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Approximation zones of the Saint-Venant equations for flood routing with overbank flow

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Abstract

The classification of river waves as gravity, diffusion or kinematic waves, corresponds to different forms of the momentum equation in the Saint-Venant system. This paper aims to define approximation zones of the Saint-Venant equations for flood routing in natural channels with overbank flow in the flooded area. Using linear perturbation theory, the different terms in the Saint-Venant equations were analysed as a function of the balance between friction and inertia. Then, using non-dimensionalised variables, flood waves were expressed as a function of three parameters: the Froude number of the steady uniform flow, a dimensionless wave number of the unsteady component of the motion and the ratio between the flooded area zone width and the main channel width. Finally, different theoretical cases, corresponding to different flooded area zone widths were analysed and compared. Results show that, when the width of the flooded area increases, the domain of application of the diffusive wave and the kinematic wave models is restricted.

Keywords: Saint-Venant equations; river waves; overbank flow

Introduction

Many hydraulic and hydrological problems involve the computation of the propagation of flood waves in open channels based on the solution of the well-known Saint-Venant (1871) equations. The Saint-Venant equations are coupled hyperbolic partial differential equations that cannot be solved analytically. It was not until Stoker (1957) that approximate numerical solutions were obtained using an explicit finite difference method. Since Stoker's work, a great number of different numerical schemes have been proposed including the method of characteristics (Abbott, 1966), a variety of sophisticated finite difference methods (Remson *et al*, 1971; Cunge *et al.*, 1980; Dooge *et al.*, 1982) and finite element schemes (Cooley and Moin, 1976; Fread, 1985; Szymkiewicz, 1991, 1993). Comparisons of numerical solutions for the Saint-Venant equations have been published by Price (1974) and Greco and Panattoni (1977).

The complexity inherent in the calculations has led to the development of approximate methods. Several authors have examined the approximation zone of the Saint-Venant equation in the particular case of one main channel river (Dooge and Harley, 1967; Woolhiser and Liggett, 1967; Weinmann and Laurenson, 1974; Ponce and Simons, 1977; Bocquillon, 1978; Daluz Vieira, 1983; Ferrick, 1985; Moussa and Bocquillon, 1996). Within this basic model, river waves may be classified as gravity, diffusion or kinematic waves, corresponding to different forms of the momentum equation.

However, the major parts of research in this field have studied the case of a channel with one section corresponding to the main channel, and little attention was given to overbank flow during flood events (Rutschmann and Hager, 1996). The cross section of a channel may be composed of several distinct sub-sections with each sub-section different in roughness from the others (Chow, 1959; Carlier, 1980). For example, an alluvial channel subject to seasonal floods generally consists of a main channel and two side channels.

The object of this paper is to develop a quantitative method for identifying river wave types in the case of flood events with overbank flow. The analysis presented herein endeavours to apply the theory of linear stability to the set of the Saint-Venant equations in a non-dimensionalised space as proposed by Ponce and Simons (1977) and Moussa and Bocquillon (1996). Then, different theoretical cases, corresponding to different values of the ratio between the main channel width and the flooded area zone width, were analysed and compared.

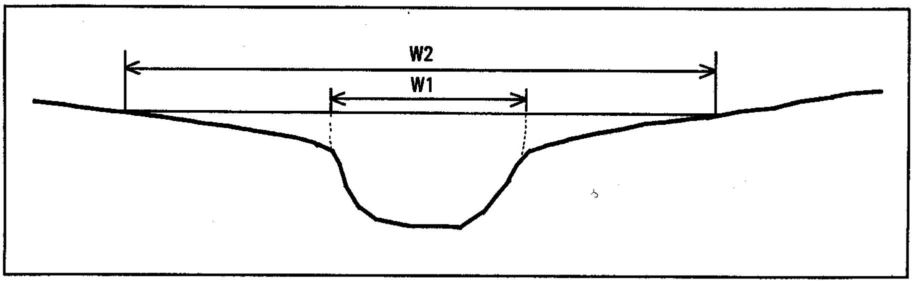

Fig. 1. *A channel consisting of one main section and two side sections.*

The Saint-Venant equations for channels of compound section

The dynamic modelling of a one-dimensional, gradually varied, unsteady flow in open channels is based on the numerical solution of the Saint-Venant equations. In the case of a river with a flooded area, let W_1 and W_2 be respectively the width of the main channel and the flooded area zone respectively (Fig. 1). The two equations, describing mass and momentum, can be written as follows (Chow, 1959; Henderson, 1963, 1966; Abbott, 1979)

$$W_2 \frac{\partial y}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} + g \cdot (S_f - S) = 0 \tag{2}$$

where y is the flow depth (m), V is the flow velocity ($m\ s^{-1}$), g is the acceleration due to gravity ($m\ s^{-2}$), S is the river bed slope, S_f the slope of energy line, Q the discharge ($m^3\ s^{-1}$), x is longitudinal distance (m) and t is time (s). The basic assumption made to derive this system is that the flow is one-dimensional in the main channel and the flooded area, and that there are no lateral inflows or outflows.

