &= -\frac{R^3}{r^3\sin u}\left(\frac{\mathrm{d}P'}{\mathrm{d}\lambda} + \frac{R}{r}\cdot\frac{\mathrm{d}P''}{\mathrm{d}\lambda} + \frac{R\,R}{r\,r}\cdot\frac{\mathrm{d}P'''}{\mathrm{d}\lambda}\ \text{etc.}\right) \\ Z &= \frac{R^3}{r^3}\left(2\,P' + \frac{3RP''}{r} + \frac{4R\,R\,P'''}{r\,r}\ \text{etc.}\right).\end{aligned}$$

On the surface of the Earth, X and Y are the same horizontal components, which we have designed above by these symbols. And Z is the vertical component, positive downward. Thus, the expressions for these forces on the surface of the Earth are

$$\begin{aligned}X &= -\left(\frac{\mathrm{d}P'}{\mathrm{d}u} + \frac{\mathrm{d}P''}{\mathrm{d}u} + \frac{\mathrm{d}P'''}{\mathrm{d}u} + \text{etc.}\right), \\ Y &= -\frac{1}{\sin u}\left(\frac{\mathrm{d}P'}{\mathrm{d}\lambda} + \frac{\mathrm{d}P''}{\mathrm{d}\lambda} + \frac{\mathrm{d}P'''}{\mathrm{d}\lambda} + \text{etc.}\right), \\ Z &= 2P' + 3P'' + 4P''' + \text{etc.}\end{aligned}$$

20.

Let us combine the above with the known theorem that every function of λ and u, which, for all values of λ between 0° and 360° and u between 0° and 180°, has a definite value, can be expanded into a series of the form

$$P^0 + P' + P'' + P''' + \text{ etc.}$$

The general term $P^{(n)}$ satisfies the above partial differential equation. Noting that such an expansion is unambiguous and that this series always converges, we obtain the following remarkable propositions.

I. The knowledge of the value of V at all points of the Earth's surface is sufficient to deduce the general expression of V for all external space. Thus, we can determine the forces X, Y, and Z not only on the surface of the Earth, but also for the entire external space. Obviously, for this one only needs to expand V/R into a series applying the mentioned theorem[38].

[36]T: We found that the series expansion originally presented by Gauss as well as that one used in the translation provided by Elizabeth Sabine contains a misprint. In the original print of the *Theory*, the numerator of the second expansion coefficient incorrectly reads $(n-m)\cdot(n-m+1)$. The correct expression is $(n-m)\cdot(n-m-1)$ (e.g., Schmidt, 1935).

[37]T: With the magnetic elements defined in this way, Gauss is not using a right-handed system. For a right-handed system Y needs to be counted positive towards the east. For a more detailed discussion of the different coordinate systems used in the field of terrestrial magnetism, see Bigelow (1897).

[38]T: Gauss refers here to the series expansion in Chapter 17. Modern terminology would not call this a theorem, but an ansatz.

In the following, if not stated otherwise, the symbol V is always taken to be limited to the surface of the Earth, or as that function of λ and u derived from the general expression if $r = R$. Thus,

$$V = R\,(P' + P'' + P'' + \text{ etc.}).$$

II. The knowledge of the value of X at all points of the Earth's surface is sufficient to obtain all that has been referred to in Lemma I. In fact, according to Chapter 15, the integral expression $\int_0^u X\mathrm{d}u = (V^0 - V)/R$ holds with V^0 denoting the value of V at the north pole. And the expansion of $\int_0^u X du$ into a series of the form referred to must necessarily be identical with

$$V^0 - P' - P'' - P''' - \text{etc.}$$

III. In a similar manner, and under the considerations in Chapter 16, it is clear that the knowledge of Y on the whole Earth, combined with the knowledge of X at all points along a line reaching from one pole to the other, is sufficient for the foundation of the *complete* theory of terrestrial magnetism.

IV. Finally, it is clear that a complete theory is also deducible from the mere knowledge of the value of Z on the whole surface of the Earth. In fact, if Z is expanded into a series,

$$Z = Q^0 + Q' + Q'' + Q''' + \text{etc.}$$

such that the general term satisfies the often-mentioned partial differential equation; it is required that $Q^0 = 0$, and $P' = \frac{1}{2}Q'$, $P'' = \frac{1}{3}Q''$, $P''' = \frac{1}{4}Q'''$, etc.[39]

21.

Because of the simple nature of the dependence of the several forces X, Y, Z on a single function V, and the simple relationship that they have to each other, they are far better suited to serve as a foundation for the theory than the usual expression of the magnetic force given by three elements, the total intensity, the inclination, and the declination. Although the latter description, based on observational facts, seems to be natural, it cannot immediately be applied as the basis of the theory until it has been transformed into the alternative form. From this viewpoint, it would be very desirable that a general graphical representation be made of the horizontal intensity, partly because it would be more useful for the theory than the total intensity, partly because in most cases the horizontal intensity was the original observation and the total intensity was derived from it and the dip angle. It is therefore advisable to keep the elements of the horizontal force as they can be determined with extreme accuracy with present means. At any rate the observed horizontal intensity should never be suppressed when publishing the deduced total intensity without at least giving the dip angle employed in the calculations. If this is done, a person who wishes to use the horizontal intensity for the theory may either have, or will be able to reproduce, in an unbiased way, the originally observed numbers.

As interesting as it would be to base the theory of terrestrial magnetism on only horizontal needle observations, and thereby predict the vertical part or the inclination, it is by far too soon at the present time to do this. The deficiency of the currently available data does not allow one to omit the use of the vertical component. Basically, the theory has already been shown to be correct by demonstrating that the entire set of elements is described under the same principal approach.

22.

Although we are a priori certain that the series V, X, Y, and Z converge, nothing can be stated as to the degree of convergence. If the locations of the magnetic forces are limited to a moderate region near the center of the Earth, or if there were an equivalent distribution of the magnetic fluids in the Earth, the series would converge very rapidly. However, the closer the magnetic forces are to the Earth's surface, or the more irregular the distribution of the sources are, the more one needs to be prepared for a slow convergence.

In practice, absolute exactness is not attainable. One merely requires a degree of approximation that fits the circumstances. The slower the convergence, the greater the number of data points that will have to be used to obtain a certain level of accuracy.

Now, P' contains three terms and requires the knowledge of three coefficients $g^{1,0}$, $g^{1,1}$, $h^{1,1}$; P'' requires five coefficients, P''' seven, P^{IV} nine, etc. As we consider P', P'', P''', etc. as terms of the first, second, third order, etc., it is clear that if the calculation is to be extended to terms of order n, inclusive, the values of $n\,n + 2n$ coefficients must be determined. Thus, for example, 24 coefficients would be required for the fourth-order expansion.

Every given value of X, Y, or Z for given values of u and λ provides us an equation involving the coefficients. Thus, complete knowledge of the magnetic elements for each position on the Earth provides three equations. If one can assume that only terms up to the fourth order are important, then complete observations from eight points would be sufficient for the determination of all the coefficients, theoretically speaking. But one can hardly assume this, and the errors that are present in all observations together with neglecting

[39]T: These propositions are most interesting as they demonstrate that Gauss was right in claiming that measuring the horizontal component of the geomagnetic field is sufficient to describe the field. But it should be noted that this only holds if there are no external contributions.

the higher order terms would corrupt the results[40]. To decrease these unfavorable effects, the number of series of observations from stations well-distributed over the whole globe should be much greater than that of the unknown values. The unknown values should be derived from the observations by the least squares method. Although this is a simple and monotonous task, all equations are only linear; the amount of effort due to the great number of unknown values and equations will deter even the most courageous computer[41] from doing it in this form. This is especially true because the accuracy may be undermined by the presence of either incorrect observations or by accidental errors of calculation.

23.

There is another way to proceed, free from part of the above-mentioned difficulties and seemingly better adapted for a first attempt. We shall develop this procedure here, not withholding the caveats to its application. This method assumes knowledge of all three elements at points on a sufficient number of parallels, grouped in such a way that each parallel is divided into a sufficient number of equal parts.

One first needs to derive the numerical values of X, Y, and Z from the given elements in the usual form. The values of X, Y, and Z are then converted on each parallel into the forms

$$\begin{aligned} X &= k + k'\cos\lambda + K'\sin\lambda + k''\cos 2\lambda + K''\sin 2\lambda, \\ &\quad + k'''\cos 3\lambda + K'''\sin 3\lambda + \text{etc.} \end{aligned}$$

$$\begin{aligned} Y &= l + l'\cos\lambda + L'\sin\lambda + l''\cos 2\lambda + L''\sin 2\lambda, \\ &\quad + l'''\cos 3\lambda + L'''\sin 3\lambda + \text{etc.} \end{aligned}$$

$$\begin{aligned} Z &= m + m'\cos\lambda + M'\sin\lambda + m''\cos 2\lambda + M''\sin 2\lambda \\ &\quad + m'''\cos 3\lambda + M'''\sin 3\lambda + \text{etc.} \end{aligned}$$

One then obtains as many values for each of the coefficients k, l, m, k', etc. as there are parallels of latitude under consideration. According to theory on each parallel, $l = 0$; therefore the values of l resulting from this calculation furnish a measure of the degree of uncertainty still associated with the numbers taken as a basis.

From the equations[42]

$$\begin{aligned} k &= -g^{1,0}\,\frac{\mathrm{d}P^{1,0}}{\mathrm{d}u} - g^{2,0}\,\frac{\mathrm{d}P^{2,0}}{\mathrm{d}u} - g^{3,0}\,\frac{\mathrm{d}P^{3,0}}{\mathrm{d}u} - \text{etc.}, \\ m &= 2\,g^{1,0}P^{1,0} + 3\,g^{2,0}P^{2,0} + 4\,g^{3,0}P^{3,0} + \text{etc.}, \end{aligned}$$

the total number of which is double the number of the parallels used, we have to obtain (after substituting in $\mathrm{d}P^{1,0}/\mathrm{d}u$, etc. and in $P^{1,0}$, etc. the corresponding numerical values of u) by the least squares method as many of the coefficients $g^{1,0}$, $g^{2,0}$, $g^{3,0}$, etc. as are intended to be used.

In a similar manner the equations

$$\begin{aligned} -k' &= g^{1,1}\,\frac{\mathrm{d}P^{1,1}}{\mathrm{d}u} + g^{2,1}\,\frac{\mathrm{d}P^{2,1}}{\mathrm{d}u} + g^{3,1}\,\frac{\mathrm{d}P^{3,1}}{\mathrm{d}u} + \text{etc.}, \\ L' &= g^{1,1}\,\frac{P^{1,1}}{\sin u} + g^{2,1}\,\frac{P^{2,1}}{\sin u} + g^{3,1}\,\frac{P^{3,1}}{\sin u} + \text{etc.}, \\ m' &= 2\,g^{1,1}P^{1,1} + 3\,g^{2,1}P^{2,1} + 4\,g^{3,1}P^{3,1} + \text{etc.}, \end{aligned}$$

the number of which is three times larger than the number of parallels, serve to determine the coefficients $g^{1,1}$, $g^{2,1}$, $g^{3,1}$, etc., and the following

$$\begin{aligned} -K' &= h^{1,1}\,\frac{\mathrm{d}P^{1,1}}{\mathrm{d}u} + h^{2,1}\,\frac{\mathrm{d}P^{2,1}}{\mathrm{d}u} + h^{3,1}\,\frac{\mathrm{d}P^{3,1}}{\mathrm{d}u} + \text{etc.}, \\ -l' &= h^{1,1}\,\frac{P^{1,1}}{\sin u} + h^{2,1}\,\frac{P^{2,1}}{\sin u} + h^{3,1}\,\frac{P^{3,1}}{\sin u} + \cdots, \\ M' &= 2\,h^{1,1}P^{1,1} + 3\,h^{2,1}P^{2,1} + 4\,h^{3,1}P^{3,1} + \text{etc.}, \end{aligned}$$

determine the coefficients $h^{1,1}$, $h^{2,1}$, $h^{3,1}$, etc.

Furthermore, to determine the coefficients $g^{2,2}$, $g^{3,2}$, $g^{4,2}$ etc., the equations[43]

$$\begin{aligned} -k'' &= g^{2,2}\,\frac{\mathrm{d}P^{2,2}}{\mathrm{d}u} + g^{3,2}\,\frac{\mathrm{d}P^{3,2}}{\mathrm{d}u} + g^{4,2}\,\frac{\mathrm{d}P^{4,2}}{\mathrm{d}u} + \text{etc.}, \\ L'' &= 2\,g^{2,2}\,\frac{P^{2,2}}{\sin u} + 2\,g^{3,2}\,\frac{P^{3,2}}{\sin u} + 2\,g^{4,2}\,\frac{P^{4,2}}{\sin u} + \text{etc.}, \\ m'' &= 3\,g^{2,2}P^{2,2} + 4\,g^{3,2}P^{3,2} + 5\,g^{4,2}P^{4,2} + \text{etc.} \end{aligned}$$

are used. The coefficients of the succeeding higher orders are obtained in a similar manner.

24.

The main advantage of this method over that given in Chapter 22 is that the unknown values are separated into groups,

[40] In such a limited method, the effect would be least injurious if the eight points were distributed symmetrically on the surface of the Earth, that is to say, if they coincided with the corners of a cube inscribed inside the globe or represent a similar spatial distribution.

[41] T: Not an electronic computer but a human computer is meant here, a person knowledgable in doing the necessary calculations to determine the coefficients, for example; see Grier (2005) for more information on human computers.

[42] T: There is a misprint in Sabine's translation in the expression for m; her text reads $\rho^{3,0}$. The correct expression is $P^{3,0}$.

[43] T: There is a misprint in Gauss' original text that reads $\mathrm{d}P^{2,2}$ in the expression for L''. In Sabine's translation the correct expression $P^{2,2}$ is already used.

each of which is determined by itself. Thus, the calculation is greatly simplified. In the other method, intermingling all the unknown quantities makes their separation extremely difficult. On the other hand, there are disadvantages of this new method in that it is not based on direct observations. It relies on graphical representations, representing them only approximately in areas where we do not possess observations at all. In areas where observations are lacking, the representations are only conjectural and, to a certain extent, arbitrary, deviating far from reality. However, we must either postpone all trial calculations until we have a far more complete and accurate data set, or, with our present very sparse data, make a trial calculation. We should only expect a rough approximation, nothing more. A clear measure of success provides a comparison of the results of the calculations with those of actual observations. If these trial calculations show that this attempt has positive results, it will encourage future attempts by either method.

25.

Several years ago I attempted these calculations repeatedly. However, because of the inadequacy of the data, I was forced to step back. Nevertheless, I would have tried to finish an attempt provided my often-expressed wish for a representation of the horizontal intensity in general had been fulfilled. This missing map could not be substituted by a combination of existing incomplete general maps of dip and total intensity.

The publication of Sabine's Map of the total Intensity (in the *Seventh Report of the British Association for the Advancement of Science*) (Sabine, 1838) has now stimulated me to undertake and finish a new attempt, by the way only using the concepts mentioned in the previous chapter.

The data employed in the calculations are for the intensity from the above-mentioned map, for the declination from Barlow's map (*Phil. Trans., 1833*) (Barlow, 1833), and for the inclination from the map drafted by Horner[44] (*Physikalisches Wörterbuch, Volume VI*) (Muncke, 1845); data from 12 points on 7 parallels were used. Gaps in these maps could only be filled in a very delicate way.

Throughout the calculations it was noticed that the calculations needed to be extended to at least the fourth order, making the number of coefficients to be 24. In all probability the fifth-order terms might also be important[45]. However, in a first trial the values of k, m, k', etc. remain to be affected by the unavoidable influence of the many uncertainties of the data. The introduction of a still greater number of unknown values in the process of expansion would most likely not be profitable.

It should be mentioned that the intensities in Sabine's map are given in units that are in common use, for which the total intensity in London is 1.372. This unit is changed here for the determination of the coefficients and the supporting table given further down[46] in such a way that all values have been increased by a factor of 1 thousand. Thus, the intensity in London is 1372[47]. By the way, it is obvious that units for the intensity may be taken arbitrarily as the unit for μ may be considered arbitrary as well. They need to be made consistent. If further considerations are needed requiring μ to be determined in absolute values, it will only be necessary to multiply all the coefficients by the factor that is used to correct the intensities to absolute values.

26.

The numerical values of the 24 coefficients obtained by the first calculation, counting the longitude λ east of Greenwich, are as follows:

$g^{1,0}$	=	+925.782	$g^{2,2}$	=	+0.493
$g^{2,0}$	=	−22.059	$g^{3,2}$	=	−73.193
$g^{3,0}$	=	−18.868	$g^{4,2}$	=	−45.791
$g^{4,0}$	=	−108.855	$h^{2,2}$	=	−39.010
$g^{1,1}$	=	+89.024	$h^{3,2}$	=	−22.766
$g^{2,1}$	=	−144.913	$h^{4,2}$	=	+42.573
$g^{3,1}$	=	+122.936	$g^{3,3}$	=	+1.396
$g^{4,1}$	=	−152.589	$g^{4,3}$	=	+19.774
$h^{1,1}$	=	−178.744	$h^{3,3}$	=	−18.750
$h^{2,1}$	=	−6.030	$h^{4,3}$	=	−0.178
$h^{3,1}$	=	+47.794	$g^{4,4}$	=	+4.127
$h^{4,1}$	=	+64.112	$h^{4,4}$	=	+3.175.

These numbers, which may be considered as the *Elements of the Theory of Terrestrial Magnetism*, are used both here and in the supporting table to be introduced later. They were directly derived from the calculations, keeping the decimals. For anyone familiar with calculations, they will understand that these fractional parts are not significant, since we are far from being able to determine with certainty even the integers. However, it is important that the observations should be closely compared with one and the same definite system of elements. Thus there was no reason to truncate the numbers to integer values, as nothing would be gained in terms of easing the comparison between the computational results and observations.

[44]T: Johann Kaspar Horner (1774–1834), Swiss theologian, physicist, and astronomer. In an editorial note to volume XI of the *Physikalisches Wörterbuch*, Georg Wilhelm Muncke (1772–1847) mentioned that Horner was unable to finish his contribution on the magnetism of the Earth. The article was finished by Muncke himself (Muncke, 1845).

[45]T: Ludwig Friedrich Kämtz (1801–1867) provided a calculation of the fifth-order terms. In a letter to Edward Sabine, he claims that extending the calculations to the fifth order provides better results (Kämtz, 1854).

[46]T: These supporting tables are reproduced in the Appendix.

