Hydrology and Earth System Sciences, 3(2), *197–203* (1999) © EGS

Hydrology & Earth System Sciences

The FORGEX method of rainfall growth estimation II: Description

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Abstract

The Focused Rainfall Growth Extension (FORGEX) method produces rainfall growth curves focused on a subject site. Focusing allows the incorporation of rainfall extremes observed regionally while respecting local variations in growth rates. The starting point for the analysis is an extensive set of annual maximum rainfalls, with values at each gauged site standardized by the median. Following the philosophy of the earlier FORGE method, a strongly empirical approach is adopted. The rainfall growth curve is represented by linear segments on a Gumbel scale, and is fitted by a least-squares criterion. The selection of data points is intricate and includes both the traditional pooling of regional extremes and the incorporation of *network maximum* events. The latter comprise the largest events from successive hierarchical networks of gauges, focused on the site for which estimates are required. Their treatment takes account of interdependence using the Dales and Reed model of spatial dependence in rainfall extremes.

Introduction

Very large rainfall depths in short durations of a few hours, or a few days, occur only rarely. Resultant intense flooding can damage property and engineered structures, disrupt communications and livelihoods, and—in catastrophic cases—lead to loss of life. Rainfall growth estimation is concerned with assessing the depth of very rare rainfalls, such as that which has a probability of exceedance in any year of only 0.001: the so-called 1000-year event. The T-year rainfall growth factor is the ratio of the T-year maximum rainfall depth to a reference rainfall depth, such as the median of annual maxima. Generally, the rainfall growth curve is conditioned on a particular duration, such as one hour or one day.

The paper presents the Focused Rainfall Growth Extension (FORGEX) method. This is a development of the Focused Rainfall Growth Estimation (FORGE) method (Reed and Stewart, 1989; Reed and Stewart, 1994; Stewart *et al.*, 1995). The extension of growth curves to long return periods is accomplished by combining traditional methods of pooling extreme values with a special treatment of the largest annual maximum rainfall events observed across successive hierarchical networks of gauges focused on the subject site. The work forms part of a programme of research to develop a UK Flood Estimation Handbook (Reed, 1994; IH, 1999). The specification is to produce a method for estimating rainfall depth-duration-frequency at any site, for any duration between one hour and eight days, and for return periods up to 1000 years or longer.

The FORGEX method uses local rainfall data where possible and exploits data from farther afield when estimating rarer growth factors. Key features of the method are:

- the use of annual maximum rainfall data;
- standardization by the median of annual maxima;
- focusing: the analysis is tailored to the subject site, with precedence given to the use of local data;
- focusing: avoids boundary problems associated with fixed regions;
- the use of shifted network maximum rainfalls, accounting for inter-site dependence in rainfall extremes;
- the seamless extension of the growth curve to long return periods;
- formulation of the growth curve as linear segments on a Gumbel scale, avoiding an explicit distributional assumption.

Description of the FORGEX method is divided into two parts. The first section presents an overview of the solution method to obtain the rainfall growth curve, illustrating its graphical foundation. Intricate features of the method are described in a second section. Further examples of the method are presented in the accompanying paper by Faulkner and Jones (1999), which also describes a method for constructing distribution-free confidence intervals.

Formulation as a graphical method

The raw materials for the rainfall growth curve estimation are a tabulated series of annual maximum rainfalls for durations such as one clock-hour or one measurement-day. Let the *i*th annual maximum rainfall at site *j* be denoted by R_{ij}. Record lengths are seldom long enough to estimate rainfall frequency reliably from a direct analysis of extreme values observed at the subject site. In analogous situations, many hydrologists favour the pooling of records from several sites which collectively can be judged homogeneous or quasi-homogeneous in terms of the frequency distribution of the extreme values of the variable under study. This regional approach is a compromise between respecting local observations of extreme events and avoiding gross extrapolation based on relatively few data.