The side channels are usually found to be rougher than the main channel. So the mean velocity, V, in the main channel is greater than the mean velocities in the side channels. In such a case, the Manning formula may be applied separately to each sub-section in determining the mean velocity of the sub-section. Then, the discharges in the sub-sections can be computed. The total discharge is, therefore, equal to the sum of these discharges. As the velocity in the main channel is greater than the velocity in the flooded area, the part of discharge in the flooded area sub-section is small in comparison to the discharge in the main channel, so the discharge Q is expressed as

$$Q \approx A_1 \cdot V \tag{3}$$

where A_1 is the cross-sectional area of the flow (m^2) defined as a function of x and y

$$A_1 = W_1 \cdot y \tag{4}$$

Differentiating Eqns. (3) and (4) gives

$$\frac{\partial A_1}{\partial y} = W_1 \tag{5}$$

$$\frac{\partial Q}{\partial x} = V\frac{\partial A_1}{\partial x} + A_1\frac{\partial V}{\partial x} = V\frac{\partial A_1}{\partial y}\frac{\partial y}{\partial x} + A_1\frac{\partial V}{\partial x} \tag{6}$$

Substituting Eqns. (5) and (6) into Eqn. (1) gives

$$W_2\frac{\partial y}{\partial t} + V \cdot W_1 \cdot \frac{\partial y}{\partial x} + A_1\frac{\partial V}{\partial x} = 0 \tag{7}$$

Let η be the ratio between the flooded area zone width and the main channel width

$$\eta = \frac{W_2}{W_1} \tag{8}$$

The two Eqns. (1) and (2) of the Saint-Venant system can be written as follows

$$\begin{array}{cccc} & \eta\frac{\partial y}{\partial t} + & y\frac{\partial V}{\partial x} + & V\frac{\partial y}{\partial x} = 0 \\ \text{Term} & \text{(I)} & \text{(II)} & \text{(III)} \end{array} \tag{9}$$

$$\begin{array}{cccccc} & \frac{\partial V}{\partial t} + & V\frac{\partial V}{\partial x} + & g\frac{\partial y}{\partial x} + & g \cdot (S_f - S) = 0 \\ \text{Term} & \text{(IV)} & \text{(V)} & \text{(VI)} & \text{(VII) (VIII)} \end{array} \tag{10}$$

The term S_f is usually calculated using the Manning formula. As the velocity, V, in the main channel is greater than the velocity in the flooded area, the term of the Manning formula applied to the flooded area is small in comparison to the term in the main channel

$$S_f = n \cdot V^2 \cdot R^{-m} \tag{11}$$

where V is the mean velocity ($m\ s^{-1}$), R the hydraulic radius (m), n the coefficient of roughness and m a constant ($m \approx 4/3$). For the main channel

$$R = \frac{W_1 \cdot y}{W_1 + 2 \cdot y} \tag{12}$$

For wide river sections ($y \ll W_1$), the Manning formula can be written

$$S_f \approx n \cdot V^2 \cdot y^{-m} \quad (13)$$

The two Eqns. (9) and (10) give the generalised form of the Saint-Venant system with a flooded area and where the side channels are rougher than the main channel. In this case, the Saint-Venant system depends on the parameter η that appears in the mass equation. The particular case $\eta = 1$, corresponding to flood routing without overbank flow, was studied extensively by Moussa and Bocquillon (1996). The object of this paper is to generalise this method for different values of the parameter η corresponding to different widths of the flooded area and then to analyse the ability to use the same wave model when the flooded area width varies during flood events.

Analysis of river wave types

The analysis presented herein endeavours to apply the theory of linear stability to the set of equations governing the motion in open channel flow as proposed by Ponce and Simons (1977) and Napiorkowski (1992) and then to define parameter ranges representing each wave type in the Saint-Venant system for different values of η. The analysis is based on the principle that the balance between friction and inertia determines river wave behaviour. The Saint-Venant equations are written in dimensionless form, the system equation provides parameters that quantify the magnitudes of all terms in the equation and indicate the relative importance of friction and inertia.

SMALL PERTURBATION ANALYSIS OF WAVE PROPAGATION

In the usual manner of stability calculations, Eqns. (9) and (10) must satisfy the unperturbed steady uniform flow for which $y = y_0$ and $V = V_0$ as well as the perturbed flow. The problem may be represented as the superposition of two regimes, a permanent regime and a small perturbation to the steady uniform flow. Let $y_0(x)$, $V_0(x)$, $\varepsilon.y'(x,t)$ and $\varepsilon.V'(x,t)$ be the values of $y(x,t)$ and $V(x,t)$ respectively for the permanent regime and for the small perturbation

$$V(x,t) = V_0(x) + \varepsilon \cdot V'(x,t) \quad (14)$$

$$y(x,t) = y_0(x) + \varepsilon \cdot y'(x,t) \quad (15)$$

For the steady uniform flow, Eqns. (9) and (10) lead to

$$V_0 \frac{\partial y_0}{\partial x} + y_0 \frac{\partial V_0}{\partial x} = 0 \quad (16)$$

$$V_0 \frac{\partial V_0}{\partial x} + g \frac{\partial y_0}{\partial x} = g(S - S_{f0}) \quad (17)$$

with S_{f0} the slope of the energy line

$$S_{f0} \approx n \cdot V_0^2 \cdot y_0^{-m} \quad (18)$$

This unperturbed flow is characterised by the Froude number (F_0) of the unperturbed flow

$$F_0^2 = \frac{V_0^2}{g y_0} \quad (19)$$

Substitution of perturbed variables (14) and (15) and Eqns. (16) and (17) into (9) and (10), and by neglecting the terms with ε^2, and by neglecting the derivatives of permanent terms (V_0 and y_0) as they are small in comparison to the derivatives of the perturbation terms (V' and y'), gives

$$\eta \frac{\partial y'}{\partial t} + V_0 \frac{\partial y'}{\partial x} + y_0 \frac{\partial V'}{\partial x} = 0 \quad (20)$$

$$\frac{\partial V'}{\partial t} + V_0 \frac{\partial V'}{\partial x} + g \frac{\partial y'}{\partial x} + g \cdot S_{f0} \cdot \left(2 \frac{V'}{V_0} - m \frac{y'}{y_0}\right) = 0 \quad (21)$$

