Special Issue

HYREX: the HYdrological Radar EXperiment

Preface

The HYdrological Radar EXperiment (HYREX) was a Natural Environment Research Council (NERC) Special Topic Research Programme which ran from 1993 for three years. NERC Special Topics aimed to 'pump prime' or boost research in selected areas, bringing together a community of researchers working under a common theme. The broad aim of HYREX was to gain a better understanding of rainfall variability, as sensed by weather radar, and how this variability impacts on river flow at the catchment scale.

The impetus for HYREX arose from the work of the NERC Steering Committee on the Hydrological Applications of Weather Radar, which later evolved into the Interagency Committee on the Hydrological Use of Weather Radar. This Committee serves as a UK forum for the exchange of ideas on the hydrological use of weather radar. Working under the chairmanship of Dr Peter Walsh, the Committee developed a research review document entitled 'Opportunities for Radar Hydrometeorology during the 1990s: Proposals for Strategic Research to address UK requirements'. The review of requirements for basic research identified the pressing need for a radar hydrology facility which would complement the operational radar network with a dense raingauge network, experimental radars and a variety of hydrometeorological sensors. This gave birth to the concept of a focused community hydrological radar experiment to which the acronym HYREX was applied.

The Brue catchment in Somerset, south-west England, was chosen to be the experimental catchment for HYREX because river flows were well gauged and it enjoyed good radar coverage from the Wardon Hill, Cobbacombe Cross and Chilbolton radar installations. A vital part of the experiment was the provision of a dense network of 49 raingauges for the 135 km^2 catchment to the gauging station at Lovington. This was managed as a National Rivers Authority (NRA) R and D Project by Ms Linda Aucott.

A NERC HYREX Steering Committee was set up under the chairmanship of Professor Mike Hall (International Institute for Infrastructural, Hydraulic and Environmental Engineering, IHE-Delft), with the following members and affiliations (at that time): Dr Keith Browning (University of Reading), Professor Ian Cluckie (University of Salford), Mr Bob Hatton (NRA), Dr Anthony Illingworth (UMIST), Mr Bob Moore (Institute of Hydrology), Dr John Tyson (North West Water Ltd) and Dr Howard Wheater (Imperial College). Mrs Lisa Stewart (Institute of Hydrology) and Mrs Anne Roberts (NERC) served on the NERC Secretariat for the Committee. The Committee was responsible for reviewing applications for grants, monitoring progress, stimulating initiatives such as workshops and seminars, and reporting on the Programme to NERC.

HYREX benefited from the support of the NRA, the Met Office, the Ministry of Agriculture, Fisheries and Food (MAFF), North West Water Ltd. and the Rutherford Appleton Laboratory (RAL). These organisations supported the infrastructure costs whilst NERC provided grants to the researchers involved in HYREX. The NRA provided the dense raingauge network, installed and operated under contract by AllWater Technology, and river flow data from the Brue catchment. The Met Office provided radar data from Wardon Hill and Cobbacombe, radiosondes and C130 aircraft flights. MAFF funded a HYREX community database at the Institute of Hydrology (IH). North West Water Ltd. provided funding for a Vertically-Pointing Radar operated by the University of Salford. Doppler dual-polarisation radar data from the Chilbolton radar was provided by RAL. The NERC Equipment Pool provided an Automatic Weather Station, and a Soil Water Station plus disdrometer were supplied by IH.

The HYREX Programme posed six major scientific questions:

- What are the structures in rainfall systems and can the parameterisation of mesoscale and cloud physics models be improved by using radar?

- What is the spatial structure of rainfall fields and can this be established by comparisons of point and spatial estimates of rainfall using dense raingauge networks and radar?

- Which is the most effective radar system for the discrimination of precipitation (rain, snow, hail, graupel) and for the subsequent use of precipitation estimation in hydrological forecasting?

- Can the outputs from global satellite-based rainfall measurements be improved by using radar/rainfall networks for calibration?

- Do improved spatial rainfall estimates from radar lead to better estimates of runoff and is there any scale dependency?

- Can integrated atmospheric/hydrological models for rainfall and flow forecasting be developed with the support of weather radar?

These questions along with the experimental facilities were used as an invitation to researchers to bid into the Programme. This led to the selection of six projects addressing aspects of all of these questions, with the exception of that relating to satellite measurements. The six projects, subdivided into three broad themes, were:

Rainfall measurement

Design of radar/raingauge networks for hydrological use: Institute of Hydrology

Radar hydrometeorology using a vertically pointing radar: University of Salford

Verification of polarisation radar techniques for improving estimates of rainfall: University of Reading and Rutherford Appleton Laboratory

Stochastic rainfall models

Spatial-temporal rainfall fields: modelling and statistical aspects: University of London (Imperial College, University College) and Nuffield College Oxford

The development of a stochastic space-time rainfall forecasting system for real-time flow forecasting: University of Newcastle upon Tyne

Physically-based rainfall models

Methods for short period precipitation and flow forecasting incorporating radar data: University of Reading and Institute of Hydrology

The sponsors of HYREX placed particular emphasis on the development of a 'community' to further hydrological research using weather radar. To this end, workshops attended by all the recipients of the awards, the Steering Committee and invited guests were held at regular intervals. For the first two workshops in 1993 and 1994, venues were chosen close to the Brue catchment and the Chilbolton radar installation so that site visits could be undertaken by all participants. The remaining three workshops (two in 1995 and one in 1996) were held at IH Wallingford. An important by-product of the discussions at these workshops was the agreement between the award holders to concentrate their attention on specific 'intensive observation periods' so that the maximum possible benefit could be derived from the available instrumentation.

A final conference reporting on the six projects marked the end of the Programme. It was convened by the British Hydrological Society, NERC and the Royal Meteorological Society, on the 6 November 1996 at the Institution of Civil Engineers in London. The results from each project are presented in this Special Issue of Hydrology and Earth System Sciences.

Whilst formal support for HYREX as a NERC Special Topic ended in April 1997, the NRA and its successor, the Environment Agency, continued to maintain the dense raingauge network over the Brue catchment until spring 2000. MAFF supported IH's work on the quality control and data management of the HYREX archive and the Met Office agreed to supply data from the Wardon Hill and Cobbacombe Cross radars for this extended period. IH, which became the Centre for Ecology and Hydrology (CEH) Wallingford in April 2000, also maintained the automatic weather and soil stations.

NERC Seedcorn funding has supported web access to the archive at the British Atmospheric Data Centre. The HYREX dataset and metadata are now available to the international research community via the web address: http://www.badc.rl.ac.uk/data/hyrex

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Hydrology & Earth
System Sciences

Design of the HYREX raingauge network

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Abstract

Dense raingauge experiments in the past have experienced difficulties in the automated recording of rainfall amount and timing which with the benefit of modern instrument technology are now less problematic. The HYdrological Radar EXperiment, HYREX, provided a timely opportunity to design and implement a dense raingauge network in support of rainfall measurement and modelling research studies concerned with the use of weather radar in hydrology. The principles and random function theory underlying the design of this raingauge network over the Brue catchment in south-west England are detailed in this paper.

Keywords: raingauge, design, network, rainfall, flood, spatial correlation

Context

There has not been a dense recording raingauge network experiment in the UK since the two investigations at Cardington and Winchcombe over the period 1954 to 1967. The Cardington experiment in Bedfordshire was based on a core set of 16 gauges at 1 km spacing over a flat 10 km² area: it was "bedevilled by instrument troubles" (Holland, 1967). A hilly area in the vicinity of Winchcombe in the Cotswolds provided a natural extension of the Cardington experiment. Lewis (1986) provides an incisive review of the problems besetting both experiments and concludes "that if the instrumental technology of 25 years later had been available the results of the investigations would have been much more reliable." Notable advances include the replacement of strip charts by digital logging devices and the development of quartz clocks, ensuring time-synchronised rainfall measurements. Weather radars were absent from both experiments and their relevance to radar rainfall measurement accuracy can only be pursued indirectly, for example as done in the work of Kitchen and Blackall (1992). Huff (1970) provides an example outside the UK of an experiment using 50 recording raingauges over 260 km² of east Central Illinois in the USA. Carried out from 1952 to 1953, it employed 6-hour charts from which 1-minute rainfall amounts were extracted for analysis of the spatial distribution of rainfall rates. The density of network required to measure accurately the large gradients in rainfall rate argued for the combined use of radar and raingauges in future investigations.

The Dee Weather Radar and Real Time Hydrological Forecasting Project (CWPU, 1977) utilised a special network of raingauges for weather radar assessment purposes. Commissioned from 1972 to 1976, the network comprised at its peak some 80 tipping-bucket raingauges over the 1000 km² catchment of the Dee to Erbistock in North Wales. However, tipping bucket recording gauges were still at the development stage and problems were experienced especially with the magnetic tape recording of data. It "required constant monitoring to maintain the ratio of data returned to data potential above the minimum of 70% that was considered acceptable". Related studies have, or are being done, in other climatic conditions (e.g. Lebel et al., 1991) but their results cannot be generalised to conditions in the UK. The ability to analyse the joint sampling characteristics of raingauge and radar measurements of rainfall fields through a properly designed radar/raingauge experiment benefiting from modern instrument technology has been long overdue.

The Experiment

The HYdrological Radar EXperiment (HYREX) was a three year research programme starting in 1993. HYREX aimed to advance hydrological science through gaining a better understanding of rainfall variability in space and time, as sensed by weather radar, and how this variability impacts on flow regimes at the catchment scale (Moore et al., 1994). The community research investigation, funded by the
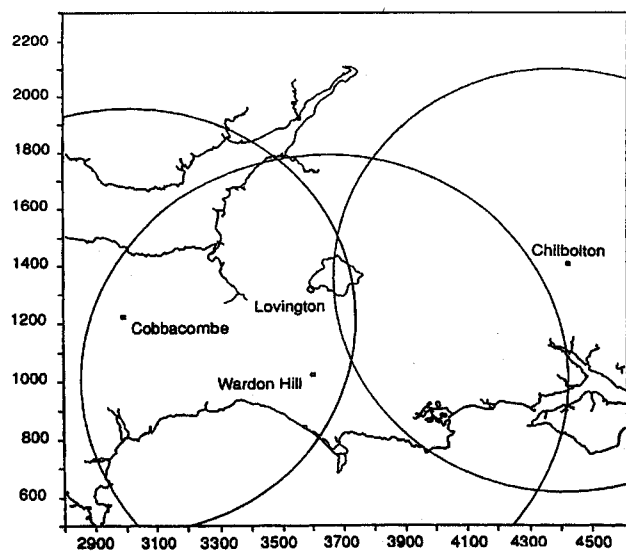
Fig. 1. *The Brue catchment gauged at Lovington and the associated scanning radars (75 km radar circles indicated).*

Natural Environment Research Council under its Special Topic Programme, supported six research projects based at the University of London (Imperial and University Colleges), Newcastle, Reading and Salford together with the Institute of Hydrology (IH). Themes of research ranged from improved precipitation measurement using polarisation and vertical pointing radars, through network design of radar/raingauge networks and spatial-temporal modelling of rainfall fields, to rainfall forecasting based on stochastic and meteorological concepts.

An important focus to the project was a set of experimental facilities centred on the Brue catchment in Somerset, south-west England. This comprised a new network of 49 recording raingauges over the 135 km^2 catchment, with river flows measured at Lovington, and scanned by three radars: a new Doppler C-band radar at Cobbacombe Cross, a conventional C-band radar at Wardon Hill and an experimental Doppler dual-polarisation S-band radar at Chilbolton (Fig. 1). Additional facilities included a mobile vertical-pointing X-band radar, a line network of rapid response raingauges aligned from Chilbolton towards the Brue, automatic weather and soil moisture stations, a disdrometer, radiosonde ascents from various locations and access to the Met Office Research Flight C130 aircraft. Support for this infrastructure came from the National Rivers Authority (now the Environment Agency), the Met Office, the Ministry of Agriculture Fisheries and Food and a water utility (North West Water).

