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Comparative analysis of some methods for deriving the expected flood reduction curve in the frequency domain

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Abstract

This article compares the results of three different models, namely empirical, geomorphoclimatic and stochastic, proposed in the literature for synthesising the reduction curve of average river discharges, of given frequency, over different durations. The analysis used observed reduction ratios inferred for twelve recording gauge stations with known rating curves, situated on central Italian watercourses all of which flow into the Adriatic sea. Particular emphasis was laid on the difficulties encountered in the parameterisation of the models, on the relations between the different formulations and on the existence of a link between the model parameters and the characteristic response times of the basins.

Introduction

The design of hydraulic flood protection structures requires the estimation of discharge volumes of pre-defined return period for durations from several hours to several days. In some cases, (e.g. design of flood control reservoirs or retarding basins, flood damage studies requiring information on depth, area, and duration of flooding), knowledge of the actual shape of the discharge hydrograph is not necessary, as the crucial information is the total volume of water to be stored (NERC, 1975, chapter 5). In these cases, the probability study of the discharge volumes becomes, in a manner perfectly analogous to that of flood peaks, the search for the frequency law of the variable $V_{D,T}$, where V is the discharge volume over the duration D with return period T.

If the discharge hydrograph shape is not of interest, the volume $V_{D,T}$ can be expressed as $V_{D,T} = Q_{D,T} \cdot D$, where $Q_{D,T}$ is the mean discharge, over duration D, with an assigned return period T. Consequently, the probability study of the flood volumes $V_{D,T}$ leads to an estimation of the discharge quantiles $Q_{D,T}$, which can be expressed as:

$$Q_{D,T} = K_{D,T} \cdot \mu_{Q_D} \tag{1}$$

where μ_{Q_D} is the mean of the annual maximum values of the average discharges over a duration D (*scale factor*), and $K_{D,T}$ is the *growth factor*. However, estimation of the growth factor $K_{D,T}$ requires an amount of data frequently not easily available even in the instrumented river sections.

To by-pass this difficulty, the United Kingdom NERC (1975) suggests estimating $Q_{D,T}$ by relating it to the flood peak Q_T of equal return period by means of the *reduction ratio* (or *flow duration reduction curve*), defined as:

$$r_{D,T} = Q_{D,T}/Q_T \tag{2}$$

The quantile Q_T can also be expressed as $Q_T = K_T \cdot \mu_Q$ where K_T is the relevant growth factor and μ_Q (i.e. the scale factor) is the mean of the annual maximum flood peaks. As more flood peak data are generally available than average discharges Q_D, the estimation of K_T is easier and more precise than that of $K_{D,T}$.

Thus, the quantile $Q_{D,T}$ is formulated as:

$$Q_{D,T} = r_{D,T} \cdot Q_T = r_{D,T} \cdot K_T \cdot \mu_Q \tag{3}$$

where the focus is now on the flow duration reduction curve $r_{D,T}$. In scientific literature, many studies are dedicated to the models and techniques for estimating both the growth factor K_T and scale factor μ_Q, while, to the authors' knowledge, only three models have been proposed for representing the reduction curve $r_{D,T}$.

The first of the these models, which can be classified as *empirical*, is presented in the NERC report itself (1975). It is a two-parameter equation, selected solely on the basis of its capability of reproducing the trend in the observed reduction ratios. The second model, proposed by Fiorentino *et al.* (1987), is classified by its authors as *geomorphoclimatic*. In this model, the rainfall is represented through the well known two-parameter intensity duration curve while the basin response is represented through a Geomorphological Instantaneous Unit Hydrograph (GIUH) (e.g. Rosso, 1982; Troutman and Karlinger, 1985, 1986). The third model, proposed by Bacchi *et al.* (1992), can be classified as *stochastic* and is based on the analysis of the extremes of two Gaussian stationary processes representing the continuous time series of the discharges through a river section and the corresponding average discharges relevant to time windows of generic duration *D*.

These models are characterised by different basic hypotheses, structures and parameters whose estimation requires different techniques and data sets. This paper presents a critical review of each model and analyses the ability of each to reproduce the observed flow duration reduction curves using data from twelve river sections situated in central Italy. Particular emphasis is laid on the difficulties encountered in the parameterisation of these models, on the possible relations between the three different formulations and on the existence of a link between the model parameters and the characteristic response time of the basins.

The flow duration reduction curve

Equation 2 shows a formal dependence of the flow reduction curve on the return period *T;* it can be useful to re-write Eqn. 2 as:

$$r_{D,T} = \frac{Q_{D,T}}{Q_T} = \frac{\mu_{Q_D} \cdot K_{T,D}}{\mu_Q \cdot K_T} = \frac{\mu_{Q_D} \cdot (1 + \kappa_{T,D} \cdot CV_D)}{\mu_Q \cdot (1 + \kappa_T \cdot CV)} \quad (4)$$

where κ_T is the *frequency factor* (Chow, 1951), which is a function of the return period and the appropriate probability distribution used, while *CV* is the coefficient of variation of the annual maximum peak floods. The same applies to $\kappa_{T,D}$ and CV_D.

Typical probability distributions used for the frequency factors are the Extreme Value Type I (EV1) model or the Generalised Extreme Value (GEV) model (Jenkinson, 1955). For example, in the case of the EV1 model used for both variables Q_T and $Q_{D,T}$, the reduction curve becomes:

$$r_{D,T} = \frac{\mu_{Q,D} \cdot \left\{1 - \frac{\sqrt{6}}{\pi}\left\{0.5772 + ln\left[ln\left(\frac{T}{T-1}\right)\right]\right\} \cdot CV_D\right\}}{\mu_Q \cdot \left\{1 - \frac{\sqrt{6}}{\pi}\left\{0.5772 + ln\left[ln\left(\frac{T}{T-1}\right)\right]\right\} \cdot CV\right\}} \quad (5)$$

which shows the dependency of the reduction curve on the return period *T*. Nevertheless, this dependency becomes weaker as *T* increases. In fact, Eqn. 5, for $T \rightarrow \infty$, tends to:

$$r_D = \frac{\mu_{Q,D} CV_D}{\mu_Q CV} \quad (6)$$

where the limit is reached quickly (say $T \cong 30$–50), given that the usual values observed of the coefficients of variation remain in the range *0.2–0.4* (note that in Eqn. 6 the symbol used for the reduction curve is r_D to highlight the independence from the return period).

However, as long as the *CV* remains constant at different durations, the reduction curve does not vary with the return period and reduces to the ratios of the mean values of the annual maxima, i.e.:

$$r'_D = \frac{\mu_{Q,D}}{\mu_Q} \quad (7)$$

Equation 7 can be obtained directly from Eqn. 5 when the return period $T = 2.33$, i.e. when the return period of the mean of a Gumbel variate is considered, even if the *CV* can vary over different durations.

Similar results can be obtained using different probability distributions and are confirmed by case studies (NERC, 1975; Bacchi *et al.*, 1992; Franchini and Galeati, 1998).

To summarise, the reduction curve is weakly dependent on the return period *T* so that, in cases in the real world, it can be considered completely independent and the symbol r_D could be used instead of $r_{D,T}$. Furthermore, the reduction curve coincides with the ratios of the mean annual floods when the growth factor does not depend on the durations *D*: for this case the symbol r_D' is used.

In this last case, real world data show a tendency for CV_D to decrease slowly as the time interval *D* increases. However, as the coefficient of variation has a large sampling error, it is often assumed constant over the different durations. This assumption implies the use of Eqn. 7, although average discharges over longer durations may be slightly overestimated (*5–10%* considering the range of the usual values of the *CV*).

In the design of hydraulic flood protection structures, the overestimation from the use of the reduction curve expressed by Eqn. 7 can be accepted easily because it errs on the side of safety; thus it compensates for other sources of uncertainty in dimensioning the structure. For this reason, r_D' represents the most common and preferred form of the reduction curve (NERC, 1975).

These considerations are fundamental to understanding correctly, the three models described below. The first two models of the reduction curve refer directly to r_D' (Eqn. 7); the third of Bacchi *et al.* (1992) is based on the general definition of the reduction curve $r_{D,T}$ (Eqn. 2) but can be transformed easily into an expression which is an estimator of r'_D.

Empirical model

The empirical model (EM) was presented in the United Kingdom NERC report (1975). Its original expression can be re-arranged as:

$$r'_D = \left(1 + \frac{D}{\alpha}\right)^{\beta - 1} \tag{8}$$

where α and β are two parameters to be estimated; parameter α has a time dimension, while β is dimensionless and less than one. Its shape is justified solely by the capability of representing the observed reduction ratios expressed as ratios of the mean annual maximum values over different durations.

Traditionally, the parameters of Eqn. 8 are estimated using the least squares method. However, the parameter values obtained are poorly correlated with catchment characteristics, as shown in NERC (1975), which makes it difficult to regionalise Eqn. 8. To by-pass this problem, a modified estimation approach may be used; it depends on the type of data available and will be presented directly in the section dedicated to the numerical application.