The solution for a small perturbation in the depth of flow is postulated as a sinusoidal upstream input with zero lateral inflow, in the following exponential form as proposed by Katopodes (1982) and Hunt (1987)

$$y' = e^{i\omega t} \cdot e^{-\lambda x} \quad (22)$$

$$V' = \mu \cdot e^{i\omega t} \cdot e^{-\lambda x} \quad (23)$$

where ε is a small real value, ω the frequency and λ and μ complex ($i = \sqrt{-1}$) with

$$\lambda = \lambda_r + i\lambda_i \quad \text{and} \quad \mu = \mu_r + i\mu_i \quad (24)$$

with λ_r, λ_i, μ_r and μ_i real numbers.

The small perturbation, in the form of a harmonic oscillation, is characterised by the wave period T

$$T = \frac{2\pi}{\omega} \quad (25)$$

ANALYSIS OF THE PROBLEM USING NON-DIMENSIONALISED VARIABLES

This problem is studied by using non-dimensionalised variables y_+, V_+, x_+ and t_+ with

$$y' = y_+ \cdot y_0 \ ; \ V' = V_+ \cdot V_0 \ ; \ x = \frac{x_+ \cdot y_0}{S_{f0}} \ ; \ t = \frac{t_+ \cdot y_0}{V_0 \cdot S_{f0}} \quad (26)$$

These transformations reduce Eqns. (20) and (21) to

$$\eta \cdot \frac{\partial y_+}{\partial t_+} + \frac{\partial y_+}{\partial x_+} + \frac{\partial V_+}{\partial x_+} = 0 \quad (27)$$

$$\frac{V_0^2}{g \cdot y_0} \left(\frac{\partial V_+}{\partial t_+} + \frac{\partial V_+}{\partial x_+}\right) + \frac{\partial y_+}{\partial x_+} + 2 \cdot V_+ - m \cdot y_+ = 0 \quad (28)$$

Then, it is convenient to non-dimensionalise the input perturbation with the non-dimensionalised variables ω_+, λ_+ and μ_+

$$\omega = \omega_+ . \frac{V_0 \cdot S_{f0}}{y_0} \qquad \lambda = \lambda_+ . \frac{S_{f0}}{y_0} \qquad \mu = \mu_+ \tag{29}$$

Let T_+ be the non-dimensionalised period; combining Eqns. (26) and (29) gives

$$\omega \cdot t = \omega_+ t_+ \qquad \lambda x = \lambda_+ x_+ \qquad T_+ = \frac{2\pi}{\omega_+} = \frac{T \cdot V_0 \cdot S_{f0}}{y_0} \tag{30}$$

Substituting (26) and (30) into (22) and (23) gives

$$y_+ = e^{i \cdot \omega_+ . t_+} e^{-\lambda_+ . x_+} \tag{31}$$

$$V_+ = \mu_+ \cdot e^{i \cdot \omega_+ . t_+} e^{-\lambda_+ . x_+} \tag{32}$$

Differentiating Eqns. (31) and (32) gives

$$\frac{\partial y_+}{\partial t_+} = i \cdot \omega_+ \cdot y_+ \qquad \frac{\partial V_+}{\partial t_+} = i \cdot \omega_+ \cdot V_+ = i \cdot \omega_+ \cdot \mu_+ \cdot y_+ \tag{33}$$

$$\frac{\partial y_+}{\partial x_+} = -\lambda_+ \cdot y_+ \qquad \frac{\partial V_+}{\partial x_+} = -\lambda_+ \cdot V_+ = -\lambda_+ \cdot \mu_+ \cdot y_+ \tag{34}$$

Substituting (32), (33) and (34) into Eqns. (27) and (28) gives

$$i \cdot \eta \cdot \omega_+ - \lambda_+ - \mu_+ \cdot \lambda_+ = 0 \tag{35}$$

$$F_0^2 \cdot \mu_+ . (i \cdot \omega_+ - \lambda_+) - \lambda_+ + 2 \cdot \mu_+ - m = 0 \tag{36}$$

Eliminating μ_+ between the two Eqns. (35) and (36) leads to

$$\begin{aligned}(F_0^2 - 1) \cdot \lambda_+^2 - \left[i \cdot \omega_+ \cdot F_0^2 (1+\eta) + 2 + m\right] \cdot \lambda_+ \\ - \eta \cdot \omega_+^2 \cdot F_0^2 + 2 \cdot i \cdot \eta \cdot \omega_+ = 0\end{aligned} \tag{37}$$

Finally the solution obtained is

$$\begin{aligned}\lambda_+ = {} & \frac{1}{F_0^2 - 1} \\ & \left\{1 + \frac{m}{2} + i \cdot \omega_+ \cdot \left(\frac{1+\eta}{2}\right) F_0^2 \pm \right. \\ & \left[\left(\left(1 + \frac{m}{2}\right)^2 - \eta \cdot \omega_+^2 F_0^2 - \omega_+^2 \cdot F_0^4 \cdot \left(\frac{1-\eta}{2}\right)^2\right)\right. \\ & \left.\left. + \omega_+ \cdot \left(F_0^2 \cdot \frac{m(1+\eta) + 2 - 2\eta}{2} + 2 \cdot \eta\right) \cdot i\right]^{1/2}\right\}\end{aligned} \tag{38}$$

Equation (35) gives

$$\mu_+ = \frac{i \cdot \omega_+ \cdot \eta}{\lambda_+} - 1 \tag{39}$$

In the non-dimensionalised space, the Saint-Venant system can be expressed as a function of λ_+ and μ_+. The two non-dimensionalised terms λ_+ and μ_+ are two functions of three dimensionless numbers, the Froude number F_0^2 that describes the hydraulic characteristics of the channel reach, the period of the sinusoidal perturbation T_+ and the ratio between the flooded area and the main channel widths η. So, the analysis of the contribution of each term in the Saint-Venant system (Eqns. 9 and 10) can be made a function of the three parameters F_0^2, T_+ and η.