One of the six research projects, carried out by IH, concerned the "Design of radar/raingauge networks for hydrological use". This aimed to review the requirements for rainfall field estimates in the hydrological sciences and to establish how these can be best met using networks of radars and raingauges. Data collected from the radars and dense

network of raingauges were to be subject to statistical analysis and physical interpretation to improve understanding of (i) the accuracy of different sensors for measuring rainfall and (ii) the sensitivity of catchment flow models to rainfall uncertainty and variability. An initial step in the HYREX Programme was the design of the raingauge network itself. A design group was set up including representation from IH, the University of London (University College and Imperial College), the University of Oxford (Nuffield College) and the NRA. The chosen design comprised 22 gauges located at the centre of 2 km radar grid squares, two SW-NE lines of four squares each containing two gauges and two squares having dense sub-networks of 8 gauges in areas of low and high relief. This gave a total of 52 gauges. The arrangement of the 8 gauge-within-a-square sub-networks was chosen so that the mean of their values would provide the "best" estimate of the mean rainfall over the square: this resulted in a diamond-within-a-square configuration with sides of length 0.778 and 1.38 km respectively. This "design requirement" was turned into an "operational design" following site visits and discussions with landowners. Installation began in September 1993 and by December was largely complete. The "as constructed" design, incorporating 49 gauges, is shown in Fig. 2. The principles used to underpin the detailed design of this network are the main subject of this paper.

Equipment

The raingauges used were typical of those deployed by the Environment Agency in support of flood forecasting, being Casella tipping-bucket raingauges with a bucket size of 0.2 mm and mounted vertically on a concrete paving slab set flush with the ground with the gauge aperture of 400 cm^2 at a height of 36 cm above this. Ground-level (pit) gauges were not used as these would depart from standard Met Office operational practice and would prove difficult to install. Each gauge was afforded the protection of a 4 by 4 m wooden fenced and gated compound, particularly necessary where large farm animals grazed. Technolog Newlog data loggers were used to record the time-of-tip and data were down-loaded at monthly intervals using a Husky portable computer; telemetry proved too expensive an option for this research experiment. A typical raingauge installation is shown in Fig. 3. Wood et al. (2000) describe some of the problems encountered in practical use and the data quality control procedures followed in forming an archive of raingauge data under HYREX.

The Catchment

The Brue catchment to Lovington drains an area of 135 km^2 rising from 20 m at the Crump weir gauging station to 260 m AOD on its eastern boundary. Land use is dominated
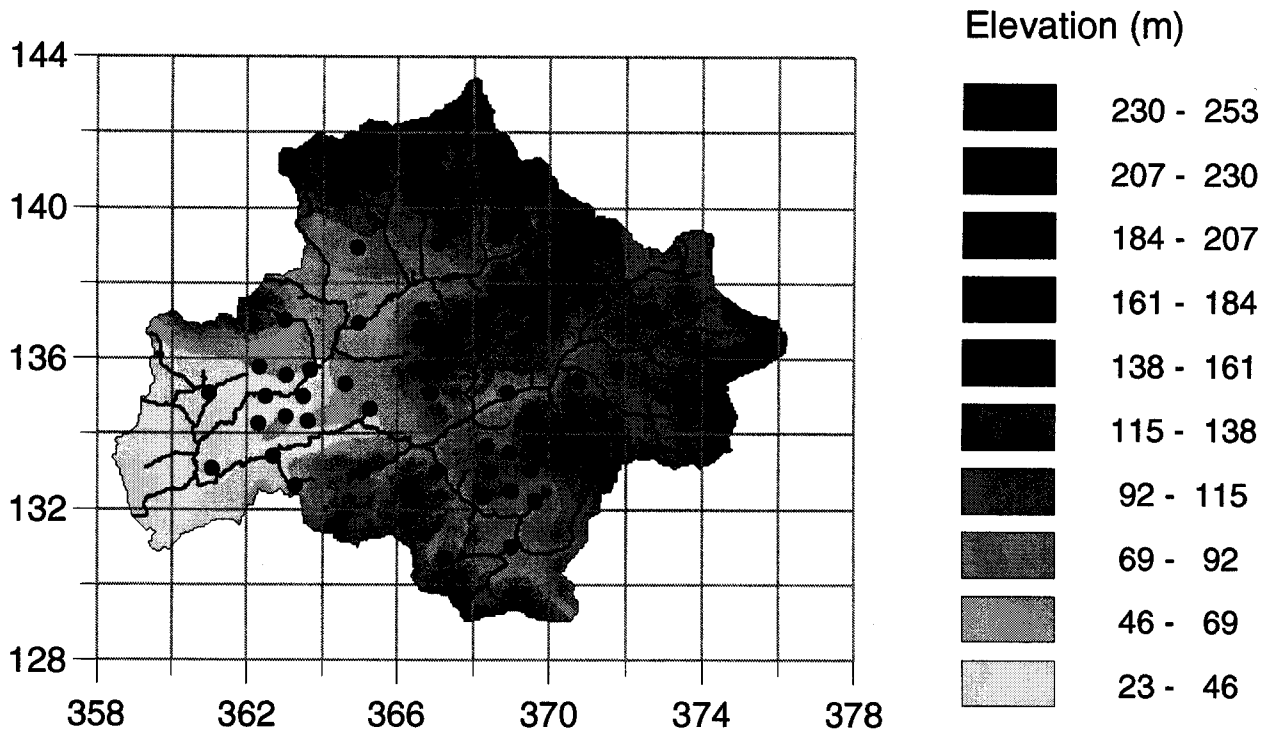
Fig. 2. *The HYREX network of 49 raingauges within the Brue catchment.*

by pasture land on clay soils with some patches of woodland in the higher relief eastern half of the catchment. Average annual rainfall for the standard period 1961 to 1990 is 867 mm. River flow has a mean of $1.88 \text{ m}^3 \text{ s}^{-1}$ and reached an instantaneous recorded maximum since 1965 of $96 \text{ m}^3 \text{ s}^{-1}$. The catchment has experienced exceptional storms and flooding as reported by Clark (1996), notably affecting the town of Bruton. Until the Martinstown (Dorset) storm of 1955, Bruton held the national record for the highest one-day rainfall at 243 mm on 28 July 1917; flooding was even greater in 1768 (Clark, 1999). A flood-detention reservoir now affords protection to Bruton, providing storage when inflows exceed about $8 \text{ m}^3 \text{ s}^{-1}$ from a catchment area of 30 km^2. The design study for the dam suggests that a 100 year return period flood of $70 \text{ m}^3 \text{ s}^{-1}$ will be reduced to $20 \text{ m}^3 \text{ s}^{-1}$.

The Brue catchment, overlapped by 52 of the 2 km squares which form the radar data grid, was chosen as the focus of HYREX because of the proximity to two national network radars at Wardon Hill, 40 km to the south, and at Cobbacombe Cross, 70 km to the west; also the experimental radar at Chilbolton lies some 70 km to the east. From a hydrological perspective, its moderate relief was seen as preferable to a completely flat area, as was its previous history of notable storms and floods. The size and relief of the catchment were seen as representative of many catchments in the UK requiring flood warning using rainfall-runoff modelling methods. Although the headwaters are fed by Mendip and Salisbury Plain springs, the water balance is not affected significantly by external influences. The gauging station at Lovington provides reliable measurements of flow up to high levels.

Detailed Design of the Raingauge Network

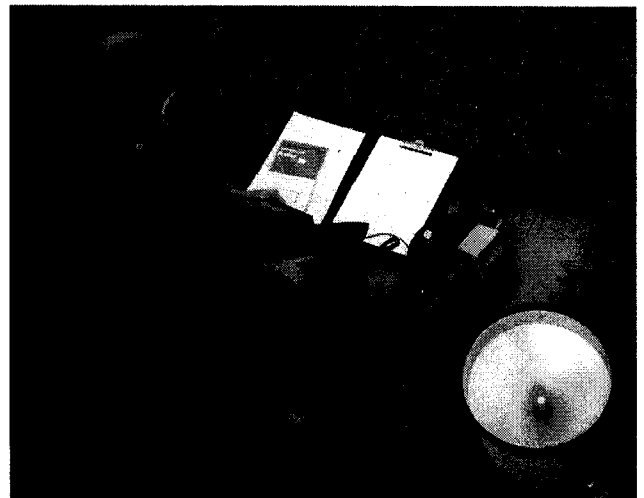The HYREX design group considered how best to design a

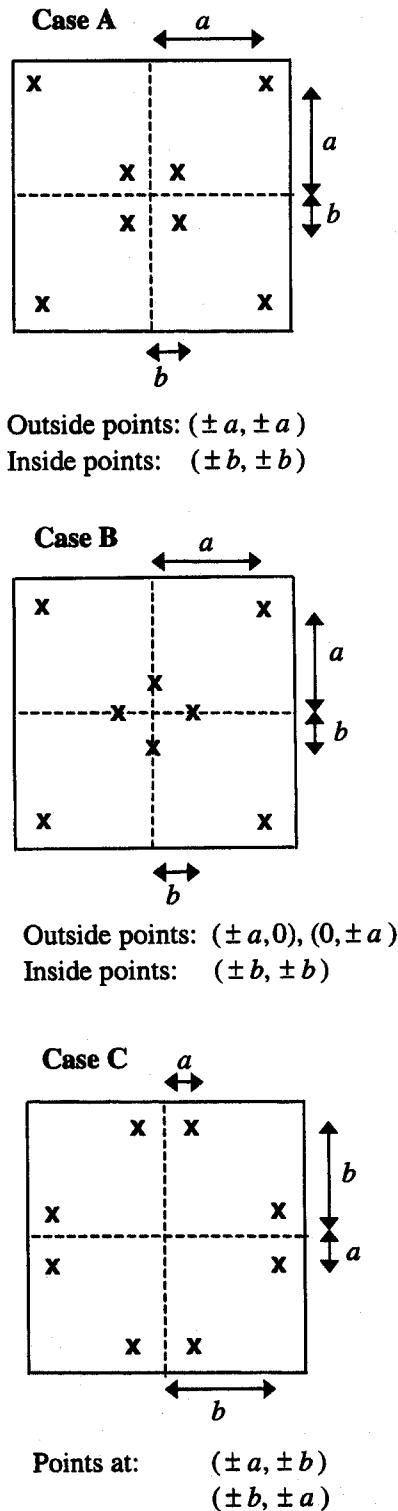
Fig. 3. *Downloading data from a raingauge installation.*

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Case A

Outside points: $(\pm a, \pm a)$
Inside points: $(\pm b, \pm b)$

Case B

Outside points: $(\pm a, 0), (0, \pm a)$
Inside points: $(\pm b, \pm b)$

Case C

Points at: $(\pm a, \pm b)$
                  $(\pm b, \pm a)$

Fig. 4. *Alternative raingauge network design configurations for 8 gauges within a square.*

principles applied were four-fold. First was the need for full catchment coverage required especially for catchment-scale water balance purposes. Second was the need for a network with a range of gauge spacings to support studies concerned with rainfall field structure and the representativeness of gauges and radars. The importance of radar to the research programme led naturally to the design network being aligned with the radar squares. In addition, there was a need to provide very good estimates of surface rainfall over 2 km squares to compare directly with estimates from radar. Third was the need to have a configuration capable of supporting studies of the orographic influence on rainfall rates and its relation to the prevailing storm direction. In this context, the orography of the Brue catchment has a northwest-southeast alignment with the prevailing storm direction orthogonal to this. Fourth was the need to constrain the network to an affordable level. These guiding principles led to a "design network" comprising of 52 raingauges configured on 2 km square regions encompassed by the catchment as follows:

(i) *'Catchment coverage' network*: 22 regions with 1 gauge
(ii) *'Dense' networks*: 2 regions with 8 gauges
(iii) *Two SW-NE 'line' networks*: 7 regions with 2 gauges

The design of the two special dense sub-networks required special attention. Two had been chosen to allow both areas of high and low relief to be represented. The Brue catchment divides roughly equally between an area of low relief to the west and an area of high relief to the east (Fig. 2.). The best siting of 8 gauges within a 2 km sided square was interpreted as that giving the best estimate of mean rainfall over the area. Other features, such as the ability to examine the local structure of the rainfall field were also of interest. The possibilities of introducing an element of randomisation in the design and of changing locations of gauges part way through the experiment were considered and thought not to be appropriate. Intuitively one would expect the best locations of 8 gauges within a square to have a symmetric arrangement of one of the three types presented in Fig. 4. For a given arrangement two parameters, a and b $(a, b < 1)$, define the x- and y-coordinates of each gauge position in a square $[-1, +1]$ by $[-1, +1]$ centred on $(0, 0)$. For Case A, the ratio $\lambda = b/a = 3$ gives a network in which the four diagonal points are equally spread. Choosing $a = \frac{1}{4}$ gives a network in which no two gauges are within $\frac{1}{2}$ km of each other. In all three cases, a network providing more information on the local structure of the rainfall field might have $a = \frac{1}{8}$ and $b = \frac{3}{4}$ ($\lambda = 6$). The arrangement that provides the best estimate of areal average rainfall within a square is investigated in the next section. Bras and Rodríguez-Iturbe (1993) provide valuable background to the use of random function theory for raingauge network design and this forms the basis of the approach followed here.