[47]T: A proper conversion factor to the SI system for this new unit is 34.9412 nT. That is, the magnetic intensity at London in the middle of the 19th century was 47 939 nT. For further details on the magnetic units used by Gauss, see Chapter 31 of the *Theory*.

27.

The expression for V, deduced from the above numbers, is as follows (for the sake of brevity e stands for cos u, and f for sin u)[48]:

$$V/R =$$

$$\begin{aligned}
&-1.977+937.103\,e+71.245\,ee-18.868\,e^3-108.855\,e^4\\
&+(64.437-79.518\,e+122.936\,ee+152.589\,e^3)\,f\cos\lambda\\
&+(-188.303-33.507\,e+47.794\,ee+64.112\,e^3)\,f\sin\lambda\\
&+(7.035-73.193\,e-45.791\,ee)\,ff\cos 2\lambda\\
&+(-45.092-22.766\,e-42.573\,ee)\,ff\sin 2\lambda\\
&+(1.396+19.774\,e)\,f^3\cos 3\lambda\\
&+(-18.750-0.178\,e)\,f^3\sin 3\lambda\\
&+4.127\,f^4\cos 4\lambda\\
&+3.175\,f^4\sin 4\lambda.
\end{aligned}$$

Further, the completely developed expressions for the three components of the magnetic force are sufficiently important to be presented here.

$$X =$$

$$\begin{aligned}
&(937.103+142.490\,e-56.603\,ee-435.420\,e^3)f\\
&+(-79.518+181.435\,e-298.732\,ee-368.808\,e^3\\
&\quad+610.357e^4)\cos\lambda\\
&+(-33.507+283.892\,e+259.349\,ee\\
&\quad-143.383\,e^3-256.448e^4)\sin\lambda\\
&+(-73.193-105.652\,e+219.579\,ee\\
&\quad+183.164\,e^3)f\cos 2\lambda\\
&+(-22.766+175.330\,e+68.098\,ee\\
&\quad-170.292\,e^3)f\sin 2\lambda\\
&+(19.774-4.188\,e-79.096\,ee)\,ff\cos 3\lambda\\
&+(-0.178+56.250\,e+0.716\,ee)\,ff\sin 3\lambda\\
&-16.508\,e\,f^3\cos 4\lambda\\
&-12.701\,e\,f^3\sin 4\lambda
\end{aligned}$$

$$Y =$$

$$\begin{aligned}
&(188.303+33.507\,e-47.794\,ee-64.112\,e^3)\cos\lambda\\
&+(64.437-79.518\,e+122.936\,ee-152.589\,e^3)\sin\lambda\\
&+(90.184+45.532\,e-85.146\,ee)\,f\cos 2\lambda\\
&+(14.070-146.386\,e-91.582\,ee)\,f\sin 2\lambda\\
&+(56.250+0.534\,e)\,ff\cos 3\lambda\\
&+(4.188+59.322\,e)\,ff\sin 3\lambda\\
&-12.701\,f^3\cos 4\lambda\\
&+16.508\,f^3\sin 4\lambda
\end{aligned}$$

[48]T: Höppner (2013) has pointed out that a sign error occurred in this series expression for the magnetic potential. We have corrected the sign in front of the value 42.573.

$$Z =$$

$$\begin{aligned}
&-24.593+1896.847\,e+400.343\,ee\\
&-75.471\,e^3-544.275e^4\\
&+(79.700-107.763\,e+491.744\,ee\\
&\quad-762.946\,e^3)\,f\cos\lambda\\
&+(-395.724-155.473\,e+191.176\,ee\\
&\quad+320.560\,e^3)\,f\sin\lambda\\
&+(34.187-292.772\,e-228.955\,ee)\,f\cos 2\lambda\\
&+(-147.439-91.064\,e+212.865\,ee)\,ff\sin 2\lambda\\
&+(5.584+98.870\,e)\,f^3\cos 3\lambda\\
&+(-75.000-0.890\,e)\,f^3\sin 3\lambda\\
&+20.635\,f^4\cos 4\lambda\\
&+15.876\,f^4\sin 4\lambda.
\end{aligned}$$

After these components have been calculated at a given point, we determine the basic components of the magnetic force in the usual form. Let δ be the declination, i the inclination, ψ the total, and ω the horizontal intensity. One first determines δ and ω by means of the formulas

$$X=\omega\cos\delta,\quad Y=\omega\sin\delta,$$

and then i and ψ by means of the following expressions:

$$\omega=\psi\cos i,\quad Z=\psi\sin i.$$

28.

As the formulas for X, Y, and Z together contain 71 terms, their immediate calculation is a considerable effort. Doing this for a large number of places is even more daunting, as without doing the same calculation twice there is no hope to avoid calculational errors. Little would be gained by dropping terms where the coefficients are less than 1 or even less than 10 units, for there would still be 65 terms. As the value of this work would be uncertain if it were not tested by a considerable number of actual observations, I did not shy away from constructing a supporting table, facilitating and shortening the calculations and also helping to reduce errors[49].

For the construction of the table, the values of the coefficients are expressed in the following form:

[49]Part of the calculations for this supporting table were performed by Doctor Goldschmidt. T: Carl Wolfgang Benjamin Goldschmidt (1807–1851) was a professor of astronomy at the University of Göttingen. He was a student and later the assistant to Gauss at the astronomical observatory in Göttingen. Goldschmidt was also one of the academic teachers of Bernhard Riemann.

$$X = a^0 + a'\cos(\lambda + A') + a''\cos(2\lambda + A'') + a'''\cos(3\lambda + A''') + a^{\mathrm{IV}}\cos(4\lambda + A^{\mathrm{IV}})$$

$$Y = b'\cos(\lambda + B') + b''\cos(2\lambda + B'') + b'''\cos(3\lambda + B''') + b^{\mathrm{IV}}\cos(4\lambda + B^{\mathrm{IV}})$$

$$Z = c^0 + c'\cos(\lambda + C') + c''\cos(2\lambda + C'') + c'''\cos(3\lambda + C''') + c^{\mathrm{IV}}\cos(4\lambda + C^{\mathrm{IV}}).$$

The first table contains those parts of X and Z that are independent of λ. In the four following tables are given the values of the auxiliary angles A', A'', etc. and the logarithms of a', a'', etc., in each case for several degrees of latitude $\phi = 90° - u$. The table is placed at the end of this article[50].

As an example the calculation for Göttingen is placed here.

For latitude $+51°32'$ one finds the following from the tables:

a^0	=	+500.8				c^0	=	+1465.2
$\log a'$	=	2.28980	$\log b'$	=	2.1890	$\log c'$	=	2.20204
$\log a''$	=	1.79403	$\log b''$	=	2.03220	$\log c''$	=	2.12777
$\log a'''$	=	1.32522	$\log b'''$	=	1.46845	$\log c'''$	=	1.43199
$\log a^{\mathrm{IV}}$	=	0.59391	$\log b^{\mathrm{IV}}$	=	0.70016	$\log c^{\mathrm{IV}}$	=	0.59091
A'	=	249° 30′	B'	=	358° 24′	C'	=	105° 44′
A''	=	311 45	B''	=	64 50	C''	=	165 15
A'''	=	234 10	B'''	=	318 13	C'''	=	42 22
A^{IV}	=	142 26	B^{IV}	=	232 26	C^{IV}	=	322 26

For the longitude 9° 56.5′, the contributions to X, Y, and Z are found as follows:

X	Y	Z
+500.8		+1465.2
−35.71	+152.89	−68.99
+54.76	+9.92	−133.67
−2.21	+28.77	+8.27
−3.92	+0.19	+3.90
$X = +513.72$	$Y = +191.77$	$Z = +1274.71$

The further calculation then results in

$\delta = +20°28'$ $\quad \log \omega = 2.73907$

$i = +6643$

$\psi = 1387.6$ or in the unit commonly used

$\psi = 1.3876.$

29.

The following table[51] compares the results of our formulas with observations at 91 stations taken from all parts of the Earth. As the three maps from which we have taken the data for our calculation are intended to represent the phenomena for the most recent epoch, we have included in our comparison only very recent observations. By preference we have taken observations at those stations where all three elements of magnetism were measured. We are not presently requiring that the observations are taken simultaneously as this would reduce our priceless data[52] to a very small number.

Concerning the observations used in the comparison, I add the following notes:

The determinations of the intensity are taken mostly from Sabine's *Report on the Variations of the Magnetic Intensity* (from the above-mentioned *Seventh Report of the British Association for the Advancement of Science*).

The large number of observations from the Russian Empire and neighboring parts of China we owe to Hansteen[53] (*Poggendorff's Annals*) (Hansteen, 1833), Erman[54] (*Reise um die Erde* and manuscript communications) (Erman, 1841), von Humboldt[55] (*Voyage aux régions équinoxiales*, Part 13) (Humboldt and Bonpland, 1831), Fuss[56] (*Mémoires de l'Academie des Sciences de St. Petersbourg, Sixième serie*) (von Fuss, 1838), Fedor[57] (Communicated in manuscript through Struve) (Fedorov, 1838), Reinke[58] (*Observations Météorologiques et Magnétiques, faties dans l'étendue de l'Empire de Russie, redigées par* A. T. Kupffer Nr. II) (Reinke, 1837).

At the following places we use mean values from the determinations of several observers. The differences between them are sometimes greater than would be caused by annual changes:

[50]T: The table mentioned here is part of the Appendix of the 1839 issue of the *Resultate*.

[51]T: In the original paper four tables are used, not just one as mentioned by Gauss here.

[52]T: The German word reads *Besitz*. Gauss regarded the magnetic observations as a real treasure here.

[53]T: Christopher Hansteen (1784–1873), Norwegian astronomer and physicist.

[54]T: Georg Adolf Erman (1806–1877), German physicist; son of Paul Erman and father of Johann Peter Adolf Erman, a well-known Egyptologist.

[55]T: Alexander von Humboldt (1769–1859), German scientist and diplomat.

[56]T: Georg Albert von Fuss (1806–1854), Russian astronomer; son and grandson of the mathematicians Paul Heinrich and Nicolaus von Fuss.

[57]T: Vasilij Fedorovie Fedorov (1802–1855), Russian astronomer; Friedrich Georg Wilhelm Struve (1793–1864) was a German astronomer and is well known for his work on double stars.

[58]T: Julii Maksimovich Reinke (1811–1865), Russian mining engineer; Reinke graduated from the St. Petersburg Mining Institute in 1833 and became the first director and observer (1836–1838) of the Catherinenburg (now Yekaterinburg) "magnetic house".

		Latitude	Longitude	Computed	Declination observed	Difference
1	Spitzbergen	+79°50′	11°40′	+26°31′	+25°12′	+1°19′
2	Hammerfest	70 40	23 46	+12 23	+10 50	+1 33
3	Magn. Pol. n. Ross	70 05	263 14	−22 23		
4	Reikiavik	64 08	338 05	+40 12	+43 14	−3 02
5	Jakutsk	62 01	129 45	+0 05	+5 50	−5 45
6	Porotowsk	62 01	131 50	+0 04	+4 46	−4 42
7	Nochinsk	61 57	134 57	−0 03	+2 11	−2 14
8	Tschernoljes	61 31	136 23	0 00	+3 30	−3 30
9	Petersburg	59 56	30 19	+6 47	+6 44	+0 03
10	Christiania	59 54	10 44	+19 55	+19 50	+0 05
11	Ochotsk	59 21	143 11	−0 18	+2 18	−2 36
12	Tobolsk	58 11	68 16	−7 19	−10 29	+3 10
13	Tigil Fluss	58 01	158 15	−4 20	−4 06	−0 14
14	Sitka	57 03	224 35	−28 45	−28 19	−0 26
15	Tara	56 54	74 04	−7 44	−9 36	+1 52
16	Catharinenburg	56 51	60 34	−5 20	−6 18	+0 58
17	Tomsk	56 30	85 09	−7 21	−8 34	+1 13
18	Nishny Nowgorod	56 19	43 57	+1 10	−0 27	+1 37
19	Krasnojarsk	56 01	92 57	−5 49	−6 40	+0 51
20	Kasan	55 48	49 07	−1 07	−2 22	+1 15
21	Moskwa	55 46	37 37	+4 26	+3 02	+1 24
22	Königsberg	54 43	20 30	+14 15	+13 22	+0 53
23	Barnaul	53 20	83 56	−7 00	−7 25	+0 25
24	Uststretensk	53 20	121 51	+1 29	+4 21	−2 52
25	Gorbizkoi	53 06	119 09	+1 05	+2 54	−1 49
26	Petropaulowsk	53 00	158 40	−3 34	−4 06	+0 32
27	Uriupina	52 47	120 04	+1 16	+4 04	−2 48
28	Berlin	52 30	13 24	+18 31	+17 05	+1 26
29	Pogromnoi	52 30	111 03	−0 38	+0 18	−0 56
30	Irkuzk	52 17	104 17	−2 27	−1 38	−0 49
31	Stretensk	52 15	117 40	+0 54	+2 52	−1 58
32	Stepnoi	52 10	106 21	−1 52	−1 08	−0 44
33	Tschitanskoi	52 01	113 27	0 00	+1 13	−1 13
34	Nerchinsk Stadt	51 56	116 31	+0 42	+2 53	−2 11
35	Werchneudinsk	51 50	107 46	−1 26	−0 24	−1 02
36	Orenburg	51 45	55 06	−2 48	−3 22	+0 34
37	Argunskoi	51 33	119 56	+1 22	+3 44	−2 22
38	Göttingen	51 32	9 56	+20 28	+18 38	+1 50
39	London	51 31	359 50	+25 37	+24 00	+1 37
40	Nerchinsk Bergw.	51 19	119 37	+1 20	+4 06	−2 46
41	Tschindant	50 34	115 32	+0 34	+2 14	−1 40
42	Charazaiska	50 29	104 44	−2 09	−2 27	+0 18
43	Zuruchaitu	50 23	119 03	+1 18	+3 11	−1 53
44	Troizkosawsk	50 21	106 45	−1 34	−0 12	−1 22
45	Abagaitujewskoi	49 35	117 50	+1 08	+2 54	−1 46
46	Altanskoi	49 28	111 30	−0 16	+0 48	−1 04
47	Menschinskoi	49 26	108 55	−0 56	+0 12	−1 08
48	Paris	48 52	2 21	+24 06	+22 04	+2 02
49	Chunzal	48 13	106 27	−1 30	−1 06	−0 24
50	Urga	47 55	106 42	−1 26	−1 16	−0 10

		Inclination			Intensity	
	Computed	observed	Difference	Computed	observed	Difference
1	+82°1′	+81°11′	+0°50′	1.599	1.562	+0.037
2	77 19	77 15	+0 04	1.545	1.506	+0.039
3	88 48	90 00	−1 12	1.717		
4	80 40	77 00	+3 40	1.527		
5	74 36	74 18	+0 18	1.661	1.697	−0.036
6	74 27	74 00	+0 27	1.658	1.721	−0.063
7	74 12	73 37	+0 35	1.653	1.713	−0.060
8	73 48	73 08	+0 40	1.648	1.700	−0.052
9	70 25	71 03	−0 38	1.469	1.410	+0.059
10	72 04	72 07	−0 03	1.456	1.419	+0.037
11	71 36	70 41	+0 55	1.621	1.615	+0.006
12	70 13	71 01	−0 48	1.575	1.557	+0.018
13	69 55	68 28	+1 27	1.583	1.577	+0.006
14	76 30	75 51	+0 39	1.697	1.731	−0.034
15	69 46	70 28	−0 42	1.586	1.575	+0.011
16	68 24	69 16	−0 52	1.535	1.523	+0.012
17	70 33	70 55	−0 22	1.613	1.619	−0.006
18	67 09	68 41	−1 32	1.469	1.442	+0.027
19	70 24	71 00	−0 36	1.638	1.657	−0.019
20	67 13	68 25	−1 12	1.477	1.433	+0.044
21	66 45	68 57	−2 12	1.446	1.404	+0.042
22	67 19	69 26	−2 07	1.410	1.365	+0.045
23	67 50	68 10	−0 20	1.591	1.605	−0.014
24	68 32	68 11	+0 21	1.609	1.656	−0.047
25	68 32	68 22	+0 10	1.611	1.660	−0.049
26	65 31	63 50	+1 41	1.521	1.489	+0.032
27	68 17	67 53	+0 24	1.612	1.667	−0.055
28	66 45	68 07	−1 22	1.391	1.367	+0.024
29	68 25	68 08	+0 17	1.616	1.640	−0.024
30	68 17	68 14	+0 03	1.616	1.647	−0.031
31	67 55	67 38	+0 17	1.606	1.649	−0.043
32	68 12	68 10	+0 02	1.615	1.663	−0.048
33	67 56	67 42	+0 14	1.609	1.668	−0.059
34	67 43	67 11	+0 32	1.604	1.635	−0.031
35	67 55	68 06	−0 11	1.612	1.657	−0.045
36	63 14	64 44	−1 30	1.461	1.432	+0.029
37	67 10	66 54	+0 16	1.595	1.655	−0.060
38	66 43	67 56	−1 13	1.388	1.357	+0.031
39	68 54	69 17	−0 23	1.410	1.372	+0.038
40	66 59	66 33	+0 26	1.593	1.617	−0.024
41	66 35	66 32	+0 3	1.592	1.650	−0.058
42	66 45	66 56	−0 11	1.599	1.643	−0.044
43	66 12	66 13	−0 01	1.584	1.626	−0.042
44	66 38	66 19	+0 19	1.597	1.642	−0.045
45	65 33	64 48	+0 45	1.577	1.583	−0.006
46	65 46	65 20	+0 26	1.585	1.619	−0.034
47	65 48	65 31	+0 17	1.587	1.630	−0.043
48	66 45	67 24	−0 39	1.389	1.348	+0.041
49	64 42	64 29	+0 13	1.574	1.612	−0.038
50	64 25	64 04	+0 21	1.571	1.583	−0.012