Choice of flood routing method

In most natural open channels, the movement of flood waves is governed predominantly by the balance of the various forces included in the two equations of mass (9) and motion (10). Numerous studies have addressed the question of which flood routing method might be appropriate for different circumstances. The object is to study the evolution of the approximation zones of the Saint-Venant system for different values of the parameter η corresponding to different widths of the flooded area.

THE DIFFERENT TERMS OF THE SAINT-VENANT SYSTEM

In most practical applications, some terms in the Saint-Venant Eqns. (9) and (10) can be neglected since they are small in comparison to others. Let τ_1, τ_2 and τ_3 be respectively the moduli of the three terms of Eqn. (35) that correspond respectively to the terms (I), (II) and (III) of the mass Eqn. (9)

$$\tau_1 = |i \cdot \eta \cdot \omega_+| \qquad \tau_2 = |-\lambda_+| \qquad \tau_3 = |-\mu_+ \cdot \lambda_+| \tag{40}$$

and let τ_4, τ_5, τ_6, τ_7 and τ_8 be respectively the moduli of the five terms of Eqn. (36) that correspond respectively to the terms (IV), (V), (VI), (VII) and (VIII) of the momentum Eqn. (10)

$$\begin{aligned}&\tau_4 = |i \cdot F_0^2 \cdot \mu_+ \cdot \omega_+| \qquad \tau_5 = |-F_0^2 \cdot \mu_+ \cdot \lambda_+| \\ &\tau_6 = |-\lambda_+| \qquad \tau_7 = |2 \cdot \mu_+| \qquad \tau_8 = |m|\end{aligned} \tag{41}$$

Note that for a complex number a + bi, where a and b are real numbers and $i = \sqrt{-1}$, the modulus is

$$|a + b \cdot i| = (a^2 + b^2)^{1/2} \tag{42}$$

$$|\lambda_+| = (\lambda_{r+}^2 + \lambda_{i+}^2)^{1/2} \tag{43}$$

$$|\mu_+| = \left(\frac{\omega_+^2 \cdot \eta^2}{|\lambda_+|^2} - \frac{2 \cdot \omega_+ \cdot \eta \cdot \lambda_i}{|\lambda_+|^2} + 1\right)^{1/2} \tag{44}$$