Geomorphoclimatic model

The geomorphoclimatic model (GM) proposed by Fiorentino *et al.* (1987) for representing the reduction curve r_D' considers a simple event-based rainfall-runoff transformation whose main aspects can be subdivided into three parts: a) the net rainfall input, b) the rainfall-runoff transformation and c) the technique for estimating μ_Q and μ_{Q_D}.

a) The net rainfall input. It is assumed that, for any storm duration d, the areal rainfall intensity $i_A(d)$ is constant in time and decreasing with d (Fig. 1a). The *net* areal precipitation is expressed simply as $\varphi \cdot i_A(d)$, where φ is the runoff coefficient assumed to be constant over time and independent of the duration and intensity of the rainfall.

b) The rainfall-runoff transformation. The basin is considered as a lumped linear system; hence, the output, i.e. the basin outlet discharge $q(t,d)$, relevant to a net input $\varphi\ i_A(d)$, can be expressed through the convolution integral:

$$q(t,d) = \varphi \cdot i_A(d) \cdot \int_{\phi}^{t} u(\tau)d\tau \cdot A;$$

$$\phi = \begin{vmatrix} t-d & \text{if} \quad t-d>0 \\ 0 & \text{if} \quad t-d \le 0 \end{vmatrix} \tag{9}$$

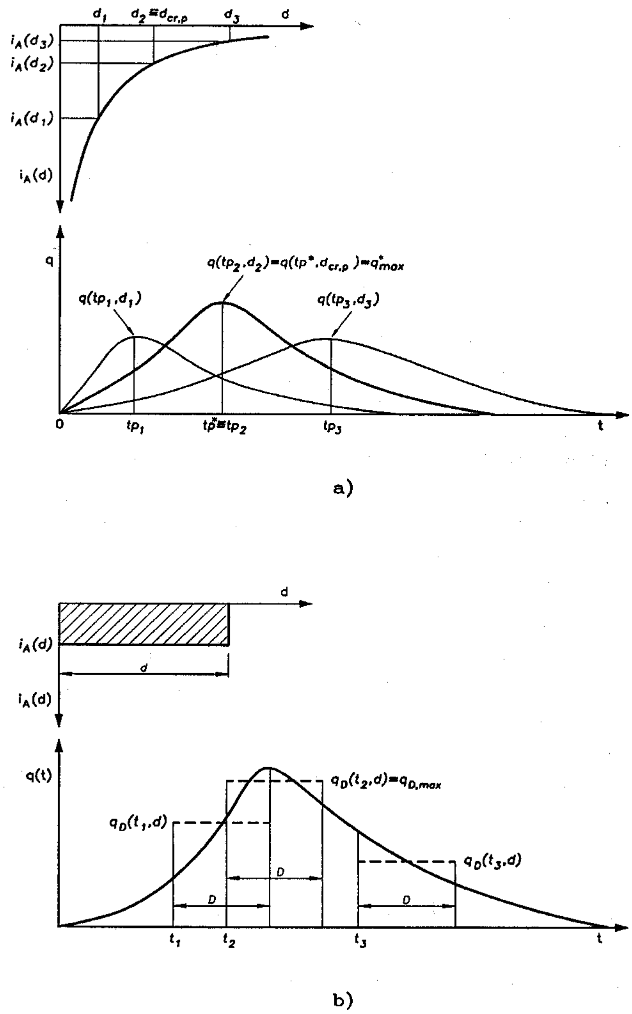

Fig. 1. *Schematic representation of the of the geomorphoclimatic model: (a) identification of the critical rainfall duration $d_{cr,p}$ with reference to the flood peak; (b) different average values $q_D(t,d)$ for a given rainfall duration d.*

where $u(\tau)$ is the IUH of the basin. With:

$$S(t) = \int_0^t u(\tau)d\tau \quad ;$$

$$\Delta S(t,d) = \begin{vmatrix} [S(t) - S(t-d)] & \text{if } t > d \\ S(t) & \text{if } t \le d \end{vmatrix} \tag{10}$$

Eqn. 9 becomes:

$$q(t,d) = \varphi \cdot i_A(d) \cdot \Delta S(t) \cdot A \tag{11}$$

For an assigned rainfall duration d, the maximum value $q_{\max}$ of $q(t,d)$ is obtained at the instant t_p when $\Delta S(t,d)$ is a maximum. Denoting the so-called *peak function* as $\sigma(d) = \Delta S(t_p,d) = S(t_p) - S(t_p - d)$ (Wood and Hebson, 1986), the maximum discharge value can be expressed as:

$$q(t_p, d) = q_{\max} = \varphi \cdot i_A(d) \cdot \sigma(d) \cdot A \qquad (12)$$

As the duration of the precipitation d increases, the peak function tends towards *1*, while the areal rainfall intensity $i_A(d)$ tends towards zero[1]. Thus, there is a critical precipitation duration $d \equiv d_{cr,p}$ at which the product $i_A(d) \cdot \sigma(d)$ is a maximum. Indicating with t_p^* the instant at which $q_{\max}^*$ is obtained, i.e. the maximum value of $q_{\max}$ (see Fig. 1a), Eqn. 12 can be rewritten as:

$$q(t_p^*, d_{cr,p}) = q_{\max}^* = \varphi \cdot i_A(d_{cr,p}) \cdot \sigma(d_{cr,p}) \cdot A \qquad (13)$$

In a similar manner, the maximum value of the discharge $q_D(t,d)$, which represents the mean discharge over the time interval of duration D (Fig. 1b) starting at the instant t and referring to a flood wave produced by a net areal rainfall $\varphi \cdot i_A(d)$, can be written as:

$$q_D(t_D^*, d_{cr,v}) = q_{D,\max}^* = \varphi \cdot i_A(d_{cr,v}) \cdot \Xi(d_{cr,v}) \cdot A \qquad (14)$$

where generally $d_{cr,v} \neq d_{cr,p}$ and $\Xi(d_{cr,v})$ is a function similar to the peak function defined above (see Appendix A).

c) The technique for estimating μ_Q and μ_{Q_D}. The reduction ratios r_D' are based on the average values of the annual maximum floods over different durations D. Equations 13 and 14 allow for the estimation of maximum discharge values for a given intensity duration curve $i_A(d)$. The assumption made by Fiorentino *et al.* (1987) is that the values of μ_Q and μ_{Q_D} can be estimated through Eqns. 13 and 14, respectively, when the intensity duration curve $i_A(d)$ is parameterised to represent the average values of the annual maximum intensities, i.e. $\mu_{i_A}(d)$. On the basis of this assumption, the reduction curve r_D can be written as:

$$r_D' = \frac{\mu_{Q_D}}{\mu_Q} = \frac{\mu_{i_A(d_{cr,v})} \cdot \Xi(d_{cr,v})}{\mu_{i_A(d_{cr,p})} \cdot \sigma(d_{cr,p})} \qquad (15)$$

This last equation shows that the reduction curve obtained using the GM model is defined after the attribution of the areal intensity duration curve (representing the average values of the annual maximum intensities of different rainfall durations) and the IUH (representing the rainfall-runoff transformation process, present in the functions $\sigma(d_{cr,p})$ and $\Xi(d_{cr,v})$). The definition of the areal intensity duration curve is related to the type of data available and will be discussed in the section dedicated to the numerical application.

[1] In the case of an IUH with a finite base time equal to the time of concentration T_c of the basin, the peak function $\sigma(d)$ is 1 for $d \geq T_c$; otherwise, in the case of an IUH with an infinite base time (e.g. Gamma or Weibull type IUH), the peak function $\sigma(d)$ tends towards 1 as d tends towards infinity.

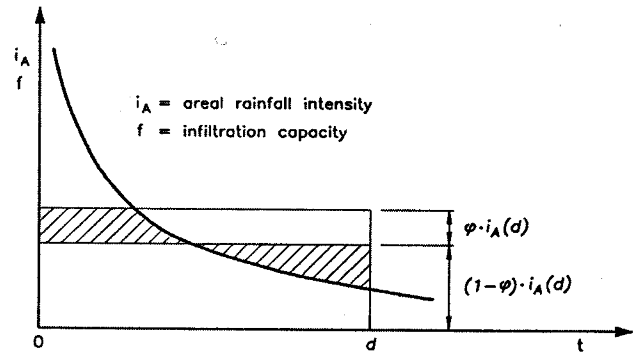

Fig. 2. *Schematic representation of the compensation effect in the infiltration process.*

The main assumptions characterising the GM model can be summarised as follows: (a) the runoff coefficient φ is independent of the duration and the intensity of the precipitation, (b) the GM model considers only the surface component of the flood events and (c) the average values of the annual maximum flood discharges over different durations are one-to-one related to the average values of the annual maximum intensities.

With reference to the first assumption, it is known that the infiltration capacity of the soil is not constant but decreases with time, during a storm event, following a law which is affected by rainfall intensity. The assumption of a constant coefficient φ can, however, be justified (Rodriguez-Iturbe *et al.*, 1982) by assuming a compensating effect as shown in Fig. 2 where the shadow areas are equal so that the total amount of water stored in the soil is given by the area of the rectangle $d \cdot (1-\varphi) \cdot i_A(d)$.

The second assumption may be justified when peak discharges or those relevant to short durations are considered. However, in the case of average discharges over long durations, the base discharge component may not be disregarded, being of the same magnitude of the average discharge itself. This aspect, considered on its own, should produce an underestimation of the reduction ratios on long durations. However, the other model simplifications combined with the calibration process mask this negative influence, as will be shown in the numerical application.

The third assumption is acceptable only in the very narrow frame of a lumped linear system when the areal rainfall input hyetograph can have a rectangular shape *alone* and the runoff coefficient is *constant* for all storm events. Indeed, it is well known that the flood characteristics, produced by a rainfall of given duration and total depth, depend on the space-time storm distribution and the soil water content at the beginning of the storm event. Thus, an assigned flood peak value can be produced by very different storm events and an assigned total rainfall depth can produce very different flood peaks (Franchini *et al.*, 1996).

In conclusion, the three assumptions produce a marked

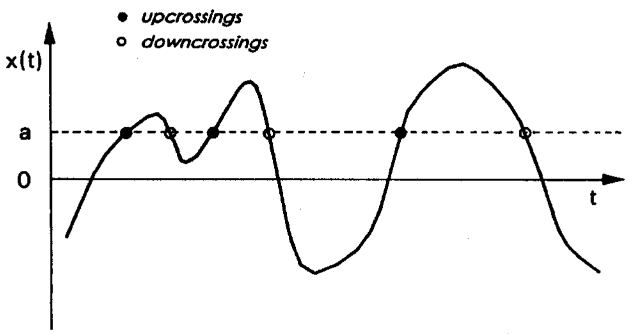

Fig. 3. *Schematic representation of the crossings of a continuous process in relation to a set threshold.*

gap between the model and the real physical phenomenon of the rainfall runoff transformation. However, the GM produces fairly good quality results, and a physical interpretation of the main characteristics of the IUH $u(t)$, used to represent the rainfall runoff transformation, is still possible.