		Latitude	Longitude	Computed	Declination observed	Difference
51	Astrachan	+46°20′	48°0′	+1°40′	+1°12′	+0°28′
52	Chologur	46 00	110 34	−0 20	+0 49	−1 09
53	Ergi	45 32	111 25	−0 06	+1 07	−1 13
54	Mailand	45 28	9 09	+20 56	+18 33	+2 23
55	Sendschi	44 45	110 26	−0 20	+0 30	− 0 50
56	Batchay	44 21	112 55	+0 16	+0 59	−0 43
57	Scharabudurguna	43 13	114 06	+0 32	+0 46	−0 14
58	Neapel	40 52	14 06	+18 53	+15 20	+3 33
59	Chalgan	40 49	114 58	+0 42	+1 13	−0 31
60	Pekin	39 54	116 26	+0 58	+1 48	−0 50
61	Terceira	38 39	332 47	+25 17	+24 18	+0 59
62	San Francisco	37 49	237 35	−16 22	−14 55	−1 27
63	Port Praya	14 54	336 30	+16 17	+16 30	−0 13
64	Madras	13 04	80 17	−4 01		
65	Galapagos Insel	−0 50	270 23	−8 57	−9 30	+0 33
66	Ascension	7 56	345 36	+14 37	+13 30	+1 07
67	Pernambuco	8 04	325 09	+5 58	+5 54	+0 04
68	Callao	12 04	285 46	−9 06	−10 00	+0 54
69	Keeling Insel	12 05	96 55	+0 23	+1 12	−0 49
70	Bahia	12 59	321 30	+3 12	+4 18	−1 06
71	St. Helena	15 55	354 17	+18 48	+18 00	+0 48
72	Otaheite	17 29	210 30	−5 45	−7 34	+1 49
73	Mauritius	20 09	57 31	+11 09	+11 18	−0 09
74	Rio de Janeiro	22 55	316 51	−1 11	−2 08	+0 57
75	Valparaiso	33 02	288 19	−13 45	−15 18	+1 33
76	Sydney	33 51	151 17	−7 51	−10 24	+2 33
77	Vorg. d. g. Hoffn.	34 11	18 26	+27 24	+28 30	−1 06
78	Monte Video	34 53	303 47	−11 23	−12 00	+0 37
79	K. Georgs Sund	35 02	117 56	+5 12	+5 36	−0 24
80	Neu Seeland	35 16	174 00	−11 10	−14 00	+2 50
81	Concepcion	36 42	286 50	−14 43	−16 48	+2 05
82	Blanco Bay	38 57	298 01	−12 57	−15 00	+2 03
83	Valdivia	39 53	286 31	−16 13	−17 30	+1 17
84	Chiloe	41 51	286 04	−16 56	−18 00	+1 04
85	Hobarttown	42 53	147 24	−5 51	−11 06	+5 15
86	Port Low	43 48	285 58	−17 32	−19 48	+2 16
87	Port San Andres	46 35	284 25	−19 04	−20 48	+1 44
88	Port Desire	47 45	294 05	−16 52	−20 12	+3 20
89	R. Santa Cruz	50 07	291 36	−18 23	−20 54	+2 31
90	Falkland Insel	51 32	301 53	−15 16	−19 00	+3 44
91	Port Famine	53 38	289 02	−20 28	−23 00	+2 32

	Computed	Inclination observed	Difference	Computed	Intensity observed	Difference
51	+56°59′	+59°58′	−2°59′	1.358	1.334	+0.024
52	62 31	61 54	+0 37	1.545	1.580	−0.035
53	61 58	61 22	+0 36	1.539	1.559	−0.020
54	62 13	63 48	−1 35	1.331	1.294	+0.037
55	61 15	60 42	+0 33	1.529	1.530	−0.001
56	60 46	60 18	+0 28	1.520	1.553	−0.033
57	59 32	59 03	+0 29	1.502	1.538	−0.036
58	56 26	58 53	−2 27	1.271	1.271	0.000
59	56 51	56 17	+0 34	1.465	1.459	+0.006
60	55 43	54 49	+0 54	1.448	1.453	−0.005
61	68 34	68 06	+0 28	1.469	1.457	+0.012
62	64 14	62 38	+1 36	1.592	1.591	+0.001
63	45 51	46 03	−0 12	1.168	1.156	+0.012
64	4 14	6 52	−2 38	1.038	1.031	+0.007
65	13 24	9 29	+3 55	1.085	1.069	+0.016
66	5 32	1 39	+3 53	0.813	0.873	−0.060
67	13 02	13 13	−0 11	0.909	0.914	−0.005
68	−3 23	−7 03	+3 40	0.994		
69	−39 19	−38 33	−0 46	1.161		
70	+3 59	+5 24	−1 25	0.883	0.871	+0.012
71	−14 55	−18 01	+3 06	0.808	0.836	−0.028
72	−27 26	−30 26	+3 00	1.113	1.094	+0.019
73	−54 08	−54 01	−0 07	1.060	1.144	−0.084
74	−14 49	−13 30	−1 19	0.879	0.878	+0.001
75	−37 56	−39 07	+1 11	1.094	1.176	−0.082
76	−58 11	−62 49	+4 38	1.667	1.685	−0.018
77	−51 04	−52 35	+1 31	0.981	1.014	−0.033
78	−35 34	−35 40	+0 06	1.022	1.060	−0.038
79	−62 39	−64 41	+2 02	1.658	1.709	−0.051
80	−54 46	−59 32	+4 46	1.616	1.591	+0.025
81	−42 49	−44 13	+1 24	1.147	1.218	−0.071
82	−42 01	−41 54	−0 07	1.103	1.113	−0.010
83	−46 13	−46 47	+0 34	1.145	1.238	−0.093
84	−48 14	−49 26	+1 12	1.227	1.313	−0.086
85	−66 57	−70 35	+3 38	1.894	1.817	+0.077
86	−50 04	−51 20	+1 16	1.257	1.326	−0.069
87	−53 00	−54 14	+1 14	1.310		
88	−51 22	−52 43	+1 21	1.263	1.359	−0.096
89	−53 49	−55 16	+1 27	1.321	1.425	−0.104
90	−52 46	−53 25	+0 39	1.276	1.367	−0.091
91	−57 38	−59 53	+2 15	1.424	1.532	−0.108

	(12) Tobolsk	
Declination	Hansteen, 1828	−9°58′
	Erman, 1828	−9 47
	Fuss, 1830	−11 52
	Fedor, 1833	−10 20
Inclination	Erman, 1828	71 07
	von Humboldt, 1829	70 56
	Fuss, 1830	71 01
	Fedor, 1833	71 02
	(16) Catharinenburg	
Declination	Hansteen, 1828	−6°27′
	Erman, 1828	−7 23
	Reinke, 1836	−5 05
Inclination	Erman, 1828	69 24
	von Humboldt, 1829	69 06
	Füs, 1830	69 19
	Fedor, 1832	69 15
	(17) Tomsk	
Declination	Hansteen, 1828	−8°32′
	Erman, 1829	−8 36
Inclination	Erman, 1829	70 59
	Fuss, 1830	70 51
	(18) Nishny Novogorod	
Declination	Erman, 1828	−0°46′
	Fuss, 1830	−0 08
	(19) Krasnojarsk	
Declination	Hansteen, 1829	−6°43′
	Erman, 1829	−6 37
	Fedor, 1835	−7 26
Inclination	Erman, 1829	70 53
	Fedor, 1835	71 08
	(20) Kasan	
Inclination	Erman, 1828	68°21′
	von Humboldt, 1829	68 27
	Fuss, 1830	68 26
	(21) Moskwa	
Declination	Hansteen, 1828	+3°03′
	Erman, 1828	+3 01
Inclination	Erman, 1828	68 58
	von Humboldt, 1829	68 57
	(30) Irkuzk	
Declination	Hansteen, 1829	−1°37′
	Erman, 1829	−1 52
	Fuss 1830	−1 25
Inclination	Erman, 1829	68 07
	Fuss, 1830	68 15
	Fuss, 1832	68 20
	(36) Orenburg	
Inclination	von Humboldt, 1829	64°41′
	Fedor, 1832	64 47
	(44) Troizkosawsk	
Declination	Hansteen, 1829	+0°05′
	Erman, 1829	+0 33
	Fuss, 1830	−0 01
Inclination	Erman, 1829	66 14
	Fuss, 1830	66 24

Most of the measurements from the Southern Hemisphere are from Captains King[59] and FitzRoy[60], taken from a short paper by Sabine (*Magnetic Observations made during the Voyages of H. B. M.'s ships Adventure and Beagle*, 1826–1836) (Sabine, 1838).

The determinations for the remaining single stations are taken partly from the above-named sources; from the remaining I still mention the following:

1. Spitsbergen. Observer Sabine 1823 (from his *Account of Experiments to determine the Figure of the Earth*).

2. Hammerfest. The declination and inclination are the means of the determinations of Sabine 1823 (from the referenced works) and of Parry[61] 1827 (from his *Narrative of an Attempt to reach the North Pole*) (Parry, 1828).

3. Magnetic Pole, after Ross 1831 (*Philosophical Transactions* 1834) (Ross, 1834).

4. Reykiavik after observations by Lottin[62] 1836 (*Voyage en Islande*) (Lottin, 1838).

28. Berlin after Encke 1836 (*Astronomisches Jahrbuch* 1839) (Encke, 1837).

38. Göttingen. The declination is for 1 October 1835 (*Resultate für 1836*, page 39) (Gauss and Weber, 1837a); the inclination is reduced to the same epoch by interpolation between von Humboldt's observation in 1827 and Forbes'[63] in 1837 (Encke, 1840).

[59]T: Phillip Parker King (1791–1856), English seafarer and surveyor; commander of the HMS *Adventure*.

[60]T: Robert FitzRoy (1805–1865), English meteorologist and seafarer; he commanded the HMS *Beagle* during Charles Darwin's voyage.

[61]T: William Edward Parry (1790–1855), English polar researcher.

[62]T: Victorien Pierre Lottin de Laval (1810–1903), French archeologist and traveler.

[63]T: James David Forbes (1809–1869), Scottish physicist.

39. London, based on observations communicated in manuscript by Captain Ross for the declination; for the inclination by Phillips, Fox, Ross, Johnson[64], and Sabine; the mean epoch for the declination April 1838, for the inclination May 1838 (Sabine, 1839).

48. Paris. For 1835 from the *Annuaire* for 1836 (Le Bureau des Longitudes, 1836).

54. Milan. 1837 from Kreil[65], communicated by him in manuscript (Kreil, 1839).

58. Naples. 1835 from observations by Sartorius and Listing[66]. The intensity, an absolute measure, has been reduced to the common unit by the application of the factor given in Chapter 31.

64. Madras. 1837 from observations by Taylor[67], taken from the *Journal of the Asiatic Society of Bengal*, May 1837 (Taylor, 1837).

30.

When judging the differences between calculation and observation shown in the tabular comparison, one should take into account that almost all the observations have both errors of measurements and of accidental anomalies of the magnetic force itself. All the measurements were also not made in the same year[68]. On the other hand, our formulas do not include components beyond the fourth order, and those of the following orders may still be significant. Given these circumstances, the agreement between calculation and observation appears to be as satisfactory as might be expected from a first effort.

Our expression for V/R may be regarded as being realistic, at least for its more important contributions. It appears worthwhile to form a graphical representation of the course of the numerical values of this function for matters of visualization. A map has been drawn by Dr. Goldschmidt consisting of three parts. The first uses a Mercator projection representing the whole globe between the parallels 70° northern and 70° southern latitude. The other two maps are polar projections, extending to latitude 65°. Corrections and additions, which will undoubtedly arise from a new calculation based on more perfect observations, will cause alterations of these lines, particularly in the high southern latitudes. However, no important changes to the general form of the system of lines are expected without major changes in the expression for V/R. We are thus led to the important result that the system of lines of equal values of V on the surface of the Earth is actually predicted by the simplest type described in Chapter 13, and consequently there are only *two magnetic poles* on the Earth, apart from the possible case of local exceptions discussed in Chapter 13.

Exact computation, based on our magnetic elements, provides these two pole positions:

1. At 73°35′ northern latitude, 264°21′ longitude east from Greenwich; the value of the total intensity is 1.701 in the units in common usage.

2. At 72°35′ southern latitude, 152°30′ longitude, the total intensity is 2.253.

At the first of these points V/R reaches its largest value, +895.86, at the second the smallest value −1030.24.

According to Ross's observations, the north magnetic pole is located 3°30′ to the south of the position resulting from our calculations. The calculation also indicates, as inspection of the comparing table shows, that at this place the direction of the magnetic force differs by 1°12′ from the observation. We expect a considerably greater displacement of the position of the south magnetic pole. As at Hobart, which is the nearest station to this pole, the calculations give too low of a dip angle by 3°38′, as far as the observations can be relied upon. It therefore seems probable that the actual south magnetic pole is considerably northward of the position given by our calculation. It should be looked for at about 66° latitude and 146° longitude.

[64]T: John Phillips (1800–1874), English geologist; Robert Were Fox (1789–1877), British geologist; Edward John Johnson (1784–1853), Captain and first Superintendent of the Royal Navy Compass Department.

[65]T: Karl Kreil (1798–1862), Austrian meteorologist and astronomer.

[66]T: Wolfgang Sartorius Baron of Waltershausen (1809–1876), German geologist; he was a close friend and collaborator of Gauss. Sartorius published the first biography on Gauss. Besides his work on geological and mineralogical studies, he is well known as the translator of Adam Smith's *Wealth of Nations* into German. Johann Wolfgang von Goethe was his godfather. Johann Benedict Listing (1808–1882) was a German mathematician, who made important discoveries in mathematical topology, inspired by his mentor Carl Friedrich Gauss. We have not been able to trace down any document with the mentioned observations.

[67]T: Thomas Glanville Taylor (1804–1848), director of the Madras Observatory and independent discoverer of the Great Comet of 1831. Taylor was astronomer for the Honourable East India Company.

[68]Examples on the important disagreement between different observers at one and the same place are already given in the previous chapter. Some more can be added here with differences much larger than accountable to calculation, but indicating regular yearly changes. In 1829 the dip at Valparaiso was −40°11′ according to King, in 1835 −38°3′ according to FitzRoy. In Mauritius the intensity was 1.096 in 1818, according to Freycinet, 1.192 in 1836, according to FitzRoy. The differences are still greater at Otaheite, where Erman found an intensity of 1.172 in 1830, but FitzRoy 1.017 in 1835. Otaheite is thus a station of the highest importance for the future improvement of the elements as the difference exceeds the greatest difference found between computed and observed intensities in our 86 comparisons. T: Louis Claude Desaulces de Freycinet (1779–1842), French explorer.

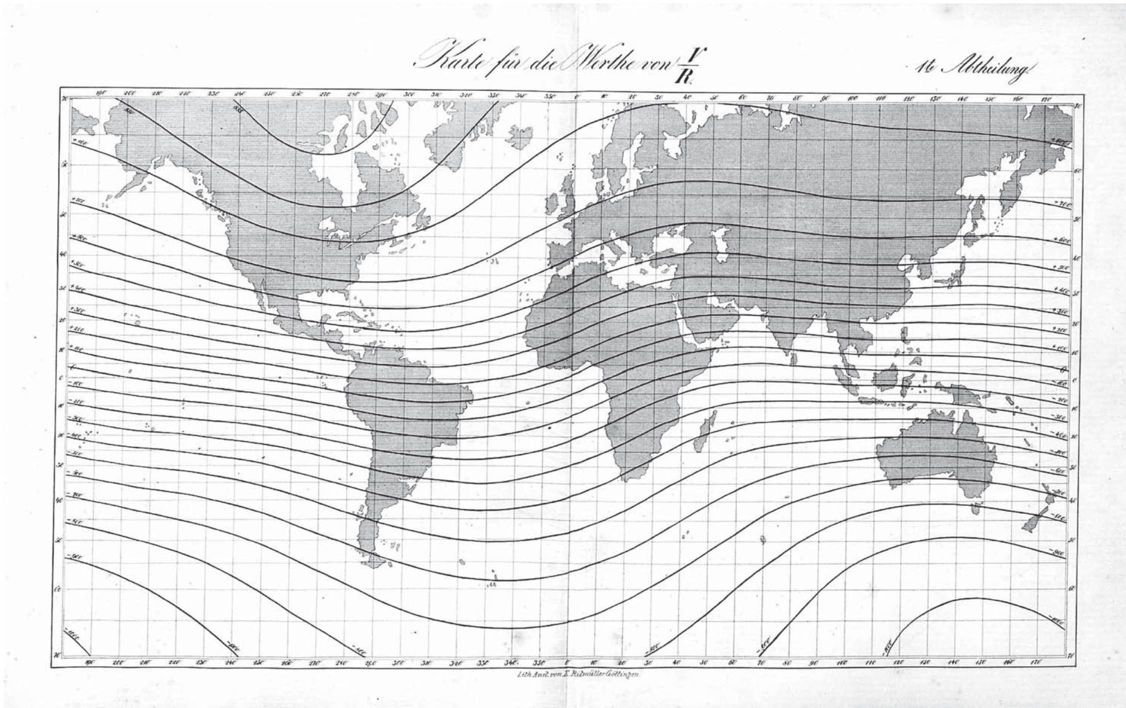

Figure 3. Isocontour lines of the ratio V/R. A Mercator projection of the Earth's surface is used between latitudes 70° north and 70° south. This figure is referenced in the original text, but it was not included in the article itself. The lithography was prepared by Johann Eduard Ritmüller (1805–1869), a well-known illustrator and lithographer, who founded the *lithographische Anstalt* in Göttingen in 1831, where many of the lithographs of C. F. Gauss and Wilhelm Weber were produced. The figure was published in the annex volume *10 Tafeln zu Gauss und Weber, Resultate: Jahrgang 1838*, Weidmannsche Buchhandlung, Leipzig, 1839. The heading reads: "Map of the values of V/R. First part". At the bottom of the figure a hint to the lithographer company Ritmüller is given. Source: Library of the Technische Universität Braunschweig.

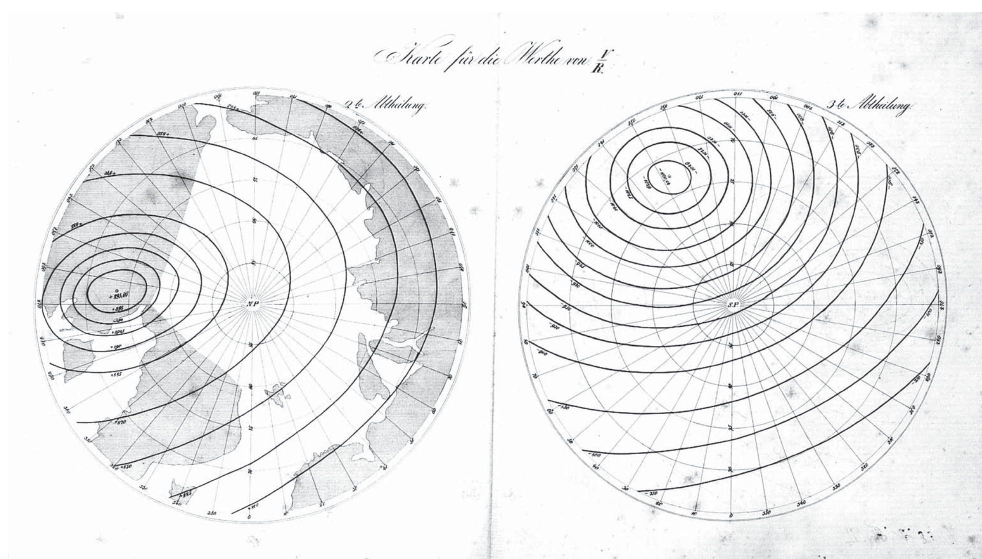

Figure 4. The system of isocontour lines in the northern (left) and southern (right) polar regions. Like Fig. 3 this figure is referenced in the original text, but it was not included in the article itself. The figure was published in the annex volume *10 Tafeln zu Gauss und Weber, Resultate: Jahrgang 1838*, Weidmannsche Buchhandlung, Leipzig, 1839. The heading reads: "Map of the values of V/R. Second part, third part". Source: Library of the Technische Universität Braunschweig.