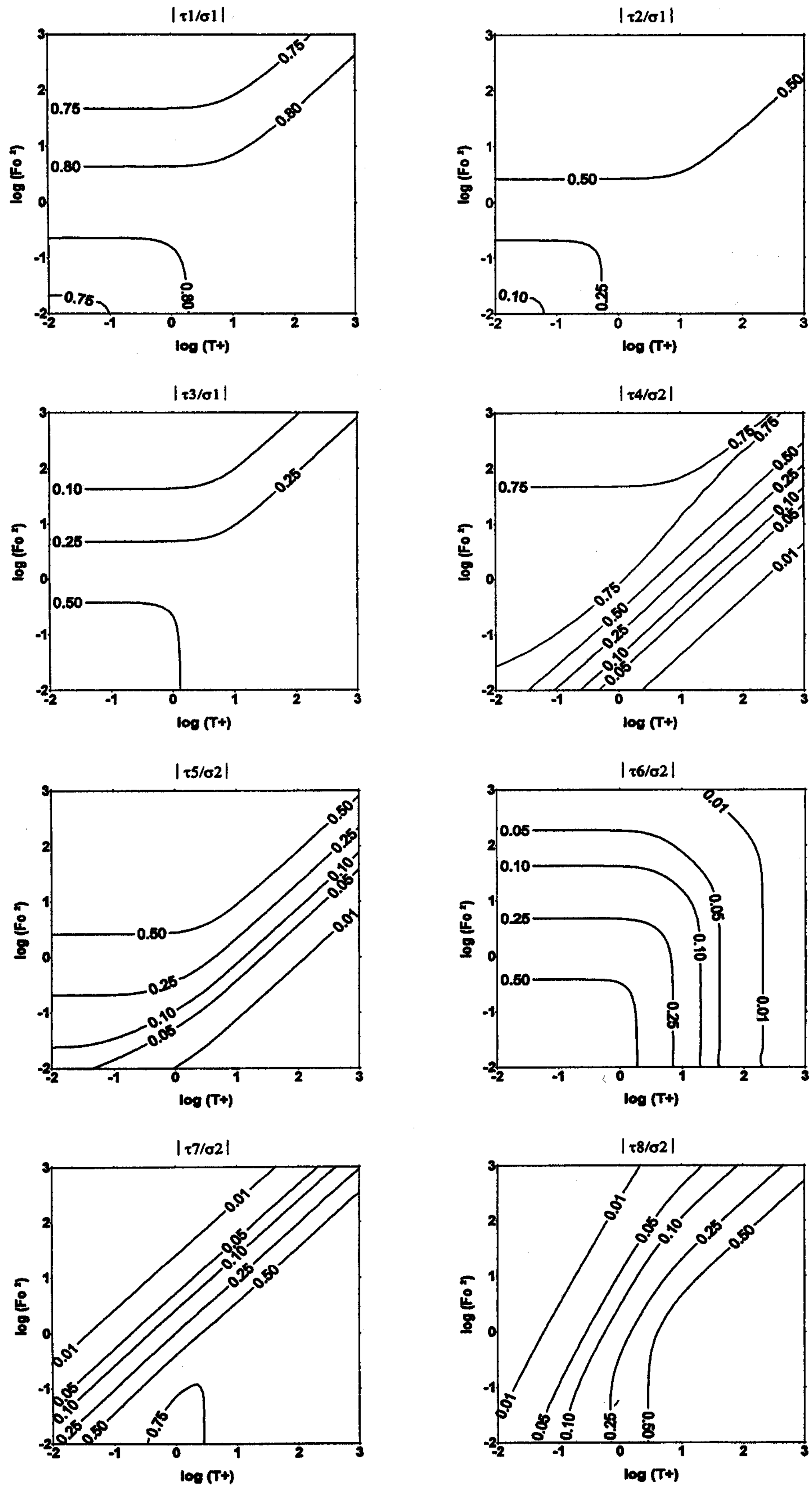

Fig. 2. *Values of the terms* $|\tau_1/\sigma_1|$, $|\tau_2/\sigma_1|$ *and* $|\tau_3/\sigma_1|$ *of the mass equation and* $|\tau_4/\sigma_2|$, $|\tau_5/\sigma_2|$, $|\tau_6/\sigma_2|$, $|\tau_7/\sigma_2|$ *and* $|\tau_8/\sigma_2|$ *of the momentum equation for* $\eta = W_2/W_1 = 1$.

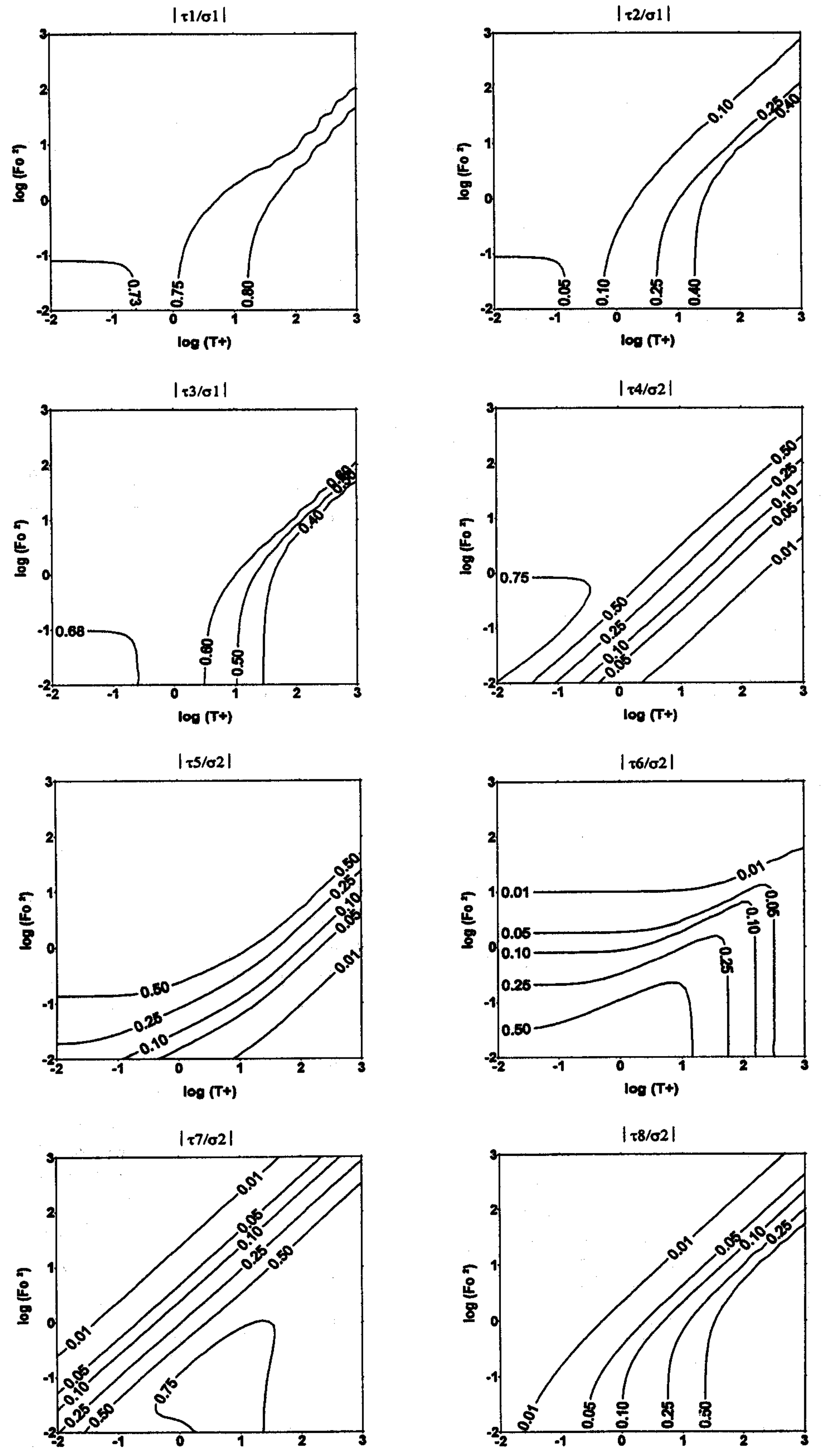

Fig. 3. *Values of the terms* $|\tau_1/\sigma_1|$, $|\tau_2/\sigma_1|$ *and* $|\tau_3/\sigma_1|$ *of the mass equation and* $|\tau_4/\sigma_2|$, $|\tau_5/\sigma_2|$, $|\tau_6/\sigma_2|$, $|\tau_7/\sigma_2|$ *and* $|\tau_8/\sigma_2|$ *of the momentum equation for* $\eta = W_2/W_1 = 8$.