raingauge network for the Brue catchment which would best serve the broad goals of the programme. At this stage, the six specific research projects had still to be agreed. The

Estimation of areal average rainfall within a square

Let $R(x)$ denote the rainfall field at location x and p_j the position of the jth gauge in a network of N raingauges. Then the true areal average rainfall over an area A

$$\overline{R} = \frac{1}{|A|} \int_A R(x)\mathrm{d}x \qquad (1)$$

can be estimated as

$$\hat{R} = \frac{1}{N} \sum R(p_j) \qquad (2)$$

with error $\overline{R} - \hat{R}$. Under the stationarity assumptions that

$$E[R(x)] = \text{const} \qquad (3a)$$

$$\mathrm{cov}[R(x_1), R(x_2)] = \sigma(x_1 - x_2) \qquad (3b)$$

the expected mean squared estimation error (MSE) is

$$E[(\overline{R} - \hat{R})^2] = \frac{1}{|A|^2} \int_A \int_A \sigma(x - y)\mathrm{d}x\mathrm{d}y$$

$$- \frac{2}{|A|N} \sum_{j=1}^{N} \int_A \sigma(x - p_j)\mathrm{d}x$$

$$+ \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma(p_i - p_j). \qquad (4)$$

Of the three terms on the right hand side, only the second and third depend on the location of gauges. Note also that the first term

$$c = E\left[(\overline{R} - E(\overline{R}))^2\right] = \frac{1}{|A|^2} \int_A \int_A \sigma(x - y)\mathrm{d}x\mathrm{d}y \qquad (5)$$

is the mean square deviation between the notional mean over a continuous network of gauges and the assumed mean of the underlying stochastic process. The optimal locations of raingauges for a given covariance function σ, are the same as those for any covariance function of the form $\sigma^* = f\sigma + g + h\delta$ for any feasible constants f, g and h and where δ is the delta function located at zero distance.

The above theory can be used to choose the optimal placement of N gauges within an area, where optimal is understood in the special sense of achieving minimum mean square error of estimated areal average rainfall. Consider the example of placing 8 raingauges ($N = 8$) in a 2 km square ($A = 4$ km^2). Suppose that the rainfall field has unit variance with an isotropic Gaussian spatial correlation function, giving

$$\sigma(p_i - p_j) = \exp(-k\, d_{ij}^2) \qquad (6)$$

where $d_{ij} = (p_i - p_j)$ is the Euclidean distance between positions p_i and p_j and the parameter k measures the decay of the correlation. Then the estimation error is

$$E[(\overline{R} - \hat{R})^2] = c - \frac{2}{4 \times 8} \sum_{1}^{8} \int_A \exp(-k\|x - p_j\|^2)\mathrm{d}x$$

$$+ \frac{1}{8^2} \sum_{i,j}^{8} \exp(-k\, d_{ij}^2). \qquad (7)$$

The gauge position set $\{p_i\}$ is then chosen to minimise the above.

Selection of the gauge positions, for the three network designs depicted in Fig. 4, as functions of the parameters a and b introduced in the preceding section has been done using a 2-D shrinking grid search procedure. Results are presented in Table 1 for both the Gaussian spatial correlation function of Eqn. (6) and the exponential spatial correlation function for which

$$\sigma(p_i - p_j) = \exp(-k\, d_{ij}). \qquad (8)$$

Use of a final step size of 0.005 in the search means that the locations are close to the optimal ones. The best locations are identified by the parameters a and b for different values of the decay of the correlation, k. A weaker correlation with steeper decay (increasing k) is seen to reduce the standard deviation of the true mean rainfall for the square. The root mean square error of the 8 gauge average rainfall as an estimate of the true mean for the square is presented for the optimal locations. It is seen that this can be larger than the standard deviation of the true mean when the spatial correlation is very low: in these cases the long-term mean (if known) would provide a better estimate of the true mean for the square than using an average of the 8 gauges.

The results indicate that the arrangement for Case B is always best, irrespective of the type and strength of the spatial correlation function used. Figure 5 provides an indication of the sensitivity of the root mean square error (rmse) criterion to the gauge positions and to the assumed correlation function for Case B. The parameters a and b defining gauge location are used as the axes and the rmse contours are drawn for Gaussian and exponential correlation functions with the correlation parameter k set to 1. Contour levels are at 5, 10, 25, 50 and 100% above the minimum for the Gaussian function, and at 0.5, 1, 2.5, 5 and 10% above the minimum for the exponential function.

The correlation function of rainfall was not known when the network was being designed, and was a subject of the research programme itself. However, experience at the time suggested that most weight should be given to low to moderate values of k when choosing an arrangement based on the results presented in Table 1. Thus one would expect there to be both a moderately large correlation at a distance of 2–3 km and a moderately large drop from near one, as the distance increases from zero to 2–3 km. This has been confirmed by subsequent analyses, presented in Fig. 4 of

Table 1. Best gauge locations for an 8 gauge within a square sub-network.

(a) Exponential correlation function

k	St. dev. of true mean	Case A		Case B		Case C	
		a b	rmse	a b	rmse	a b	rmse
0.25	0.881	.335 .725	.0784	.54 .69	.0723	.315 .69	.0795
0.5	0.782	.335 .725	.110	.54 .69	.102	.315 .69	.112
1.0	0.630	.335 .72	.155	.54 .685	.143	.31 .685	.156
2.0	0.440	.33 .715	.211	.53 .675	.198	.305 .68	.212
4.0	0.267	.315 .705	.270	.53 .65	.261	.30 .675	.271
8.0	0.153	.29 .675	.316	.49 .635	.313	.285 .67	.316

(b) Gaussian correlation function

k	St. dev. of true mean	Case A		Case B		Case C	
		a b	rmse	a b	rmse	a b	rmse
0.25	0.861	.405 .70	0.00476	.575 .70	0.00319	.40 .69	0.00543
0.5	0.764	.395 .695	0.0159	.565 .695	0.0105	.385 .68	0.0177
1.0	0.637	.375 .695	0.0453	.55 .690	0.0286	.36 .665	0.0492
2.0	0.516	.345 .70	0.100	.535 .69	0.0591	.325 .655	0.107
4.0	0.381	.325 .715	0.161	.52 .685	0.106	.30 .675	0.169
8.0	0.282	.30 .695	0.216	.50 .635	0.187	.30 .685	0.218

Wheater *et al.* (2000), which suggests values of k in the range 0.25 to 0.5 as appropriate for the Brue catchment. The overall arrangement chosen, shown in Fig. 6, has four gauges about 550 m east, north, west and south of the centre, with the other four on the diagonals with one example being about 690 m east and 690 m north of the centre (that is 0.69 of the way from the centre to the corner at a distance of 975 m). This choice, as well as being "best" in terms of the 8 gauges estimating the mean rainfall over the square, also provides a variety of inter-gauge distances for better estimation of the spatial correlation function of the rainfall field.

The "best" sub-network of 8 gauges, configured as a diamond within a square arrangement, as shown in Fig. 6, was used as the ideal design to be followed within HYREX. Deviations from this ideal pattern were allowed only to accommodate practical siting factors. The overall design network for the Brue catchment is shown in Fig. 7, which can be compared with the "as constructed" network in Fig. 2. Difficulties with landowner permission left one 2 km square region devoid of raingauges in the northeast part of the catchment whilst other departures reflected local siting details. There was a tendency for gauges to be located towards the edge of a field but a review process ensured that the gauges were not located too near to hedges forming the field boundary. Installation of the raingauge network began
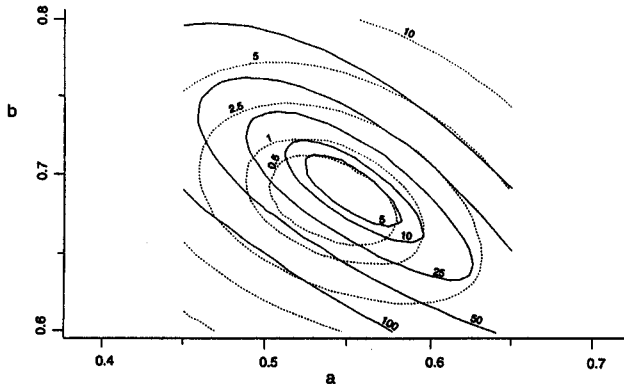
Fig. 5. *Sensitivity of root mean square error criterion to the gauge position (as indicated by the values for a and b) and to the assumed correlation function. Full lines: contours at 5, 10, 25, 50 and 100% above the minimum for the Gaussian function. Dashed lines: contours at 0.5, 1, 2.5, 5 and 10% above the minimum for the exponential function.*
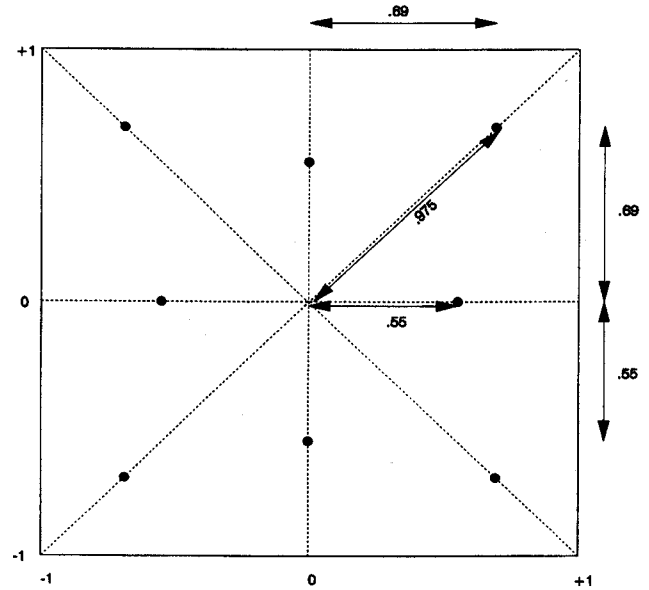
Fig. 6. *Recommended 8 gauge sub-network design with a = 0.69 and b = 0.55.*

in September 1993 and by the end of the year all 49 gauges were operational. The network continued to operate until spring 2000.

Conclusion

HYREX provided a timely opportunity to install a dense raingauge network over a catchment to support research

studies concerned with rainfall field structure, with rainfall measurement using radars and raingauges, and with relating rainfall to the associated runoff response. The principles of catchment-wide coverage, variety of gauge spacing, taking into account physical influences relating to orography and prevailing storm direction, and affordable cost underpinned
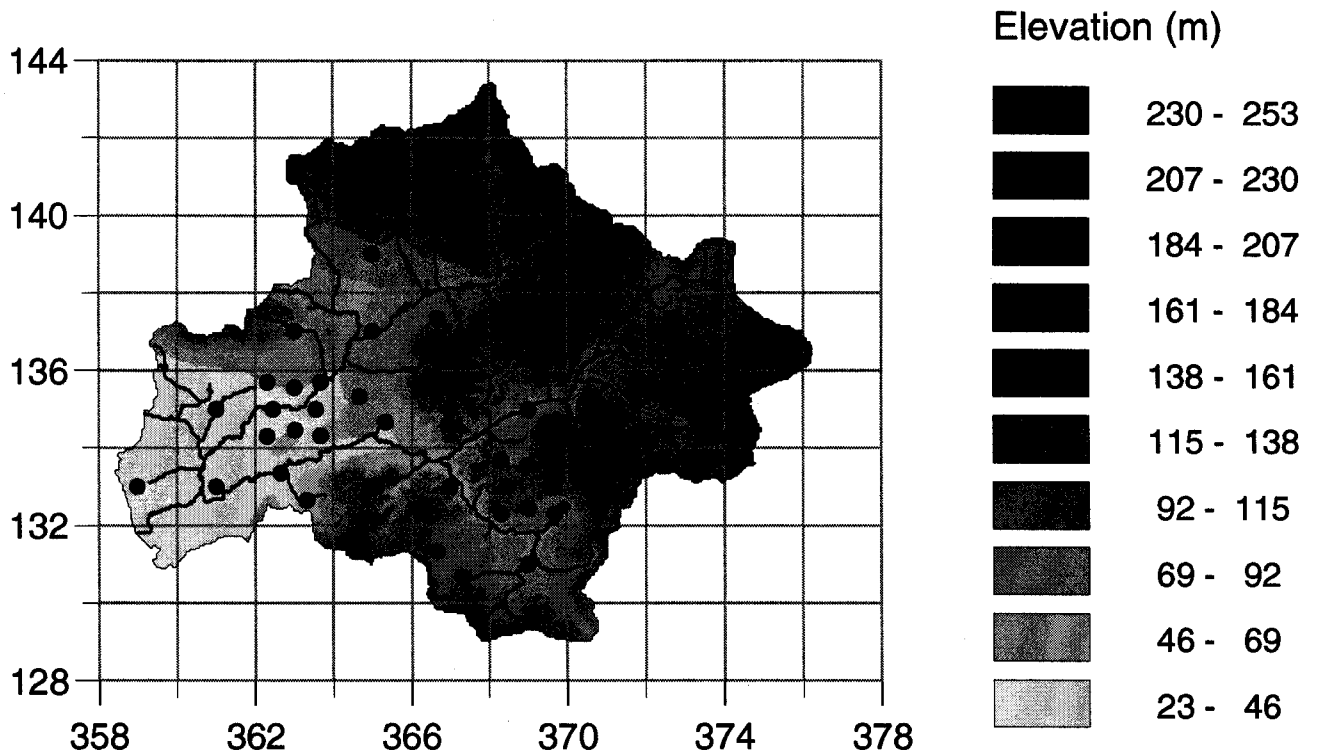
Elevation (m)

