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Mixed rectangular pulses models of rainfall

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Abstract

A stochastic rainfall model, obtained as the superposition of independent Neyman-Scott Rectangular Pulses (NSRP), is proposed to provide a flexible parameterisation and general procedure for modelling rainfall. The methodology is illustrated using hourly data from Auckland, New Zealand, where the model is fitted to data collected for each calendar month over the period: 1966–1998. For data taken over the months April to August, two independent superposed NSRP processes are fitted, which may correspond to the existence of mixtures of convective and stratiform storm types for these months. The special case of the superposition of an independent NSRP process and a Poisson rectangular pulses process fits the data for January to March, whilst the original NSRP model (i.e. without superposition) fits the data for September to November. A simulation study verifies that the model performs well with respect to the distribution of annual totals, the proportion of dry periods, and extreme values.

Keywords: stochastic processes, point processes;, rainfall time series, Poisson cluster models

Introduction

Considerable research has been undertaken in developing mathematical models for rainfall based on stochastic point processes. In particular, statistical properties for models based on a Poisson cluster process, such as the Bartlett-Lewis or Neyman-Scott process, have been derived to enable these models to be fitted to historical data (Rodriguez-Iturbe *et al*., 1987; Rodriguez-Iturbe *et al*., 1988; Cowpertwait, 1998). Spatial-temporal extensions of these models have also been developed (Northrop, 1998; Cowpertwait, 1995); in particular, a fitting procedure for a spatial-temporal model based on a Neyman-Scott point process was developed and used to fit the model to multisite data extracted from the Arno Basin, Italy (Cowpertwait *et al*., 2002).

The work described here improves the fitting procedure developed in Cowpertwait *et al*., 2002 and develops a more flexible and general model parameterisation. It is focused on fitting a temporal model, based on superposed Neyman-Scott processes, for which spatial parameters can be fitted in a secondary procedure (see Section 3 in Cowpertwait *et al*. 2002).

In summary, the objectives are:

1. To propose the use of the superposition of more than one independent NSRP process (or a mixture of independent NSRP and Poisson rectangular pulses processes (PRP)) to provide a flexible model parameterisation and fitting procedure.

2. To further develop the fitting procedure of the temporal process developed further in Cowpertwait *et al*., 2002 by:
(a) including the NSRP third moment function in the fitting procedure at time scales greater than 1h;
(b) including sample properties taken at the 6h level of aggregation (i.e. using a greater range of time scales for model fitting);
(c) using harmonic curves to smooth the sample estimates over the calendar months (to reduce 'between month' sampling error);
(d) using bootstrap standard errors as weights in the optimisation algorithm to allow for the sampling error in the estimated coefficient of variation at different sampling intervals.

3. To illustrate the fitting of the extended model and provide empirical support for the methodology.

Mixed NSRP process

Suppose n temporal Neyman-Scott Rectangular Pulses (NSRP) processes are denoted $NSRP_i$, i = 1, ..., n. For a stationary period, each process is summarised by the following random variables and model parameters: (i) the time between adjacent storm origins is an independent exponential random variable with parameter λ_i (so storm origins arrive in a Poisson process); (ii) the waiting time for a cell origin after a storm origin is an independent exponential random variable with parameter β_i; (iii) the lifetime of a cell is an independent exponential random variable with parameter η_i; (iv) the number of cells per storm C_i is an independent random variable, which is taken here to be geometric with mean μ_{C_i}; (v) the intensity X_i of a cell is an independent random variable that remains constant throughout the cell lifetime, and is taken here to be a Weibull random variable with survivor function $P(X > x) = e^{-(x/\theta_i)^{\alpha_i}}$.

An NSRP process is equivalent to a Poisson rectangular pulses processes if the number of cells C per storm is always one. The cell dispersion parameter β then becomes redundant and the single cell can be attached to the storm origin without affecting the properties of the Poisson process. Further details of the Poisson rectangular pulses model and mathematical properties are given in Rodriguez-Iturbe *et al*. (1987).

Let $\{Y_{ij}^{(h)} : j = 1, 2, ...\}$ be the resultant rainfall time series over a sampling interval of width h due to the $NSRP_i$ process, so that the total rainfall in the j-th time interval, due to the superposition of the n independent NSRP processes, is the sum $Y_j^{(h)} = \sum_{i=0}^{n} Y_{ij}^{(h)}$.

The following statistical properties of the rainfall series of the aggregated $NSRP_i$ process are known from previous work (e.g. see Rodriguez-Iturbe *et al*., 1987; Cowpertwait, 1998):

$$\mu_i(h) = E[Y_{ij}^{(h)}] = \mu_i \mu_{C_i} \mu_{X_i} / \eta_i \qquad (1)$$

$$\begin{aligned}\gamma_i(h,l) = Cov[Y_{i,j}^{(h)}, Y_{i,j+l}^{(h)}] = \lambda_i \eta_i^{-3} A(h,l)[2\mu_{C_i} E(X_i^2) \\ + \mu_{X_i}^2 \beta_i^2 E(C_i^2 - C_i)]/(\beta_i^2 - \eta_i^2) \\ - \lambda_i \mu_{X_i}^2 B(h,l) E(C_i^2 - C_i)/\beta_i(\beta_i^2 - \eta_i^2)\end{aligned} \qquad (2)$$