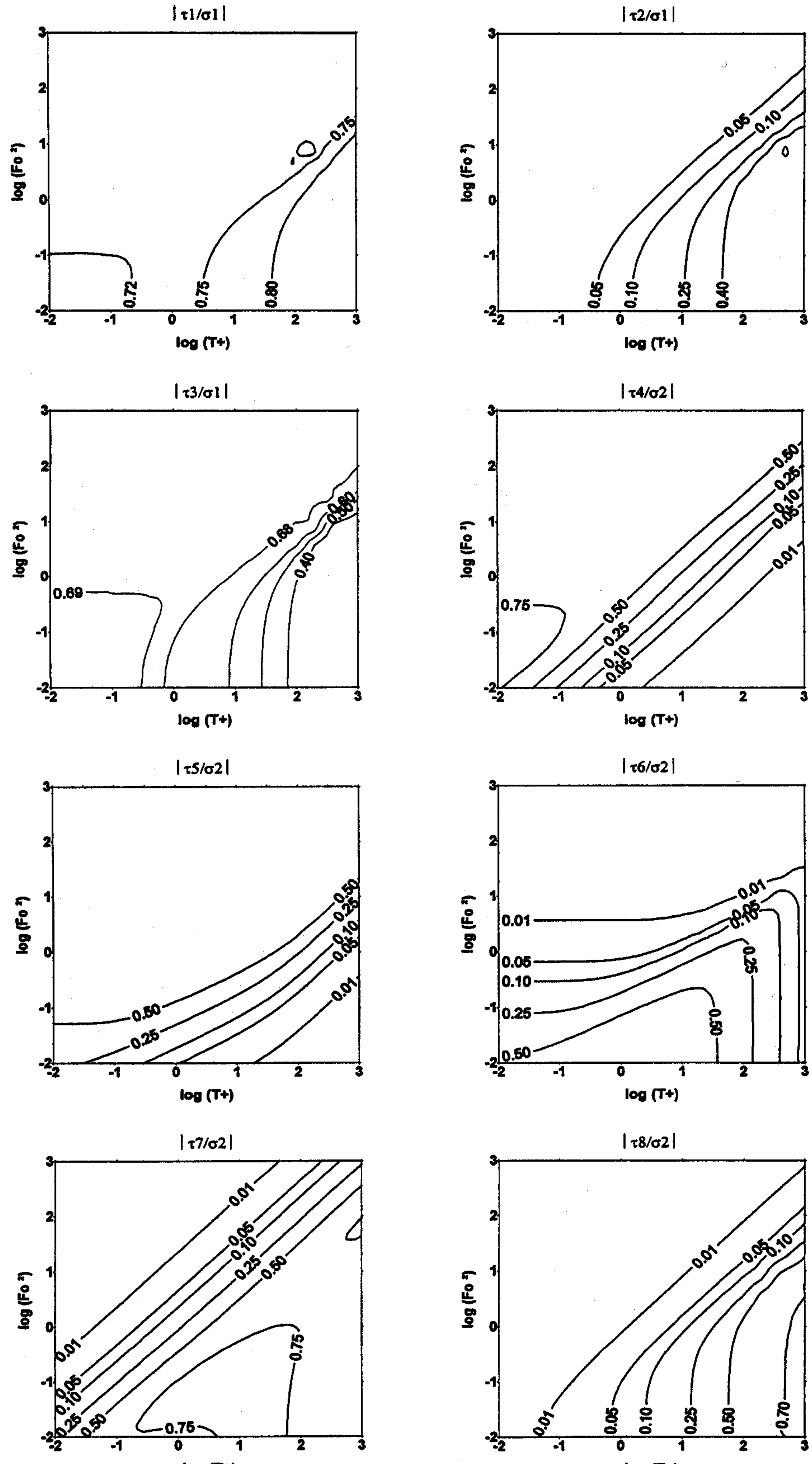

Fig. 4. *Values of the terms* $|\tau_1/\sigma_1|$, $|\tau_2/\sigma_1|$ *and* $|\tau_3/\sigma_1|$ *of the mass equation and* $|\tau_4/\sigma_2|$, $|\tau_5/\sigma_2|$, $|\tau_6/\sigma_2|$, $|\tau_7/\sigma_2|$ *and* $|\tau_8/\sigma_2|$ *of the momentum equation for* $\eta = W_2/W_1 = 20$.