Stochastic model

The stochastic model (SM) presented by Bacchi *et al.* (1992) aims at characterising the extremes of the continuous series $q(t)$ of discharges flowing through the section under study, and the extremes of the associated integral process $q_D(t)$ defined as:

$$q_D(t) = \frac{1}{D}\int_t^{t+D} q(\tau)d\tau \tag{16}$$

Assuming that the processes $q(t)$ and $q_D(t)$ can be reduced, by standardisation, to the Gaussian and stationary processes $X(t)$ and $X_D(t)$ respectively, Bacchi *et al.* (1992) derive an expression for the reduction curve $r_{D,T}$ analytically by analysing the crossing properties of the two standardised processes with reference to a given threshold value (Fig. 3).

The reduction curve, with reference to its general definition given by Eqn. 2, has the following expression:

$$r_{D,T} = \frac{\mu_q}{Q_T} + \sqrt{\Psi(D)}\sqrt{\left(1-\frac{\mu_q}{Q_T}\right)^2 - 2\left(\frac{\sigma_q}{Q_T}\right)^2 ln\left(\frac{\Omega_2}{\Omega_{2,D}}\right)} \tag{17}$$

where μ_q and σ_q^2 are the mean and the variance of the process $q(t)$, Ω_2 and $\Omega_{2,D}$ are the second-order characteristic frequencies of the spectral density function of the standardised process $X(t)$ and relevant integral process $X_D(t)$ respectively, while $\Psi(D)$ is the process variance function, i.e. the law which expresses the attenuation of the variance $\sigma^2_{X_D}$ of $X_D(t)$ versus the increase in D. Its form is linked strictly to the auto-correlation function $\rho(\tau)$ of the process $X(t)$ (which, of course, coincides with that of the process $q(t)$) via the relation (Vanmarke, 1983):

$$\Psi(D) = \frac{1}{D^2}\int_0^D d\tau \int_0^D \rho(t-\tau)d\tau \tag{18}$$

In turn, the auto-correlation function $\rho(\tau)$ is linked to the parameter θ, which represents the "scale of fluctuation", i.e. the characteristic time interval of the fluctuation of the process $X(t)$ (and so of the process $q(t)$), via the following relation (Vanmarcke, 1983):

$$\theta = 2\int_0^\infty \rho(\tau)d\tau \tag{19}$$

This parameter is particularly useful in characterising the behaviour of the auto-correlation function; in fact it gives a measure of the rate of decrease of the auto-correlation.

Furthermore, Bacchi *et al.* (1992, p. 2776) observe that $\mu_q/Q_T \ll 1$ and $\sigma_q/Q_T \ll 1$ even for return periods T which are not particularly high (say T greater than 5) and therefore Eqn. 17 can be written as:

$$r_{D,T} \cong \sqrt{\Psi(D)}, \quad \forall T \tag{20}$$

confirming the general weak dependency of the reduction curve in relation to the return period. However, recalling that the return period of the mean annual peak flood is of the order of 2–4 years, Eqn. 20 can be also considered an estimator of r'_D.

To give an explicit estimation of the function $\Psi(D)$ (and thus of the reduction curve according to Eqn. 20), the form of the auto-correlation function $\rho(\tau)$ must be specified. Bacchi *et al.* (1992) suggest that the standardised process $X(t)$ can be considered as an autoregressive process of order n (identified below as AR(n)), whose auto-correlation function is:

$$\rho(D) = exp(-D/k) \sum_{k=0}^{n-1} \frac{(n-1)!(n+k-1)!}{(2n-2)!k!(n-k-1)!}(2D/k)^{n-k-1} \tag{21}$$

where k is a temporal scale parameter (Gelb, 1974, pp. 42–44). Consequently, on the basis of Eqn. 19, the scale of fluctuation is:

$$\theta = k\frac{2^{2n-1}[(n-1)!]^2}{(2n-2)!} \tag{22}$$

Table 1 sets out the various expressions for the auto-correlation function for $n = 1,2,3,4$ obtained by combining Eqns. 21 and 22. Table 2 contains the corresponding expressions of the reduction curve obtained by replacing the equations of Table 1 in Eqn. 18 and eventually in Eqn. 20. It can be seen that the stochastic model is completely defined by the scale of fluctuation θ alone.

Table 1. Stochastic model: autocorrelation function $\rho(D)$ expressed as a function of the fluctuation scale parameter θ for different orders of the $AR(n)$ process.

n	$\rho(D) = exp(-D/k)\sum_{k=0}^{n-1}\frac{(n-1)!(n+k-1)!}{(2n-2)!k!(n-k-1)!}(2D/k)^{n-k-1}; \quad \theta = k\frac{2^{2n-1}[(n-1)!]^2}{(2n-2)!}$
$n=1$	$\rho(D) = exp\left(-2\frac{D}{\theta}\right)$
$n=2$	$\rho(D) = \frac{(4D+\theta)\cdot exp\left(-4\frac{D}{\theta}\right)}{\theta}$
$n=3$	$\rho(D) = \frac{(256\cdot D^2 + 144\cdot D\cdot\theta + 27\cdot\theta^2)\cdot exp\left(-\frac{16}{3}\frac{D}{\theta}\right)}{27\cdot\theta^2}$
$n=4$	$\rho(D) = \frac{(32768\cdot D^3 + 30720\cdot D^2\cdot\theta + 12000\cdot D\cdot\theta^2 + 1875\cdot\theta^3)\cdot exp\left(-\frac{32}{5}\frac{D}{\theta}\right)}{1875\cdot\theta^3}$

On the basis of the general theory of linear systems, the autoregressive process AR(n) can be interpreted as the output of a linear system whose «kernel function» is a Gamma with parameters n and k and whose input is a «shot noise» process with finite variance and Poisson occurrences. Using this interpretation, the discharge process $q(t)$, in its standardised formulation $X(t)$, can be read as the result of a rainfall-runoff transformation in which the *net* rainfall is represented by the «shot noise» process and the basin response by a Gamma-type IUH with parameters n and k (Brath *et al.*, 1992). It follows that $q(t)$ is seen as the *surface runoff component* while the parameter θ, linked to the parameters n and k by Eqn. 22, can be expressed as a function of the lag time t_L typical of the basin. For a Gamma IUH $t_L = n \cdot k$. Finally, the expression of θ versus the lag time t_L, for $n = 1,2,3,4$, is shown in Table 3.

In short, the interpretation of the process $q(t)$ suggested by Brath *et al.* (1992) allows the parameter θ, which is purely statistical in nature, to be linked to the parameter t_L whose physical significance is clearly understood. Nevertheless, even if a physical interpretation of the parameter θ is then possible, the SM is derived starting from, two statistical hypotheses, i.e. the process representing the river discharges $q(t)$ can be reduced to a (1) stationary and (2) Gaussian process.

The river discharges $q(t)$ are interpreted as a realisation of a stochastic process in continuous time. Usually this process is periodic on account of the intrinsic seasonality of the phenomenon. It is, however, possible to obtain an

Table 2. Stochastic model: estimator of the reduction curve r'_D according to the SM for different orders of the $AR(n)$ process.

n	$\hat{r}'_D = \sqrt{\Psi(D)} = \sqrt{\frac{1}{D^2}\int_0^D d\tau \int_0^D \rho(t-\tau)dt}$
$n=1$	$\hat{r}'^2_D = \frac{\theta}{D} - \frac{1}{2}\left(\frac{\theta}{D}\right)^2 + \left[\frac{1}{2}\left(\frac{\theta}{D}\right)^2\right] exp\left(-2\frac{D}{\theta}\right)$
$n=2$	$\hat{r}'^2_D = \frac{\theta}{D} - \frac{3}{8}\left(\frac{\theta}{D}\right)^2 + \left[\frac{3}{8}\left(\frac{\theta}{D}\right)^2 + \frac{1}{2}\frac{\theta}{D}\right] exp\left(-4\frac{D}{\theta}\right)$
$n=3$	$\hat{r}'^2_D = \frac{\theta}{D} - \frac{45}{128}\left(\frac{\theta}{D}\right)^2 + \left[\frac{2}{3} + \frac{45}{128}\left(\frac{\theta}{D}\right)^2 + \frac{7}{8}\frac{\theta}{D}\right] exp\left(-\frac{16}{3}\frac{D}{\theta}\right)$
$n=4$	$\hat{r}'^2_D = \frac{\theta}{D} - \frac{175}{512}\left(\frac{\theta}{D}\right)^2 + \left[\frac{8}{5} + \frac{175}{512}\left(\frac{\theta}{D}\right)^2 + \frac{19}{16}\frac{\theta}{D} + \frac{64}{75}\frac{D}{\theta}\right] exp\left(-\frac{32}{5}\frac{D}{\theta}\right)$

Table 3. Stochastic model: relation between the fluctuation scale parameter θ and the lag time t_L

n	$\theta = \frac{t_L}{n} \frac{2^{2n-1}[(n-1)!]^2}{(2n-2)!}$
$n = 1$	$\theta = 2 \cdot t_L$
$n = 2$	$\theta = 2 \cdot t_L$
$n = 3$	$\theta = 16/9 \cdot t_L$
$n = 4$	$\theta = 8/5 \cdot t_L$

approximate second-order stationary series by means of the standardisation:

$$X'(t) = \frac{q(t) - \mu_{\tau^*}}{\sigma_{\tau^*}} \tag{23}$$

where μ_{τ^*} and σ_{τ^*} are the periodic mean and standard deviation, respectively. This possibility can justify, partially, the mathematical developments producing the reduction curve 17, which, however, are theoretically relevant to a fully stationary process. Moreover, the flow process $q(t)$ is usually non-Gaussian. It might be reduced to a process which is approximately Gaussian by logarithmic or other appropriate transformations. However, Bacchi *et al.* (1992) do not apply them on account of the uncertainties in the preservation of the statistical properties. In any case, their numerical applications show that although the marginal distribution of river flow processes is usually non-Gaussian, the reduction curve obtained with the Gaussian hypothesis provides a good approximation of the observed reduction ratios, irrespective of the assumed underlying distribution and, thus, is valid at least from a practical point of view.