where $A(h,0) = (h\eta_i + e^{-\eta_i h} - 1)$, $B(h,0) = (h\beta_i + e^{-\beta_i h} - 1)$, and, for l a positive integer, $A(h,l) = \frac{1}{2}(1 - e^{-\eta_i h})^2 e^{-\eta_i h(l-1)}$, $B(h,l) = \frac{1}{2}(1 - e^{-\beta_i h})^2 e^{-\beta_i h(l-1)}$;

$$\begin{aligned}\xi_i(h) = E[\{Y_{ij} - \mu_i(h)\}^3] = 6\lambda_i \mu_{C_i} E(X_i^3) \\ (\eta_i h - 2 + \eta_i h e^{-\eta_i h} + 2e^{-\eta_i h})/\eta_i^4 \\ + 3\lambda_i \mu_{X_i} E(X_i^2) E(C_i^2 - C_i) p(\eta_i, \beta_i, h)/\{2\eta_i^4 \beta_i (\beta_i^2 - \eta_i^2)^2\} \\ + \lambda_i \mu_{X_i}^3 E\{C_i(C_i - 1)(C_i - 2)\} \\ \times q(\eta_i, \beta_i, h)/\{2\eta_i^4 \beta_i (\eta_i^2 - \beta_i^2)(\eta_i - \beta_i)(2\beta_i + \eta_i)(\beta_i + 2\eta_i)\}\end{aligned} \qquad (3)$$

where p and q are functions containing high-order powers in η_i and β_i and are given in Cowpertwait (1998). In the above equations, for X_i is a Weibull random variable, $E(X_i^{\,r}) = \theta_i^r \Gamma(1 + r/\alpha_i)$ for r = 0, 1, 2, ..., and, for C_i a geometric random variable, $E(C_i^2 - C_i) = 2\mu_{C_i}(\mu_{C_i} - 1)$ and $E\{C_i(C_i - 1)(C_i - 2)\} = 6\mu_{C_i}(\mu_{C_i} - 1)^2$.

Thus, each $NSRP_i$ process is summarised by a parameter set $\mathbf{P}_i = \{\lambda_i, \mu_{C_i}, \beta_i, \eta_i, \alpha_i, \theta_i\}$, and statistical properties {ì $_i$(h), ã$_i$(h), î $_i$(h)} for i = 1, ..., n. Hence, the superposition of n independent NSRP processes, denoted as *SNSRP(n)*, is summarised by a parameter set: $\mathbf{P}^n = \bigcup_{i=1}^{n} \mathbf{P}_i$. Note that *SNSRP*(1) is equivalent to the original NSRP process. For a general discussion on the superposition of point processes refer to Cox and Isham (1980).

As the $NSRP_i$ process is independent, the statistical properties of the superposed process *SNSRP*(n) (abbreviated below to S_n) are the sum of the equivalent properties for each NSRP process, i.e.

$$\mu_{S_n} = \mu_1(h) + ... + \mu_n(h) \qquad (4)$$

$$\gamma_{S_n} = \gamma_1(h,l) + ... + \gamma_n(h,l) \qquad (5)$$

$$\xi_{S_n} = \xi_1(h) + ... + \xi_n(h) \qquad (6)$$

For notational convenience, for a sampling interval of width h, let $\sigma_i^2(h) = \gamma_i(h,0)$ be the variance and $\gamma_i(h) = \gamma_i(h,1)$ be the lag 1 autocovariance. Following Cowpertwait *et al*. (2002), dimensionless functions are preferred when fitting the model. For the superposed process, these are given by:

Coefficient of variation, $\nu_{S_n}(h) = \sigma_{S_n}(h)/\mu_{S_n}(h)$ (7)

Autocorrelation (lag 1), $\rho_{S_n}(h) = \gamma_{S_n}(h)/\sigma_{S_n}^2(h)$ (8)

Coefficient of skewness, $\kappa_{S_n}(h) = \xi_{S_n}(h)/\sigma_{S_n}^3(h)$ (9)

The set of dimensionless model functions for an *SNSRP*(n) process is $\mathbf{S}_n = \{\nu_{S_n}(h), \rho_{S_n}(h), \kappa_{S_n}(h)\}$. The set $\bigcup_i [\mathbf{S}_i \bigcup \{\mu_{S_i}\}]$, with equivalent sample estimates, can be used to estimate the parameters in $\mathbf{P}^n$.

Fitting Procedure

The fitting procedure is illustrated using a 33-year record (1966–1998) of hourly data taken from the North Shore of Auckland, New Zealand. The fitting procedure proceeds in stages as follows.

1. A subset $\tilde{\mathbf{S}}_1 \subset \mathbf{S}_1$ of dimensionless model functions for the $NSRP_1$ process is selected to represent the rainfall process over a range of time scales. The choice made when fitting to the Auckland data was the coefficient of variation, lag 1 autocorrelation, and coefficient of skewness, each sampled over 1-, 6-, and 24-hour time intervals, i.e. $\tilde{\mathbf{S}}_1 = \{\nu(h), \rho(h), \kappa(h) : h = 1, 6, 24\}$.

2. For each calendar month i, sample properties (denoted as $f'(i)$, using the notation 'prime' to mean 'estimate'), which are the ensemble sample equivalents to the functions in $\tilde{\mathbf{S}}_1$ given in step 1, are estimated by pooling all available data taken over the month. This also assumes that the ensemble properties are approximately stationary over the period of a calendar month. Further details on the evaluation of the sample properties are given in Cowpertwait *et al*. (2002).