STANDARDIZATION

Growth curve construction is concerned with estimating the *T*-year extreme value relative to a reference extreme value. The use of a reference, or standardizing, value insulates the regional analysis from site-specific factors which influence the typical magnitude of extreme values but may have little systematic effect on their relative variability and skewness if the assumption of quasi-homogeneity holds. Although more complex approaches to regionalization can be considered (e.g. Rossi and Villani, 1994), the annual maximum series is typically standardized by dividing by a reference value (e.g. NERC, 1975). Sometimes referred to as the *index flood* method, the approach has been extensively used by flood hydrologists subsequent to Dalrymple (1960). The index variable used in the FORGEX method is the median. Standardized annual maxima for site *j* are thus defined by:

$$X_{ij} = \frac{R_{ij}}{RMED_j} \qquad (1)$$

where $RMED_j$ denotes the median of annual maxima at site *j*. A required minimum record length of ten years ensures that RMED is reasonably estimated. Adoption of the median, rather than the mean, as the standardizing value is preferred because the distribution of annual maximum rainfalls is typically skewed, and individual large values can sometimes exert a strong effect on the arithmetic mean. A further advantage is that the median value corresponds to a fixed return period of two years on the annual maximum scale. This point of reference simplifies and unifies growth curve construction.

GAUGE SELECTION

The FORGEX method pools data from a hierarchy of raingauge networks. Each network comprises gauges—lying within a certain distance of the subject site—for which the 'ten annual maxima or more' criterion is met. As successive networks are constructed, the radius of the defining circle is increased and annual maximum rainfall data are drawn together from larger numbers of sites (see Fig. 1). The name of the method is taken from this idea of focusing the analysis on the site for which estimates are required.

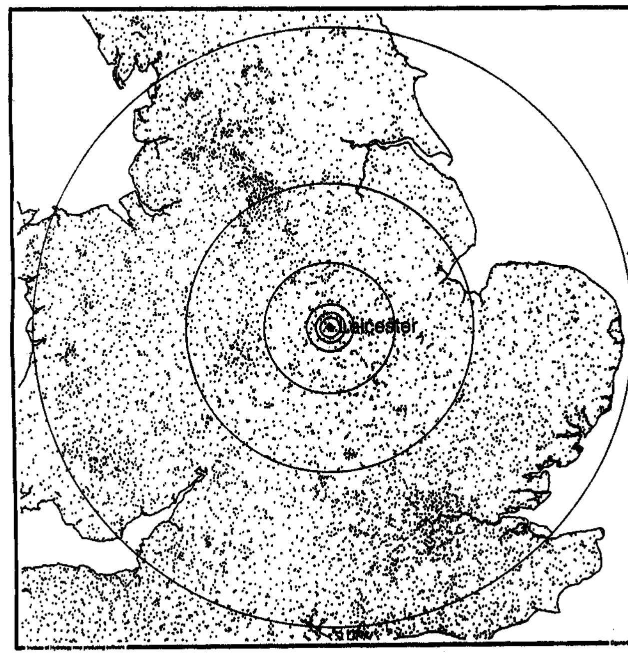

Fig. 1. *Networks of daily raingauges focused on Leicester; the largest network has a radius of 200 km.*

An upper limit is placed on the distance over which data can contribute to growth curve estimation at the subject site. This maximum distance, *distmax*, guards against the pooling of data from sites of radically different climate. While the FORGEX method pools data isotropically (i.e. according only to distance from the subject site), a variant could be developed to respect particular climatological features. For example, networks might be defined by ellipses to encourage pooling along coasts. Other schemes might be devised to pool data from sites which share a characteristic exposure to, or shelter from, typical rain-producing weather systems. Restraint in pooling in FORGEX is a compromise between respecting climatic differences (by choosing a short maximum range, *distmax*, and/or using a more selective network expansion strategy than concentric circles) and gaining access to enough observations to support the definition of the rainfall growth curve at long and very long return periods. A maximum range of 200 km is adopted in the UK applications reported here.

PLOTTING POSITIONS

The treatment of individual annual maximum records follows established practice (e.g. NERC, 1975; Cunnane, 1989). The annual maxima are ranked and allocated positions on a variate versus Gumbel reduced variate plot according to the Gringorten (1963) formula:

$$F_i = \frac{i - 0.44}{N + 0.12} \quad (2)$$

where F_i is the non-exceedance probability, i the rank in increasing order, and N the number of annual maxima. The Gumbel reduced variate is defined by:

$$y = -\ln(-\ln F_i) \quad (3)$$

The pooling of standardized annual maxima from stations in a defined network or region can be presented graphically in several ways. A natural method is simply to superpose extreme value plots from each site. NERC (1975) introduced the *y-slice* method, whereby pooled data which fall within a given class interval (or slice) of the Gumbel reduced variate are grouped together. NERC used class intervals of 0.5 when pooling flood peak data. Although FORGEX uses y-slices, it does so in a different manner.