31.

Though one should pay some attention to the two points on the Earth's surface where the horizontal force vanishes and are called the magnetic poles, because of their importance in shaping the appearance of the horizontal force on the Earth's surface, one must be careful not to attribute too much significance to them. The chord that connects these two points has no significance, and it would be a great mistake to call this straight line the *magnetic axis*. The only way of giving a generally valid meaning to the idea of a magnetic axis of a body was discussed in Chapter 5 of the *Intensitas Vis Magneticae*, where it is understood to denote the straight line on which the moment of the free magnetism contained in the body maximizes. In order to determine the position of the thus defined magnetic axis of the Earth and as well the moment of the Earth's magnetism in relation to this same axis, we only require a knowledge of the elements of the first order of V, as noted above in Chapter 17. According to our terms in Chapter 26, $P' = +925.782 \cos u + 89.024 \sin u \cos \lambda - 178.744 \sin u \sin\lambda$, and thus $-925.782\,R^3$, $-89.024\,R^3$, $+178.744\,R^3$ are the moments of terrestrial magnetism with respect to the axis of the Earth and the two radii for longitudes 0° and 90°. The direction of Earth's axis is assumed towards the north pole, and the negative sign of the corresponding moment implies that the magnetic axis makes an obtuse angle with it, or that the magnetic north pole points towards the south. The direction of the magnetic axis is found parallel to the Earth's diameter at 77° 50′ northern latitude, and 296° 29′ longitude to 77° 50′ southern latitude, 116° 29′ longitude. The magnetic moment in relation to this axis is = 947.08 R^3. It should be remembered that our elements are based on the unit of intensity that is a thousandth part of the unit in common use. In order to obtain the reduction to the absolute unit established in the *Intensitas Vis Magneticae*, we must remark that in the latter work the horizontal intensity in Göttingen on 19 July 1834 was = 1.7748. This combined with the dip 68° 1′ gives a total intensity of 4.7414, while the total intensity according to the unit employed above was 1357. Thus, the reducing factor is = 0.0034941, and the magnetic moment of the Earth, expressed in absolute units, is

$$= 3.3092\,R^3.$$

As the millimeter is the unit of length employed in the above absolute unit for the Earth's magnetic force, R must also be given in millimeters. As the ellipticity of the Earth is not taken into account here anyhow, it is sufficient to assume R to be the radius of a circle whose circumference is 40 000 millions of millimeters. Hence the above magnetic moment will be expressed by a number whose logarithm is 29.93136, or is 853 800 quadrillion. Using the same absolute unit, the magnetic moment of a pound weight magnetic bar was found by experiments made in the year 1832 (*Intensitas*, Chapter 21) to be 100 877 000. The magnetic moment of the Earth is therefore 8464 trillion times greater. Thus 8464 trillions of such magnetic bars, with aligned magnetic axes, would be required to replace the magnetic effect of the Earth in the exterior space. If the magnetism of the Earth were uniformly distributed throughout its volume, this would correspond to eight such bars (more exactly 7.831) on every cubic meter. Described in this way this result preserves its meaning even if not considering the Earth to be an actual magnet, but attributing the terrestrial magnetism to persistent galvanic currents within the Earth. But if we consider the Earth as a real magnet, we are obliged to ascribe, *on the average*, to each portion of it with the size of an eighth of a cubic meter at least[69] as great a force of magnetism as that contained in one of the above-mentioned bars. Such a result would be unexpected by any physicist[70].

32.

The actual distribution of the magnetic fluids in the Earth necessarily remains unknown. In fact, according to a general theorem that has been already mentioned in Chapter 2 of the *Intensitas*, and will be discussed in greater detail on another occasion, we may substitute any distribution of the magnetic fluids in the interior of a body by a distribution on the surface of this physical body. This will leave the effect on every point of the external space precisely the same, whereby it may be easily concluded that *one and the same* action on all external space may be deduced from an infinite number of *different* distributions of the magnetic fluids in the interior.

In contrast to this we can specify that fictitious distribution on the surface of the Earth, which will be equivalent to the actual distribution within the interior with respect to the corresponding forces in the exterior. And because of the spherical form of the Earth we can do this in a very simple manner. That is to say the density of the magnetic fluid in each point on the Earth's surface, i.e., the quantum[71] of the fluid that corresponds to the unit of surface is expressed by

[69]In as far as we should not assume the magnetic axes to be aligned to each other everywhere, the more random the situation, the greater the average force must be to produce the same total magnetic moment.

[70]T: The magnetic moment derived by Gauss, 0.94708 R^3 in *Humboldt unit*, corresponds to a dipole moment of 8.5382×10^{15} Tm^3 or 8.5382×10^{22} Am^2 when using the conversion factor 3.49412×10^4 nT and an Earth radius of 6366.2 km as Gauss did. The mean magnetization is thus 79 A/m, corresponding to a specific magnetic moment of 1.4×10^{-2} Am^2 kg^{-1}. For comparison, a modern industrial bar magnet has a magnetization of 10–100 Am^2 kg^{-1}; the magnetization of asteroids is estimated as $10^{-7} - 10^{-2}$ Am^2 kg^{-1} (e.g., Richter et al., 2001; Acuña et al., 2002; Auster et al., 2010; Richter et al., 2012). The trillion and quadrillion expressions used here are those of the long-scale system. That is, a trillion (quadrillion) is equivalent to 10^{18} (10^{24}).

[71]T: Using the expression *quantum* does not mean that Gauss already had in mind the 20th-century quantum concept. The expression *quantum* was a common expression to denote an amount required.

the formula

$$\frac{1}{4\pi}\,(V/R-2Z),$$

or by

$$-\frac{1}{4\pi}(3P'+5P''+7P'''+9P^{\mathrm{IV}}\text{etc.}).$$

The importance of this formula will hereafter be exhibited by graphical representation; here it is only noted that it is negative in the Northern Hemisphere, positive in the southern half of the Earth, but such that the dividing line cuts the equator twice (in longitudes 6° and 186°) and deviates from it on both sides by about 15° north and 15° latitude. Furthermore, in the Northern Hemisphere there are two minima, but in the Southern Hemisphere only one maximum exists. Cursory computation gives these minima and this maximum:

−209.1	in	55° N. latitude	263° longitude
−200.0	in	71° N. latitude	116° longitude
+277.7	in	70° S. latitude	154° longitude

These values are based on our units used for the elements, and therefore need to be multiplied by 0.00343941 if to be expressed in absolute values.

33.

Our elements, as already stated above, should be regarded only as a first approximation. And as such their agreement with the observations presented in Chapter 29 is sufficiently satisfactory. It is not doubted that much greater agreement would be obtained from an improved calculation with these present observations. And such a calculation would not be of further difficulty beside its length, still being deterring even if abridged by the introduction of skillful methods as used by the astronomers for the improvement of terms of planetary and cometary orbits. Although this difficulty might be easily overcome by dividing the work among a number of computers[72], it does not appear advisable to do this now, as there is still so much uncertainty in the data from many places whose usage is essential. It will be best, at the present time, to pursue a further comparison between the terms and the observations, thereby improving the reliability of the general maps as compared with the exclusive empirical method used so far. But perhaps we will be permitted a glance at the future progress of the theory, the full realization of which may indeed be far away.

34.

For a satisfactory refinement and completion of the elements, more stringent requirements need to be applied to observational data than have been done up to now. These should exhibit an accuracy at all points, which has been obtained so far only at a few points. They should be corrected for any irregular motion. They should be made all at the same instant of time. It will probably be a long time before such demands are realized. But most essential is the availability of a *complete* set of observations (i.e., including all three elements), particularly from places from those parts of the Earth where such observations are still totally missing. Indeed, a new data point will have an increasing importance to the general theory the further its distance is from those we already have in our possession.

After a sufficient interval of time has passed, the elements need to be determined again in order to deduce their secular changes. It will be essential for this purpose to abandon the present measurements of the intensities altogether, and to substitute them by absolute measurements.

In the course of future centuries, these changes will no longer appear uniform, and the examination of the way the elements progress will offer to natural scientists[73] inexhaustible materials for research.

35.

But the future will also shed light on interesting points of the theory.

In our theory it is assumed that in every determinate magnetized part of the Earth, precisely equal quantities of positive and negative fluids are contained. If magnetic fluids in reality did not exist, but only represented a fictitious surrogate for galvanic currents instead, this equality would be necessarily part of the substitution. If, on the other hand, we attribute reality to magnetic fluids, one could doubt without inconsistency the equality of the quantities of the two fluids. With respect to single magnetic bodies (natural or artificial magnets), the question as to whether they do or do not contain an excess of either magnetic fluid could be decided by dedicated experiments. In case of the existence of any such magnetic excess in a body of this nature, a plumb line should deviate from the true vertical position in the direction of the magnetic meridian. If experiments of this kind are made with a great number of artificial magnets and in a locality sufficiently far away from iron, and if they do not show the slightest deviation (which we expect), the equality of the two fluids might be inferred for the whole Earth with the highest degree of probability. But this would not wholly exclude the possibility of some inequality, however.

In our theory the existence of such an inequality would not cause any difference beside that P^0 (Sect. 17) would no longer = 0. The consequence of this would be that for all external space it would be necessary to add to the expression for Z the series member RRP^0/rr, so that on the surface of the Earth, a (constant) term P^0 should be added. The X and

[72]T: See Grier (2005) on human computers.

[73]T: The German word *Naturforscher* was translated as *men* by Mrs. Sabine. The term *scientist* had only been coined by William Whewell in 1833 and was not yet in common English usage.

Y components would not at all be affected. Once the future has provided a more extensive opulence of precise observations than currently offered, one might determine whether their precise representation requires a non-vanishing value of P^0 or not[74]. Based on the current state of the data, such an undertaking would be completely unsuccessful.

36.

Another part of our theory for which there could be questions is the assumption that the agents of the terrestrial magnetic force are located exclusively in the interior of the Earth.

Should the main causes be located solely or completely outside the Earth, and if we do not allow for groundless fantasy, confining ourselves to scientifically known facts, we can only think of galvanic currents. But the atmosphere is no conductor of such currents, nor is empty space. Thus, we go beyond our knowledge in trying to find any source of galvanic currents in the upper regions. However, the enigmatical phenomenon of the Aurora Borealis, in which there is the eerie appearance that electricity in motion may perform an important role, does not give us the right to discount absolutely the possibility of such currents just because of our lack of knowledge. It will therefore be interesting to determine the type of magnetic action formed by these at the surface of the Earth[75].

37.

Let us then assume the existence of constant galvanic currents in a vault-like or bowl-shaped sphere S, encompassing the Earth[76], denote by S' all the space included by S, and by S'' all the external space that includes S and S'. Whatever the configuration of the galvanic currents may be, we substitute for them a fictitious distribution of the magnetic fluids in the space S, the magnetic action for which will be exactly similar to that of the currents in all the remaining spaces S' and S''. This important proposition has already been mentioned in Chapter 3. It rests on the following grounds: first, that these currents may be resolved into an infinite number of elementary currents (i.e., such that may be considered to be linear). Secondly, the well-known theorem, first demonstrated, I believe, by Ampere[77] is that in place of each linear current bounding an arbitrary surface, one may substitute a distribution of the magnetic fluid on both sides of this surface, at immeasurably small distance from it, with the same action. Thirdly, there is the evident possibility of assigning to every closed line inside S a surface bounding it and lying entirely inside S.

If one designates by $-v$ the aggregate of all the quotients produced by dividing all the elements of the imaginary magnetic fluid by the distance to an indeterminate point O in S' or S'', needless to say that the elements of the southern fluid are to be considered as negative, then the partial differential quotients of v (just like those of V in our above theoretical considerations) express the components of the magnetic force that the galvanic currents produce at O.

38.

Although the detailed development of the theory on which the proposition used in the last chapter is based needs to be done at another occasion, there is an important point related to it that deserves to be mentioned here. If one constructs two *different* surfaces, F and F', each bounded by the same linear current G, and taking the simplest case for the sake of brevity, and those surfaces having no other point in common besides that borderline, they will include a portion of space. Now, if O is situated outside this space, one obtains, for that part of v that belongs to G, one and the same value, independent of the magnetic fluids distributed among F or F'. This value is equal to the product of the intensity of the galvanic current G (measured by a proper unit) multiplied by the solid angle, the vertex of which is at O, and which is included by straight lines, drawn from point O to the points of G, or, which is the same thing, multiplied by that portion of the spherical surface with radius 1 around O, which is the common projection of both F and F'. If, on the other hand, O is situated inside the space enclosed by F and F', the two respective values of the part of v in question will not be the same, depending on whether one assigns the magnetic fluids to F or to F', because different parts of the spherical surface mentioned correspond to them, and those ones taken together make up the whole spherical surface. But since the directions of the galvanic current towards F and F' are different, the intensity of the current needs to have opposite sign in the multiplication into the parts of the spherical surface. The consequence is that the algebraic difference between the values of the part

[74]T: Gauss is discussing here the possible existence of magnetic monopoles. It is remarkable how important experimental results are for this mathematician.

[75]T: The Swedish astronomers and physicists Anders Celsius (1701–1744) and Olav Peter Hiorter (1696–1750) were the first to conduct systematic studies on the relation between magnetic field variations and auroral activity. Between 19 January 1741 and 19 January 1742 Hiorter made 6638 hourly observations of the variation of his compass needle and auroral activity (Hiorter, 1749). These observations indicate a very close relationship between both phenomena. Later, in 1808, Alexander von Humboldt discovered magnetic storms by observing auroras and oscillating magnetic needles (e.g., Tsurutani et al., 1997). Gauss was aware of these observations (Gauss and Weber, 1837b).

[76]T: It is noteworthy that here and elsewhere in the text Gauss did not make use of drawings to make his text more readable. He solely based his explanations on words. We refrain from adding our own drawings to avoid changing the original spirit of Gauss' text too much. It should be noted that the bowl-shaped sphere is nowadays called the ionosphere.

[77]T: André-Marie Ampére (1775–1836), French mathematician and physicist; Gauss refers to the work by Ampère (1826) on what is presently called the magnetic double sheet approach.

of v in question is equal to the product of the intensity of the current multiplied by the whole spherical surface, or by 4π.

Hence it may easily be deduced, that if O is situated in S'', the value of v remains independent of the choice of the connecting surface. On the other hand, if O is situated in S', the absolute value of v does indeed depend on that choice, but the differential of v does not.

By the way, the most fruitful theorem touched upon here in relation to the magnetic action of a linear galvanic current, whereby the product of the intensity of that current, into the portion of spherical surface that is bounded by the line of current from O outwards, has the same relation to attracting or repelling forces as the parts of the mass divided by the distance from O; this theorem still requires in its generality further detailed explanations, saved for later detailed treatment.

39.

The value of v, which in general is a function of r, u, and λ, transforms on the surface of the Earth into a function of u and λ alone, and

$$-\frac{\mathrm{d}v}{R\,\mathrm{d}u},\quad -\frac{\mathrm{d}v}{R\sin u\,\mathrm{d}\lambda}$$

are the horizontal components of the magnetic force resulting from the galvanic currents, directed respectively towards the north and west. Thus it is evident that the remarkable propositions mentioned in Chapters 15 and 16 are likewise correct in this case. But for the third component, the vertical magnetic force, the case will be somewhat different if the sources are situated above, not situated in the interior. To determine the vertical magnetic force resulting from the former, v must first be considered as a function of r, u, and λ, differentiated with respect to r, and then $r = R$ must be substituted. Now, for the inner space S', to which the surface of the Earth belongs, v can only be expanded into a series according to ascending powers of r. If we make

$$\frac{v}{R} = p^0 + \frac{r}{R}\cdot p' + \frac{rr}{RR}\cdot p'' + \frac{r^3}{R^3}\cdot p''' + \text{etc.},$$

then p^0 is a constant, namely, the value of v/R at the center of the Earth; p', p'', p''', etc., on the other hand, are functions of u and λ, which satisfy the same partial differential equations as P', P'', P''', etc. above. From this it follows, in a similar manner to Chapter 20, that knowledge of the value of v at every point of the Earth's surface is sufficient to enable us to deduce the general expression for the whole space S'. It also follows that this value, with the exception of a constant part, or, stated in a different way, that knowledge of the coefficients p', p'', p''', etc., can be achieved by the knowledge of the horizontal forces on the surface of the Earth. But it follows that the value of the vertical force on the same surface is not (as it would be if the forces acted from the interior of the Earth)

$$= 2p' + 3p'' + 4p''' + \text{etc.},$$

but is

$$= -p' - 2p'' - 3p''' - \text{etc.}$$

Now, as our numerical components (Chapter 26), determined under the supposition of the former formula, already give a very satisfactory representation of the entirety of the phenomena, and whereas these are wholly incompatible with the second formula, the fallacy of the hypothesis, placing the causes of terrestrial magnetism into the space external to the Earth, must be viewed as being proved.

40.