3. Harmonic curves are fitted through the pooled sample estimates obtained in step 2 using least squares regression, i.e. if $f'(i)$ is the estimate for the ith calendar month (i = 1, ..., 12), and μ_i is random error, then the harmonic model:

$$f'(i) = c_0 + \sum_{j=1}^{5} \{c_j \cos(2\pi ij/12) + s_j \sin(2\pi ij/12)\} + \varepsilon_i$$

is fitted using stepwise regression to ensure only those terms (c_j, s_j) of significance are included in the final model. (The S 'step' function discussed by Venables and Ripley (2002) on p175 was used for this purpose.) This procedure assumes that the ensemble properties should have a seasonal variation that varies smoothly over the calendar months, and, therefore, reduces 'between month' sampling error. The fitted harmonic value for the ith month is denoted as $f''(i)$ to distinguish it from the pooled sample estimate $f'(i)$. The final estimate, i.e. that predicted by the fitted stochastic model is then denoted $f'''(i)$.

4. For each calendar month (i = 1, ..., 12), λ_i, μ_{C_i}, β_i, η_i, α_i for the original NSRP process ($SNSRP(1) \equiv NSRP_1$) are estimated by minimising the following sum of squares (with n = 1):

$$SS(n) = \sum_{h=1,6,24} w_h \cdot \left\{ \left(1 - \frac{\nu''_{S_n}(h)}{\nu_{S_n}(h)}\right)^2 + \left(1 - \frac{\nu_{S_n}(h)}{\nu''_{S_n}(h)}\right)^2 + \left(1 - \frac{\rho''_{S_n}(h)}{\rho_{S_n}(h)}\right)^2 + \left(1 - \frac{\rho_{S_n}(h)}{\rho''_{S_n}(h)}\right)^2 + \left(1 - \frac{\kappa''_{S_n}(h)}{\kappa_{S_n}(h)}\right)^2 + \left(1 - \frac{\kappa_{S_n}(h)}{\kappa''_{S_n}(h)}\right)^2 \right\} \quad (10)$$

$SS(1)$ is not a function of θ, so the above minimisation routine is used to provide the estimates: $\{\lambda'_{1,i}, \mu'_{C_{1,i}}, \beta'_{1,i}, \eta'_{1,i}, \alpha'_{1,i}\}$ for i = 1, ..., 12. Bounded optimisation is used to reduce the parameter space in the search routine. In particular, $\mu_{C_i} \geq 1$ so a lower bound of 1 is needed on this parameter when minimising SS.
Non-parametric bootstrap standard errors can be used for the weights w_h in equation 10.
For the Auckland data, the weights w_1, w_6 and w_{24} were taken as: 0.30^{-1}, 0.20^{-1}, and 0.13^{-1} respectively, which corresponded to the reciprocals of the bootstrap standard errors for the coefficient of variation (CV) sampled at 1-, 6-, and 24-hour time intervals (a median value was used to obtain one value for all the months). This indicates that properties sampled at 24-hour time intervals are likely to be more accurate than those sampled at smaller time intervals and therefore should receive a higher weight in the minimisation routine.
For the Auckland data the minimum was $SS(1)$ = 3.26.

5. The parameters for a superposed $NSRP_2$ process are estimated, by minimising $SS(2)$, i.e. equation 10 with n = 2, and taking $\theta_1 = \theta_2$, ensuring that θ_1 and θ_2 do not appear in $SS(2)$. Thus, partly for convenience, the scale factor is taken to be the same in both processes. A further possibility, which would also ensure that θ_1 and θ_2 do not appear in $SS(2)$, would be to take $\theta_{,1} = k\theta_2$ for some constant k, but this would introduce a further parameter, and so was not investigated further. The parameter estimates obtained for the $NSRP_1$ process in step 4 are retained with the exception of λ_1 which is re-estimated. (λ_1 and λ_2 are both given small lower bounds to allow the possibility of the special cases $\lambda_1 \to 0$ or $\lambda_2 \to 0$ arising in the estimation procedure.)

Care is needed to avoid over-parameterisation and over-fitting because the superposition of several NSRP processes could lead to large numbers of parameters. Some of the estimates obtained for $NSRP_1$ in step 4 could be used in the superposed process $NSRP_2$. Hence, it is judicious to consider a range of subsets of $\mathbf{P}_1$ for inclusion in $\mathbf{P}_2$ to avoid over-parameterisation.

For the Auckland data, the following parameter set was selected for the $SNSRP(2)$ model: $\{\lambda_1, \mu_{C_1}, \beta_1, \eta_1, \alpha_1, \theta_1, \lambda_2, \mu_{C_2}, \eta_2\}$, which has $\beta_2 = \beta_1$, $\alpha_2 = \alpha_1$, and $\theta_1 = \theta_2$. With the exception of θ_1 which is estimated last (see step 6), these parameters were estimated for each month by minimising $SS(2)$ in equation 10. An improved fit to the sample properties was obtained, as the minimum $SS(2) = 1.24 < 3.26 = SS(1)$ in step 4. This is over a 50% reduction in the minimum, though currently there is no formal procedure to assess this against the increase in model parameters.

6. Finally, the scale parameter $\theta_{1,i}$ for the ith month ($i = 1, ..., 12$) is estimated directly from the sample mean taken over 1h time intervals.