	230 - 253
	207 - 230
	184 - 207
	161 - 184
	138 - 161
	115 - 138
	92 - 115
	69 - 92
	46 - 69
	23 - 46

Fig. 7. *The idealised design for the HYREX network of 52 raingauges over the Brue catchment.*

the basic design of the HYREX raingauge network. Random function theory has been utilised to choose a configuration for the two sub-networks of 8 gauges within a 2 km square so as to meet a criterion of best estimating the areal average rainfall for each square region. This led to a diamond within a square pattern for each network of 8 gauges, one sited in a square of low relief and the other in a square of high relief. The idealised design network for the catchment was turned into an "as constructed" network, with modifications reflecting local siting issues and landowner permissions. Roberts *et al.* (2000) describe how data collected from the dense raingauge network, along with ancillary measurements from radars and other instrumentation, are now available to the international research community via a HYREX web site at the British Atmospheric Data Centre.

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Wood, S.J., Jones, D.A. and Moore, R.J., 2000. Accuracy of rainfall measurement for scales of hydrological interest. *Hydrol. Earth Syst. Sci.*, 4, 531–543.

Hydrology & Earth
System Sciences

Accuracy of rainfall measurement for scales of hydrological interest

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Abstract

The dense network of 49 raingauges over the 135 km^2 Brue catchment in Somerset, England is used to examine the accuracy of rainfall estimates obtained from raingauges and from weather radar. Methods for data quality control and classification of precipitation types are first described. A super-dense network comprising eight gauges within a 2 km grid square is employed to obtain a "true value" of rainfall against which the 2 km radar grid and a single "typical gauge" estimate can be compared. Accuracy is assessed as a function of rainfall intensity, for different periods of time-integration (15 minutes, 1 hour and 1 day) and for two 8-gauge networks in areas of low and high relief. In a similar way, the catchment gauge network is used to provide the "true catchment rainfall" and the accuracy of a radar estimate (an area-weighted average of radar pixel values) and a single "typical gauge" estimate of catchment rainfall evaluated as a function of rainfall intensity. A single gauge gives a standard error of estimate for rainfall in a 2 km square and over the catchment of 33% and 65% respectively, at rain rates of 4 mm in 15 minutes. Radar data at 2 km resolution give corresponding errors of 50% and 55%. This illustrates the benefit of using radar when estimating catchment scale rainfall. A companion paper (Wood et al., 2000) considers the accuracy of rainfall estimates obtained using raingauge and radar in combination.

Keywords: rainfall, accuracy, raingauge, radar

Introduction

Hydrological modelling and forecasting require rainfall data as one of their most important inputs. Often this will be required as an areal average estimate, the two most relevant scales of interest being the catchment scale and the size of the square grid used by weather radar (here, 2 km). In general, these areal average rainfall values will be estimated from a point measurement, for example using a raingauge, or an equivalent areal observation such as can be obtained from weather radar. The accuracy of all forms of rainfall measurement is of importance, particularly to hydrological modelling applications (Larson and Peck, 1974; Peck, 1980; Moore, 1998), and this work aims to quantify the accuracy of areal rainfall measurement by raingauges and by radar. For small flashy catchments a small time-step is necessary to model the flood response and even for larger catchments benefits can be gained from frequent model updating using flow observations (Austin and Moore, 1996). A time-step of 15 minutes forms the focus of attention here since this is commonly used in the UK for flow forecasting purposes. However, it is of fundamental importance to know how accurately rainfall can be measured over different time-scales and this is also investigated.

The investigation of rainfall accuracy reported here is made possible through the use of the specially designed HYREX raingauge network and associated weather radars. This dense network of 49 tipping-bucket raingauges installed across the Brue catchment, in Somerset, South-West England, is shown in Fig. 1. Continuously scanning C-band radars at Wardon Hill, 30 km south of the catchment, and at Cobbacombe Cross, 70 km to the west, are used here as sources of radar rainfall data. The configuration of the raingauge network is such that 19 of the gauges are located singly within 2 km radar grid squares. There are two SW-NE lines of 4 squares each containing at least two gauges, with the alignment chosen to be along the prevailing storm direction and orthogonal to the topography. In addition, there are two higher density grid squares, each containing 8 gauges, in areas of low and high relief. The two high density grid squares are designed to give best estimates of mean rainfall over a 2 km pixel. The size of the tipping-bucket is 0.2 mm and the time-of-tip is recorded to the nearest ten seconds. In addition, the Bridge Farm raingauge site (Gauge 8) also includes an automatic weather station and a soil moisture station on a permanent basis. The associated relief is shown in Fig. 2 which indicates that the higher ground is both to the east and to the north of the catchment. This higher ground lies in the path of weather systems which move in a north-easterly direction and can induce low level growth of precipitation, sometimes referred to as orographic enhancement.
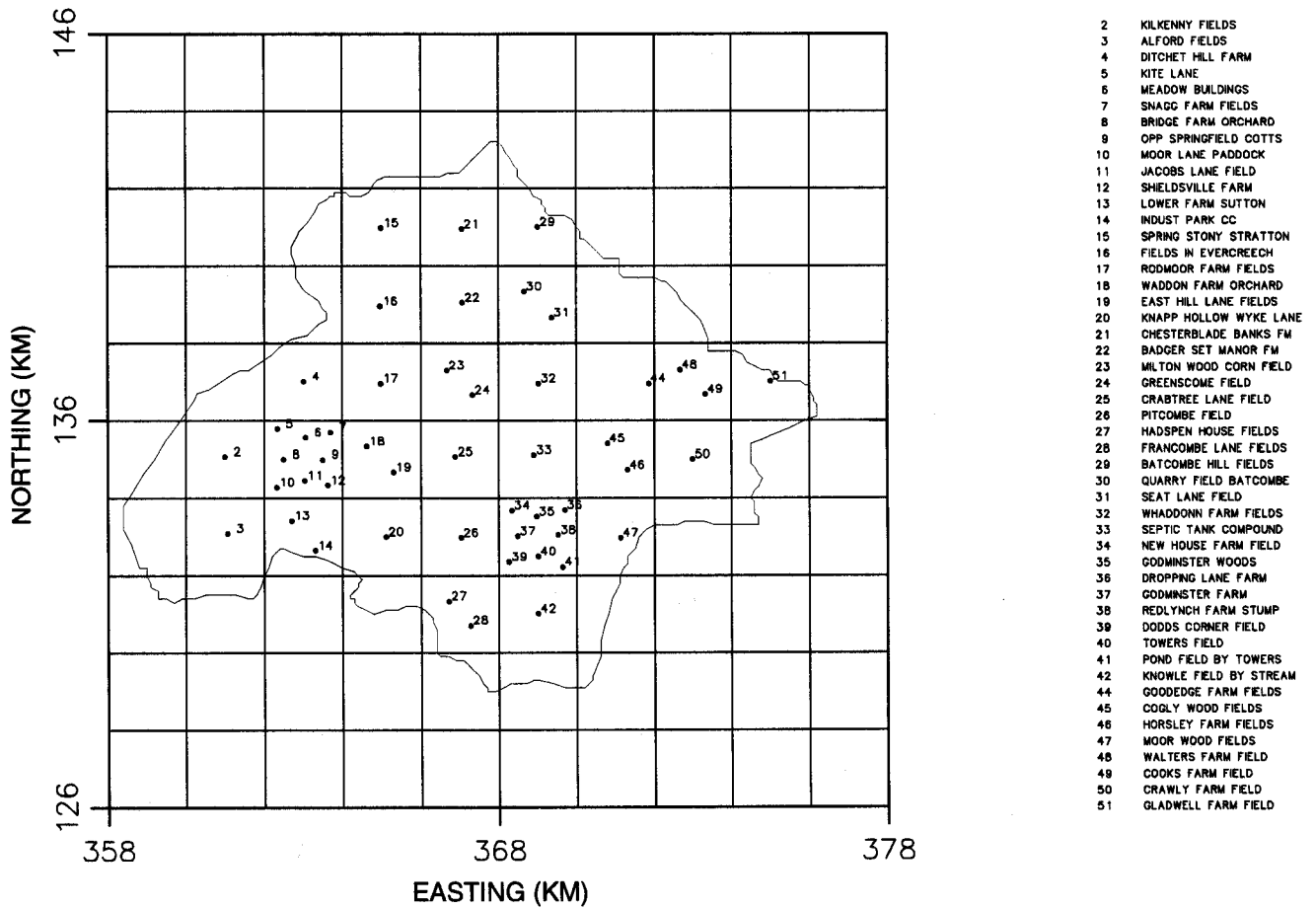
2	KILKENNY FIELDS
3	ALFORD FIELDS
4	DITCHET HILL FARM
5	KITE LANE
6	MEADOW BUILDINGS
7	SNAGG FARM FIELDS
8	BRIDGE FARM ORCHARD
9	OPP SPRINGFIELD COTTS
10	MOOR LANE PADDOCK
11	JACOBS LANE FIELD
12	SHIELDSVILLE FARM
13	LOWER FARM SUTTON
14	INDUST PARK CC
15	SPRING STONY STRATTON
16	FIELDS IN EVERCREECH
17	RODMOOR FARM FIELDS
18	WADDON FARM ORCHARD
19	EAST HILL LANE FIELDS
20	KNAPP HOLLOW WYKE LANE
21	CHESTERBLADE BANKS FM
22	BADGER SET MANOR FM
23	MILTON WOOD CORN FIELD
24	GREENSCOME FIELD
25	CRABTREE LANE FIELD
26	PITCOMBE FIELD
27	HADSPEN HOUSE FIELDS
28	FRANCOMBE LANE FIELDS
29	BATCOMBE HILL FIELDS
30	QUARRY FIELD BATCOMBE
31	SEAT LANE FIELD
32	WHADDONN FARM FIELDS
33	SEPTIC TANK COMPOUND
34	NEW HOUSE FARM FIELD
35	GODMINSTER WOODS
36	DROPPING LANE FARM
37	GODMINSTER FARM
38	REDLYNCH FARM STUMP
39	DODDS CORNER FIELD
40	TOWERS FIELD
41	POND FIELD BY TOWERS
42	KNOWLE FIELD BY STREAM
44	GOODEDGE FARM FIELDS
45	COGLY WOOD FIELDS
46	HORSLEY FARM FIELDS
47	MOOR WOOD FIELDS
48	WALTERS FARM FIELD
49	COOKS FARM FIELD
50	CRAWLY FARM FIELD
51	GLADWELL FARM FIELD

Fig. 1. *The HYREX dense raingauge network over the Brue catchment.*

An important step in the use of the HYREX dataset for investigating the accuracy of rainfall measurements from raingauges and radar is the initial quality control of the data. Indeed, instrument malfunction can be viewed as an im-portant facet of the overall rainfall measurement accuracy problem. Data quality control is discussed in detail in the next section before proceeding to the main part of the paper dealing with the quantification of rainfall measurement
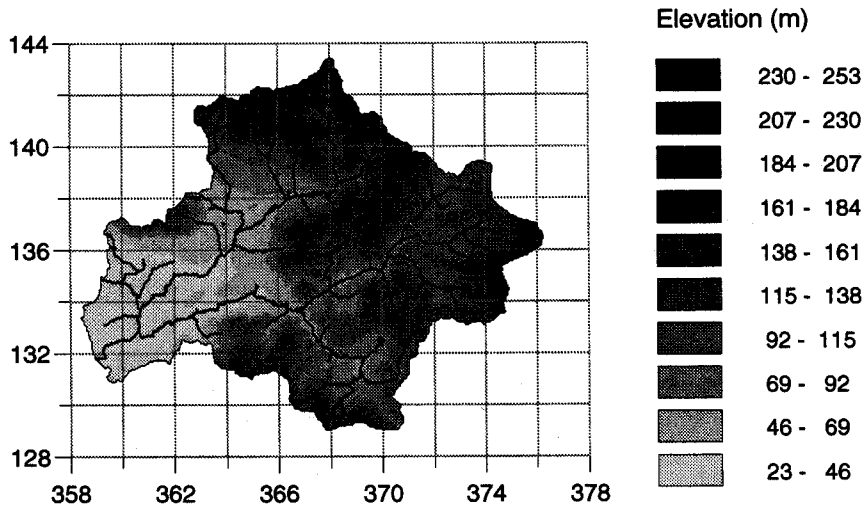

Fig. 2. *Relief over the Brue catchment.*

accuracy at different scales in space and time. Quality control extends to the identification of precipitation type, to allow later analyses to differentiate accuracy in stratiform and convective rain, and to suppress the use of data at times when precipitation falls in solid form.