Nevertheless, the possibility that part of the terrestrial magnetic force, even if only a relatively minor contribution, is generated from above cannot be regarded as being disproved. A far more complete and accurate knowledge of the phenomena will in the future shed light on this important point of the theory. If, under the supposition of mixed causes, we attach the same meaning as before to the characters V, P^0, P', P'', etc. and v, p^0, p', p'', applying the former to the internal sources, and the latter to the causes acting from the outside, and if further $V + v = W$, $P^0 + p^0 = \Pi^0$, $P' + p' = \Pi'$, $P'' + p'' = \Pi''$, etc. are defined, then on the surface of the Earth,

$$\frac{W}{R} = \Pi^0 + \Pi' + \Pi''\text{etc.},$$

where $\Pi^{(n)}$ satisfies the same partial differential equation as $P^{(n)}$ (Chapter 18). And the two components of the horizontal magnetic force existing there are expressed by

$$-\frac{\mathrm{d}W}{R\,\mathrm{d}u},\quad -\frac{\mathrm{d}W}{R\,\mathrm{d}\sin u\,\mathrm{d}\lambda}.$$

The propositions mentioned in Chapters 15 and 16 therefore retain their validity in this case, and one can determine the magnitudes Π', Π'', Π''', etc. simply from the knowledge of the horizontal forces; however, this does not enable one to conclude on the existence of mixed causes. But, if we consider the vertical force by itself, and bring it into the form $Q^0 + Q' + Q'' + Q'''+$, etc., such that $Q^{(n)}$ satisfies the above-mentioned partial differential equations, then

$$\begin{aligned} Q^0 &= P^0, \\ Q' &= 2P' - p', \\ Q'' &= 3P'' - 2p'', \\ Q''' &= 4P''' - 3p''', \end{aligned}$$

etc., and consequently

$$\begin{aligned} 3\,P' &= \Pi' + Q', & 3\,p' &= 2\,\Pi' - Q' \\ 5\,P'' &= \Pi'' + Q'', & 5\,p'' &= 3\,\Pi'' - Q'' \\ 7\,P''' &= \Pi''' + Q''', & 7\,p''' &= 4\,\Pi''' - Q''' \end{aligned}$$

and so on.

Thus, by combination of the horizontal force with the vertical, one obtains the means of dividing W into its constituent parts V and v, and thus one can learn whether a sensible value may be assigned to the latter. Only the constant part of v, namely, p^0, is left entirely undetermined by the observations, the reason of which is evident from Chapter 38.

In view of this interesting aspect, it appears important to consider the horizontal magnetic force by itself, and we see here an additional reason for the recommendations above (Chapter 21).

41.

Sufficient data for the investigation outlined above probably will be missing for a long time. But it is worthwhile noting that the variations of the magnetic force, which manifest themselves simultaneously at different places on the Earth's surface, are susceptible to an identical treatment. The necessary data might be available much earlier, both with respect to the regular changes with daily and annual variations as well as irregular changes. Some general remarks concerning these future studies should be granted some place here.

After bringing the observed simultaneous changes for each place into the form of variations of the components of the magnetic force ΔX, ΔY, ΔZ, it must first be determined whether the variations of the two horizontal components are in accord with our theory, whereby $-\Delta X$ and $-\sin u \cdot \Delta Y$ must be values of the partial differential quotients as a function of u and λ, according to these variables. In the positive case the conclusion will be that the causes either are actual galvanic currents, or at least they act in the same manner as such currents or as separated magnetic fluids. In the opposite case, it would be proved that the causes cannot be galvanic currents. One notices that highly important conclusions may be derived even from the knowledge of the changes in the horizontal force only, assuming that the determinations are sufficiently accurate, numerous, and extensive. If one has available simultaneous variations of the vertical force, then, *supposing the former case*, the method of the preceding chapter will inform us on whether the causes are situated above or below the surface of the Earth. Furthermore, as they are probably situated in a sheet of small thickness compared to the whole body of the Earth, it may be possible to determine the mode of their distribution[78], at least approximatively.

Concerning the second possibility discussed above, it certainly appears to me that this is less probable with regards to regular changes in the terrestrial magnetic force depending on the time of the year or of the day. However, as to the irregular changes occurring in short intervals, I do not venture a guess on their sources at the present time. If these irregular changes arise from great electric movements above the atmosphere, it would be difficult to place these in the category of galvanic currents. Although everything indicates that galvanic currents are electricity in motion, every movement of electricity is not a galvanic current, but only if the movement forms a circle returning back into itself. As it is only under this condition that it is allowable to make the often-mentioned substitution of separated magnetic fluids instead of galvanic currents, then, in the hypothesis mentioned, our relations between the components would no longer apply. That is to say, the second case would actually be present. Only the establishment of this important case would already be of great interest. And with suitable extensive and accurate observations, it would not be beyond our reach to trace both the places and the nature of such motions.

G.[79]

VIII. Addendum to the article: General Theory of the Terrestrial Magnetism[80]

After printing, a small error in the two compared tables on pages 36–39[81] was noted at two places, caused at Callao by incorrect latitude information in the mentioned paper, and at St. Helena by a calculation error. I am using this opportunity to add to the corrected calculation a further comparison between theory and observations at eight other places, information that I recently received[82].

The observations in Stockholm are from Rudberg[83]; intensity and inclination 1832, declination 1833 (*Poggendorff's Annalen*, Volume 37) (Rudberg, 1836). In Brussels the observations are from 1832; for the declination and inclination from Quetelet[84] (Bulletins de l'Academie de Bruxelles T. VI) (Académie Royale des Sciences, des Lettres et des Beaux-Arts de Belgique, 1836), for the intensity from Rudberg (in Sabine's work cited on page 40 top). The measured values for the other remaining places as well as the determination of of the intensity and a newer value of the declination for Callao are courtesy of Sabine[85]. The observations

[78]T: Elizabeth Sabine translated the German word *Verbreitung* into the English word *propagation*, a non-suitable translation, which may have hinted at an unstated theory held by her husband.

[79]T: By this capital letter, abbreviating his family name, Carl Friedrich Gauss finished his most important contribution.

[80]T: The Latin number indicates that this addendum is the eighth article in the *Resultate* for 1838.

[81]T: These tables are part of Chapter 29.

[82]T: In the following tables the station numbers of these new stations are identical to those of stations already listed earlier and being closest with respect to latitude, but marked by an asterisk.

[83]T: Frederik Rudberg (1800–1839), Swedish physicist and professor in Uppsala.

[84]T: Lambert Adolphe Jacques Quetelet (1796–1874), Flemish astronomer and sociologist. The reference we found does not exactly match the information given by Gauss, but the data published in the referenced work correspond with that used by Gauss.

[85]T: These observations were probably made available to Gauss during the Little Magnetic Congress, which he organized mid-October 1839 in Göttingen (Biermann, 1990; Wolfschmidt, 2009). This congress was attended by Edward Sabine, Humphrey Lloyd,

		Latitude	Longitude	Computed	Declination observed	Difference
8*	Port Etches	+60°21′	213°19′	-28°-33′	31°38′	+3°05′
8**	Lerwick	+60 09	358 53	+27 10	+27 16	−0 06
11*	Stockholm	+59 20	18 04	+15 22	+14 57	+0 25
34*	Valentia	+51 56	349 43	+30 02	+28 43	+1 19
40*	Brüssel	+50 52	4 50	+23 23	+22 19	+1 04
54*	Montreal	+45 27	286 30	+5 23	+7 30	−2 07
62*	Oahu	+21 17	202 00	−12 19	−10 40	−1 39
64*	Panama	+8 37	280 31	−06 44	−07 37	+0 53
68	Callao	−12 04	282 52	−9 32	−10 00	+0 28
71	St. Helena	−15 55	354 17	+19 27	+18 00	+1 27

	Computed	Inclination observed	Difference	Computed	Intensity observed	Difference
8*	+76°25′	+76°03′	+0°22′	1.678	1.75	−0.072
8**	+73 46	+73 45	+0 01	1.469	1.421	+0.048
11*	+70 52	+71 40	+0 48	1.451	1.382	+0.069
34*	+71 25	+70 52	+0 33	1.448	1.409	+0.039
40*	+67 29	+68 49	−1 20	1.393	1.369	+0.024
54*	+77 24	+76 19	+1 05	1.713	1.805	−0.092
62*	+37 36	+41 35	−3 59	1.125	1.14	−0.015
64*	+34 40	+31 55	+2 45	1.238	1.19	+0.048
68	−4 39	−6 14	+1 35	1.003	0.97	+0.033
71	−14 52	−18 01	+3 09	0.811	0.836	−0.025

from Lerwick and Valencia were made by Captain James Ross in 1833, those in Port Etches, Panama, and Oahu in 1837 by Captain Belcher[86], and those in Callao 1838 by him as well. Finally, the inclination and intensity in Montreal was observed by Major Estcourt[87] in 1838. The declination, however, is from 1834, the observer unnamed[88].

Two further minor points need to be improved. The latitude of Naples is by 10 min too small, due to a misprint, but the calculation itself was done with the correct latitude 14°16′. FitzRoy's observation of the declination in Otaheite is noted on page 41 as 7°34′ and at another place as 7°54′ E. But not that one used in the comparing table is the correct one, but the other, and the difference of the calculation is therefore +2°9′.

Furthermore one should note the following misprints in the article. Page 4, line 29 reads 14 instead of 12. Page 21, line 10 from bottom reads $\int T'\, r^0\, \mathrm{d}\mu$ instead of $\int T'\, \mathrm{d}\mu$, and $\int T''\, r^0\, r^0\, \mathrm{d}\mu$ instead of $\int T''\, \mathrm{d}\mu$. On page 22, line 1 and 2 instead of three times $\int$ is written $\int r^0$. And in the supporting tables[89] for $\phi = +45°$ $\log a' = 2.29796$, for $\phi = +36°$ $\log a''' = 1.35513$, for $\phi = -43°$ $\log a' = 1.33836$, for $\phi = -13°$ $\log c^{\mathrm{IV}} = 1.37047$.

In regard to the figures used to illustrate the studies educed in Chapter 12, it must be mentioned that the skillful lithographer Mr. Ritmüller made an attempt to illustrate the differences of the intensity in a twofold way, using differing line thicknesses and varying shading in between.

Due to the delayed final printing of this issue, it was possible to add in addition to the map for the values of V two further tables. The first one, showing the calculated values of the declination, the reader is indebted to my respected friend, the

Adolph Theodor Kupffer, and Carl August Ritter von Steinheim (1801–1870), a German physicist and pioneer of magnetic measurements in Bavaria. Steinheim also constructed the first printing telegraph.

[86]T: Edward Belcher (1799–1877), British naval officer and explorer.

[87]T: James Bucknall Bucknall Estcourt (1803–1855), English military person; the observations were made while Estcourt was on a mission in the province of New Brunswick during the Upper Canada Rebellion.

[88]T: This observation was probably made by Henry Wolsey Bayfield (1795–1885), Royal Navy Surveyor in Canada. Sabine (1849) lists Bayfield as the observer of the declination in Montreal in 1834. However, the given declination deviates by a half degree from what Gauss used.

[89]T: Carl Friedrich Gauss here refers to tables published in a further addendum. These supporting tables provide an extensive list of numbers to support the actual calculation of the direction and intensity of the magnetic field at the surface of the Earth, following the *Theory*. These supporting tables are presented in an appendix to this contribution and are reproduced from the original publication.

Hülfstafeln

zur Berechnung

der Richtung und Stärke

der magnetischen Kräfte

auf der Oberfläche der Erde

aus den Elementen der Theorie.

Figure A1. Cover page of the supporting tables as part of the *Resultate aus den Beobachtungen des Magnetischen Vereins im Jahre 1838*.

co-editor of the *Resultate*[90]. To improve the readability of the rather complex system of lines of equal declination, points where the declination has a maximum as well as those where two lines of equal declination cross (or where the same line crosses itself) have been calculated with special care. Two points of the first kind are found, of the second kind four. The common character of such points is the vanishing of the first differential of the declination in every direction. By the way, it is unnecessary to remark that, in such regions where the declination only varies slowly in every direction, such as in the southern and south-east part of Asia, minor changes in the values of the declination can cause very large changes in the construction of the line system.

The same is true in regard to the map constructed by Doctor Goldschmidt for the intensity, using the tables. Two maxima and one crossing point in the Northern Hemisphere and a maximum in the Southern Hemisphere as well as two minima and two crossing points in the middle zone were found.

Based on the theory, similar maps of the inclination, the horizonal intensity, the three components of the terrestrial magnetic force, and that distribution of the magnetic fluids on the surface of the Earth, which may be regarded as a representative for the actual one in the interior (see page 47), are under construction. And we hope to publish them in the next issue of the *Resultate*.

G.

[90]T: Wilhelm Weber is meant here.

Appendix: Gauss' supporting tables

In a set of supporting tables Carl Friedrich Gauss provides numerical values of coefficients necessary for the determination of the direction and intensity of the magnetic forces at the surface of the Earth as derived from his spherical harmonic expansion coefficients. Gauss refers to these tables in his contribution, but the tables represent an independent part of the *Resultate aus den Beobachtungen des Magnetischen Vereins im Jahre 1838*. Here these tables are reproduced from the original printing.

Tafel 1. | **Tafel 1.**

φ	X a°	Z c°	φ	X a°	Z c°
+ 90°	+ 0,0	+ 1652,9	+ 45°	+ 605,0	+ 1354,1
89	10,3	1652,8	44	620,7	1334,2
88	20,5	1652,7	43	636,2	1313,6
87	30,8	1652.4	42	651,5	1292,1
86	41,2	1652,1	41	666,6	1270,0
85	51,6	1651,7	40	681,5	1247,1
84	62,1	1651,1	39	696,2	1223,5
83	72,8	1650,5	38	710,6	1199,2
82	83,5	1649,7	37	724,7	1174,1
81	94,3	1648,8	36	738,5	1148,4
80	105,3	1647,7	35	752,0	1122,0
79	116,5	1646,4	34	765,2	1094,9
78	127,8	1645,0	33	777,9	1067,2
77	139,3	1643,3	32	790,3	1038,9
76	151,0	1641,4	31	802,3	1009,9
75	162,9	1639,3	30	813,9	980,5
74	175,0	1637,0	29	825,0	950,4
73	187,4	1634,3	28	835,7	919,9
72	199,9	1631,3	27	845,9	888,9
71	212,6	1628,0	26	855,7	857,4
70	225,6	1624.4	25	864,9	825,5
69	238,9	1620,3	24	873,7	793,2
68	252,3	1615,9	23	882,0	760,5
67	266,0	1611,0	22	889,8	727,5
66	279.9	1605,7	21	897,0	694,1
65	294,0	1600,0	20	903,8	660,5
64	308,3	1593,7	19	910,0	626,7
63	322,8	1586,9	18	915,8	592,6
62	337,6	1579,6	17	921,0	558,4
61	352,5	1571,7	16	925,7	523,9
60	367,6	1563,2	15	929,8	489,4
59	382,9	1554,1	14	933,5	454,8
58	398,3	1544,4	13	936,7	420,1
57	413,9	1534,0	12	939,4	385,4
56	429,6	1523,0	11	941,6	350,7
55	445,4	1511,2	10	943,3	316,0
54	461,3	1498,9	9	944,6	281,3
53	477,2	1485,8	8	945,4	246,7
52	493,3	1471,9	7	945,7	212,3
51	509,3	1457,4	6	945,7	177,9
50	525,4	1442,1	5	945,2	143,7
49	541,4	1426,0	4	944,3	109,6
48	557,4	1409,2	3	943,0	75,8
47	573,4	1391,6	2	941,4	42,1
46	589,2	1373,2	+ 1	939,4	+ 8,6
45	605,0	1354,1	0	937,1	− 24,6

Figure A2. Supporting table, listing the values for the parameters a^0 and c^0 for different values of the latitude ϕ. It should be noted that in the text the co-latitude u with $\phi = 90^0 - u$ is used. The latitude range $+90^0$ to 0^0 is covered in this table.

Tafel 1.			Tafel 1.		
φ	X a^0	Z c^0	φ	X a^0	Z c^0
0°	+ 937,1	− 24,6	− 45°	+ 680,2	− 1275,1
− 1	934,5	57,6	46	672,0	1299,5
2	931,5	90,3	47	663,5	1323,9
3	928,3	122,8	48	654,8	1348,1
4	924,8	154,9	49	645,9	1372,3
5	921,0	186,9	50	636,7	1396,2
6	917,0	218,5	51	627,2	1420,0
7	912,8	249,8	52	617,3	1443,7
8	908,4	280,8	53	607,2	1467,1
9	903,8	311,6	54	596,8	1490,3
10	899,1	342,0	55	586,0	1513,2
11	894,1	372,1	56	574,9	1536,1
12	889,1	402,0	57	563,5	1558,6
13	883,9	431,6	58	551,7	1580,8
14	878,6	460,8	59	539,6	1602,7
15	873,2	489,8	60	527,0	1624,2
16	867,7	518,6	61	514,1	1645,4
17	862,1	547,0	62	500,9	1666,1
18	856,4	575,3	63	487,2	1686,5
19	850,7	603,2	64	473,2	1706,4
20	844,9	631,0	65	458,8	1725,9
21	839,1	658,5	66	444,0	1744,9
22	833,2	685,7	67	428,9	1763,3
23	827,3	712,8	68	413,3	1781,2
24	821,4	739,7	69	397,4	1798,6
25	815,4	766,4	70	381,2	1815,3
26	809,3	792,9	71	364,6	1831,4
27	803,2	819,3	72	347,6	1846,9
28	797,1	845,5	73	330,3	1861,6
29	790,9	871,6	74	312,7	1875,7
30	784,7	897,5	75	294,8	1889,1
31	778,5	923,3	76	276,6	1901,7
32	772,1	949,0	77	258,1	1913,5
33	765,7	974,6	78	239,3	1924,6
34	759,3	1000,1	79	220,3	1934,8
35	752,7	1025,5	80	201,0	1944,2
36	746,1	1050,9	81	181,6	1952,8
37	739,3	1076,1	82	161,9	1960,5
38	732,5	1101,2	83	142,1	1967,3
39	725,5	1126,3	84	122,1	1973,3
40	718,4	1151,3	85	101,9	1978,3
41	711,1	1176,2	86	81,7	1982,5
42	703,7	1201,0	87	61,3	1985,7
43	696,0	1225,8	88	40,9	1988,0
44	688,2	1250,5	89	20,5	1989,5
45	680,2	1275,1	90	0	1989,9

Figure A3. Supporting table, listing the values for the parameters a^0 and c^0 for the latitude range 0^0 to -90^0.

Tafel 2.