   Thus, for the Auckland data and the parameterisation given in step 5, $\theta_{1,i}$ was estimated from:

$$\theta'_{1,i} = \bar{x}_i \left\{ \frac{\lambda'_{1,i}\mu'_{C_{1,i}}\Gamma(1+1/\alpha'_{1,i})}{\eta'_{1,i}} + \frac{\lambda'_{2,i}\mu'_{C_{2,i}}\Gamma(1+1/\alpha'_{2,i})}{\eta'_{2,i}} \right\}^{-1} \quad (11)$$

   where $\bar{x}_1$ was the estimated mean rainfall for the ith month. This procedure gives an exact fit to the sample mean for each month.

Fitted model

The parameter estimates for the Auckland data are given in Table 1. Discussion is focused on January and July as representative of summer and winter respectively.

For January, there are two storm types, one represented by an NSRP process and the other by a Poisson Rectangular Pulses (PRP) process which is the special case of $\mu_C \equiv 1$ in an NSRP process. The PRP process is the more frequent of the two ($0.017 = \lambda'_{2,1} > \lambda'_{1,1} = 0.0041$), and represents isolated convective cells, whilst the NSRP process represents clusters of convective cells. When compared with the PRP process, the NSRP process has a slightly smaller value of η indicating that the clusters of cells tend to have longer lifetimes, which might correspond to stratiform rain.

For July (winter), there are two storm types both with clustering, i.e. there are two superposed NSRP processes (Table 1). The more frequent (type 2) has shorter expected cell lifetimes ($1/\eta'_{2,7} \approx 0.4$ h) compared with the less frequent (type 1) storms ($1/\eta'_{1,7} \approx 1$ h) occurring in July. These estimates may reflect the mixture of stratiform and convective storm types that occur in July.

Comparing the estimates for January with those for July, in January there are fewer but more intense cells, because $\lambda'_{j,1}\mu'_{C_{j,1}} < \lambda'_{j,7}\mu'_{C_{j,7}}$ and $\theta'_{1,1}\Gamma(1+1/\alpha'_{1,1}) < \theta'_{1,7}\Gamma(1+1/\alpha'_{1,7})$, which characterises summer convective rainfall compared with winter stratiform rainfall.

The additional parameterisation, i.e. the superposed mixed model, was not needed from September to November. (Possibly a single NSRP model would also suffice for December, which had a small estimate of λ_2) Consequently, the fitting procedure and model parameterisation provide additional flexibility without forcing in additional parameters when they are not needed.

The fitted and sample properties are plotted in Figs. 1–4. There is some slight underestimation of 1 h skewness and overestimation of 6 h skewness over the summer months (see Fig. 3), which may have some effect on summer extreme values. However, overall, the model fits the data well and the results support the use of the superposed process.

Validation of fitted model

For validation, it is appropriate to simulate data using the fitted model and to compare simulated and historical properties that were not used in the fitting procedure but which are likely to be important in hydrological applications.

Table 1. Fitted Model: Parameter Estimates for Auckland Data

Month i	$\lambda'_{1,i}$	$\mu'_{C_{1,i}}$	$\beta'_{1,i}$	$\eta'_{1,i}$	$\alpha'_{1,i}$	$\lambda'_{2,i}$	$\mu'_{C_{2,i}}$	$\eta'_{2,i}$	$\theta'_{1,i}$
1	0.00409	14.4	0.0721	1.98	0.645	0.0173	1.0	2.43	1.86
2	0.00367	20.4	0.081	1.94	0.617	0.0185	1.0	2.01	1.65
3	0.00660	15.3	0.107	1.64	0.626	0.00608	1.0	1.24	1.24
4	0.00704	13.9	0.109	1.33	0.57	0.00457	13.2	2.42	0.954
5	0.00523	30.8	0.0947	1.13	0.433	0.0133	20.7	2.78	0.222
6	0.00485	32.2	0.104	0.755	0.391	0.0156	37.0	2.22	0.114
7	0.0125	15.2	0.163	0.837	0.457	0.0142	26.2	2.63	0.219
8	0.0229	6.24	0.145	1.09	0.65	0.00507	11.7	2.98	0.871
9	0.0197	7.56	0.0956	1.3	0.667	0.0	.	.	1.09
10	0.0158	10.1	0.0954	1.41	0.617	0.0	.	.	0.711
11	0.0151	8.14	0.109	1.54	0.641	0.0	.	.	1.12
12	0.00977	7.84	0.0889	1.75	0.659	0.000682	7.84	1.75	2.0

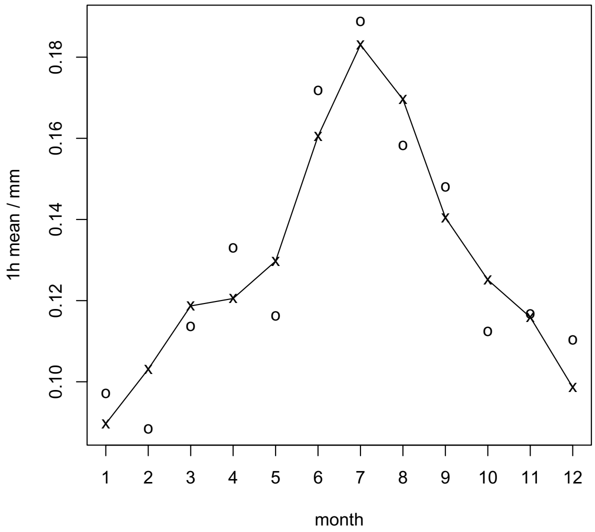

Fig. 1. *Mean rainfall for 1-hour data: o Sample estimate* μ'_{S_2} *from historical data; — harmonic estimate* μ''_{S_2} *; × fitted value* μ'''_{S_2} *(an exact fit is obtained to the harmonic estimate).*

Hence, five records of length equal to the historical record (33-years) were simulated using the fitted model (Table 1). (Five records were used so that an informal assessment of sampling variability in the extreme values could be made.)