Data quality control

The dense network of raingauges over the Brue catchment, whilst providing an excellent experimental facility in support of rainfall research, presents a formidable challenge in terms of data quality control. The integrity of data obtained from HYREX will be of vital importance to subsequent analyses. Periods of invalid data, when a gauge is not performing properly, are difficult to identify. Diagnostic reports were made by field staff when the data were downloaded from each gauge approximately once a month. Typical problems reported were funnels being blocked by debris and mouse damage to electrical cables. The exact time that the fault occurred within the month is thus difficult to determine. If a gauge was found to be blocked, then the corrective procedure followed was first to remove the blockage and then to let the remaining water run through and be recorded. This reduces the error in the total amount of rain recorded between successive download dates although it can cause difficulties when the data need to be analysed on a finer time-scale.

A conservative estimate of the periods of invalid data could be obtained simply by neglecting all data prior to the time when a fault was found at a particular raingauge, back-dated to when the gauge was previously visited and found to be working satisfactorily. However, this assumes that all the faults are both found and recorded: this may not always be possible, especially if the gauge rectifies itself before the next download. The use of cumulative rainfall totals over a month to highlight malfunctioning gauges has provided the best method for data quality control. Data from clusters of 10 gauges, all relatively close to one another, have been plotted together as cumulative hyetographs over a month to attempt to pinpoint any anomalous gauge behaviour. This approach, combined with the diagnostic reports and knowledge of when each gauge was visited, can lead to accurate (nearest day) assessments of periods of inoperation. It is worth noting that fewer than 50% of the faults discovered in this way were recorded in the diagnostic reports. Since the quality control method is somewhat subjective, experience has indicated that it is important not to be over-zealous in data removal: local fluctuations are to be expected, especially in convective rainfall situations. The cumulative hyetograph of Fig. 3 reveals a host of unusual features for the month of September 1994. In particular, the gauges at Towers Field (Gauge 40) and Dropping Lane Farm (Gauge 36) are seen to record apparent dramatic rainfall on the 4th and 6th respectively. Both gauges were visited on the 6th, although diagnostic reports have no

mention of faults with either gauge. The plots expose the obvious presence of gauge blockages in both cases: these periods were therefore defined as having invalid data for these gauges. The gauge at Redlynch Farm Stump (Gauge 38) illustrates a period where the trace has the gradually rising behaviour associated with a blocked gauge with water "trickling" through. This is clearly evident on and around 15 September. On this occasion a blockage is reported in the diagnostic report, dated 4 October 1994, and the data period is again defined as having invalid data.

Potential timing errors were investigated by calculating correlation coefficients of 15 minute gauge totals with either coincident radar or neighbouring raingauge totals, for a range of time shifts. In nearly all cases, a zero time shift produced the highest correlation coefficient serving to confirm the absence of longstanding timing errors in the data. However, the procedure did identify a case where timing errors had arisen, seemingly because the internal clock of a gauge had begun to "drift" over a period of several months. The availability of tip-times in the raw data allowed a correction to be made for this drift.

Figure 4 provides a summary of the valid data periods (as determined by the above quality control procedures) for each raingauge, plotted as a bar chart over time with the vertical axis indicating the gauge number. The period of record analysed here is from 1 September 1993 to 30 June 1996. For each gauge the start date of operation is taken to be the date *after* the date that the first tip was recorded. It is thus seen that the first gauges came into operation on and around 19 September 1993. The two super-dense networks of 8 gauges within a 2 km square – gauges 5 to 12 (low relief) and 34 to 41 (high relief) – are highlighted on the chart. August 1994 through September 1994 is a period which was particularly "patchy" in terms of coverage by valid gauges. Residue from the mowing of the grass in the summer months within the gauge compound is a known cause of problems, which was rectified in later years by clearing grass manually.

Snow and hail can be expected to activate the tipping mechanism of a raingauge in a different way to regular rainfall. In addition, the response of the radar will be sensitive to the type of hydrometeors present. As a consequence, part of the quality control process aimed to identify unusual weather events. Using the UK Met Office's Daily Weather Summaries, hourly weather identifiers were created to distinguish convective and stratiform type rainfall as well as solid precipitation such as snow and hail. Periods where solid forms of precipitation were detected are, in the main, omitted from the analysis carried out here since the concern is to quantify *rainfall* measurement accuracy. Figure 5 details an event on 22 February 1994 when snow fell throughout the night, stopping around mid-morning. It shows the 15 minute rainfall estimates from the radars at Cobbacombe and Wardon Hill scanning over the Brue catchment, as well as the raingauge at Bridge Farm (Gauge 8). The presence of snow storms is shown clearly by both
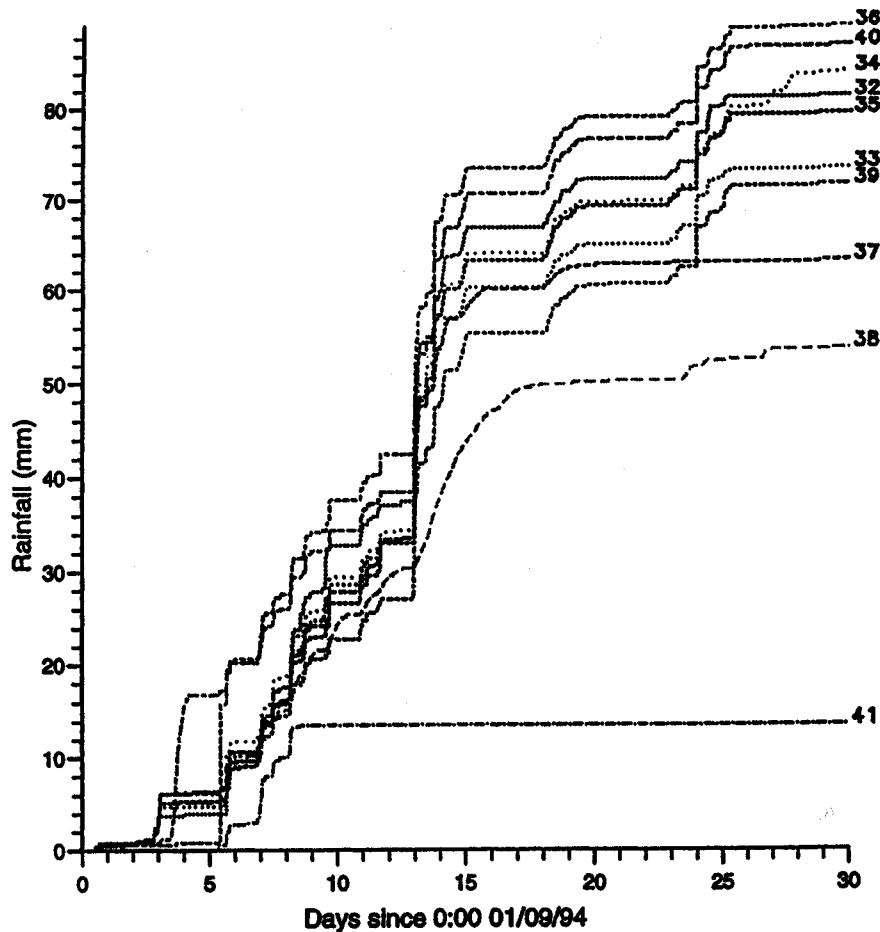
Fig. 3. *Cumulative hyetographs used for quality control of the HYREX dataset: raingauges 32 to 44 over the period 1 to 30 September 1994.*

radars although there is no response at the raingauge. The choice of the Bridge Farm gauge is due to its proximity to the Automatic Weather Station although its behaviour is typical of all other gauges in the catchment. Around 0900 the gauge begins to register a number of tips, probably signalling the melting of snow collected in the gauge funnel overnight. This is in agreement with the temperature at the automatic weather station rising above 0°C at approximately 0900. Rain which fell from midday onwards appears to be recorded satisfactorily by both radar and raingauge.

Another form of unusual precipitation is freezing rain. Only one event of this nature was identified in the HYREX dataset analysed here. This event, discussed by Pike (1996), occurred late in the evening of 30 December 1995. Figure 6 shows how both radars recorded the presence of the storm. The anomalous spike recorded by the Bridge Farm raingauge is representative of similar occurrences at many other gauges in the network but the cause of this behaviour is unknown.

Having addressed the important issue of data quality control, the resulting HYREX raingauge dataset can be used with confidence to address the central question of rainfall measurement accuracy at different space and time scales.

The substantial periods of missing data resulting from what is a carefully maintained set of raingauges would seem to have important consequences for other networks of operational raingauges. These will not enjoy typically the benefit of the spatial gauge density which has allowed the quality control procedures used here to be applied with confidence.

Besides the exclusion of snowfall events identified from Meteorological Daily Weather summaries, certain limited quality-control procedures were applied to the radar data. These were based on a comparison of monthly total rainfalls as measured by the radar and by raingauges at a pixel-scale. The analysis did not show any sudden change in the relative amounts recorded by the two sources, although it did reveal a consistent bias in the radar data treated as an estimate of the rainfall that would be recorded by a raingauge on the ground. It also revealed a small number of periods of anomalous propagation affecting recorded radar values over the Brue catchment. Values from these periods do not affect the results presented here since the estimated "true" rainfall is zero in these cases and such cases are excluded from the analysis. This does highlight the fact that the comparison between radar and raingauge-based estimates excludes cases
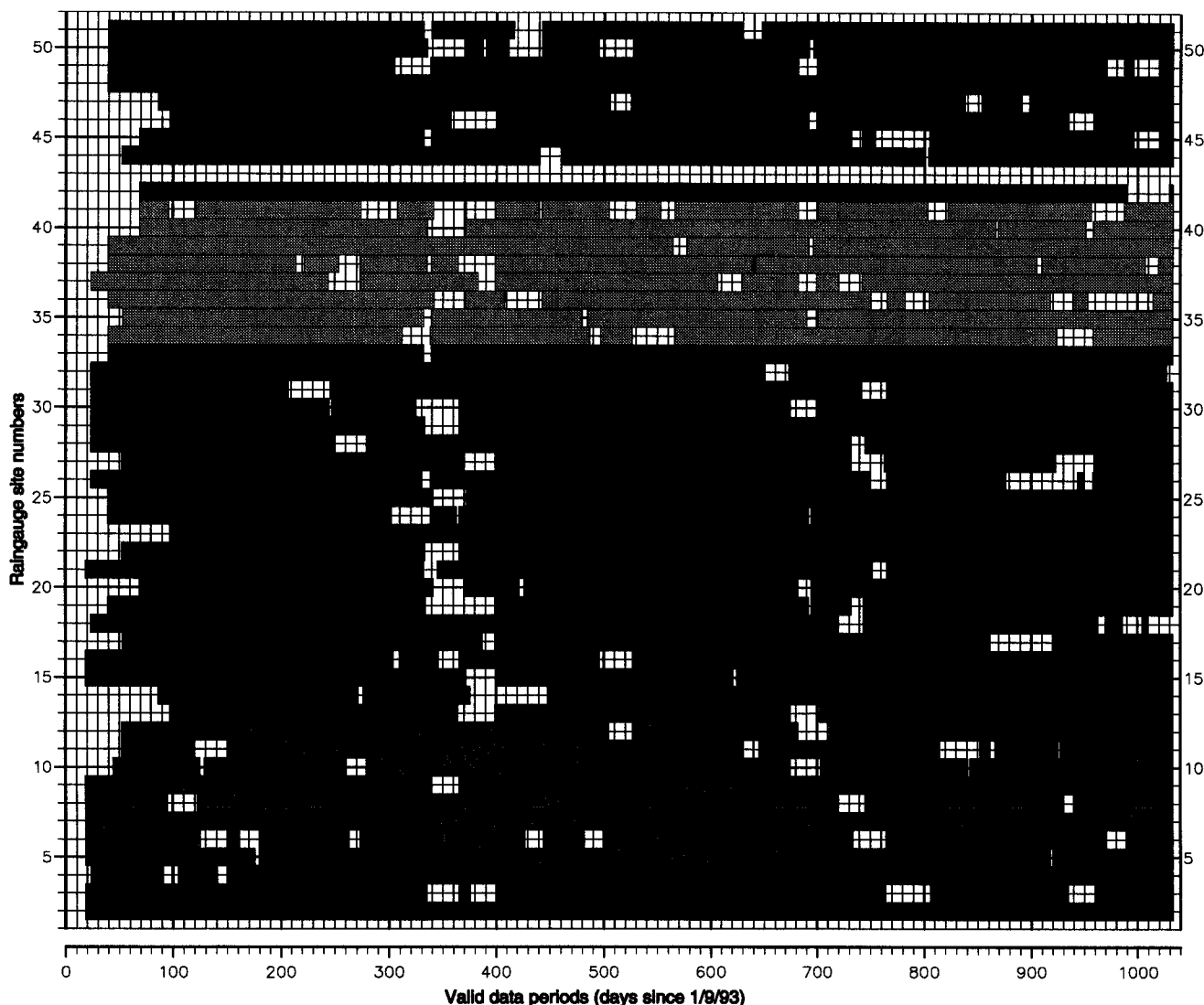
Fig. 4. *Bar chart indicating the periods of valid data for each raingauge in the HYREX network for the period 1 September 1993 to 30 June 1996; the two super-dense networks of 8 gauges within a 2 km square – gauges 5 to 12 (low relief) and 34 to 41 (high relief) – are highlighted.*