φ	X A^{I}	X log a^{I}	Y B^{I}	Y log b^{I}	Z C^{I}	Z log c^{I}
+ 90°	292° 9′	2,07430	22° 9′	2,07430	172° 29′	— ∞
89	292. 4	2,07444	22. 7	2,07437	172. 27	0,72139
88	291. 50	2,07488	22. 2	2,07458	172. 20	1,02153
87	291. 26	2,07563	21. 54	2,07493	172. 8	1,19615
86	290. 52	2,07669	21. 43	2,07543	171. 51	1,31904
85	290. 10	2,07811	21. 29	2,07607	171. 30	1,41333
84	289. 19	2,07990	21. 11	2,07686	171. 3	1,48952
83	288. 20	2,08211	20. 51	2,07781	170. 31	1,55192
82	287. 14	2,08477	20. 28	2,07891	169. 54	1,60623
81	286. 0	2,08791	20. 2	2,08017	169. 11	1,65259
80	284. 41	2,09156	19. 33	2,08160	168. 22	1,69305
79	283. 16	2,09573	19. 2	2,08320	167. 28	1,72868
78	281. 46	2,10046	18. 28	2,08498	166. 27	1,76027
77	280. 13	2,10574	17. 52	2,08693	165. 20	1,78844
76	278. 37	2,11157	17. 14	2,08906	164. 6	1,81369
75	276. 59	2,11794	16. 34	2,09138	162. 45	1,83641
74	275. 20	2,12481	15. 52	2,09388	161. 16	1,85697
73	273. 41	2,13215	15. 9	2,09658	159. 41	1,87567
72	272. 3	2,13991	14. 24	2,09945	157. 57	1,89278
71	270. 25	2,14803	13. 37	2,10252	156. 6	1,90856
70	268. 50	2,15646	12. 50	2,10577	154. 6	1,92325
69	267. 17	2,16512	12. 2	2,10920	151. 59	1,93709
68	265. 46	2,17394	11. 13	2,11280	149. 44	1,95028
67	264. 19	2,18288	10. 24	2,11658	147. 21	1,96304
66	262. 56	2,19183	9. 34	2,12052	144. 51	1,97558
65	261. 36	2,20074	8. 44	2,12461	142. 15	1,98809
64	260. 19	2,20954	7. 55	2,12885	139. 33	2,00074
63	259. 7	2,21816	7. 5	2,13322	136. 46	2,01369
62	257. 58	2,22656	6. 15	2,13772	133. 55	2,02708
61	256. 53	2,23468	5. 26	2,14232	131. 2	2,04101
60	255. 52	2,24246	4. 38	2,14703	128. 8	2,05556
59	254. 55	2,24986	3. 50	2,15183	125. 15	2,07077
58	254. 1	2,25686	3. 3	2,15669	122. 22	2,08665
57	253. 11	2,26339	2. 17	2,16162	119. 33	2,10318
56	252. 24	2,26944	1. 32	2,16659	116. 48	2,12032
55	251. 40	2,27497	0. 48	2,17159	114. 8	2,13799
54	250. 59	2,27996	0. 5	2,17661	111. 35	2,15610
53	250. 21	2,28439	359. 23	2,18164	109. 7	2,17456
52	249. 46	2,28822	358. 43	2,18666	106. 47	2,19326
51	249. 13	2,29145	358. 3	2,19166	104. 34	2,21210
50	248. 43	2,29406	357. 25	2,19662	102. 29	2,23098
49	248. 15	2,29603	356. 49	2,20155	100. 32	2,24979
48	247. 49	2,29734	356. 13	2,20641	98. 42	2,26848
47	247. 25	2,29799	355. 39	2,21121	96. 59	2,28692
46	247. 3	2,39796	355. 6	2,21593	95. 24	2,30508
45	246. 43	2,29724	354. 34	2,22057	93. 56	2,32288

Figure A4. Supporting table, listing the values for the parameters $A', \log a', B', \log b', C'$ and $\log c'$ for the latitude range $+90^0$ to 45^0.

Tafel 2.

φ	X: A^I	X: log a^I	Y: B^I	Y: log b^I	Z: C^I	Z: log c^I
+ 45°	246° 43′	2,29724	354° 34′	2,22057	93° 56′	2,32288
44	246. 24	2,29581	354. 4	2,22512	92. 34	2,34027
43	246. 6	2,29367	353. 35	2,22956	91. 18	2,35721
42	245. 49	2,29080	353. 7	2,23389	90. 9	2,37367
41	245. 34	2,28719	352. 40	2,23811	89. 5	2,38961
40	245. 19	2,28282	352. 14	2,24221	88. 6	2,40502
39	245. 5	2,27770	351. 50	2,24618	87. 12	2,41988
38	244. 52	2,27179	351. 26	2,25002	86. 23	2,43417
37	244. 39	2,26510	351. 4	2,25372	85. 39	2,44789
36	244. 25	2,25760	350. 43	2,25728	84. 58	2,46103
35	244. 12	2,24928	350. 22	2,26071	84. 22	2,47360
34	243. 58	2,24012	350. 3	2,26398	83. 48	2,48558
33	243. 44	2,23010	349. 44	2,26711	83. 19	2,49699
32	243. 28	2,21920	349. 27	2,27009	82. 52	2,50782
31	243. 10	2,20742	349. 10	2,27292	82. 28	2,51808
30	242. 51	2,19471	348. 54	2,27560	82. 7	2,52779
29	242. 30	2,18107	348. 38	2,27813	81. 48	2,53693
28	242. 5	2,16647	348. 23	2,28052	81. 32	2,54554
27	241. 37	2,15089	348. 9	2,28275	81. 18	2,55360
26	241. 4	2,13431	347. 55	2,28483	81. 6	2,56113
25	240. 26	2,11671	347. 41	2,28677	80. 55	2,56815
24	239. 41	2,09807	347. 28	2,28856	80. 47	2,57465
23	238. 49	2,07839	347. 15	2,29021	80. 39	2,58066
22	237. 49	2,05768	347 3	2,29171	80. 33	2,58618
21	236. 37	2,03595	346 50	2,29309	80. 29	2,59121
20	235. 13	2,01326	346. 38	2,29433	80. 25	2,59578
19	233. 35	1,98970	346. 26	2,29544	80. 22	2,59991
18	231. 39	1,96540	346. 14	2,29642	80. 20	2,60356
17	229. 23	1,94057	346. 2	2,29728	80. 19	2,60679
16	226. 45	1,91553	345. 49	2,29802	80. 18	2,60959
15	223. 41	1,89072	345. 36	2,29865	80. 17	2,61198
14	220. 9	1,86675	345. 23	2,29917	80. 16	2,61397
13	216. 7	1,84438	345. 10	2,29958	80. 15	2,61556
12	211. 35	1,82457	344. 56	2,29990	80. 15	2,61677
11	206. 34	1,80835	344. 42	2,30014	80. 13	2,61761
10	201. 12	1,79678	344. 27	2,30028	80. 11	2,61809
9	195. 33	1,79064	344. 11	2,30035	80. 9	2,61822
8	189. 50	1,79046	343. 55	2,30035	80. 5	2,61802
7	184. 15	1,79621	343. 37	2,30029	80. 0	2,61750
6	178. 56	1,80737	343. 19	2,30018	79. 54	2,61667
5	174. 3	1,82310	343. 0	2,30002	79. 46	2,61554
4	169. 39	1,84235	342. 40	2,29983	79. 37	2,61414
3	165. 47	1,86409	342. 18	2,29961	79. 25	2,61246
2	162. 26	1,88741	341. 56	2,29938	79. 12	2,61054
1	159. 34	1,91156	341. 32	2,29914	78. 56	2,60839
0	157. 9	1,93596	341. 7	2,29890	78. 37	2,60603

Figure A5. Supporting table, listing the values for the parameters A', $\log a'$, B', $\log b'$, C' and $\log c'$ for the latitude range $+45^0$ to 0^0.

Tafel 2.

	X		Y		Z	
φ	A'	log a'	B'	log b'	C'	log c'
0°	157° 9'	1,93596	341° 7'	2,29890	78° 37'	2,60603
—1	155. 7	1,96018	340. 40	2,29869	78. 15	2,60347
2	153. 26	1,98393	340. 12	2,29850	77. 50	2,60075
3	152. 3	2,00702	339. 42	2,29836	77. 22	2,59789
4	150. 55	2,02930	339. 11	2,29827	76. 50	2,59491
5	150. 0	2,05070	338. 38	2,29824	76. 14	2,59185
6	149. 16	2,07116	338. 3	2,29830	75. 34	2,58874
7	148. 41	2,09068	337. 27	2,29846	74. 50	2,58562
8	148. 14	2,10923	336. 49	2,29873	74. 1	2,58252
9	147. 54	2,12683	336. 10	2,29912	73. 8	2,57949
10	147. 39	2,14348	335. 29	2,29965	72. 11	2,57658
11	147. 28	2,15919	334. 46	2,30033	71. 8	2,57383
12	147. 22	2,17398	334. 1	2,30118	70. 1	2,57129
13	147. 18	2,18785	333. 15	2,30222	68. 49	2,56902
14	147. 16	2,20083	332. 27	2,30345	67. 32	2,56707
15	147. 16	2,21292	331. 37	2,30489	66. 11	2,56549
16	147. 18	2,22413	330. 47	2,30655	64. 45	2,56435
17	147. 19	2,23446	329. 54	2,30845	63. 15	2,56368
18	147. 22	2,24391	329. 1	2,31059	61. 42	2,56354
19	147. 24	2,25250	328. 6	2,31298	60. 5	2,56397
20	147. 25	2,26022	327. 11	2,31564	58. 26	2,56499
21	147. 26	2,26706	326. 14	2,31856	56. 44	2,56664
22	147. 25	2,27302	325. 16	2,32176	55. 1	2,56893
23	147. 23	2,27809	324. 18	2,32523	53. 17	2,57187
24	147. 19	2,28227	323. 20	2,32899	51. 32	2,57546
25	147. 13	2,28554	322. 21	2,33302	49. 47	2,57966
26	147. 4	2,28790	321. 22	2,33733	48. 3	2,58447
27	146. 52	2,28932	320. 22	2,34191	46. 20	2,58984
28	146. 37	2,28978	319. 23	2,34675	44. 39	2,59572
29	146. 18	2,28928	318. 24	2,35186	43. 0	2,60207
30	145. 55	2,28780	317. 25	2,35722	41. 24	2,60883
31	145. 27	2,28530	316. 27	2,36281	39. 51	2,61593
32	144. 54	2,28177	315. 30	2,36863	38. 21	2,62331
33	144. 15	2,27720	314. 33	2,37467	36. 55	2,63090
34	143. 30	2,27156	313. 37	2,38091	35. 32	2,63864
35	142. 37	2,26483	312. 42	2,38733	34. 13	2,64646
36	141. 36	2,25701	311. 48	2,39392	32. 58	2,65430
37	140. 25	2,24809	310. 56	2,40066	31. 46	2,66210
38	139. 4	2,23808	310. 4	2,40754	30. 38	2,66980
39	137. 30	2,22701	309. 14	2,41454	29. 34	2,67736
40	135. 43	2,21492	308. 25	2,42163	28. 33	2,68471
41	133. 40	2,20190	307. 37	2,42882	27. 36	2,69181
42	131. 20	2,18809	306. 51	2,43606	26. 42	2,69862
43	128. 39	2,17367	306. 6	2,44336	25. 52	2,70510
44	125. 37	2,15891	305. 23	2,45069	25. 4	2,71121
45	122. 10	2,14420	304. 41	2,45804	24. 19	2,71691

Figure A6. Supporting table, listing the values for the parameters $A', \log a', B', \log b', C'$ and $\log c'$ for the latitude range 0^0 to -45^0.

Tafel 2.

φ	X: A^{I}	X: $\log a^{I}$	Y: B^{I}	Y: $\log b^{I}$	Z: C^{I}	Z: $\log c^{I}$
— 45°	122° 10′	2,14420	304° 41′	2,45804	24° 19′	2,71691
46	118. 16	2,13005	304. 1	2,46539	23. 37	2,72218
47	113. 56	2,11708	303. 22	2,47272	22. 58	2,72698
48	109. 7	2,10605	302. 44	2,48003	22. 21	2,73129
49	103. 53	2,09781	302. 8	2,48730	21. 47	2,73508
50	98. 16	2,09320	301. 33	2,49451	21. 14	2,73833
51	92. 24	2,09289	301. 0	2,50166	20. 44	2,74100
52	86. 25	2,09739	300. 28	2,50873	20. 16	2,74307
53	80. 27	2,10679	299. 57	2,51571	19. 49	2,74453
54	74. 40	2,12081	299. 28	2,52260	19. 25	2,74534
55	69. 11	2,13887	299. 0	2,52937	19. 1	2,74550
56	64. 5	2,16018	298. 33	2,53603	18. 40	2,74495
57	59. 25	2,18391	298. 7	2,54256	18. 20	2,74370
58	55. 12	2,20923	297. 43	2,54895	18. 1	2,74169
59	51. 25	2,23544	297. 20	2,55521	17. 43	2,73892
60	48. 4	2,26198	296. 57	2,56131	17. 26	2,73535
61	45. 4	2,28840	296. 36	2,56727	17. 11	2,73094
62	42. 26	2,31436	296. 16	2,57306	16. 57	2,72566
63	40. 5	2,33963	295. 57	2,57868	16. 43	2,71948
64	38. 1	2,36405	295. 39	2,58413	16. 31	2,71235
65	36. 10	2,38751	295. 22	2,58941	16. 19	2,70421
66	34. 32	2,40996	295. 5	2,59451	16. 8	2,69503
67	33. 5	2,43134	294. 50	2,59942	15. 58	2,68474
68	31. 47	2,45165	294. 35	2,60415	15. 49	2,67328
69	30. 37	2,47088	294. 22	2,60868	15. 40	2,66056
70	29. 35	2,48904	294. 9	2,61302	15. 32	2,64650
71	28. 40	2,50615	293. 57	2,61716	15. 24	2,63100
72	27. 50	2,52223	293. 45	2,62111	15. 17	2,61395
73	27. 5	2,53729	293. 35	2,62485	15. 11	2,59520
74	26. 25	2,55136	293. 25	2,62839	15. 5	2,57459
75	25. 49	2,56447	293. 16	2,63172	14. 59	2,55193
76	25. 17	2,57662	293. 7	2,63484	14. 54	2,52699
77	24. 48	2,58784	292. 59	2,63776	14. 50	2,49948
78	24. 23	2,59816	292. 52	2,64046	14. 45	2,46904
79	24. 0	2,60758	292. 45	2,64296	14. 42	2,43523
80	23. 40	2,61613	292. 39	2,64524	14. 38	2,39746
81	23. 22	2,62382	292. 34	2,64730	14. 35	2,35498
82	23. 7	2,63067	292. 29	2,64915	14. 32	2,30676
83	22. 53	2,63668	292. 25	2,65079	14. 30	2,25136
84	22. 42	2,64187	292. 21	2,65220	14. 28	2,18665
85	22. 32	2,64624	292. 18	2,65340	14. 26	2,10937
86	22. 25	2,64981	292. 16	2,65439	14. 25	2,01401
87	22. 19	2,65258	292. 14	2,65515	14. 24	1,89028
88	22. 15	2,65456	292. 13	2,65570	14. 23	1,71505
89	22. 12	2,65574	292. 12	2,65603	14. 23	1,41453
90	22. 11	2,65614	292. 11	2,65614	14. 23	— ∞

Figure A7. Supporting table, listing the values for the parameters $A', \log a', B', \log b', C'$ and $\log c'$ for the latitude range -45^0 to -90^0.

Tafel 3.

φ	X		Y		Z	
	A''	$\log a''$	B''	$\log b''$	C''	$\log c''$
+ 90°	347° 16′	— ∞	77° 16′	— ∞	176° 59′	— ∞
89	347. 15	0,60246	77. 16	0,60263	176. 59	9,17222
88	347. 13	0,90273	77. 15	0,90333	176. 58	9,77385
87	347. 8	1,07753	77. 12	1,07889	176. 56	0,12532
86	347. 2	1,20066	77. 9	1,20311	176. 53	0,37419
85	346. 54	1,29525	77. 5	1,29903	176. 49	0,56672
84	346. 44	1,37159	77. 0	1,37704	176. 45	0,72351
83	346. 32	1,43517	76. 55	1,44260	176. 40	0,83554
82	346. 19	1,48927	76. 48	1,49899	176. 34	0,96937
81	346. 3	1,53601	76. 40	1,54833	176. 27	1,06923
80	345. 45	1,57682	76. 32	1,59206	176. 19	1,15802
79	345. 25	1,61273	76. 22	1,63121	176. 10	1,23779
78	345. 3	1,64451	76. 12	1,66655	176. 1	1,31006
77	344. 39	1,67272	76. 0	1,69865	175. 50	1,37599
76	344. 13	1,69780	75. 48	1,72795	175 39	1,43647
75	343. 43	1,72012	75. 35	1,75483	175. 27	1,49222
74	343. 12	1,73995	75. 20	1,77955	175. 14	1,54381
73	342. 38	1,75753	75. 5	1,80237	175. 0	1,59171
72	342. 1	1,77302	74. 49	1,82347	174. 45	1,63630
71	341. 20	1,78662	74. 31	1,84301	174. 29	1,67772
70	340. 37	1,79844	74. 13	1,86114	174. 12	1,71684
69	339. 51	1,80860	73. 53	1,87798	173. 54	1,75329
68	339. 1	1,81720	73. 32	1,89362	173. 35	1,78747
67	338. 7	1,82433	73. 11	1,90815	173. 14	1,81956
66	337. 9	1,83005	72. 48	1,92165	172. 53	1,84971
65	336. 6	1,83444	72. 24	1,93420	172. 31	1,87806
64	334. 59	1,83756	71. 58	1,94584	172. 7	1,90472
63	333. 48	1,83947	71. 32	1,95663	171. 42	1,92979
62	332. 30	1,84022	71. 4	1,96663	171. 16	1,95338
61	331. 7	1,83986	70. 35	1,97587	170. 48	1,97557
60	329. 38	1,83845	70. 4	1,98440	170. 20	1,99642
59	328. 3	1,83604	69. 33	1,99224	169. 50	2,01601
58	326. 20	1,83270	69. 0	1,99944	169. 18	2,03440
57	324. 29	1,82850	68. 25	2,00602	168. 45	2,05165
56	322. 30	1,82350	67. 49	2,01200	168. 10	2,06780
55	320. 23	1,81779	67. 12	2,01743	167. 34	2,08291
54	318. 6	1,81148	66. 33	2,02232	166. 56	2,09694
53	315. 39	1,80465	65. 52	2,02669	166. 17	2,11015
52	313. 2	1,79747	65. 10	2,03056	165. 35	2,12237
51	310. 14	1,79005	64. 26	2,03396	164. 52	2,13370
50	307. 14	1,78257	63. 41	2,03690	164. 7	2,14417
49	304. 4	1,77522	62. 54	2,03941	163. 20	2,15372
48	300. 42	1,76818	62. 5	2,04151	162. 31	2,16267
47	297. 8	1,76168	61. 14	2,04320	161. 40	2,17076
46	293. 25	1,75593	60. 22	2,04451	160. 47	2,17810
45	289. 31	1,75115	59. 27	2,04545	159. 51	2,18474

Figure A8. Supporting table, listing the values for the parameters A'', $\log a''$, B'', $\log b''$, C'' and $\log c''$ for the latitude range $+90^0$ to 45^0.