Neither the proportion of dry intervals or the extreme values were used to fit the model. Consequently, as both of these may be important in hydrological applications, they were selected for model validation. In addition, the distribution of annual totals can be important for planning purposes. As this aggregation level is much greater than any used to fit the model, a comparison of historical and simulated distributions of annual totals also seemed appropriate for model validation.

Hence, the annual totals were found for each year in the simulated and historical data, and a quantile-quantile plot was used to compare the distributions (Fig. 5). The proportion of dry intervals for 12- and 24-hour time scales were evaluated for each month in both the historical and simulated series (Figs. 6 and 7). A bound of 1mm was used to define a 'dry' period to avoid potential discrepancies in trace values which are not likely to be of practical importance (see Cowpertwait ,1998, for a further discussion on dry bounds in a similar context). In addition, the ordered maximum rainfalls for each year over 1-, 6-, 12-, and 24-hour sampling intervals were calculated and plotted against the reduced Gumbel variate (the return period T was also plotted; Figs. 8-9).

The distribution of simulated annual totals follows closely

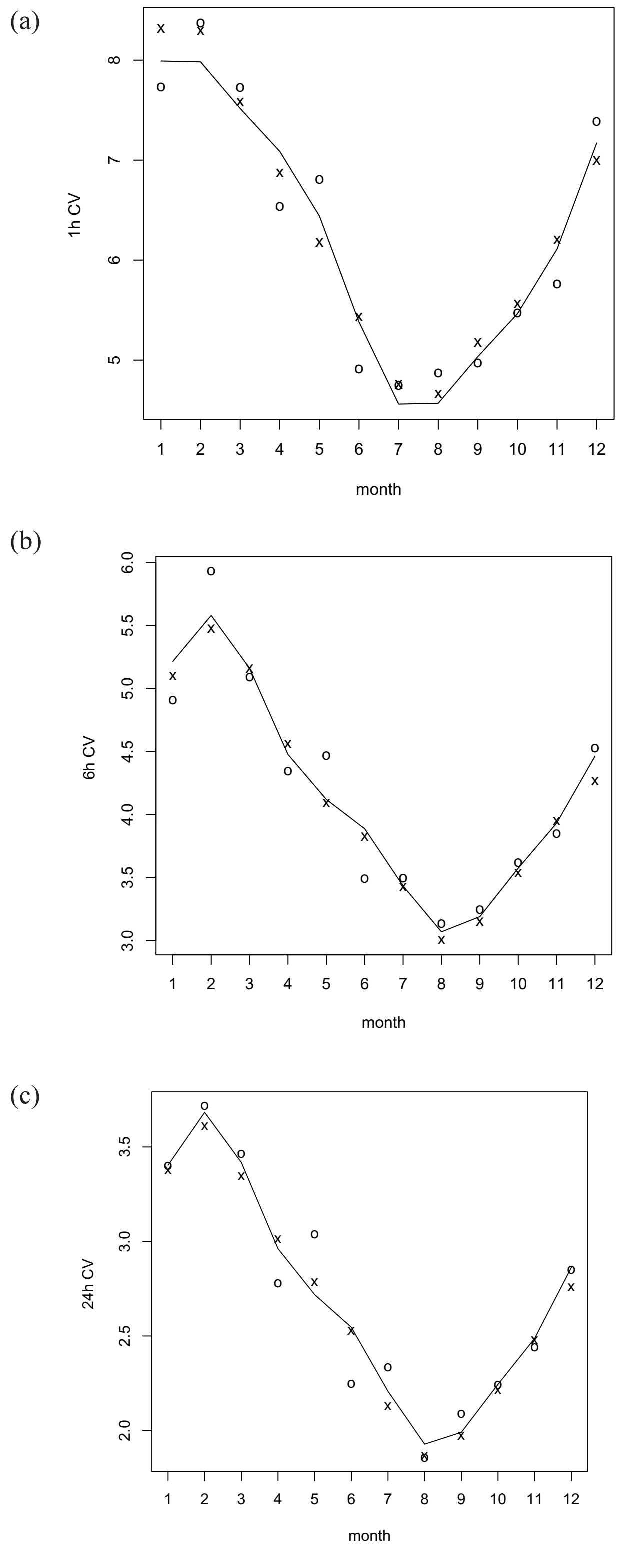

Fig. 2. *Coefficient of variation over (a) 1-, (b) 6-, and (c) 24-hour sampling intervals: o Pooled Sample Estimate* v'_{S_2} *; — Harmonic Estimate* v''_{S_2} *; × Fitted Value* v'''_{S_2} *.*