where radar performs very badly. The analyses here do not make any correction for the long-term bias in the radar data, although this is considered in the accompanying paper (Wood *et al.*, 2000). This seems reasonable in that the radar data used here are essentially the same as would have been available for operational use in flood warning centres during the period being analysed. It should be noted that no rain-gauge correction procedure has been applied to the radar data analysed here.

Evaluating the accuracy of rainfall estimates

For hydrological applications, major uses of raingauge data are to provide estimates of rainfall input to a catchment, and to provide a comparison with radar-derived estimates of rainfall, typically relating to a 2 km grid square. Measurement of rainfall at both catchment and 2 km grid-square scales will be carried out typically by a single, or at most two, raingauges in an operational network. For many studies, the position of the raingauges which perform best in measuring the areal rainfall average are of interest, since this could influence future siting of raingauges. These questions are usually addressed by a spatial correlation function (or semi-variogram) approach which can provide a formula for estimation accuracy in terms of the number and potential location of raingauges (see, for example, Morrisey *et al.*, 1995). The correlation function relates to the treatment of rainfall as a random function in space. A review of areal rainfall estimation accuracy using random function theory is provided by Bras and Rodriguez-Iturbe (1993) while papers by Anagnostou *et al.* (1999) and Ciach and Krajewski (1999) apply similar methods to comparisons of raingauges and
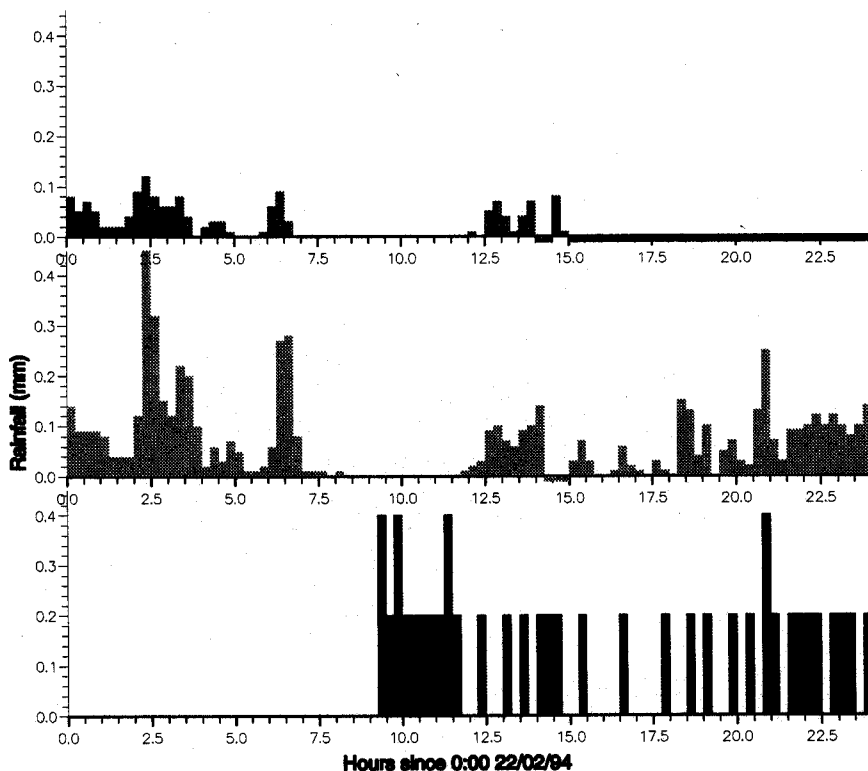
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Fig. 5. *Data quality control during unusual events using radar and raingauge hyetographs: snowfall event on 22 February 1994 as measured by the Cobbacombe and Wardon Hill radars and by the raingauge at Bridge Farm, which records only as snow starts to melt in the funnel.*
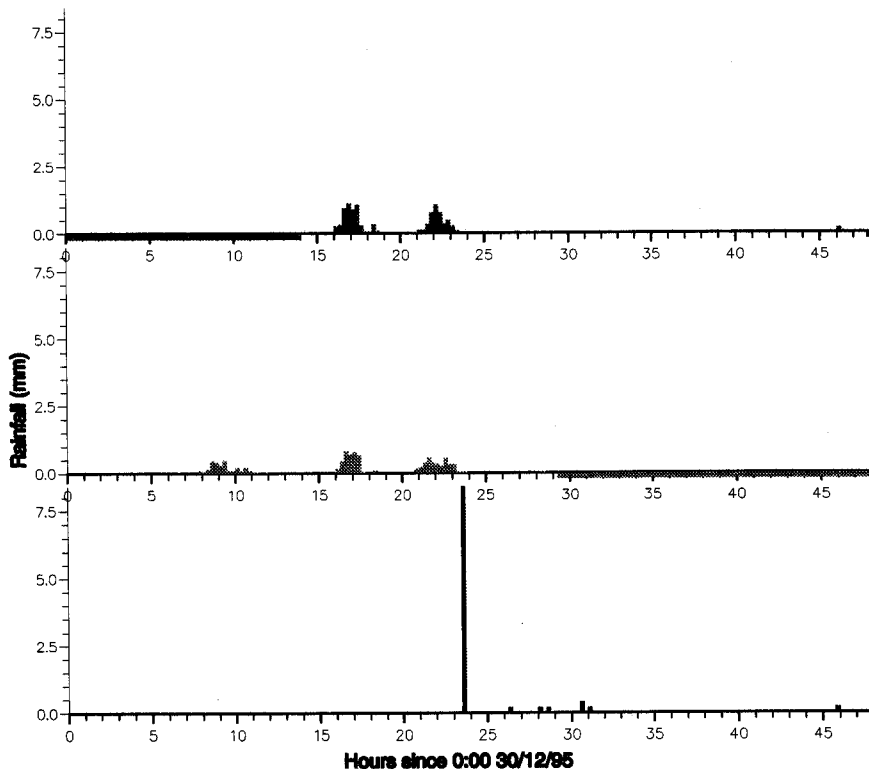

Fig. 6. *Data quality control during unusual events using radar and raingauge hyetographs: freezing rain event late in the evening of 30 December 1995 as measured by the Cobbacombe and Wardon Hill radars and by the raingauge at Bridge Farm, giving an anomalous spike.*

radar. However, a correlation-based approach has the major disadvantage that it obscures a potentially important consideration: namely, that the accuracy with which rainfall is estimated can be expected to vary with the intensity of rainfall. Anagnostou *et al.* (1999) apply the correlation theory to the logarithm of rainfall which has the effect of treating estimation accuracy as varying proportionally to rainfall intensity. A further difficulty is that the estimation accuracy depends on the strength of spatial dependence which itself may change with rainfall intensity. A correlation function approach produces an answer which is averaged across rainfall intensities. This can be overcome partly by judicious pre-selection of the time frames included in the analysis (O'Connell *et al.*, 1979). Thus different numerical answers would be obtained from correlation analyses of:

(i) all time frames;
(ii) time frames with non-zero rainfall recorded at least one raingauge;
(iii) time frames with "widespread rainfall" according to some specification; and
(iv) time frames within or close to a "rainfall event" according to a given definition.

The correlation analysis could therefore be manipulated to provide values for the accuracy of estimation for a range of cases. For example, one could define the overall intensity of rain in a time frame as the average of the intensities recorded at all raingauges and arrive eventually at values for the accuracy of estimation of rainfall over a particular area when the overall intensity is in each of a number of ranges, say 0–1 mm h^{-1}, 1–2 mm h^{-1}, 2–3 mm h^{-1}, etc. There are clearly a number of difficulties here including:

(a) specifying the ranges considered so that the data provide enough time frames for the estimation of the spatial covariance structure; and
(b) the need to frame the selection criteria so that the assumption of spatial stationarity remains reasonable.

The dense network of raingauges installed for the HYREX project enables a more direct approach to be taken; one which might be termed "empirical". In this approach the dense network is used to provide a good approximation to the true rainfall against which the accuracy, say, of a single gauge estimate may be compared for different rainfall intensities; further details are given below. The approach does have the disadvantage that the results are limited to the particular cases studied, in contrast to the correlation function approach in which, once the correlation function has been estimated, a single formula supplies the answer to how well the rainfall over any-sized area is estimated by any configuration of any number of raingauges. In addition, the results are subject to errors of unknown size, but ones which can reasonably be assumed to be small: these errors are related to the use of a reasonably good, but not perfect, estimate of what the true rainfall would be.

The empirical approach has the advantage of providing a

direct way of examining how the size of estimation error varies with the amount of rain actually observed. This approach is most appropriate when there is a large number of gauges available to form an estimate of the "true" rainfall which is itself to be estimated by a simpler procedure. It is thus reasonable to apply this procedure to radar pixels having 8 raingauges in the pixel, and in cases where the rainfall over the Brue catchment is to be estimated.

In the case of a 2 km pixel, the "ground truth" or "best estimate", T, is defined as the mean of the available raingauges (up to the maximum of 8) within the pixel. This quantity can be defined at both the high and the low relief pixels and is deemed invalid if less than 6 of the raingauges are working satisfactorily for that particular time-frame. Similarly, in the case of catchment rainfall, the network provides up to 49 raingauges which are available for use in a weighted average to form a value, T, for the catchment average rainfall. The essence of the approach is the assumption that there are so many raingauges used in calculating T that it is essentially the same as the unknown true rainfall for the pixel or catchment. It is then of interest to see how accurately T can be reproduced using either the value from a single raingauge or a radar estimate of rainfall. For a given time-frame, the estimate R can be compared with T to form a raw estimate of the mean square error

$$S^2 = (R - T)^2. \qquad (1)$$

Such raw estimates of mean square error are obviously poor and it would be standard practice to form an average across a large number of time-frames. For this study a slightly different approach is taken driven by the underlying interest in how the size of the error S varies with the amount of rainfall T (or of log S against log T), where each time-frame supplies a pair of values, and a smooth curve is fitted to estimate the relationship. This averages across time-frames, but in such a way as to reveal the dependence of the size of S on T. Effectively this estimates $E(S^2)$ as a function of log T and plots are made of $\frac{1}{2}$ log $E(S^2)$ against log T. It is convenient to think of this as log S against log T.

In this approach there is an underlying assumption that the values for T are good estimates of the true rainfall, so that values of S are a good guide to how well the simpler estimates perform. The next section considers the question of how good the values of T are for the 8 gauge radar-pixels.