Tafel 3.

φ	X		Y		Z	
	A^{II}	$log\ a^{II}$	B^{II}	$log\ b^{II}$	C^{II}	$log\ c^{II}$
+ 45°	289° 31′	1,75115	59° 27′	2,04545	159° 51′	2,18474
44	285. 30	1,74752	58. 31	2,04605	158. 53	2,19069
43	281. 22	1,74521	57. 33	2,04632	157. 53	2,19598
42	277. 9	1,74436	56. 33	2,04627	156. 50	2,20064
41	272. 54	1,74504	55. 30	2,04592	155. 44	2,20468
40	268. 38	1,74726	54. 26	2,04530	154. 36	2,20815
39	264. 24	1,75098	53. 20	2,04441	153. 25	2,21106
38	260. 15	1,75611	52. 12	2,04328	152. 11	2,21343
37	256. 10	1,76251	51. 1	2,04191	150. 55	2,21531
36	252. 13	1,77000	49. 49	2,04034	149. 35	2,21671
35	248. 23	1,77838	48. 34	2,03857	148. 12	2,21766
34	244 43	1,78746	47. 17	2,03662	146. 46	2,21819
33	241. 11	1,79704	45. 28	2,03452	145. 16	2,21834
32	237. 49	1,80692	44. 37	2,03228	143. 44	2,21813
31	234. 36	1,81694	43. 14	2,02991	142. 8	2,21759
30	231. 32	1,82693	41. 49	2,02744	140. 29	2,21677
29	228. 35	1,83676	40. 22	2,02488	138. 47	2,21568
28	225. 47	1,84632	38. 53	2,02226	137. 1	2,21438
27	223. 6	1,85551	37. 22	2,01958	135. 12	2,21287
26	220. 31	1,86425	35. 50	2,01686	133. 20	2,21123
25	218. 2	1,87248	34. 15	2,01413	131. 25	2,20947
24	215. 38	1,88014	32. 39	2,01139	129. 26	2,20762
23	213. 18	1,88721	31. 1	2,00866	127. 25	2,20572
22	211. 3	1,89364	29. 22	2,00595	125. 21	2,20380
21	208. 51	1,89942	27. 41	2,00328	123. 15	2,20189
20	206. 42	1,90455	26. 0	2,00065	121. 6	2,20002
19	204. 35	1,90900	24 17	1,99808	118. 56	2,19821
18	202. 30	1,91277	22. 33	1,99557	116. 43	2,19649
17	200. 26	1,91588	20. 48	1,99313	114. 29	2,19487
16	198. 23	1,91832	19. 3	1,99077	112. 14	2,19337
15	196. 21	1,92011	17. 17	1,98848	109. 58	2,19199
14	194. 18	1,92126	15. 31	1,98626	107. 41	2,19075
13	192. 15	1,92179	13. 44	1,98413	105. 23	2,18963
12	190. 12	1,92170	11. 57	1,98207	103. 6	2,18864
11	188. 7	1,92104	10. 11	1,98007	100. 49	2,18776
10	186. 1	1,91982	8. 24	1,97815	98. 33	2,18699
9	183. 53	1,91806	6. 38	1,97629	96. 17	2,18630
8	181. 43	1,91581	4. 52	1,97446	94. 2	2,18568
7	179. 31	1,91309	3. 7	1,97268	91. 48	2,18510
6	177. 16	1,90995	1. 22	1,97092	89. 36	2,18454
5	174. 59	1,90641	359. 37	1,96919	87. 25	2,18397
4	172. 38	1,90253	357. 54	1,96746	85. 16	2,18336
3	170. 15	1,89835	356. 11	1,96573	83. 8	2,18269
2	167. 48	1,89392	354. 29	1,96397	81. 3	2,18191
1	165. 17	1,88929	352. 48	1,96218	78. 59	2,18103
0	162. 43	1,88452	351. 8	1,96035	76. 57	2,17998

Figure A9. Supporting table, listing the values for the parameters A'', $\log a''$, B'', $\log b''$, C'' and $\log c''$ for the latitude range $+45^0$ to 0^0.

Tafel 3.

φ	X		Y		Z	
	A II	log a II	B II	log b II	C II	log c II
0°	162° 43′	1,88452	351° 8′	1,96035	76° 57′	2,17998
— 1	160. 6	1,87966	349. 29	1,95846	74. 56	2,17876
2	157. 25	1,87476	347. 50	1,95649	72. 58	2,17733
3	154. 41	1,86989	346. 13	1,95444	71. 1	2,17566
4	151. 54	1,86509	344. 36	1,95228	69. 6	2,17374
5	149. 4	1,86042	343. 1	1,95002	67. 12	2,17154
6	146. 11	1,85592	341. 26	1,94764	65. 20	2,16905
7	143. 17	1,85164	339. 53	1,94512	63. 29	2,16623
8	140. 20	1,84762	338. 20	1,94246	61. 39	2,16309
9	137. 22	1,84388	336. 47	1,93964	59. 50	2,15959
10	134. 23	1,84045	335. 16	1,93667	58. 2	2,15573
11	131. 23	1,83733	333. 45	1,93352	56. 15	2,15150
12	128. 24	1,83452	332. 14	1,93020	54. 29	2,14689
13	125. 25	1,83203	330. 45	1,92669	52. 43	2,14188
14	122. 27	1,82983	329. 15	1,92299	50. 57	2,13648
15	119. 31	1,82790	327. 47	1,91910	49. 12	2,13067
16	116. 36	1,82621	326. 18	1,91501	47. 26	2,12446
17	113. 44	1,82470	324. 50	1,91071	45. 41	2,11785
18	110. 54	1,82335	323. 22	1,90621	43. 55	3,11083
19	108. 7	1,82211	321. 54	1,90150	42. 9	2,10341
20	105. 23	1,82091	320. 26	1,89658	40. 22	2,09559
21	102. 43	1,81971	318. 58	1,89145	38. 34	2,08737
22	100. 5	1,81846	317. 30	1,88612	36. 45	2,07878
23	97. 30	1,81710	316. 2	1,88057	34. 56	2,06981
24	94. 59	1,81560	314. 34	1,87483	33. 5	2,06047
25	92. 31	1,81388	313. 5	1,86887	31. 13	2,05078
26	90. 5	1,81193	311. 37	1,86272	29. 20	2,04076
27	87. 43	1,80968	310. 8	1,85637	27. 26	2,03041
28	85. 23	1,80711	308. 38	1,84983	25. 29	2,01975
29	83. 5	1,80419	307. 8	1,84311	23. 32	2,00881
30	80. 50	1,80087	305. 38	1,83621	21. 33	1,99760
31	78. 36	1,79714	304. 7	1,82913	19. 32	1,98614
32	76. 25	1,79296	302. 35	1,82188	17. 30	1,97445
33	74. 14	1,78831	301. 3	1,81447	15. 26	1,96255
34	72. 5	1,78323	299. 31	1,80690	13. 20	1,95047
35	69. 57	1,77765	297. 58	1,79919	11. 14	1,93821
36	67. 49	1,77157	296. 25	1,79134	9. 6	1,92581
37	65. 42	1,76499	294. 51	1,78335	6. 57	1,91327
38	63. 35	1,75791	293. 16	1,77524	4. 47	1,90061
39	61. 27	1,75034	291. 41	1,76701	2. 37	1,88785
40	59. 19	1,74228	290. 6	1,75866	0. 26	1,87498
41	57. 10	1,73373	288. 31	1,75020	358. 14	1,86202
42	55. 0	1,72472	286. 55	1,74163	356. 3	1,84896
43	52. 49	1,71526	285. 19	1,73297	353. 52	1,83580
44	50. 37	1,70537	283. 43	1,72420	351. 42	1,82252
45	48. 23	1,69506	282. 7	1,71533	349. 33	1,80912

Figure A10. Supporting table, listing the values for the parameters A'', $\log a''$, B'', $\log b''$, C'' and $\log c''$ for the latitude range 0^0 to -45^0.

Tafel 3.

	X		Y		Z	
φ	A^{II}	$log\ a^{II}$	B^{II}	$log\ b^{II}$	C^{II}	$log\ c^{II}$
— 45°	48° 23′	1,69506	282° 7′	1,71533	349° 33′	1,80912
46	46. 7	1,68438	280. 31	1,70636	347. 25	1,79558
47	43. 49	1,67335	278. 56	1,69729	345. 18	1,78186
48	41. 29	1,66199	277. 21	1,68810	343. 13	1,76793
49	39. 7	1,65036	275. 47	1,67880	341. 10	1,75376
50	36. 42	1,63848	274. 13	1,66937	339. 10	1,73931
51	34. 16	1,62640	272. 40	1,65981	337. 12	1,72452
52	31. 47	1,61415	271. 8	1,65009	335. 17	1,70935
53	29. 17	1,60177	269. 37	1,64021	333. 25	1,69375
54	26. 45	1,58929	268. 7	1,63013	331. 35	1,67764
55	24. 11	1,57675	266. 39	1,61985	329. 50	1,66098
56	21. 37	1,56417	265. 12	1,60933	328. 7	1,64368
57	19. 2	1,55158	263. 47	1,59855	326. 28	1,62568
58	16. 26	1,53898	262. 23	1,58747	324. 52	1,60691
59	13. 51	1,52638	261. 2	1,57607	323. 21	1,58728
60	11. 17	1,51376	259. 42	1,56430	321. 52	1,56672
61	8. 44	1,50111	258. 25	1,55212	320. 27	1,54513
62	6. 13	1,48839	257. 9	1,53949	319. 6	1,52242
63	3. 45	1,47556	255. 56	1,52635	317. 48	1,49850
64	1. 20	1,46254	254. 46	1,51265	316. 34	1,47326
65	358. 58	1,44928	253. 37	1,49834	315. 24	1,44658
66	356. 40	1,43567	252. 31	1,48335	314. 17	1,41834
67	354. 27	1,42163	251. 28	1,46760	313. 13	1,38840
68	352. 19	1,40704	250. 27	1,45101	312. 12	1,35661
69	350. 15	1,39176	249. 29	1,43351	311. 15	1,32281
70	348. 18	1,37567	248. 34	1,41498	310. 21	1,28680
71	346. 25	1,35860	247. 41	1,39531	309. 30	1,24837
72	344. 39	1,34039	246. 51	1,37437	308. 42	1,20727
73	342. 59	1,32084	246. 3	1,35202	307. 57	1,16322
74	341. 25	1,29975	245. 18	1,32808	307. 16	1,11588
75	339. 56	1,27687	244. 36	1,30235	306. 37	1,06485
76	338. 34	1,25192	243. 57	1,27458	306. 0	1,00966
77	337. 18	1,22457	243. 21	1,24448	305. 27	0,94972
78	336. 8	1,19443	242. 47	1,21167	304. 56	0,88472
79	335. 4	1,16100	242. 16	1,17572	304. 28	0,81256
80	334. 5	1,12370	241. 47	1,13602	304. 3	0,73327
81	333. 13	1,08172	241. 22	1,09181	303. 40	0,64493
82	332. 26	1,03401	240. 59	1,04207	303. 19	0,54547
83	331. 45	0,97911	240. 39	0,98533	303. 1	0,43201
84	331. 10	0,91487	240. 21	0,91948	302. 46	0,30031
85	330. 40	0,83802	240. 6	0,84123	302. 33	0,04380
86	330. 16	0,74302	239. 54	0,74509	302. 22	9,95118
87	329. 57	0,61958	239. 45	0,62075	302. 14	9,70281
88	329. 44	0,44456	239. 38	0,44509	302. 8	9,35148
89	329. 35	0,14417	239. 34	0,14432	302. 5	8,74992
90	329. 33	— ∞	239. 33	— ∞	302. 3	— ∞

Figure A11. Supporting table, listing the values for the parameters A'', $\log a''$, B'', $\log b''$, C'' and $\log c''$ for the latitude range -45^0 to -90^0.

Tafel 4.

φ	X A'''	X $\log a'''$	Y B'''	Y $\log b'''$	Z C'''	Z $\log c'''$
+ 90°	221° 48′	— ∞	311° 48′	— ∞	36° 0′	— ∞
89	221. 48	8,41399	311. 48	8,41408	36. 0	6,83649
88	221. 50	9,01555	311. 49	9,01591	36. 1	7,73926
87	221. 52	9,36689	311. 50	9,36770	36. 2	8,26700
86	221. 54	9,61559	311. 52	9,61702	36. 4	8,64106
85	221. 58	9,80790	311. 54	9,81013	36. 6	8,93082
84	222. 2	9,96141	311. 57	9,96763	36. 8	9,16719
83	222. 8	0,09612	312. 0	0,10050	36. 11	9,36663
82	222. 14	0,20957	312. 3	0,21530	36. 15	9,53899
81	222. 21	0,30901	312. 8	0,31627	36. 19	9,69062
80	222. 29	0,39732	312. 12	0,40629	36. 23	9,82585
79	222. 37	0,47655	312. 17	0,48742	36. 28	9,94777
78	222. 47	0,54824	312. 23	0,56119	36. 34	0,05867
77	222. 57	0,61353	312. 29	0,62875	36. 40	0,16026
76	223. 9	0,67331	312. 36	0,69100	36. 46	0,25391
75	223. 21	0,72831	312. 43	0,74864	36. 53	0,34068
74	223. 34	0,77908	312. 50	0,80226	37. 1	0,42143
73	223. 49	0,82611	312. 59	0,85232	37. 9	0,49686
72	224. 4	0,86977	313. 7	0,89922	37. 17	0,56756
71	224. 20	0,91040	313. 17	0,94327	37. 26	0,63402
70	224. 38	0,94825	313. 26	0,98476	37. 36	0,69664
69	224. 56	0,98357	313. 37	1,02392	37. 46	0,75579
68	225. 16	1,01656	313. 48	1,06095	37. 57	0,81266
67	225. 37	1,04739	313. 59	1,09603	38. 8	0,86482
66	225. 59	1,07620	314. 11	1,12930	38. 20	0,91520
65	226. 22	1,10314	314. 23	1,16091	38. 32	0,96309
64	226. 47	1,12831	314. 37	1,19098	38. 45	1,00868
63	227. 13	1,15183	314. 50	1,21961	38. 59	1,05213
62	227. 40	1,17377	315. 5	1,24689	39. 13	1,09356
61	228. 9	1,19422	315. 20	1,27290	39. 28	1,13312
60	228. 39	1,21325	315. 35	1,29773	39. 43	1,17090
59	229. 11	1,23093	315. 51	1,32144	39. 59	1,20702
58	229. 45	1,24732	316. 8	1,34409	40. 16	1,24157
57	230. 21	1,26246	316. 26	1,36574	40. 34	1,27462
56	230. 58	1,27641	316. 44	1,38644	40. 52	1,30626
55	231. 37	1,28922	317. 3	1,40624	41. 11	1,33655
54	232. 19	1,30091	317. 22	1,42517	41. 30	1,36556
53	233. 2	1,31152	317. 42	1,44329	41. 51	1,39345
52	233. 48	1,32110	318. 3	1,46062	42. 12	1,41996
51	234. 36	1,32967	318. 25	1,47720	42. 34	1,44546
50	235. 26	1,33726	318. 47	1,49306	42. 57	1,46990
49	236. 19	1,34390	319. 10	1,50823	43. 20	1,49327
48	237. 15	1,34960	319. 34	1,52274	43. 45	1,51567
47	238. 14	1,35441	319. 58	1,53661	44. 10	1,53711
46	239. 16	1,35835	320. 24	1,54987	44. 36	1,55764
45	240. 21	1,36143	320. 50	1,56254	45. 3	1,57728

Figure A12. Supporting table, listing the values for the parameters A''', $\log a'''$, B''', $\log b'''$, C''' and $\log c'''$ for the latitude range $+90^0$ to 45^0.

Tafel 4.

	X		Y		Z	
φ	A^{III}	$\log a^{III}$	B^{III}	$\log b^{III}$	C^{III}	$\log c^{III}$
+45°	240° 21′	1,36143	320° 50′	1,56254	45° 3′	1,57728
44	241. 30	1,36369	321. 17	1,57464	45. 31	1,59606
43	242. 43	1,36514	321. 44	1,58619	46. 0	1,61401
42	243. 59	1,36581	322. 13	1,59721	46. 30	1,63116
41	245. 19	1,36574	322. 42	1,60771	47. 1	1,64754
40	246. 44	1,36494	323. 13	1,61772	47. 33	1,66317
39	248. 13	1,36344	323. 44	1,62725	48. 6	1,67807
38	249. 47	1,36129	324. 16	1,63631	48. 40	1,69226
37	251. 26	1,35850	324. 49	1,64493	49. 15	1,70578
36	253. 11	1,45513	325. 23	1,65311	49. 51	1,71862
35	255. 1	1,35122	325. 57	1,66087	50. 29	1,73083
34	256. 57	1,34681	326. 33	1,66822	51. 7	1,74241
33	258. 59	1,34196	327. 9	1,67518	51. 47	1,75338
32	261. 8	1,33672	327. 47	1,68175	52. 28	1,76376
31	263. 23	1,33116	328. 25	1,68796	53. 10	1,77356
30	265. 45	1,32535	329. 5	1,69380	53. 54	1,78283
29	268. 13	1,31937	329. 45	1,69930	54. 39	1,79154
28	270. 49	1,31330	330. 27	1,70446	55. 25	1,79974
27	273. 31	1,30722	331. 9	1,70930	56. 12	1,80742
26	276. 21	1,30123	331. 52	1,71382	57. 1	1,81462
25	279. 17	1,29542	332. 37	1,71804	57. 51	1,82134
24	282. 19	1,28988	333. 22	1,72197	58. 43	1,82759
23	285. 28	1,28470	334. 8	1,72561	59. 36	1,83341
22	288. 42	1,27997	334. 56	1,72898	60. 30	1,83879
21	292. 1	1,27576	335. 44	1,73208	61. 26	1,84375
20	295. 24	1,27214	336. 33	1,73493	62. 23	1,84832
19	298. 50	1,26916	337. 23	1,73754	63. 21	1,85250
18	302. 19	1,26686	338. 14	1,73991	64. 21	1,85630
17	305. 50	1,26524	339. 6	1,74206	65. 23	1,85975
16	309. 21	1,26430	339. 59	1,74399	66. 25	1,86286
15	312. 52	1,26403	340. 53	1,74570	67. 30	1,86563
14	316. 22	1,26438	341. 48	1,74722	68. 35	1,86809
13	319. 51	1,26530	342. 43	1,74855	69. 42	1,87025
12	323. 17	1,26672	343. 40	1,74969	70. 50	1,87212
11	326. 41	1,26859	344. 37	1,75065	71. 59	1,87372
10	330. 1	1,27080	345. 35	1,75145	73. 9	1,87505
9	333. 19	1,27328	346. 33	1,75208	74. 21	1,87613
8	336. 32	1,27595	347. 32	1,75255	75. 34	1,87698
7	339. 43	1,27873	348. 32	1,75287	76. 47	1,87759
6	342. 49	1,28156	349. 33	1,75305	78. 2	1,87799
5	345. 53	1,28435	350. 34	1,75309	79. 17	1,87818
4	348. 54	1,28706	351. 35	1,75299	80. 34	1,87816
3	351. 51	1,28963	352. 37	1,75276	81. 51	1,87796
2	354. 47	1,29201	353. 39	1,75241	83. 8	1,87757
1	357. 40	1,29418	354. 42	1,75193	84. 26	1,87700
0	0. 31	1,29611	355. 45	1,75132	85. 45	1,87626

Figure A13. Supporting table, listing the values for the parameters A''', $\log a'''$, B''', $\log b'''$, C''' and $\log c'''$ for the latitude range $+45^0$ to 0^0.