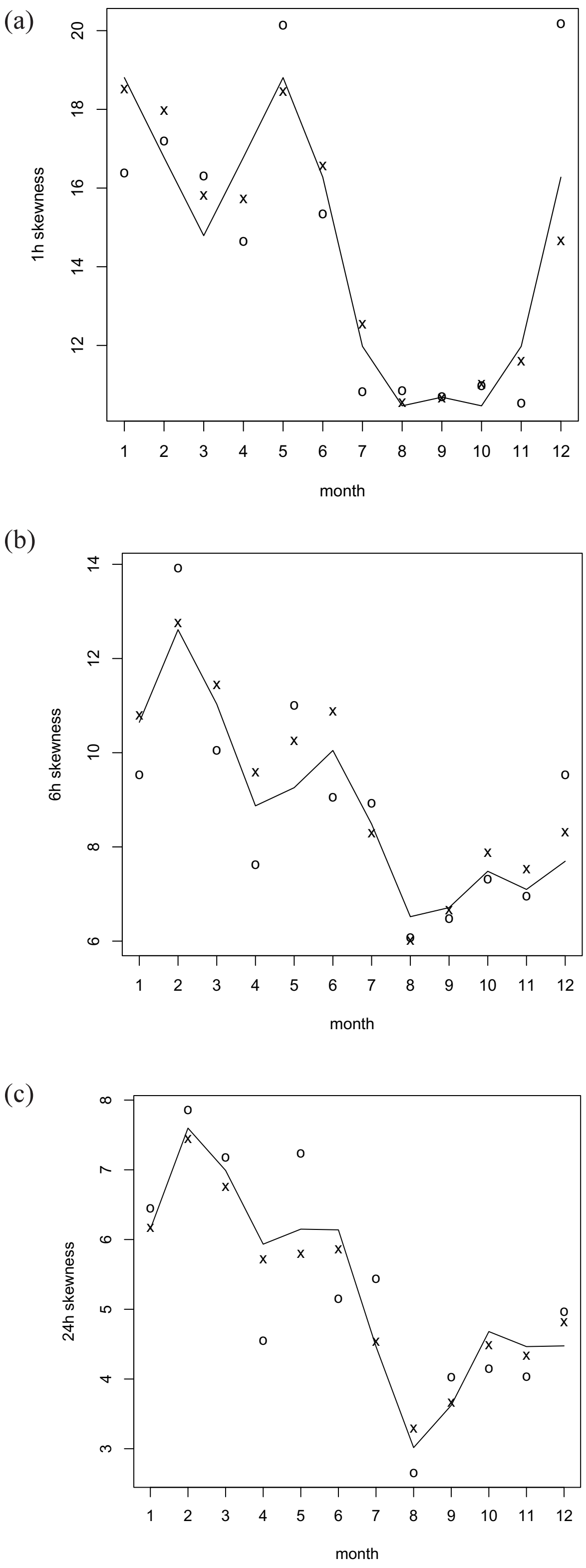

Fig. 3. *Coefficient of skewness over (a) 1-, (b) 6- and (c) 24-hour sampling intervals: o Pooled Sample Estimate* κ'_{S_2} *; —Harmonic Estimate* κ''_{S_2} *; × Fitted Value* κ'''_{S_2} *.*

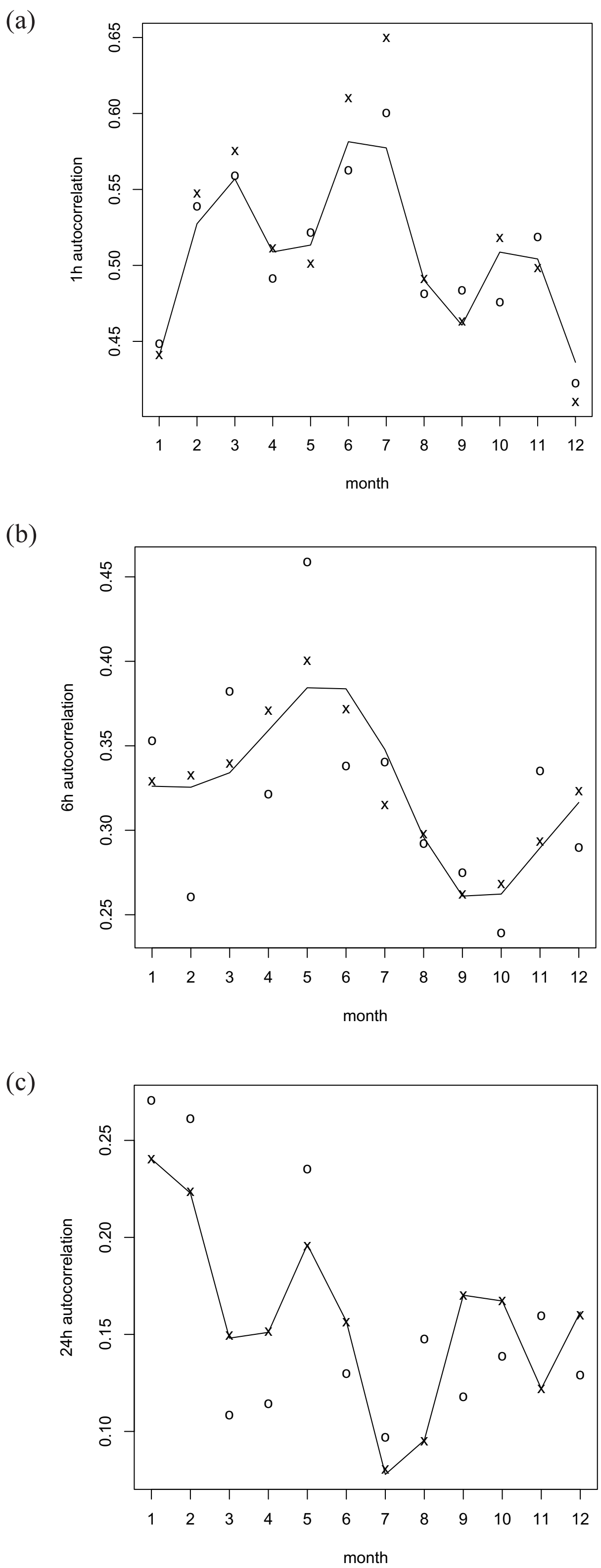

Fig. 4. *Autocorrelation over (a) 1-, (b) 6- and (c) 24-hour sampling intervals: o Pooled Sample Estimate* ρ'_{S_2} *; — Harmonic Estimate* ρ''_{S_2} *; × Fitted Value* ρ'''_{S_2} *.*