Accuracy of estimates of 2 km grid square rainfall

For the two super-dense network squares, "ground truth" is established by forming the simple arithmetic average of the 8 gauges arranged in a "diamond within a square" configuration. The HYREX gauge network was designed such that one of these squares occupies an area of low relief whilst the other lies in an area of high relief, providing the ability to
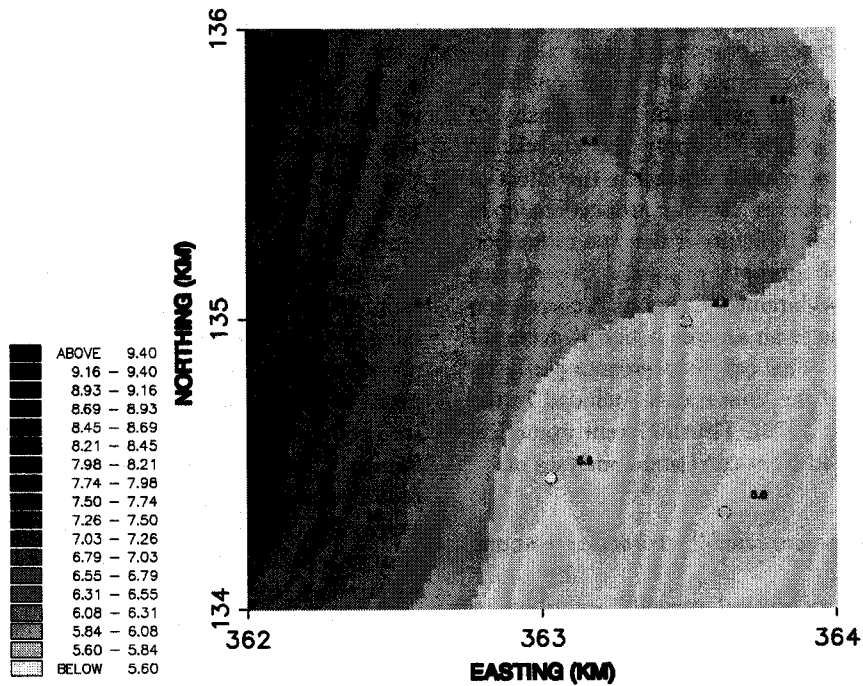
Fig. 7. *Rainfall variability within the low relief 2 km pixel, obtained by multiquadric interpolation of the HYREX raingauge network dataset, for the 15 minute interval ending 2230 3 August 1994.*

assess rainfall variability over these two area types. An example of the variability of rainfall experienced over a 2 km pixel is given in Fig. 7, for the 15 minute period ending 22:30 3 August 1994.

The accuracy with which rainfall over a 2 km square pixel can be measured is of importance for hydrological application since this spatial scale is often the basic unit of integration for weather radar data. In turn, distributed rainfall-runoff models at the catchment scale have been configured using grids of this size and using a 15 minute period of time integration (for example, Bell and Moore, 1998a,b). It is therefore important to know both how well radar estimates rainfall on a 2 km pixel basis and how well a single raingauge should agree with the radar estimate of rainfall for the square in which it lies.

The method used to quantify the accuracy of estimates of rainfall over a 2 km grid square is based on a model which assumes a spatial correlation function which is constant over the range of the pixel (that is 2.83 km), apart from a possible sharp drop over very small distances. The advantage of this model is that simple estimates for the mean square error of estimation are immediately available. The model is that the i'th raingauge value for a single time-frame, R_i, can be decomposed into three additive components. The first is a random variable variable, X, representing a *field value* common to all points within the pixel; the second is an identically distributed random variable, Y_i, representing the *measurement error* at gauge i; and the third is a random variable, Z_i, representing a *local fluctuation* in the rainfall

field at the ith gauge. Thus

$$R_i = X + Y_i + Z_i. \tag{2}$$

The *true rainfall* at any point, including non-observation points, is of the same form but excluding the measurement error term, Y. The random variables X, Y and Z are assumed independent among themselves and, apart from X, from site to site. In particular, the local fluctuations Z at different locations are assumed independent no matter how close the points considered. The inclusion of the common value X at all locations leads to the measured rainfalls at different sites having a positive correlation (and also the true rainfalls). Under these assumptions, the true average rainfall intensity for the pixel is exactly X for the time-frame in question. The data values $\{R_i, i = 1,2,..n\}$ from the n raingauges available for the time-frame may be used to construct the sample mean and sample variance

$$\bar{R} = \frac{1}{n}\sum_{i=1}^{n} R_i \tag{3}$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n} (R_i - \bar{R})^2. \tag{4}$$

In practice, a rule is introduced which leaves the mean and variance undefined if fewer than six of the eight gauges are working for the time-frame.

The quantity S^2 is capable of three interpretations. Firstly, it can be viewed as being a measure of how different

the values are likely to be of rainfalls recorded at different sites within a pixel: thus

$$E(R_i - R_j)^2 = 2E(S^2), \quad i \neq j \qquad (5)$$

which follows from the independence assumptions made in the model. Secondly, it provides a measure of how well any single gauge estimates the true pixel average rainfall (X here, or T in the more general notation used earlier)

$$E(R_i - X)^2 = E(S^2). \qquad (6)$$

This follows since S^2 is essentially an estimate of the variance of $(Y_i + Z_i)$. Thirdly, S^2 can be used to provide an estimate of how well the estimate given by the average of a number, m say, of gauges within a 2 km pixel can be expected to measure the true pixel average rainfall. Thus if $\overline{R}^{(m)}$ is the average of m gauges

$$E(\overline{R}^{(m)} - X)^2 = \frac{1}{m}E(S^2) \qquad (7)$$

and so

$$s^2 = \frac{1}{m}S^2 \qquad (8)$$

provides an estimate of the mean square error for $\overline{R}^{(m)}$.

There is a need to combine a large number of framewise pairs (T, S^2) in order to examine how the size of estimation error relates to the amount of rainfall that has actually fallen. The same procedure that was used in the section headed "Evaluating the accuracy of rainfall estimates" is used except that, here, a single value of S^2 is derived from the several raingauge values for the grid square. The adjusted averaging in Eqn. (4) is the usual formula used for estimating a variance which assures an unbiased estimate under the assumptions outlined above. The adjustment reflects the contribution made by a single gauge to the estimated value of the "true" quantity T estimated by $\overline{R}$. It is not possible to make a standard adjustment for this effect in other cases, but, when enough gauges are available to form T, the effect is small as has been found in cases where analyses have been repeated with the candidate gauge removed from contributing to T.

Values of S^2 and T calculated for each 15 minute timeframe can be used to establish an empirical relation between estimation accuracy and rainfall intensity. The mean square error of estimation, given by S, is plotted against the true rainfall estimate, given by T, as a scatter plot using logarithmic axes. Figure 8 presents an example plot calculated for the grid square in the area of low relief. A random dither factor has been added to each plotted point to emphasise the number of points which are overplotted, which arise through discretisation effects. Different rainfall types are distinguished using the symbols • and ○ to denote frontal and convective respectively. Data influenced by solid forms of precipitation are omitted from this analysis. Figure 8 includes a smoothed plot of S against T which is created
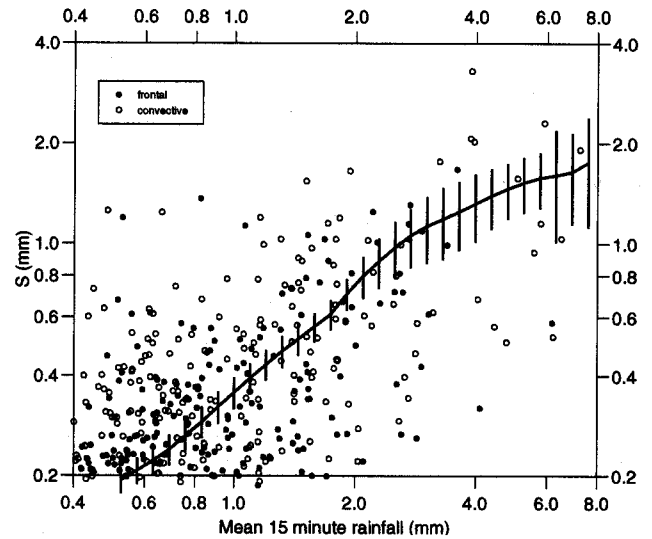

Fig. 8. *Standard error, S, of rainfall estimate from a single gauge as a function of total rainfall in a 15 minute interval over the low relief square. Vertical lines indicate the 90% range in sampling uncertainty and the symbols • and ○ denote frontal and convective rainfall types respectively.*

by the method of locally weighted least squares outlined in Appendix A: the same method is used in presenting other results. The vertical lines in Fig. 8 indicate a 90% range in the sampling distribution for the smoothed line, obtained as outlined below. The relation corresponding to Fig. 8, but for the high relief square, is very similar in overall distribution and behaviour although the confidence limits are broader.

An assessment of the uncertainty inherent in the smoothed line is obtained by a Monte Carlo bootstrap method (Efron and Tibshirani, 1986) in which the original sample of data is repeatedly resampled on the basis of sampling with replacement of blocks of 3 days. This effectively allows for both spatial and temporal correlation in the rainfall fields. Each resample is used to calculate a smoothed line by the method of Appendix A and the distribution of the resampled y values for a given x gives an indication of the sampling uncertainty. Where a comparison between the accuracies of different estimators is to be made, it is possible to apply the same combination of a smoothed line and bootstrap resampling to the differences of the squared-errors to allow an assessment of whether one estimator is better than the other. This approach automatically takes into account the statistical dependence of the errors in the estimators for the same time-point. Space limitations preclude inclusion of an example of this type of analysis.

Figure 9 shows how the accuracy of an 8-gauge estimate (derived via Eqn. 8) compares with that of a typical single gauge for the low relief square. Also shown is the accuracy associated with the 2 km radar estimate using Wardon Hill

radar. One interpretation of the values here is that, with certain assumptions, for a 15 minute rainfall of 10mm, errors in the radar estimate have an apparent mean squared error of 16 mm^2, of which 1 mm^2 may have arisen by use of the 8-gauge estimate in the assessment procedure rather than the unknown true rainfall. Thus the value of S for radar might really be 3.87mm ($\sqrt{(16-1)}$) rather than 4mm, which shows that the effect is small. A further interpretation relates to the situation with more typical networks of operational gauges where radar data can only be compared to a single raingauge in isolated radar-pixels. Figure 9 indicates that even if radar were to provide a perfect estimate of the true pixel-average rainfall, the radar data and raingauge data can be expected to have a root mean square difference of about 3mm if the true rainfall total is 10mm in 15 minutes.

Figure 10 shows the effect of temporal scale on the accuracy of a typical gauge as an estimator of 2 km grid square rainfall. Hourly accumulations of rainfall are seen to be more accurate than 15 minute ones for higher rainfalls, whilst for lower rainfalls the 15 minute totals are more accurate. Similar effects at high and low rainfalls are seen when moving to one day totals.