Tafel 4.

	X		Y		Z	
φ	A‴	log a‴	B‴	log b‴	C‴	log c‴
0°	0° 31′	1,29611	355° 45′	1,75132	85° 45′	1,87626
1	3. 21	1,29778	356. 47	1,75060	87. 3	1,87535
2	6. 10	1,29918	357. 51	1,74976	88. 22	1,87426
3	8. 58	1,30030	358. 54	1,74880	89. 41	1,87301
4	11. 46	1,30115	359. 57	1,74772	91. 0	1,87159
5	14. 34	1,30175	1. 0	1,74652	92. 19	1,87000
6	17. 22	1,30211	2. 3	1,74520	93. 38	1,86824
7	20. 11	1,30226	3. 6	1,74376	94. 56	1,86630
8	23. 0	1,30223	4. 9	1,74219	96. 14	1,86418
9	25. 51	1,30205	5. 11	1,74049	97. 31	1,86187
10	28. 43	1,30176	6. 13	1,73867	98. 48	1,85936
11	31. 36	1,30140	7. 14	1,73670	100. 4	1,85665
12	34. 30	1,30103	8. 15	1,73460	101. 19	1,85373
13	37. 26	1,30068	9. 16	1,73234	102. 33	1,85058
14	40. 23	1,30041	10. 16	1,72994	103. 47	1,84720
15	43. 21	1,30025	11. 15	1,72737	104. 59	1,84357
16	46. 20	1,30026	12. 14	1,72464	106. 10	1,83968
17	49. 19	1,30047	13. 12	1,72174	107. 20	1,83552
18	52. 19	1,30091	14. 9	1,71865	108. 29	1,83107
19	55. 18	1,30160	15. 6	1,71537	109. 36	1,82632
20	58. 16	1,30258	16. 1	1,71189	110. 42	1,82125
21	61. 14	1,30384	16. 56	1,70820	111. 47	1,81585
22	64. 9	1,30539	17. 50	1,70430	112. 51	1,81010
23	67. 3	1,30722	18. 43	1,70017	113. 53	1,80398
24	69. 54	1,30931	19. 35	1,69580	114. 53	1.79749
25	72. 42	1,31164	20. 27	1,69118	115. 53	1,78960
26	75. 27	1,31417	21. 17	1,68630	116. 51	1,78329
27	78. 8	1,31685	22. 6	1,68115	117. 47	1,77555
28	80. 45	1,31964	22. 54	1,67572	118. 42	1,76737
29	83. 17	1,32249	23. 42	1,67000	119. 36	1,75872
30	85. 45	1,32535	24. 28	1,66398	120. 28	1,74958
31	88. 7	1.32816	25. 13	1,65763	121. 19	1,73995
32	90. 25	1,33087	25. 58	1,65096	122. 8	1,72979
33	92. 38	1,33340	26. 41	1,64395	122. 56	1,71909
34	94. 46	1,33572	27. 23	1,63658	123. 43	1,70784
35	96. 49	1,33776	28. 4	1,62884	124. 28	1,69601
36	98. 46	1,33947	28. 45	1,62072	125. 12	1,68358
37	100. 39	1,34081	29. 24	1,61220	125. 54	1,67053
38	102. 27	1,34172	30. 2	1,60327	126. 36	1,65684
39	104. 10	1,34215	30. 40	1,59391	127. 16	1,64249
40	105. 49	1,34208	31. 16	1,58411	127. 55	1,62745
41	107. 24	1,34145	31. 51	1,57385	128. 32	1,61171
42	108. 54	1,34022	32. 26	1,56312	129. 9	1,59523
43	110. 20	2,33836	32. 59	1,55188	129. 44	1,57800
44	111. 42	1,33584	33. 31	1,54014	130. 18	1,55998
45	113. 0	1,33262	34. 3	1,52785	130. 52	1,54115

Figure A14. Supporting table, listing the values for the parameters A''', $\log a'''$, B''', $\log b'''$, C''' and $\log c'''$ for the latitude range 0^0 to -45^0.

Tafel 4.

	X		Y		Z	
φ	A'''	log a'''	B'''	log b'''	C'''	log c'''
— 45°	113° 0′	1,33262	34° 3′	1,52785	130° 52′	1,54115
46	114. 15	1,32867	34. 34	1,51502	131. 23	1,52147
47	115. 26	1,32395	35. 3	1,50161	131. 54	1,50092
48	116. 34	1,31844	35. 32	1,48759	132. 24	1,47945
49	117. 39	1,31210	36. 0	1,47296	132. 53	1,45705
50	118. 40	1,30491	36. 27	1,45767	133. 21	1,43365
51	119. 39	1,29681	36. 54	1,44170	133. 48	1,40924
52	120. 35	1,28780	37. 19	1,42502	134. 14	1,38376
53	121. 28	1,27783	37. 44	1,40761	134. 39	1,35716
54	122. 19	1,26686	38. 7	1,38942	135. 3	1,32940
55	123. 7	1,25486	38. 30	1,37041	135. 26	1,30043
56	123. 53	1,24178	38. 53	1,35055	135. 48	1,27017
57	124. 37	1,22759	39. 14	1,32980	136. 10	1,23857
58	125. 19	1,21223	39. 35	1,30810	136. 31	1,20556
59	125. 59	1,19566	39. 54	1,28541	136. 50	1,17106
60	126. 36	1,17782	40. 14	1,26166	137. 9	1,13498
61	127. 12	1,15865	40. 32	1,23680	137. 28	1,09724
62	127. 46	1,13808	40. 50	1,21076	137. 45	1,05774
63	128. 19	1,11603	41. 6	1,18346	138. 2	1,01635
64	128. 49	1,09244	41. 23	1,15481	138. 18	0,97296
65	129. 18	1,06719	41. 38	1,12473	138. 33	0,92742
66	129. 46	1,04019	41. 53	1,09311	138. 48	0,87957
67	130. 12	1,01132	42. 7	1,05982	139. 2	0,82925
68	130. 36	0,98045	42. 21	1,02473	139. 15	0,77624
69	130. 59	0,94743	42. 34	0,98770	139. 28	0,72031
70	131. 21	0,91208	42. 46	0,94854	139. 40	0,66122
71	131. 42	0,87421	42. 57	0,90705	139. 51	0,59864
72	132. 1	0,83357	43. 8	0,86299	140. 1	0,53223
73	132. 19	0,78990	43. 19	0,81610	140. 11	0,46157
74	132. 36	0,74286	43. 28	0,76604	140. 21	0,38618
75	132. 52	0,69208	43. 37	0,71242	140. 30	0,30547
76	133. 7	0,63709	43. 46	0,65478	140. 38	0,21874
77	133. 20	0,57730	43. 53	0,59254	140. 45	0,12512
78	133. 32	0,51202	44. 1	0,52498	140. 52	0,02356
79	133. 44	0,44034	44. 7	0,45122	140. 59	9,91270
80	133. 54	0,36110	44. 13	0,37009	141. 5	9,79081
81	134. 3	0,27280	44. 19	0,28007	141. 10	9,65560
82	134. 11	0,17337	44. 24	0,17911	141. 15	9,50400
83	134. 19	0,05992	44. 28	0,06431	141. 19	9,33165
84	134. 25	9,92822	44. 32	9,93144	141. 22	9,13223
85	134. 30	9,77171	44. 35	9,77395	141. 25	8,89588
86	134. 34	9,57941	44. 37	9,58084	141. 28	8,60613
87	134. 38	9,33071	44. 39	9,33151	141. 30	8,23208
88	134. 40	8,97937	44. 41	8,94136	141. 31	7,70435
89	134. 41	8,37781	44. 42	8,33933	141. 32	6,80158
90	134. 42	— ∞	44. 42	— ∞	141. 32	— ∞

Figure A15. Supporting table, listing the values for the parameters $A''', \log a''', B''', \log b''', C'''$ and $\log c'''$ for the latitude range -45^0 to -90^0.

Tafel 5.

	X	Y	Z
	A^{IV} = 142° 26′	B^{IV} = 232° 26′	C^{IV} = 322° 26′
φ	log a^{IV}	log b^{IV}	log c^{IV}
+ 90°	− ∞	− ∞	− ∞
89	6,04417	6,04423	4,38300
88	6,94686	6,94713	5,58686
87	7,47447	7,47507	6,29078
86	7,84836	7,84942	6,78992
85	8,13790	8,13956	7,17676
84	8,37399	8,37637	7,49252
83	8,57310	8,57635	7,75916
82	8,74509	8,74933	7,98980
81	8,89629	8,90167	8,19291
80	9,03103	9,03768	8,37426
79	9,15241	9,16047	8,53797
78	9,26271	9,27231	8,68709
77	9,36366	9,37493	8,82393
76	9,45660	9,46969	8,95028
75	9,54260	9,55766	9,06756
74	9,62252	9,63968	9,17693
73	9,69707	9,71647	9,27932
72	9,76682	9,78862	9,37551
71	9,83226	9,85659	9,46615
70	9,89381	9,92082	9,55179
69	9,95181	9,98166	9,63290
68	0,00656	0,03940	9,70988
67	0,05833	0,09430	9,78309
66	0,10734	0,14661	9,85283
65	0,15379	0,19651	9,91937
64	0,19786	0,24419	9,98295
63	0,23969	0,28981	0,04377
62	0,27943	0,33350	0,10202
61	0,31720	0,37538	0,15786
60	0,35311	0,41558	0,21146
59	0,38725	0,45419	0,26294
58	0,41972	0,49130	0,31242
57	0,45059	0,52700	0,36001
56	0,47993	0,56135	0,40583
55	0,50781	0,59444	0,44994
54	0,53428	0,62633	0,49245
53	0,55941	0,65706	0,53343
52	0,58323	0,68669	0,57295
51	0,60579	0,71528	0,61107
50	0,62713	0,74287	0,64787
49	0,64728	0,76950	0,68335
48	0,66628	0,79520	0,71762
47	0,68415	0,82002	0,75071
46	0,70092	0,84398	0,78266
45	0,71661	0,86712	0,81352

Tafel 5.

	X	Y	Z
	A^{IV} = 142° 26′	B^{IV} = 232° 26′	C^{IV} = 322° 26′
φ	log a^{IV}	log b^{IV}	log c^{IV}
+ 45°	0,71661	0,86712	0,81352
44	0,73124	0,88947	0,84332
43	0,74483	0,91105	0,87209
42	0,75740	0,93189	0,89987
41	0,76895	0,95201	0,92670
40	0,77950	0,97143	0,95260
39	0,78905	0,99018	0,97759
38	0,79761	1,00827	1,00171
37	0,80518	1,02571	1,02497
36	0,81176	1,04254	1,04741
35	0,81735	1,05876	1,06904
34	0,82195	1,07439	1,08988
33	0,82555	1,08944	1,10994
32	0,82814	1,10393	1,12926
31	0,82970	1,11786	1,14784
30	0,83023	1,13126	1,16570
29	0,82970	1,14413	1,18286
28	0,82808	1,15647	1,19932
27	0,82536	1,16831	1,21510
26	0,82149	1,17965	1,23022
25	0,81644	1,19050	1,24468
24	0,81017	1,20086	1,25850
23	0,80263	1,21075	1,27168
22	0,79374	1,22017	1,28424
21	0,78345	1,22912	1,29619
20	0,77168	1,23763	1,30752
19	0,75832	1,24568	1,31826
18	0,74327	1,25329	1,52840
17	0,72639	1,26046	1,33796
16	0,70753	1,26719	1,34695
15	0,68650	1,27370	1,35535
14	0,66306	1,27938	1,36320
13	0,63693	1,28484	1,37047
12	0,60776	1,28988	1,37720
11	0,57511	1,29451	1,38337
10	0,53839	1,29872	1,38898
9	0,49686	1,30253	1,39406
8	0,44948	1,30593	1,39859
7	0,39482	1,30892	1,40258
6	0,33075	1,31151	1,40604
5	0,25400	1,31370	1,40896
4	0,15908	1,31549	1,41134
3	0,03568	1,31688	1,41320
2	9,86069	1,31788	1,41452
1	9,56033	1,31847	1,41531
0	− ∞	1,31867	1,41558

Figure A16. Supporting table, listing the values for the parameters $A^{IV}, \log a^{IV}, B^{IV}, \log b^{IV}, C^{IV}$ and $\log c^{IV}$ for the latitude range $+90^0$ to 0^0.

Tafel 5.

	X	Y	Z
	A^{IV} = 322° 26′	B^{IV} = 232° 26′	C^{IV} = 322° 26′
φ	$\log a^{IV}$	$\log b^{IV}$	$\log c^{IV}$
0°	— ∞	1,31867	1,41558
— 1	9,56033	1,31847	1,41531
2	9,86069	1,31788	1,41452
3	0,03568	1,31688	1,41320
4	0,15908	1,31549	1,41134
5	0,25400	1,31370	1,40896
6	0,33075	1,31151	1,40604
7	0,39482	1,30892	1,40258
8	0,44948	1,30593	1,39859
9	0,49686	1,30253	1,39406
10	0,53839	1,29872	1,38898
11	0,57511	1,29451	1,38337
12	0,60776	1,28988	1,37720
13	0,63693	1,28484	1,27047
14	0,66306	1,27938	1,36320
15	0,68650	1,27350	1,35535
16	0,70753	1,26719	1,34695
17	0,72639	1,26046	1,33796
18	0,74327	1,25329	1,32840
19	0,75832	1,24568	1,31826
20	0,77168	1,23763	1,30752
21	0,78345	1,22912	1,29619
22	0,79374	1,22017	1,28424
23	0,80263	1,21075	1,27168
24	0,81017	1,20086	1,25850
25	0,81644	1,19050	1,24468
26	0,82149	1,17965	1,23022
27	0,82536	1,16831	1,21510
28	0,82808	1,15647	1,19932
29	0,82970	1,14413	1,18286
30	0,83023	1,13126	1,16570
31	0,82970	1,11787	1,14784
32	0,82814	1,10393	1,12926
33	0,82555	1,08944	1,10994
34	0,82195	1,07439	1,08988
35	0,81735	1,05876	1,06904
36	0,81176	1,04254	1,04741
37	0,80518	1,02571	1,02497
38	0,79761	1,00827	1,00171
39	0,78905	0,99018	0,97759
40	0,77950	0,97143	0,95260
41	0,76895	0,95201	0,92670
42	0,75740	0,93189	0,89987
43	0,74483	0,91105	0,87209
44	0,73124	0,88947	0,84332
45	0,71661	0,86712	0,81352

Tafel 5.

	X	Y	Z
	A^{IV} = 322° 26′	B^{IV} = 232° 26′	C^{IV} = 322° 26′
φ	$\log a^{IV}$	$\log b^{IV}$	$\log c^{IV}$
— 45°	0,71661	0,86712	0,81352
46	0,70092	0,84398	0,78266
47	0,68415	0,82002	0,75071
48	0,66626	0,79520	0,71762
49	0,64728	0,76950	0,68335
50	0,62713	0,74287	0,64785
51	0,60579	0,71528	0,61107
52	0,58323	0,68669	0,57295
53	0,55941	0,65706	0,53343
54	0,53428	0,62633	0,49245
55	0,50781	0,59444	0,44994
56	0,47993	0,56135	0,40583
57	0,45059	0,52700	0,36001
58	0,41972	0,49130	0,31242
59	0,38725	0,45419	0,26294
60	0,35311	0,41558	0,21146
61	0,31720	0,37538	0,15786
62	0,27943	0,33350	0,10202
63	0,23969	0,28981	0,04377
64	0,19786	0,24419	9,98295
65	0,15379	0,19651	9,91937
66	0,10734	0,14661	9,85283
67	0,05833	0,09430	9,78309
68	0,00656	0,03940	9,70988
69	9,95181	9,98166	9,63290
70	9,89381	9,92082	9,55179
71	9,83226	9,85659	9,46615
72	9,76682	9,78862	9,37551
73	9,69707	9,71647	9,27932
74	9,62252	9,63968	9,17693
75	9,54260	9,55766	9,06756
76	9,45660	9,46969	8,95028
77	9,36366	9,37493	8,82393
78	9,26271	9,27231	8,68709
79	9,15241	9,16047	8,53797
80	9,03103	9,03768	8,37426
81	8,89629	8,90167	8,19291
82	8,74509	8,74933	7,98980
83	8,57310	8,57635	7,75916
84	8,37399	8,37637	7,49252
85	8,13790	8,13956	7,17676
86	7,84836	7,84942	6,78992
87	7,47447	7,47507	6,29078
88	6,94686	6,94713	5,58686
89	6,04417	6,04423	4,38300
90	— ∞	— ∞	— ∞

Figure A17. Supporting table, listing the values for the parameters $A^{IV}, \log a^{IV}, B^{IV}, \log b^{IV}, C^{IV}$ and $\log c^{IV}$ for the latitude range 0^0 to -90^0.

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