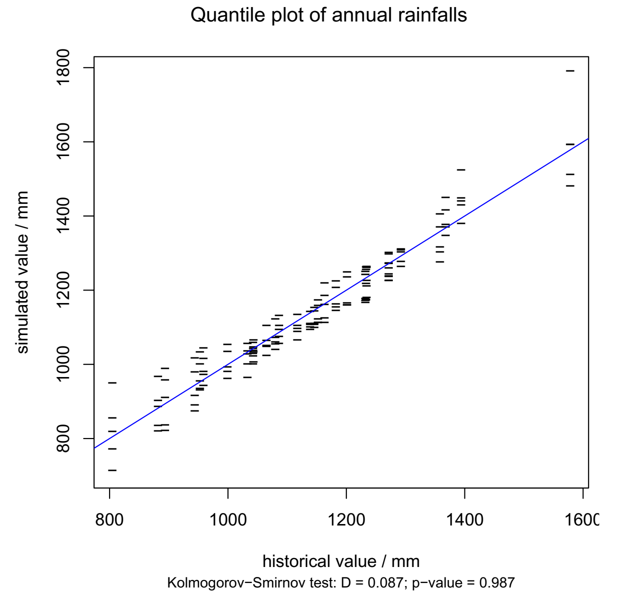

Fig. 5. *Comparison of distribution of historical and simulated annual totals (mm); Kolmogorov-Smirnov test has* D *= 0.15 and p-value 0.52 showing there is insufficient statistical evidence to reject the null hypothesis that the distributions are the same.*

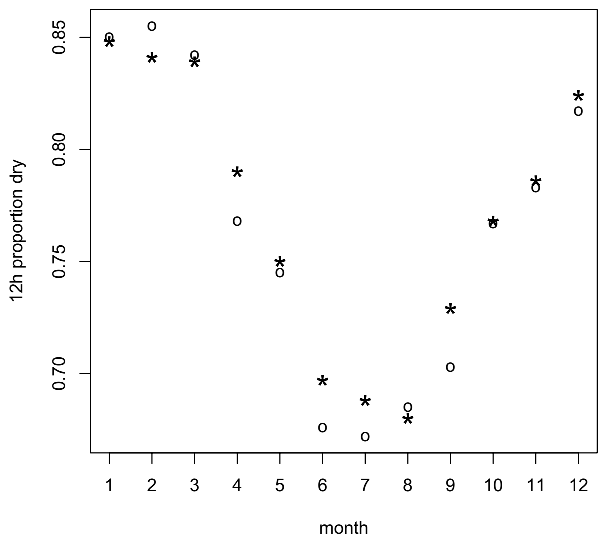

Fig. 6. *Proportion of dry rainfall (< 1mm) over 12-hour sampling intervals: o Sample estimate from historical data (33-years); * Sample estimate from simulated data (5 × 33 years).*

the distribution of annual totals in the historical data (Fig. 5). In particular, the Kolmogorov-Smirnov test showed there was no significant difference between the historical and simulated values (Fig. 5).

In general, the simulated data also show a good agreement

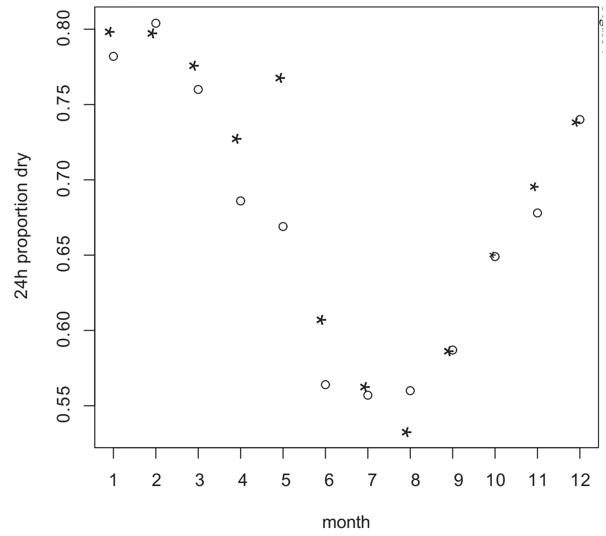

Fig. 7. *Proportion of dry rainfall (< 1mm) over 24-hour sampling intervals: o Sample estimate from historical data (33-years); * Sample estimate from simulated data (5 × 33 years).*

with the proportion of dry intervals (Figs. 6 and 7). Some over-estimation was noted for April and September. However, as the simulated values follow the seasonal pattern in the historical values closely, and the model was fitted to harmonic estimates, these were not regarded as significant discrepancies.

The model simulates representative extreme values, especially in the upper part of the distribution tail, because the historical extreme values tend to fall between the five simulated values; the figures suggest that the historical extremes are a possible realisation of the model (Figs. 8 and 9).