Let

$$\sigma_1 = (\tau_1^2 + \tau_2^2 + \tau_3^2)^{1/2} \quad (45)$$

$$\sigma_2 = (\tau_4^2 + \tau_5^2 + \tau_6^2 + \tau_7^2 + \tau_8^2)^{1/2} \quad (46)$$

The terms τ_j $(1 \le j \le 8)$ and σ_j $(1 \le j \le 2)$ depend also on the three parameters F_0^2, T_+ and η. The magnitude of each term in each equation is obtained by calculating $|\tau_1/\sigma_1|$, $|\tau_2/\sigma_1|$ and $|\tau_3/\sigma_1|$ for the mass Eqn. (35) and $|\tau_4/\sigma_2|$, $|\tau_5/\sigma_2|$, $|\tau_6/\sigma_2|$, $|\tau_7/\sigma_2|$ and $|\tau_8/\sigma_2|$ for the momentum Eqn. (36). Figures 2–4 show the values of these eight terms for m = 4/3 and for three values of η (1, 8 and 20) on a decimal log-log plot for $(10^{-2} \le T_+ \le 10^3; 10^{-2} \le F_0^2 \le 10^3)$.

The three terms of the mass equation ($|\tau_1/\sigma_1|$, $|\tau_2/\sigma_1|$ and $|\tau_3/\sigma_1|$) are of the same order (between 0.25 and 0.8) for $\eta = 1$. When η increases, the two terms $|\tau_1/\sigma_1|$ and $|\tau_3/\sigma_1|$ are still of the same order (0.4 to 0.8) but the term $|\tau_2/\sigma_1|$ can vary between 0.01 and 0.4. For the three values of η, the three terms of the mass equation are generally of the same order and no simplifications could be made in the mass Eqn. (9). In contrast, the different terms of the momentum equation may be sufficiently small to be neglected, leading to further simplifications.

SIMPLIFICATIONS OF THE MOMENTUM EQUATION OF THE SAINT-VENANT SYSTEM

In Eqn. (10), the term (IV) represents the local inertia term, the term (V) represents the convective inertia term, the term (VI) represents the pressure differential term and the terms (VII) and (VIII) account for the friction and bed slopes. Various wave models can be construed, depending on which of these four terms is assumed negligible when comparing with the remaining terms. Wave models and terms used to describe them are (Ponce and Simons, 1977)

- Gravity wave:terms (IV) + (V) + (VI)
- Diffusive wave:terms (VI) and (VII)
- Kinematic wave:term (VII)

The delimitation of the different zones is a function of the two non-dimensionalised numbers, the Froude number F_0^2 that characterises the unperturbed flow regime and T_+ that characterises the sinusoidal period of the upstream initial condition. If the terms $|\tau_j/\sigma_2|$ for $(4 \le j \le 8)$ sufficiently smaller than a threshold (here 10%) are neglected, the three approximation models of the Saint-Venant system can be defined as

- Gravity wave: $|\tau_7/\sigma_2| \le 0.1$ and $|\tau_8/\sigma_2| \le 0.1$
- Diffusive wave: $|\tau_4/\sigma_2| \le 0.1$ and $|\tau_5/\sigma_2| \le 0.1$
- Kinematic wave: $|\tau_4/\sigma_2| \le 0.1$; $|\tau_5/\sigma_2| \le 0.1$ and $|\tau_6/\sigma_2| \le 0.1$

For the three values of η, Fig. 5 shows the result with $|\tau_j/\sigma_2| \le 10\%$ for $(4 \le j \le 8)$. The contour lines of the different approximation models (gravity, diffusive and kinematic) in Fig. 5 don't have any analytical form and simple relations using the ratio F_0^2/T_+ were adjusted to give a first approximation. Table 1 gives these analytical relations as approximation zones of the gravity, diffusion and kinematic waves as a function of the two parameters F_0^2 and T_+.

DISCUSSION

The problem of flood wave propagation is considered for a simplified but typical configuration occurring during flood events, for which the channel width can change instantaneously when overbank flow occurs in the flooded area. The technique proposed herein guides the user in the choice of river wave model and specifies two types of error, the error induced by approximating the Saint-Venant system and the error due to the variation of the channel width during a flood event.

Moussa and Bocquillon (1996) showed that the results obtained for $\eta = 1$ are of the same order and confirm other results obtained by several researchers, notably Ponce *et al.* (1978) and Daluz Vieira (1983), who developed criteria for deciding the conditions under which approximate models

Table 1. Conditions of applicability of the different approximation models of the Saint-Venant system for different values of the ratio between the flooded area width and the main channel width (η)

	Gravity wave	Diffusive wave	Kinematic wave
$\eta = 1$	$\frac{F_0^2}{T_+} \ge 12.41$	$\frac{F_0^2}{T_+} \le 0.0432$	$\frac{F_0^2}{T_+} \le 0.0432$ and $T_+ \ge 39$
$\eta = 8$	$\frac{F_0^2}{T_+} \ge 2.85$	$\frac{F_0^2}{T_+} \le 0.0101$	$\frac{F_0^2}{T_+} \le 0.0101$ and $T_+ \ge 154$
$\eta = 20$	$\frac{F_0^2}{T_+} \ge 2.50$	$\frac{F_0^2}{T_+} \le 0.0037$	$\frac{F_0^2}{T_+} \le 0.0037$ and $T_+ \ge 383$