Furthermore, note that the selection of an auto-regressive process AR(n) for representing the discharge time series is motivated by the observation of Bacchi *et al.* (1992) that this type of process "has been widely used in hydrology". In other words, this assumption is certainly coherent with the stationary and Gaussian hypothesis for the time discharge series, but its selection is due mainly to its simple mathematical tractability in terms of the auto-correlation function, from which the fluctuation parameter θ (and, thus, the reduction curve) is easily derived.

A further observation may be made with reference to the interpretation given in Brath *et al.* (1992) where the process AR(n) is read as output of a rainfall-runoff transformation. While this particular point of view allows for a more physically based interpretation of the parameter θ, it remains impossible to define a link between the reduction curve, as expressed by the SM, and the rainfall regime typical of the basin upstream of the section considered. This is due to the fact that, even in the interpretation of the process AR(n) described above, the rainfall is reduced to a «shot noise» process in which the duration and intensity of the rainfall events are not characterised in any way.

Finally, a difference between the papers of Bacchi *et al.* (1992) and Brath *et al.* (1992) can be observed. In the former, the process $q(t)$ represents the river discharge; in the latter, it represents the component due to the surface runoff obtained by convolution with a Gamma IUH. However, the model application to cases in the real world is substantially insensitive to this different interpretation of $q(t)$. In fact, it is relevant to extreme events where the base component represents a very small percentage of the whole flow discharge and so the same observations, already developed for the GM, can be applied in this case.

Numerical application

DATA USED

The applicability and descriptive capability of the three models were studied with reference to 12 recording gauge stations with known rating curves, whose Apennine basins are situated in central Italy and flow into the Adriatic sea. The position of the stations examined and the geographical boundaries of the Bologna sector of the S.I.M.N. (National Hydrographic and Tide Monitoring Service) which handles the data recording and collection service, are shown in Fig. 4. These stations, listed in Table 4 together with geomorphological quantities typical of the respective basins, were selected from a series analysed in earlier regional flood regime studies (Franchini and Galeati, 1998); together they comprise a hydrologically homogeneous region.

It was possible to locate at least 10 years of continuous observations at these stations from which the required data were obtained, i.e. peak and mean annual maximum discharge values over different durations D. These data were evaluated either by examination of the original records available at the headquarters of the S.I.M.N., or by entering the recorded sets of level data and converting them to discharge values using the relevant rating curves. The maximum annual discharges over time intervals of assigned duration, the discharge values μ_Q and μ_{Q_D}, and the corresponding reduction ratios r'_D were estimated for durations up to 72 hours, since, in the basins considered, the most significant portion of the flood events builds and recedes over this period of time. The whole analysis refers to the estimation of the reduction curve in terms of ratios of the average annual maximum discharges over different durations, i.e. r'_D which, as already observed, represents the most common and preferred form in applications to cases in the real world.

From a regional analysis of the extreme rainfall depths relevant to different time intervals, d, for the whole Bologna sector of the S.I.M.N. (Franchini and Galeati, 1994) the *intensity-duration curve of the mean annual maximum values* was computed to characterise the 12 basins (*Basin Intensity Duration Curve: BIDC*) (Appendix B.)

Finally, the fit of the three models to the observed re-

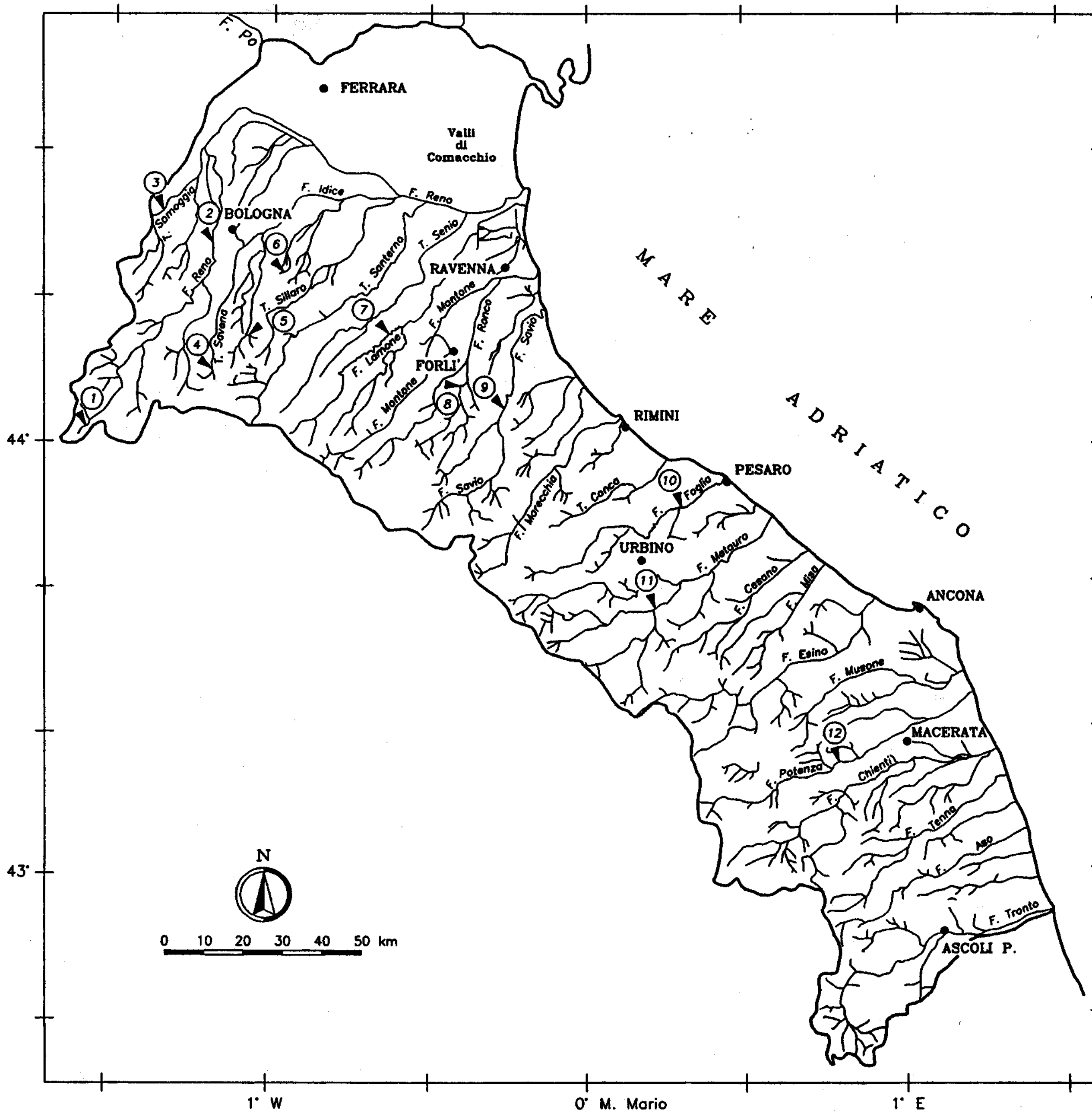

Fig. 4. *Representation of the boundaries of the Bologna SIMN sector and location of the twelve stations used.*

duction ratios was assessed by the least squares method using an optimisation algorithm available in the "Optimization" TOOLBOX from MATLAB® which is based on the SQP (Sequential Quadratic Programming) (e.g. Powell, 1983).

THE EMPIRICAL MODEL

Initially, for each station, the coefficients α and β of Eqn. 8 were estimated simultaneously, thereby obtaining the values shown in Table 5 which also includes the mean and minimum value of the coefficient of determination, R^2, for the 12 stations. The parameter β fluctuates between the values *0* and *0.524*, i.e. within a numerical range which, apart from the lowest values, recalls that of the coefficient b (see last column of Table 5) of the intensity duration curve *BIDC*.

The NERC study (1975, p. 369, Vol. I) shows that the reduction curve is not very sensitive to the coefficient β. Thus, one might assume that the coefficient β coincides with the coefficient b of the *BIDC* and then re-estimate the coefficient α, obtaining the values shown in the fourth

Table 4. Recording hydrometer stations used and associated characteristic data

n°	Station	n.d.	A	L	H_m	S	N_I	L_{med}
1	Reno at Pracchia	15	41.0	13.3	280.2	3.20	438	354
2	Reno at Casalecchio	25	1051.0	84.2	581.0	1.18	5562	476
3	Samoggia at Calcara	12	170.0	37.7	331.0	1.93	472	648
4	Savena at Castel dell'Alpi	10	11.5	6.0	275.0	7.60	87	449
5	Rio Calla Querceto	11	10.0	4.3	260.0	10.87	30	634
6	Quaderna at Palesio	14	21.9	10.2	193.4	3.47	181	360
7	Lamone at Sarna/Faenza	11	261.0	53.7	461.3	1.80	1460	433
8	Ronco at Meldola	13	442.0	57.5	512.0	2.40	1580	542
9	Savio at S. Vittore	12	597.0	66.5	483.5	1.64	1649	591
10	Foglia at Montecchio	15	603.0	80.0	345.5	1.20	1158	727
11	Candigliano at Acqualagna	11	617.0	56.3	417.0	1.38	2957	470
12	Potenza at Cannucciaro	11	439.0	58.4	449.0	2.21	1506	530

n°: station number.
n.d.: number of years available for calculating the annual maximum values of the peak and mean discharge over duration D.
A: area of the basin (km^2).
L: length of main stream (km).
H_m: mean height of the basin in relation to the outlet section (m).
S: slope of main stream (%).
N_I: number of streams of the first order (−).
L_{med}: mean length of the streams in the hydrographic network (m).

column of Table 5. With this position too, the R^2 are still very high. The parameter α obviously assumes different values from those observed previously, so as to allow Eqn. 8 to fit the sample data.