Conclusions

A stochastic model and fitting procedure based on the superposition of independent NSRP processes was developed to provide additional parameterisation, flexibility, and generality when fitting rectangular pulses models to rainfall data. In particular, the procedure can be applied to the spatial-temporal model in Cowpertwait *et al.*, 2002 before fitting cell radii parameters.

Overall, the fit to sample properties up to third order over a range of time scales was satisfactory. Furthermore, the simulated data showed good agreement with the distribution of annual totals, the proportion of dry intervals, and extreme values.

Consequently, the results support the use of the methodology in hydrological applications.

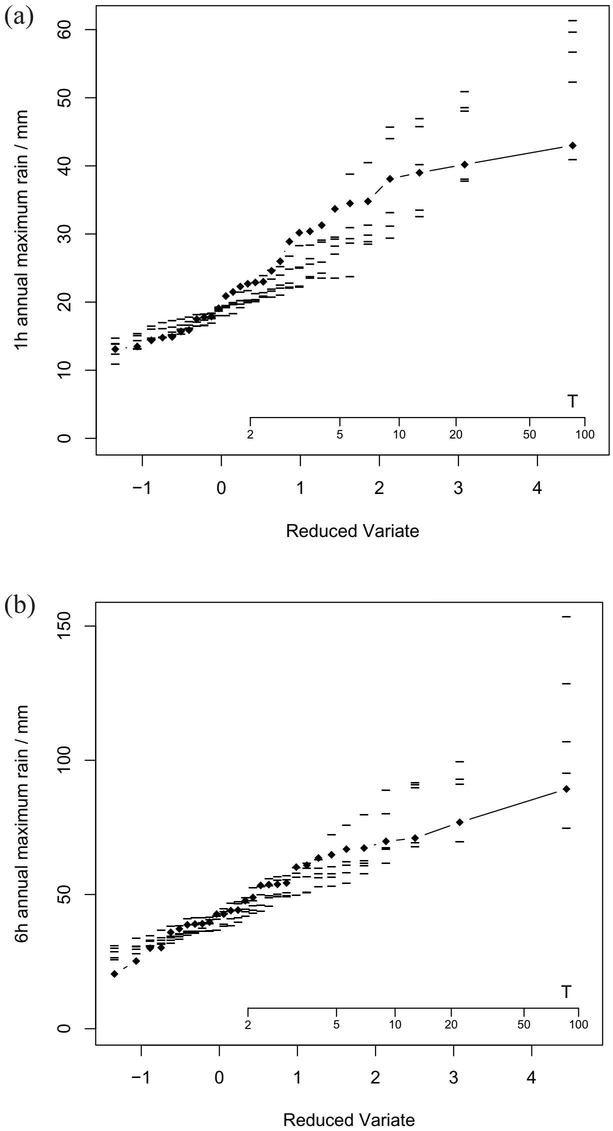

Fig. 8. *Gumbel probability plot for annual maximum rainfall over (a) 1- and (b) 6-hour sampling intervals: –•– Historical value; - Simulated values.*

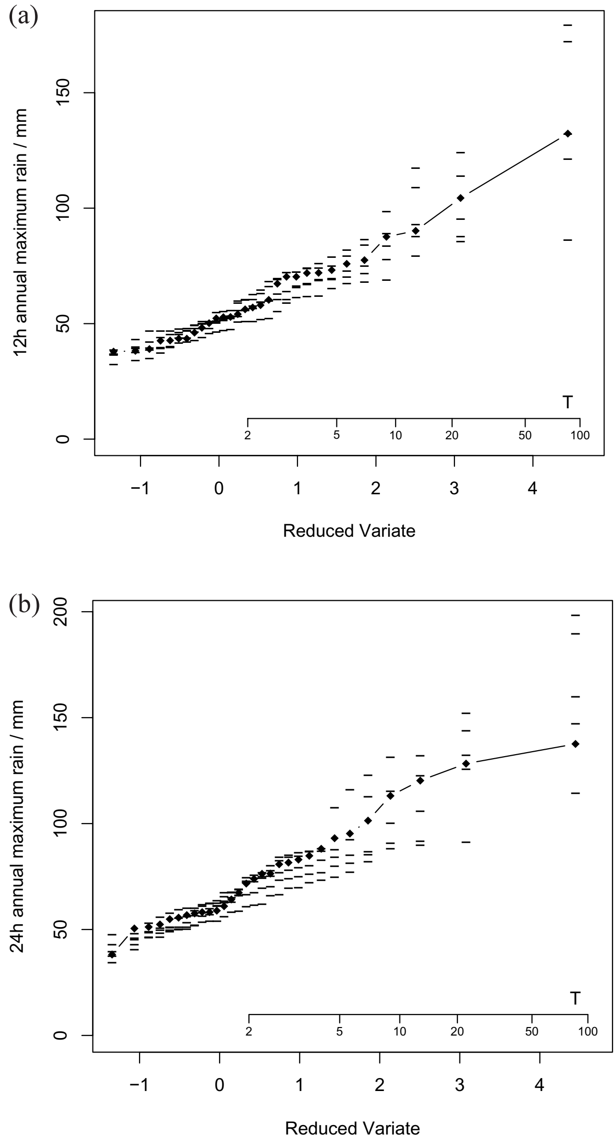

Fig. 9. *Gumbel probability plot for annual maximum rainfall over (a) 12- and (b) 24-hour sampling intervals: –•– Historical value; - Simulated values.*

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