The results obtained suggest that a straight line can be used for the accuracy relationship on a log-log graph, although further investigation might be required at low rainfall intensities. A slope of near unity for 15 minute accumulations suggests use of a logarithmic transformation would lead to almost constant accuracy on this scale. Figure 10 indicates that a power transformation with a positive exponent near zero would be needed to gain the same effect for hourly and daily accumulations.
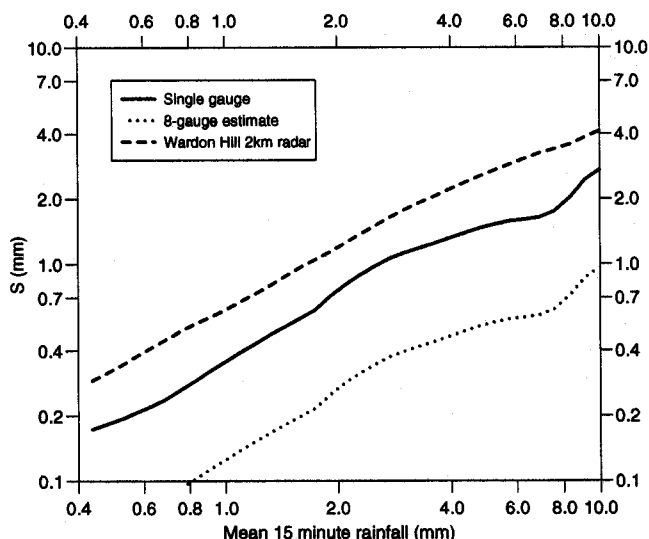

Fig. 9. *Comparison of the standard errors of estimation for 15 minute rainfall in the low relief square for a single gauge, for 8 gauges in the square and for radar estimates.*
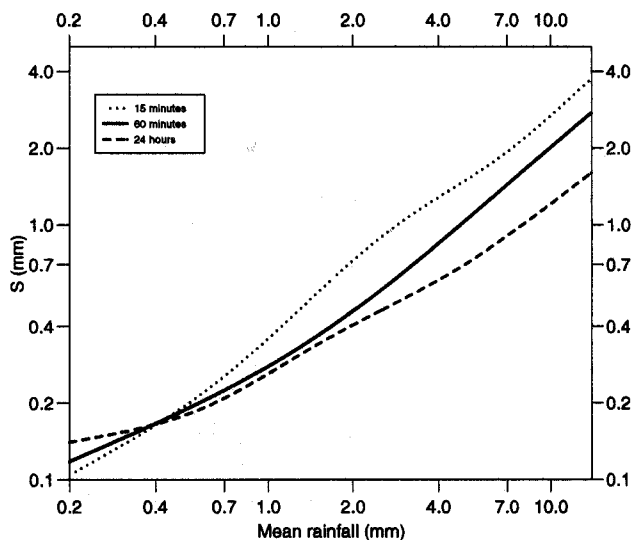

Fig. 10. *Standard error, S, of rainfall estimate obtained from a single gauge as a function of total rainfall over the Brue catchment for time periods of 15 minutes, 1 hour and 1 day.*

Accuracy of estimates of catchment average rainfall

Estimation of rainfall over a typical catchment is of importance since this spatial scale is usually taken as the basic unit for rainfall-runoff modelling, with lumped catchment models often providing the most practical solution to flow forecasting problems. It is again of interest to see how errors of estimation vary with rainfall intensity on a 15 minute timescale. An example of the variability of rainfall over the Brue catchment is given in Fig. 11, which shows the distribution of rainfall for the 15 minute interval ending 22:30 3 August 1994. When forming a value, T, to represent the true average rainfall for the Brue catchment it is clear that a simple arithmetic mean of the 49 raingauges in the HYREX network will introduce a tendency to weight the result towards values for the squares containing 8 and 2 gauges. Instead, an estimate was formed by calculating the arithmetic mean rainfall for each of the 28 squares containing one or more gauges and then taking the average of these values. Specifically the catchment average rainfall for a 15 minute interval is calculated as

$$T = \bar{R} = \frac{1}{m}\sum_{j=1}^{m}\left(\frac{1}{n_j}\sum_{i=1}^{n_j}R_{i,j}\right) \tag{9}$$

with the estimation variance for a typical gauge being estimated by

$$S^2 = \frac{1}{m}\sum_{j=1}^{m}\left(\frac{1}{n_j}\sum_{i=1}^{n_j}(R_{i,j} - \bar{R})^2\right) \tag{10}$$

where m is the number of squares containing at least one operating gauge, n_j is the number of gauges in square j with
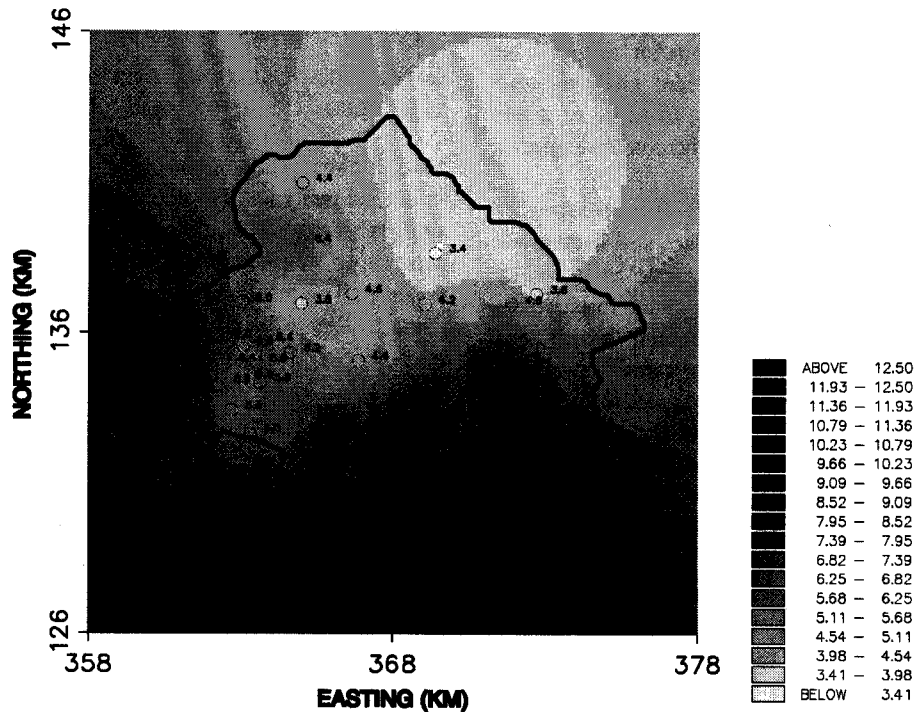
Fig. 11. *Rainfall variability over the Brue catchment, obtained by multiquadric interpolation of the HYREX raingauge network dataset, for the 15 minute interval ending 22:30 3 August 1994.*

valid data for the time-interval, and $R_{i,j}$ is the rainfall at gauge i in square j. Equation (10) is based on the same principal of not giving undue prominence to squares containing two or more gauges. Only quality controlled rainfall values are used in forming the averages.

Rainfall data for each time-interval are used to construct the sample mean, T, and sample variance, S^2, and a scatter plot of S versus T for all time-frames produced. Figure 12 shows how the accuracy of measurements from two particular raingauges compare with those from a "typical" single gauge as estimates of the catchment average rainfall. The gauge at Kilkenny Fields (Gauge 2) located on the edge of the catchment is seen to provide a poor estimate of the catchment average rainfall for all rainfall intensities. In contrast, the gauge at Crabtree Lane Field (Gauge 25) situated near the centre of the catchment provides an estimate of the catchment average rainfall that is comparable to that of a typical gauge. Also shown is the accuracy obtained using the 2 km radar data from Wardon Hill: this proves more accurate than the typical gauge estimate for all but the highest rainfalls.

Conclusions

Assimilation of a variety of data sources into the analysis of rainfall distributions, and their hydrological effects, can introduce its own problems. Intensive data quality analysis has been carried out on over 2 years of raingauge data.

Overall, this has shown that the HYREX dense raingauge network has performed well apart from some problems in the first summer (in late Summer 1994) when a large number of gauges suffered blockages, due especially to mown grass. The summers of 1995 and 1996 saw a marked improvement in this respect. Meteorological factors have
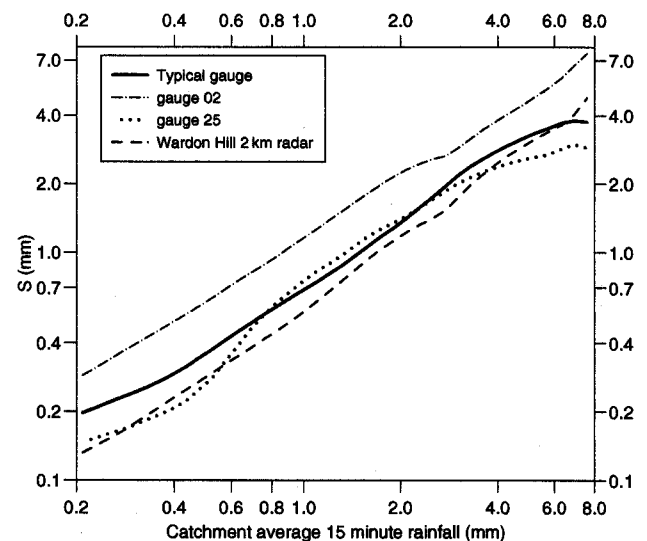

Fig. 12. *Standard error, S, of the catchment rainfall estimate obtained from a single gauge or from radar as a function of total rainfall in a 15 minute interval over the Brue catchment.*

541

been considered by studying Daily Weather Summaries. This allowed snow and other solid precipitation events to be removed from the majority of subsequent analyses, and studied separately for particular puposes. In addition, it has been possible to categorise stratiform and convective rainfall situations.

The main aim has been to quantify the accuracy of estimates of rainfall over 2 km grid square and catchment scales, focussing on the 15 minute timescale as the most relevant to rainfall-runoff modelling for flood forecasting. An empirical approach has been pursued which has involved treating each 15 minute time-frame separately. For each time-frame the "true" rainfall was set to the mean rainfall over a large number of gauges and the estimate and error were calculated. Scatter plots of these quantities allowed smoothed representative lines to be produced quantifying the relationship between measurement accuracy and rainfall intensity.

Two super-dense gauge networks each comprising of 8 gauges within a 2 km side square, one in an area of low relief and the other in high relief, were used to estimate the "true" rainfall as a simple arithmetic average. Estimation errors for individual gauge estimates, and radar estimates, of this true mean rainfall over the square have been calculated. Errors of around 33% at 4 mm were observed at the grid square situated in low relief, with larger errors of 45% observed at the high relief grid square. However, the orography in the catchment is not varied enough to allow an extensive investigation of its effect on rainfall estimation accuracy. The effect of the time-scale over which measurements are made was also investigated. As the time interval becomes larger, the measurement accuracy improves due partly to the reduction in the effects of tipping-bucket discretisation errors. Whilst convective conditions in the period analysed have led to higher intensity rainfall than during frontal events, it has not been possible to distinguish different rainfall estimation accuracies for these.

The HYREX raingauge network of 49 gauges over the Brue catchment has also been used to estimate the "true" catchment rainfall as a weighted average of the available gauge values as part of a similar assessment of the accuracy of estimates of catchment rainfall. A brief comparison of the various types of estimate can be stated as follows. For 4 mm of rain in 15 minutes the standard error of a single gauge estimate varies from 33% for the 2 km square in which it lies to 65% when treated as a catchment estimate, noting that the area ratio involved is 1:34. In these circumstances, 8 gauges in a 2 km square would provide an estimate with a standard error of 11.5%. Further, radar at a 2 km resolution has a standard error of 50% for estimating rainfall on a 2 km square and 55% for catchment rainfall. Radar at 5 km resolution has estimates of catchment rainfall with a 60% standard error. These results for radar relate to data available operationally in real-time. A companion paper (Wood et al., 2000) considers the benefit of correcting for the long-term bias present in these data.

Appendix A

SMOOTHING USING WEIGHTED LOCAL LEAST SQUARES

Scatter plots produced using the empirical approach are smoothed using locally weighted least squares. Considering a target point along the x-axis, the corresponding y value is determined as a function of the local points having similar x-values. The importance to be attached to each of the local points is determined by a weighting function $K(t)$ defined as:

$$K(t) = \begin{cases} 1 - t^2 & |t| < 1 \\ 0 & \text{otherwise} \end{cases} \tag{A.1}$$

which has the property that $K(t)$ is large only over the range $\pm 5^{-1/2}$. The value $5^{-1/2}$ arises as the standard deviation of a probability density proportional to $K(t)$. If h is the required effective smoothing width, centred on the target x value, then h^* is required such that $K(x/h^*)$ is large over the range of x from $-h$ to $+h$: hence, $h^* = \sqrt{5}\, h$. A typical value for h is (log 1 − log 0.75).

For the present smoothing application the sample points x_i and y_i denote the logarithm of the true rainfall and the mean square error of the estimate (ie. log T and S^2), respectively, for one time-frame. Smoothing proceeds as follows. For a target point x^*, and data (x_i,y_i), $i = 1...N$, weights for each point of the sample set are calculated as

$$w_i = K(\frac{x_i - x^*}{h^*}). \tag{A.2}$$

Now, the local weighted least squares regression equation is

$$y = y_w + \beta(x - x_w) \tag{A.3}$$

giving the smoothed value of y at x^* as

$$y* = y_w + \beta(x^* - x_w) \tag{A.4}$$

where

$$x_w = \frac{\sum w_i x_i}{\sum w_i} \tag{A.6}$$

$$y_w = \frac{\sum w_i y_i}{\sum w_i}. \tag{A.7}$$

$$\beta = \frac{\sum w_i (y_i - y_w)(x_i - x_w)}{\sum w_i (x_i - x_w)^2} \tag{A.8}$$

and β is treated as undefined unless the denominator $\sum w_i(x_i - x_w)^2 > 1$. The final smoothed line is obtained from the plot of $\frac{1}{2}\log y^*$ (= log $\sqrt{y^*}$ = log S) against x^* (= log T).

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