The assumption $\beta = b$ reduces the parameterisation of the reduction curve simply to the estimation of α alone, an advantage that becomes even more evident in a regionalisation process. In fact, the parameter b can be inferred easily from an analysis of the rainfall data at the stations in the basin or, as in this case, from the results of an earlier regional analysis (Franchini and Galeati, 1994).

Table 5. NERC model: estimation of the reduction curve parameters

Station	A		B	
	α	β	α	$\beta = b$
Reno at Pracchia	17.6	0.158	8.4	0.473
Reno at Casalecchio	33.6	0.000	16.8	0.390
Samoggia at Calcara	24.4	0.000	13.6	0.331
Savena at Castel dell'Alpi	5.7	0.457	6.1	0.435
Rio Calla Querceto	3.9	0.398	4.4	0.367
Quaderna at Palesio	5.6	0.455	8.0	0.340
Lamone at Sarna/Faenza	36.8	0.000	19.9	0.360
Ronco at Meldola	14.4	0.317	12.4	0.382
Savio at S. Vittore	19.2	0.289	16.4	0.359
Foglia at Montecchio	27.4	0.327	28.5	0.308
Candigliano at Acqualagna	18.9	0.183	14.2	0.327
Potenza at Cannucciaro	14.7	0.524	25.3	0.316
mean R^2	0.998		0.993	
minimum R^2	0.994		0.984	

A: estimation of parameters α and β; B: estimation of parameter α with $\beta = b$ (b exponent of the *BIDC*).

However, the assumption $\beta = b$ remains highly arbitrary; it is not justified by any statistical or physical reasoning but only by the practical advantage previously mentioned. Further justifications are thus necessary but they will become more evident after the presentation of the results relevant to the other models.

Finally, by way of example, Fig. 5a shows the reduction curve for the Reno at the Casalecchio station. The fit of Eqn. 8 with the sample points is more than satisfactory, even though at durations between 1 and 6 hours, these points reveal a downwards concavity which Eqn. 8 by its very nature cannot represent. However, the errors at these durations are extremely small.

THE GEOMORPHOCLIMATIC MODEL

This model is based on Eqn. 15 and requires for its application the definition of the areal intensity duration curve $i_A(d)$, relevant to the average annual maximum values for different time durations d, and the shape of the IUH.

The areal intensity duration curve $i_A(d)$ can be estimated by using the *BIDC*, available for each of the 12 basins considered, combined with the areal reduction factor

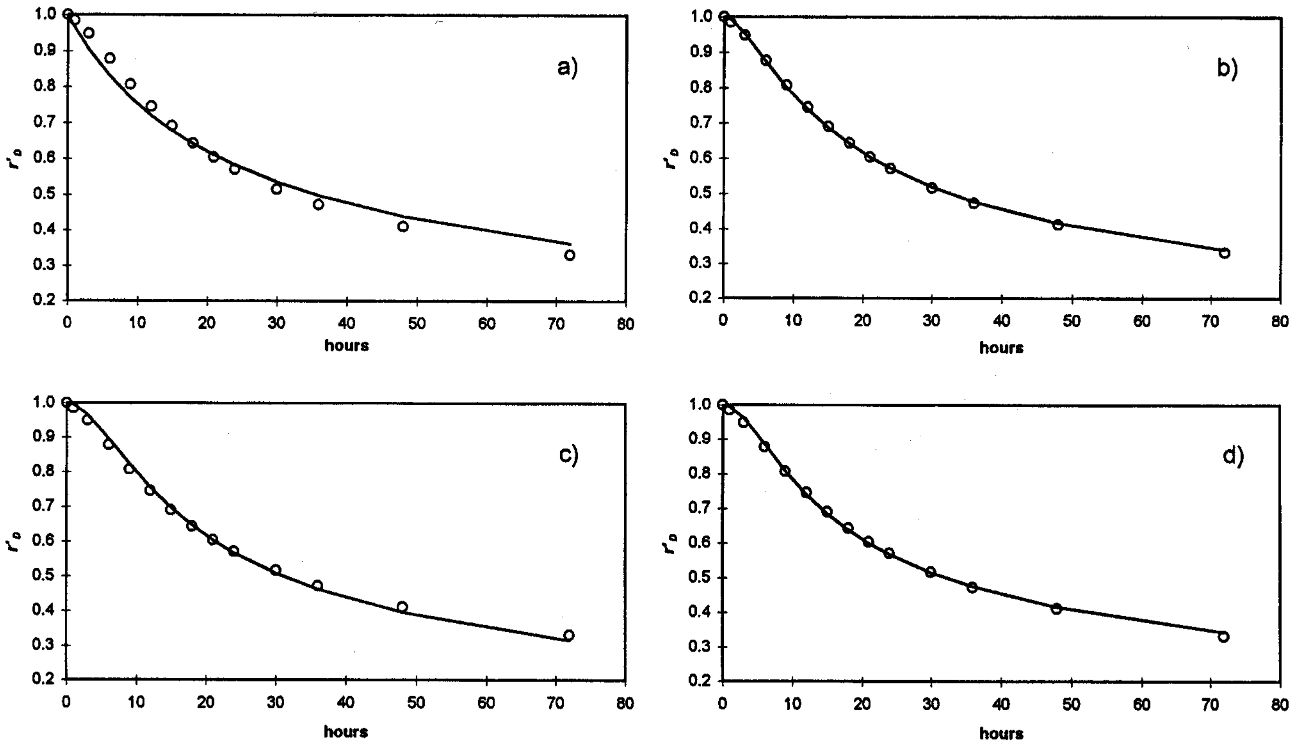

Fig. 5. *Reno at Casalecchio: comparison between the observed reduction curve (o) and the reduction curves obtained using the models. (a) EM model ($\beta = b$); (b) GM model (Gamma IUH, $n_G = 3$), (c) GM model (Weibull IUH, $n_W = 1.6$); (d) SM model ($n = 3$).*

$ARF_{d,A}$. Thus, Eqn. 15 becomes:

$$r'_D = \frac{\mu_{Q_D}}{\mu_Q} = \frac{\mu_{i_A(d_{cr,v})} \cdot \Xi(d_{cr,v})}{\mu_{i_A(d_{cr,p})} \cdot \sigma(d_{cr,p})} = \frac{ARF_{d_{cr,v},A} \cdot d_{cr,v}^{\,b-1} \cdot \Xi(d_{cr,v})}{ARF_{d_{cr,p},A} \cdot d_{cr,p}^{\,b-1} \cdot \sigma(d_{cr,p})} \qquad (24)$$

where the areal reduction factor $ARF_{d,A}$ can be expressed as:

$$ARF_{d,A} = 1 - (1 - e^{-0.013A}) \cdot e^{-0.68d^{0.33}} \qquad (25)$$

by re-parameterising the formula of the U.S. Weather Bureau for the area under study. As regards the IUH, numerous studies linking it with the geomorphological structure of the drainage network have shown that the Gamma distribution (Rodriguez-Iturbe and Valdes, 1981; Rosso, 1982) and Weibull distribution (Troutman and Karlinger, 1985, 1986) are excellent approximations of what in scientific literature is described as the geomorphological IUH (GIUH).

a) Gamma IUH

The expression of the Gamma IUH, indicated below by the symbol $u_G(t)$, is as follows:

$$u_G(t) = \frac{1}{\Gamma(n_G)} \left(\frac{t}{k_G}\right)^{(n_G - 1)} \frac{e^{-t/k_G}}{k_G} \qquad (26)$$

where $\Gamma(n_G)$ is the complete gamma function (Abramowitz and Stegun, 1965). Specifically, for $n_G = 1$, $u_G(t)$ has the exponential form characterised solely by the parameter k_G. The expected value of $u_G(t)$ is the lag time t_L of the basin linked to the parameters n_G and k_G by the relation $t_L = n_G k_G$.

To estimate the reduction curve r'_D using the GM with Gamma IUH, the following procedure was adopted:

- for a selected station, the *BIDC* with its parameter b was identified as indicated in Appendix B, while the areal reduction factor $ARF_{d,A}$ was calculated by applying Eqn. 25;
- the parameter n_G was then set at *1,2,3,4* to cover the range of values observed in cases in the real world (Liu, 1992) and, using a least squares algorithm, the parameter k_G was estimated and hence t_L. The case with n_G estimated simultaneously with k_G was also considered.

The results are shown in Table 6 together with the mean and minimum values of the coefficient of determination, R^2, for the 12 stations. Figure 5b, again with reference to the Reno at the Casalecchio section, shows the reduction curve obtained in the case of $n_G = 3$. Note the ability of the GM model to reproduce the double concavity present in the observations.

b) Weibull IUH

The expression of the Weibull IUH, indicated below by the

Table 6. Geomorphoclimatic model with Gamma IUH. Values of the lag time t_L (hours) obtained for different values of the form parameter n_G. The two columns on the right show the values of t_L and n_G estimated together.