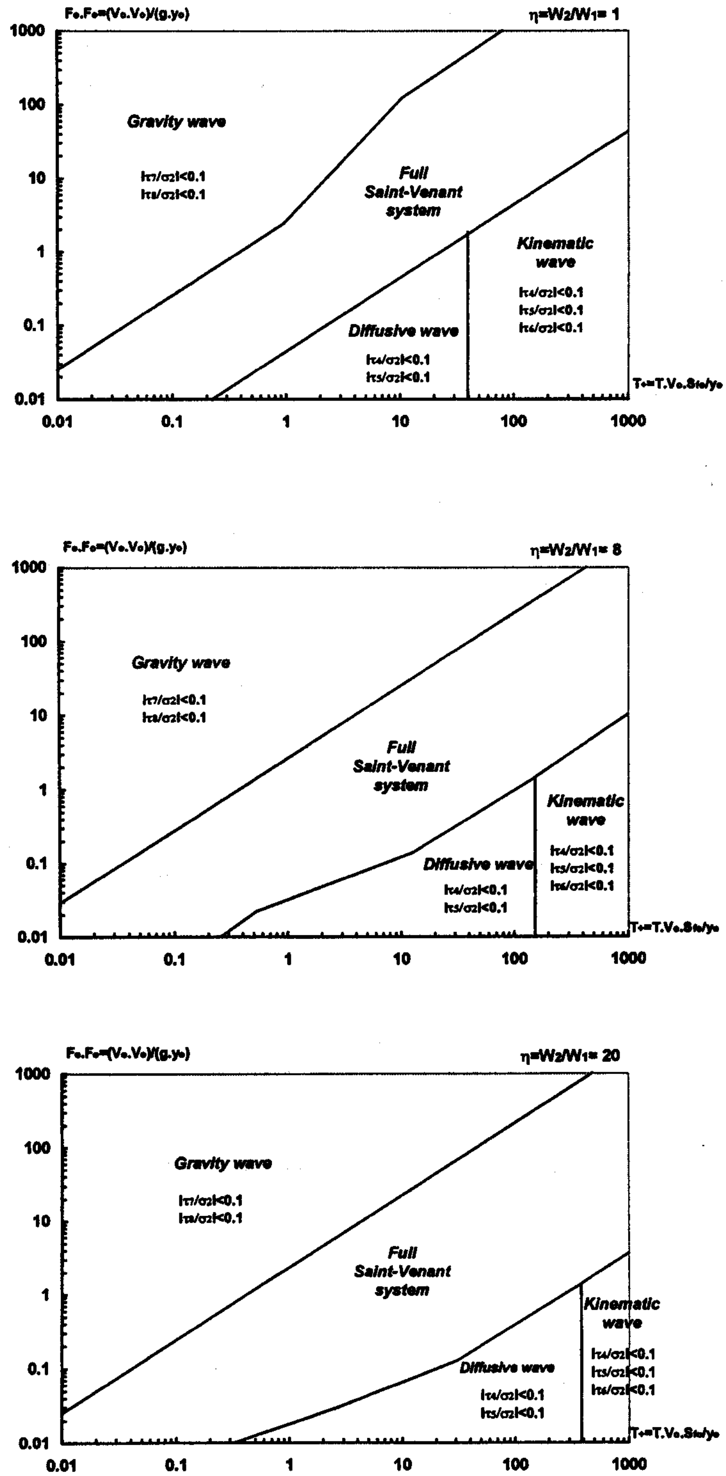

Fig. 5. *River wave approximation zones obtained from the analysis of the momentum equation of the Saint-Venant system for three values of* η = *1, 8 and 20.*

provide an acceptable representation of the momentum equation in the Saint-Venant system.

When η increases, the domain of application of the gravity wave does not change while the domain of application of the diffusive wave and the kinematic wave models is restricted and substituted by the full Saint-Venant system. The comparison of the diffusive wave domain of application for the three cases of η in Fig. 5, shows that the domain moves to the right (higher values of T_+) when η increases and substitutes for the kinematic wave zone.

Other considerations such as the computational power available or the need for real-time forecasting may also be important in the choice of technique. Practical experience suggests that the simpler diffusive wave, kinematic wave or linear methods will be adequate for many purposes. They will not, however, be suitable in flood routing modelling when η increases or varies in space (across the river channel) or in time (during a flood event). In this case, the diffusion wave model should substitute for the kinematic wave model and the full Saint-Venant system should substitute for the diffusion wave model.

In choosing a routing method the accuracy and availability of channel cross-section and roughness coefficients may have a greater effect on the predictive accuracy of a routing algorithm than the choice of the descriptive equation (Beven and Wood, 1993). In addition, both cross-sectional and reach scale roughness may be expected to vary with discharge, especially at the transition to overbank flow. Estimating roughness coefficients is a particularly difficult problem for natural channels. In fact, all routing methods will need to be calibrated to a particular site by comparing observed and predicted levels or discharges, where they are available. In general, the more complex the model, the more physical characteristics and model parameters that must be estimated or measured in the field. In this respect, the simpler routing methods with fewer parameters may have some advantages.

Conclusion

The Saint-Venant system, formed by combining the continuity and momentum equations, is controlled by the balance between friction and inertia. The case of a compound channel section is studied for flood routing problems in natural channels with overbank flow in the flooded area. The propagation characteristics of shallow water waves in open channel flow are calculated on the basis of linear stability theory. Two dimensionless parameters, the Froude number and the period of the input hydrograph enable definition of a spectrum of river waves, with continuous transitions between wave types. Gravity, diffusion and kinematic waves correspond to specific scaling parameter ranges of this spectrum. The parameter range corresponding to each wave type was studied as a function of the flooded area width. Results show that, when the width of the flooded area increases, the domain of application of the diffusive wave and the kinematic wave models is restricted.

The capability to identify wave type is necessary for constructing appropriate mathematical models of river flow and to analyse the ability to use the same wave model when the flooded area width varies in space and time. Changes in wave behaviour with the flooded area width can be addressed quantitatively using the scaling parameters.

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