Station	$n_G = 1$	$n_G = 2$	$n_G = 3$	$n_G = 4$	t_L	n_G
Reno at Pracchia	2.4	2.5	2.8	3.1	2.4	1.31
Reno at Casalecchio	3.7	3.8	4.2	4.7	4.2	2.96
Samoggia at Calcara	3.6	3.5	3.8	4.3	3.4	1.58
Savena at Castel dell'Alpi	2.2	2.2	2.4	2.6	2.2	1.10
Rio Calla Querceto	1.7	1.6	1.8	1.9	1.6	1.10
Quaderna at Palesio	3.0	2.8	3.0	3.3	2.9	1.10
Lamone at Sarna/Faenza	4.9	4.8	5.3	5.8	4.7	1.30
Ronco at Meldola	2.6	2.7	3.1	3.4	2.6	1.22
Savio at S. Vittore	3.8	3.8	4.2	4.6	3.8	1.14
Foglia at Montecchio	7.8	6.9	7.6	8.3	7.4	1.10
Candigliano at Acqualagna	3.4	3.4	3.7	4.1	3.3	1.58
Potenza at Cannucciaro	6.7	6.2	6.7	7.3	6.5	1.10
mean R^2	0.993	0.984	0.979	0.977	0.994	
minimum R^2	0.979	0.934	0.919	0.911	0.979	

symbol $u_w(t)$, is as follows:

$$u_w(t) = \frac{n_w}{k_w}\left(\frac{t}{k_w}\right)^{(n_w-1)} exp\left[-\left(\frac{t}{k_w}\right)^{n_w}\right] \qquad (27)$$

In this case too, when $n_w = 1$, $u_w(t)$ has the exponential form. Moreover, as was observed for the other type of IUH, the expected value of $u_w(t)$ is the lag time of t_L, linked to the parameters n_w and k_w by the relation $t_L = k_W \Gamma\ (1+1/n_w)$.

The estimation of the parameters n_w and k_w was performed as for the Gamma IUH. Specifically, the cases of n_w set at *1.0*, *1.2*, *1.6* and *2.0* were considered: finally, also the case in which n_w and k_w were estimated simultaneously was considered.

Table 7. Geomorphoclimatic model with Weibull IUH. Values of the lag time t_L (hours) obtained for different values of the form parameter n_w The two columns on the right show the values of t_L and n_w estimated together.

Station	$n_w = 1$	$n_w = 1.2$	$n_w = 1.6$	$n_w = 2.0$	t_L	n_w
Reno at Pracchia	2.4	2.3	2.5	2.8	2.3	1.21
Reno at Casalecchio	3.7	3.6	3.8	4.2	4.5	2.19
Samoggia at Calcara	3.6	3.3	3.4	3.8	3.3	1.29
Savena at Castel dell'Alpi	2.2	2.0	2.1	2.3	2.1	1.10
Rio Calla Querceto	1.7	1.6	1.6	1.7	1.6	1.10
Quaderna at Palesio	3.0	2.7	2.6	2.9	2.8	1.10
Lamone at Sarna/Faenza	4.9	4.6	4.7	5.2	4.6	1.18
Ronco at Meldola	2.6	2.6	2.7	3.1	2.6	1.15
Savio at S. Vittore	3.8	3.6	3.8	4.2	3.7	1.10
Foglia at Montecchio	7.8	6.8	6.7	7.4	7.2	1.10
Candigliano at Acqualagna	3.4	3.2	3.3	3.7	3.2	1.30
Potenza at Cannucciaro	6.7	6.0	6.0	6.5	6.3	1.10
mean R^2	0.992	0.990	0.980	0.975	0.994	
minimum R^2	0.969	0.959	0.923	0.905	0.979	

Table 8. Stochastic model. Estimations of the fluctuation scale parameter θ (hours) obtained for different orders of the process AR(n) and corresponding estimations of the lag time t_L (cf. Table 3).

Station	$n = 1$	t_L	$n = 2$	t_L	$n = 3$	t_L	$n = 4$	t_L
Reno at Pracchia	6.52	3.3	5.94	3.0	5.84	3.3	5.80	3.6
Reno at Casalecchio	10.23	5.1	9.08	4.5	8.88	5.0	8.80	5.5
Samoggia at Calcara	7.19	3.6	6.54	3.3	6.43	3.6	6.39	4.0
Savena at Castel dell'Alpi	4.27	2.1	3.93	2.0	3.87	2.2	3.84	2.4
Rio Calla Querceto	2.42	1.2	2.25	1.1	2.22	1.3	2.21	1.4
Quaderna at Palesio	4.19	2.1	3.86	1.9	3.80	2.1	3.78	2.4
Lamone at Sarna/Faenza	11.26	5.6	9.92	5.0	9.70	5.5	9.61	6.0
Ronco at Meldola	7.36	3.7	6.64	3.3	6.52	3.7	6.47	4.0
Savio at S. Vittore	9.26	4.6	8.23	4.1	8.06	4.5	7.99	5.0
Foglia at Montecchio	14.40	7.2	12.37	6.2	12.04	6.8	11.91	7.4
Candigliano at Acqualagna	7.41	3.7	6.69	3.3	6.57	3.7	6.52	4.1
Potenza at Cannucciaro	12.60	6.3	11.20	5.6	10.92	6.1	10.80	6.8
mean R^2	0.994		0.988		0.985		0.984	
minimum R^2	0.983		0.950		0.938		0.933	

The results are shown in Table 7. Figure 5c shows the reduction curve obtained in the case of $n_w = 1.6$.

THE STOCHASTIC MODEL

Although the parameter θ may be estimated via an analysis of the auto-correlation function with reference to the series of continuous discharge values in the section under study, the simplest way to estimate it, as suggested by Bacchi *et al.* (1992), is by the application of the least squares method, seeking the θ value which best fits Eqn. 20 to the observed values of the reduction ratios r'_D.

Table 8 shows both the values of θ obtained for different orders n of the auto-regressive process and the corresponding estimations of t_L obtained using the relations presented in Table 3. Figure 5d shows the reduction curve obtained in the case of $n = 3$. Note that this model, like the geomorphoclimatic model, also reproduces the double concavity in the observed reduction curve.

Comparison of the results produced by the various models

REPRODUCTION OF THE DOUBLE CONCAVITY IN THE OBSERVED REDUCTION CURVE

The presence of a double concavity in the observed reduction curve is due to the fact that the values μ_{Q_D} are very near to μ_Q when D is relatively small (note, in fact that $\mu_{Q_D} \underset{D\to 0}{\longrightarrow} \mu_Q$), while they decrease when D is large. Furthermore, the double concavity is more evident in the case of large basins and less evident or even absent in the case of small basins; small basins are characterised by peaked and narrow flood waves, while large basins are characterised by smoothed and long flood waves. In the former case, the difference between μ_Q and μ_{Q_D} is pronounced right from the smallest values of D and the reduction curve tends to show a dominant upward concavity. In the latter case, μ_{Q_D} tends to drop for relatively large values of D and this causes the double concavity.

The EM cannot reproduce the double concavity since it is a simple curve with only one curvature. However, the error is extremely small, at least for the basins considered in this study.

The GM is able to reproduce this double concavity because its formulation is based on a (simple) rainfall-runoff representation thus capturing the main features of the response of a basin.

Finally, the ability of the SM to reproduce the double concavity has the same explanation as for the GM because the discharge time series $q(t)$ can be interpreted as the result of a rainfall-runoff transformation. Furthermore, from a point of view of the "fitting" to a set of data, the expressions of the reduction curve given by SM and collected in Table 3 are largely flexible.

THE GEOMORPHOCLIMATIC MODEL: GAMMA IUH VS WEIBULL IUH

Table 6, which refers to the case of the Gamma IUH, shows a slight but systematic decrease in the mean value of R^2 from $n_G = 1$ to $n_G = 4$. However, the case of $n_G = 1$ gives rise to an IUH with an exponential form that is ill-equipped to represent the response of a basin to a spatially distributed rainfall event, because it produces a response hydrograph for the whole basin which does not start from zero.

Therefore, even though the optimisation procedure slightly favours the exponential IUH, it was decided to focus attention on the cases characterised by $n_G > 1$, especially as the differences in terms of R^2 are very small. Recent studies (Liu, 1992) show that limiting values of n_G fluctuate between 2 and 3 and that $n_G = 3$ is the limiting value in the case of a purely Hortonian drainage network. This latter value was, therefore, taken as representative of the results for the GM model with Gamma IUH. In this respect, the values of the lag time for $n_G = 3$ are statistically representative of those obtained with the other values of n_G (Fig. 6a).

In the case of the GM model with Weibull IUH (Table 7), a slight but systematic decrease was also observed in the mean value of R^2 from $n_w = 1$ to $n_w = 2$, in line with what was observed in the previous case. Excluding the case $n_w = 1$ (exponential IUH) on the basis of the considerations set forth above, it can be seen in Fig. 6 that the values of the lag times t_L obtained for the various values of n_w are consistent both with each other and with those obtained in the case of the Gamma IUH with $n_G = 3$.

In conclusion, these results show that the lag time is the only parameter of the IUH to be identified with the greatest accuracy, whereas the shape of the IUH remains without clear definition since the parameter n_G (n_w) can indiscriminately assume values in the range of $n_G = 1$–4 ($n_w = 1$–2). In confirmation of the above assertions, Fig. 7 shows different IUHs with reference to the Reno at the Casalecchio section: significantly different shapes of IUH

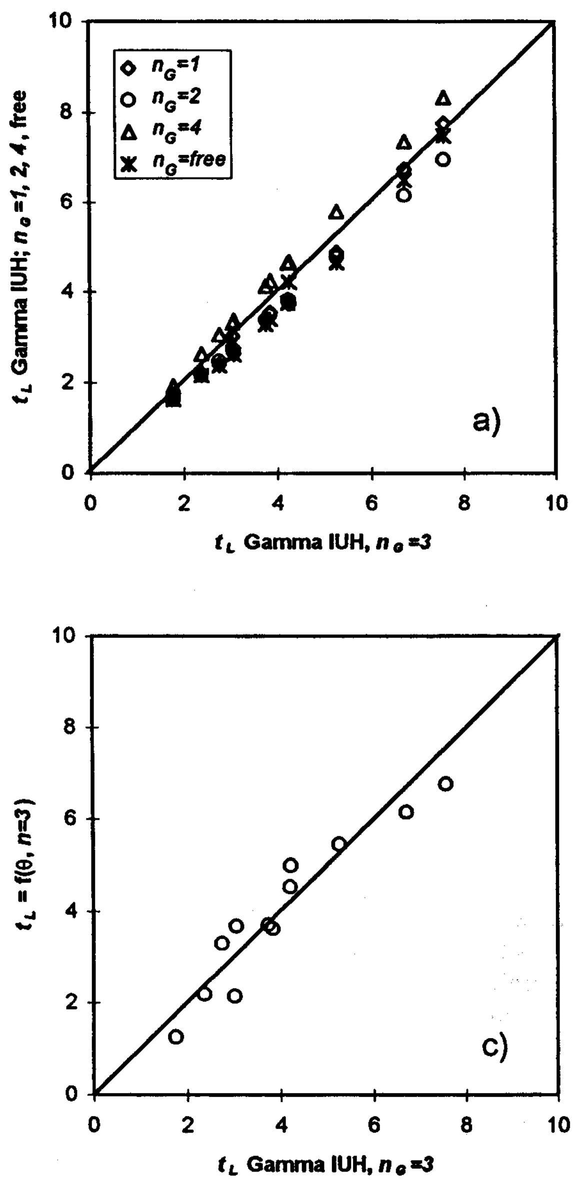

Fig. 6. *Relations between lag times t_L inferred using the various models examined. The x-axis shows the value of the lag time t_L for the GM model (Gamma IUH, $n_G = 3$). The indication "free" means that the parameters n_G and n_W are not a priori defined (see also Tables 6 and 7).*

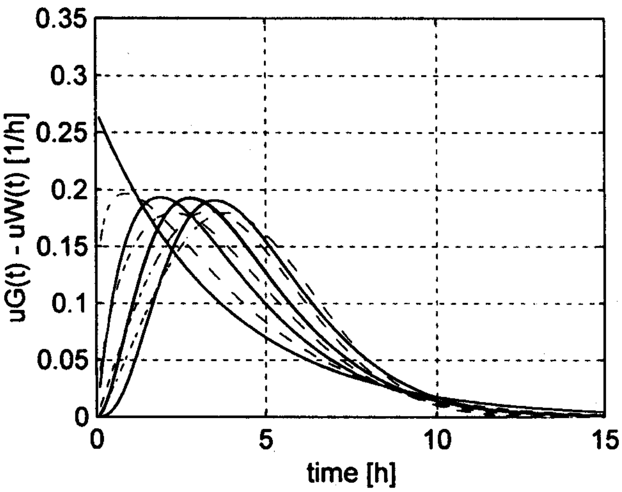

Fig. 7. *Reno at Casalecchio: graphs of the IUH's (Gamma: unbroken line; Weibull; broken line) identified with the GM model.*

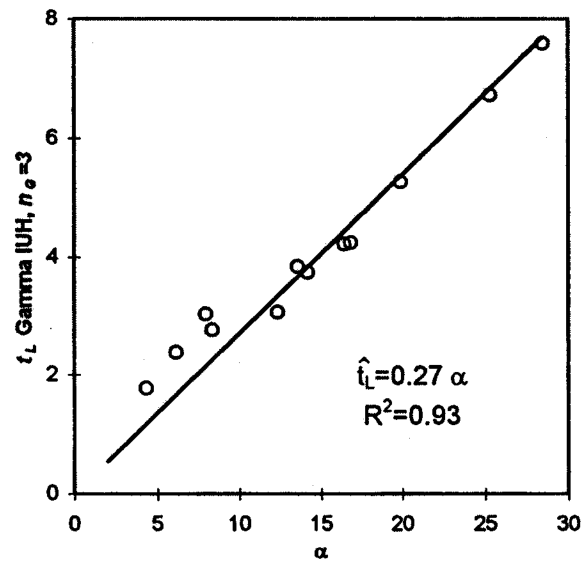

Fig. 8. *Linear regression between the lag time t_L obtained using the GM (Gamma IUH, $n_G = 3$) and the parameter α of the EM ($\beta = b$).*

correspond to virtually equivalent lag times (cf. Tables 6 and 7).

THE GEOMORPHOCLIMATIC MODEL VS THE STOCHASTIC MODEL

Table 8 shows a slight but systematic decrease in the mean value of R^2 from $n = 1$ to $n = 4$, in line with what was observed for the GM model with Gamma IUH, with which the stochastic model has a point of contact in the shape of the IUH. The validity of the interpretation provided by Brath *et al.* (1992) is confirmed by the fact that the lag times t_L, obtained using the formulae in Table 3, are statistically consistent, as proved by the tests of hypothesis made on the angular coefficient of those equations, with those obtained from the GM model with Gamma IUH. Specifically, Fig. 6c shows the values of t_L in the case of $n = 3$ versus the corresponding values obtained with the GM model (Gamma IUH, $n_G = 3$).

THE EMPIRICAL MODEL VS THE GEOMORPHOCLIMATIC MODEL

The values of the parameter α of the EM (with $\beta = b$) and the values of t_L for the GM (Gamma IUH, $n_G = 3$) reveal a linear link (Fig. 8), demonstrating that this parameter too is the expression of a characteristic basin response time.

The choice of setting $\beta = b$ and calibrating the parameter α alone is preferred because the statistical relationship between α and the time lag t_L does not exist when α is estimated simultaneously with the parameter β (see the values of α in the second column of Table 5). Indeed, the parameter α has a time dimension and, given the structure of the equation representing the EM, is expected to have small (large) values when the reduction curve decreases quickly (slowly). This behaviour of the reduction curve is, in turn, typical of the small (large) basins which are characterised by small (large) time lag t_L. Thus, a statistical correlation is expected between the parameter α and the basin response time. However, such a correlation may be masked or even lost when the parameters α and β are estimated simultaneously by an automatic and blind calibration procedure. In fact, the mutual interaction between the two parameters can drive the calibration procedure towards an unfortunate region of the two-dimensional domain of the objective function, where the values of the parameter α are completely non-correlated with the response time of the basin.

To avoid this negative consequence of the reciprocal influence of the two parameters, β should be fixed to a *reasonable* value, while only α is calibrated (see the mean of the values of β in the third column of Table 3). However, better results, i.e. a more evident correlation between α and the time response of the basin, were obtained by setting β equal to the coefficient *b* of the *BIDC*.

To summarise, the choice of setting $\beta = b$ and calibrating the parameter α alone highlights the link between α and the time response of a basin and so facilitates a possible regionalisation of the reduction curve expressed by the EM.

Summary of comparisons

INTERPRETATION OF THE t_L ESTIMATIONS OBTAINED USING THE VARIOUS MODELS

All the models, though characterised by their own distinct theoretical and formal attributes, lead systematically to the

Table 9. Comparison of lag time estimations t_L (hours) obtained using the Gamma GM with $n_G = 3$ and values obtained using the formulae proposed in the literature.

Station	1	2	3	4	5
Reno at Pracchia	2.8	1.4	2.3	4.2	2.4
Reno at Casalecchio	4.2	5.3	14.6	12.6	11.7
Samoggia at Calcara	3.8	3.0	6.4	7.7	4.6
Savena at Castel dell'Alpi	2.4	0.7	0.9	2.3	1.4
Rio Calla Querceto	1.8	0.6	0.6	1.8	1.1
Quaderna at Palesio	3.0	1.2	1.8	3.6	1.6
Lamone at Sarna/Faenza	5.3	3.4	8.7	9.2	5.4
Ronco at Meldola	3.1	3.7	8.2	8.9	7.1
Savio at S. Vittore	4.2	4.4	10.7	10.4	7.9
Foglia at Montecchio	7.6	5.8	14.0	12.2	8.1
Candigliano at Acqualagna	3.7	4.5	10.0	10.0	8.4
Potenza at Cannucciaro	6.7	4.1	8.6	9.2	6.8

Key:
1: lag time t_L (hours) for GM with Gamma IUH and $n_G = 3$
2: $t_L = 0.4\dfrac{4 \cdot \sqrt{A} + 1.5 \cdot L}{0.8 \cdot \sqrt{H_m}}$ (Giandotti, 1940) with A in km^2, L in km and H_m in m
3: $t_L = 0.000326 \cdot \left(\dfrac{L}{\sqrt{S}}\right)^{0.79}$ (Watt and Chow, 1985) with L in m and dimensionless S
4: $t_L = 2.8 \cdot \left(\dfrac{L}{\sqrt{S}}\right)^{0.47}$ (NERC, 1975) with L in km and S in m/km
5: $t_L = \dfrac{L_{med} \cdot \sqrt{\pi \cdot N_1}}{3600 \cdot c}$ (Troutman and Karlinger, 1985) with L_{med} in m and c (flood wave celerity) for all the basins assumed to be 1.5 m/s.

same values of the lag time, t_L. Since different methodological approaches produce the same estimates, it might be assumed that these estimates are at least *likely*. However, any verification of this assumption would require knowledge of the *true* values of t_L, which can only be estimated. In strictly logical terms, if the true value is not known or if criteria for appraisal such as those used for the statistical estimators are unavailable, it is impossible to say how reliable these estimates are or if they are more reliable than those obtained by applying the formulae proposed in the literature (Table 9) or rainfall-runoff models.

More specifically, with reference to this latter type of model, the lag time estimates obtained on the basis of the observed hourly areal rainfall and discharge data for the Reno at the Casalecchio and Candigliano at the Acqualagna stations, are systematically higher (about double) than those produced by the reduction curves analysis. These differences were observed both by analysing individual flood events using different methods of computing the hyetograph of net rainfall and separation of the runoff components in the observed flood wave (Chow *et al.*, 1988), and also by applying a continuous-type rainfall-runoff model (ADM model, Franchini, 1996).

Basically, the lag time values required for a good representation of the reduction curve cannot be estimated, with confidence, using the traditional formulae available in the literature, nor, in general, are they the ones which would be required to characterise the IUH of a rainfall-runoff model. Moreover, it is impossible to say whether one or the other is more or less accurate or likely. Accordingly, the lag time, t_L, referred to so far, characterising the position of the centre of mass of the transformation IUH involved directly (GM) or indirectly (EM, SM) in the formulation of the reduction curve model, must be considered in the broader sense as a "reference time" typical of the response of the generic basin.

OBSERVATIONS ON THE APPLICABILITY OF THE THREE MODELS

The geomorphoclimatic model (GM) needs the computation of the areal intensity-duration curve which, in general, requires the performance of a preliminary regionalised rainfall analysis thus providing the *BIDC*. Furthermore, knowledge of the areal reduction factor $ARF_{d,A}$ is necessary and the type of IUH (e.g. Gamma or Weibull) must be selected. However, this study established that the choice of type of IUH and the shape parameter is not significant; thus, in practical applications it is possible to assume *a priori* both the type of IUH, e.g. Gamma, and the value of the shape parameter, e.g. $n_G = 3$. The only parameter to be estimated is t_L. Where experimental data are available, t_L is determined using the optimisation procedure described; where a regionalisation process is necessary, t_L could be estimated with *statistical* formulae using the geomorphological characteristics typical of the basin as independent variables (see Tables 4 and 9). However, to obtain operationally acceptable results, these formulae *should be re-parameterised* using the values of t_L obtained from the reduction curves for the stations present in the region under study, as done in Franchini and Galeati (1998).

Lastly estimation of the parameters of the IUH (Gamma or Weibull) based on the criterion of maximising the outputs is not a simple matter and computation times can be quite considerable, even when automatic optimisation procedures are employed.

As far as the empirical model (EM) is concerned, on the assumption that $\beta = b$ (b: exponent of the *BIDC*), the only parameter to be estimated is α, which thus assumes values closely linked to the parameter t_L. If, on the other hand, both parameters are estimated, the values of α are highly variable (cf. Table 5) and the statistical link with t_L is lost. If it is necessary to carry out a regionalisation, the choice of $\beta = b$ is therefore preferable since, otherwise, there would be no possibility of defining the statistical link between α and the geomorphological characteristics of the basin.

To sum up, the geomorphological model and the empirical model always require knowledge of the *BIDC*. The geomorphological model also requires knowledge of the factor $ARF_{d,A}$, which, to be reliable, must be parameterised over the region under study, and this information is not always readily available.

Given the dependence of both models on the *BIDC*, the effect of an error in the evaluation of the coefficient b on the estimation of α was assessed in the case of the EM model, and of the lag time in the case of the GM. This coefficient b was altered by $\pm$ 10% and $\pm$ 20% in relation to the values set out in Table 5 and the parameters α (EM) and t_L (GM) were estimated. In particular, for the GM model with Gamma IUH and $n_G = 3$, an increase (decrease) of *10–20%* in b corresponds to a decrease (increase) in the lag time of approximately *12–24%*; similar variations were observed for the parameter α in the EM model. These variations can be explained by observing that an increase in the parameter, b, affects precipitation to a growing degree as its duration, d, increases. It follows that, if the lag time, t_L, is left unchanged, the reduction curve will tend to *rise* particularly in the part relevant to the longer time durations, D. Thus, for the reduction curve to remain as far as possible unchanged (so fitting the observed ratios), the lag time must be reduced. In this way the peak discharge for $D = 0$ increases, thereby offsetting the previously highlighted increases in the average discharges relevant to the longer durations.

Lastly, with regard to the stochastic model, only the parameter θ has to be estimated. Moreover, its link with t_L facilitates the search for a relation with the standard geomorphological parameters, though without diminishing the objective difficulties always encountered in such analyses and which, in any case, are independent of the model used to reconstruct the reduction curve.

Conclusions

The application of the three different models, empirical, geomorphoclimatic and stochastic, for synthesising the reduction curve, expressed as ratios of the mean of the maximum annual discharges over different durations, in twelve basins situated in central Italy, allows the following conclusions to be drawn:

- all the models allow a satisfactory construction of the observed reduction curves;
- the stochastic model is the simplest to apply since it requires only the estimation of the parameter θ which is closely linked to the "lag time" t_L;
- the empirical model is defined by just two parameters which can be estimated by fitting it to the observed reduction ratios over different durations. However, it is preferable to assume $\beta = b$, where b is the exponent of the *Basin Intensity Duration Curve (BIDC)*. This assumption implies a prior regional analysis of the precipitation, yet it facilitates the parameterisation of the model and the parameter α is thus closely linked to the "lag time" t_L;
- the geomorphological model presents the greatest application difficulties because it requires a larger amount of data (in addition to the *BIDC*, the Areal Reduction Factor of the precipitation $ARF_{d,A}$ is also required), and because of the relatively time-consuming computation method (least squares coupled with the output maximisation technique). With regard to its parameterisation, the choice of IUH type (Gamma or Weibull) and the value of the shape parameter appears to be irrelevant, as the only significant parameter is t_L;

These considerations show the importance of the "lag time", t_L, in the parameterisation of the three models. However, the values of t_L obtained using several formulae proposed in the literature generally differ from those obtained by fitting the models of the reduction curves to the observed ratios. It follows that these formulae cannot be used with confidence for parameterising the models of the reduction curves. A similar conclusion is reached where the value of t_L is obtained using rainfall-runoff models.

In other words, the parameterisation of the rainfall-runoff models does not produce results that can be used for synthesising the reduction curves and vice versa, while the formulae proposed in the literature can be used only after re-calibrating them directly on the values of t_L obtained by fitting the observed reduction ratios. This means that the values of t_L estimated through the models of the reduction curve should not be interpreted as estimates of the basin "lag time", according to the standard definition, but as a more general "reference time", characteristic of the response of the basin in the framework of the reduction curve analysis.

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Appendix A

The discharge $q_D(t,d)$, which represents the mean discharge over the time interval of duration D starting at the instant t and referring to a flood wave produced by a net areal rainfall $\varphi \cdot i_A(d)$, can be written, on the basis of Eqn. 11, as:

$$q_D(t,d) = \frac{1}{D}\int_t^{t+D} q(\tau,d)d\tau = \frac{1}{D}\int_t^{t+D} \varphi \cdot i_A(d)\cdot \Delta S(\tau,d)\cdot A \cdot d\tau \qquad \text{(A1)}$$

Assuming:

$$I\Delta S(t,d) = \frac{1}{D}\int_0^t \Delta S(\tau,d)d\tau;$$
$$\Delta(I\Delta S) = \frac{1}{D}[I\Delta S(t+D,d) - I\Delta S(t,d)] \qquad \text{(A2)}$$

Eqn. A1 becomes:

$$q_D(t,d) = \varphi \cdot i_A(d)\cdot \Delta(I\Delta S)\cdot A \qquad \text{(A3)}$$

Following the same steps as those described for the flood peak, the following statements can be made:

- for an assigned rainfall duration d, the maximum value $q_{D,\max}$ of $q_D(t,d)$ is obtained at the instant t_D when the difference $\Delta(I\Delta S)$ is at its maximum value;
- there is a critical precipitation duration $d \equiv d_{cr,v}$ (usually different from $d_{cr,p}$) at which the product $i_A(d)\cdot\Delta(I\Delta S)$ is at its maximum value: if t_D^* indicates the instant corresponding to $q^*_{D,\max}$, (i.e. the maximum value of $q_{D,\max}$), thus Eqn. A3 can be re-written as:

$$\begin{aligned} q_D(t_D^*, d_{cr,v}) &= q^*_{D,\max} \\ &= \frac{1}{D}\{\varphi \cdot i_A(d_{cr,v})\cdot [I\Delta S(t_D^*+D, d_{cr,v}) \\ &\quad - I\Delta S(t_D^*, d_{cr,v})]\cdot A\} \\ &= \varphi \cdot i_A(d_{cr,v})\cdot \Xi(d_{cr,v})\cdot A \end{aligned} \qquad \text{(A4)}$$

where $\Xi(d_{cr,v})$ is a function similar to the peak function relevant to the flood peak.

Appendix B

The intensity-duration curve is generally expressed by the following equation:

$$h = a \cdot d^b \qquad \text{(B1)}$$

To use this equation to express the mean value of the maximum annual precipitation for duration d, the following expression is more appropriate:

$$m_d = m_1 \cdot d^b \qquad \text{(B2)}$$

where m_1 and b are two parameters variable from site to site, with m_1 representative of the mean value of the maximum annual precipitation of one hour's duration. Introducing the ratio $\gamma = m_G/m_{24}$ between the mean maximum daily precipitation and the mean maximum precipitation over the 24 hour period, it follows from B2 that:

$$m_{24} = m_G/\gamma = m_1 \cdot 24^b \Rightarrow b = \frac{\ln(m_G) - \ln(\gamma) - \ln(m_1)}{\ln(24)} \quad \text{(B3)}$$

and therefore:

$$m_d = m_1 \cdot d^{\frac{\ln(m_G) - \ln(\gamma) - \ln(m_1)}{\ln(24)}} \quad \text{(B4)}$$

The estimation of m_d thus comes down to the estimation of the mean maximum daily precipitation m_G and the mean maximum hourly precipitation m_1, assuming γ to be constant over the region under study ($\gamma \cong 0.90$ across the whole of Italian national territory).

Equation B4 represents what is described in the text as the *"intensity duration curve of the mean annual maximum values of the basin" (BIDC)*. Specifically, one possible procedure for calculating the exponent b is as follows:

- the centroid of the basin is identified;
- at this point the value of the annual maximum rainfall of one day m_G and one hour m_1 is estimated, based on isolines of these quantities; Franchini and Galeati, (1994) produce these isolines with reference to the Bologna sector of the S.I.M.N.;
- formula (B3) is used to estimate b.