NETWORK EXPANSIONS

A key objective of FORGEX is that the degree of pooling should be appropriate to the rarity of event being estimated, with greater pooling at long return periods. This requirement is met by regulating the network expansions so as to obtain a near-uniform density of pooled data points along the reduced variate axis. The procedure is arranged by reference to y-slices of interval 1.0, each network in the hierarchy being associated with definition of the rainfall growth curve within a particular y-slice.

The chosen target is that there should be no fewer than 20 pooled data points within each y-slice. Thus, in a sparsely gauged region, the circle defining the jth gauge network will be larger than elsewhere, as the network expands to attain the required number of data points in the jth y-slice. Conversely, in densely gauged regions, the target of at least 20 points per y-slice can be achieved with more modest network expansions. Pooled data points in the jth y-slice come from gauges in the jth network, which subsumes all smaller networks. Thus, points from larger networks are not plotted in or to the left of the jth y-slice. This is in accordance with a guiding principle of FORGEX: that inferences from local records—about growth rates at short to medium return periods—should not be subordinated by excessive pooling.

Larger networks include greater numbers of long-record stations, and thus provide pooled data points that plot in y-slices that correspond to rarer events. However, there are few sites in the UK for which computerized annual maximum rainfall records provide a series longer than about 100 years. This means that the target density of points cannot be met from pooled points alone beyond about the fifth y-slice, which spans the interval (3.3665, 4.3665) on the reduced variate scale (see below). The pooled points are denoted by dots in the example shown in Fig. 2. The y-slices are indicated by vertical lines.

The FORGEX method uses additional data points that represent *network maximum* events. Each corresponds to the largest standardized rainfall observed in the network in a particular year. Their treatment is described in the second section of the paper. Figure 2 distinguishes the network maxima and pooled data points: for example, the digit 2 denotes a network maximum from the second smallest network. It is evident from Fig. 2 that the target density of data points is not fully met beyond the sixth y-slice. Hence, in this case, it is appropriate to adopt somewhat coarser y-slices beyond the 200-year return period marker.

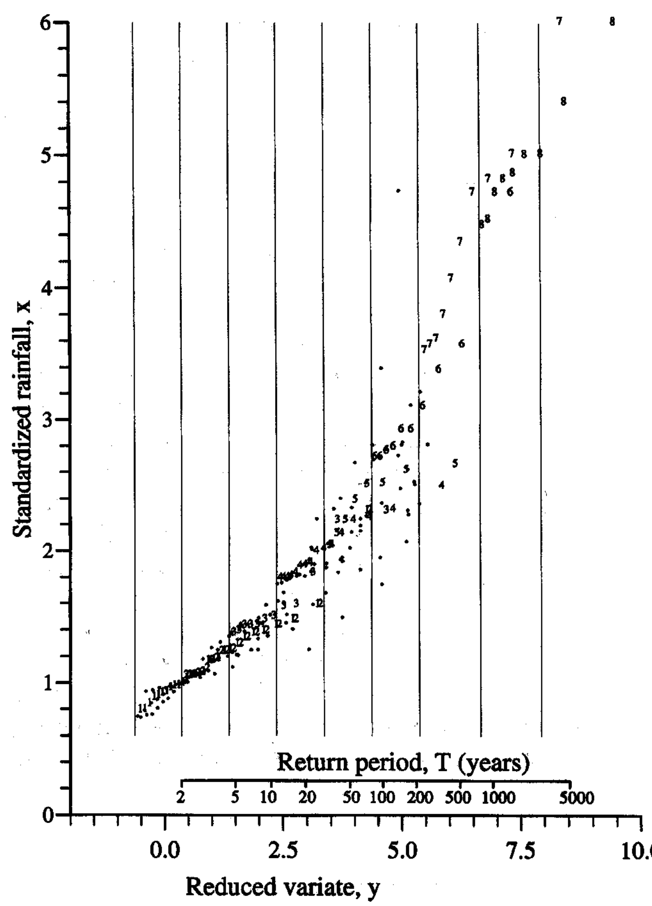

Fig. 2. *Plotting positions of pooled points (dots) and shifted netmax points (numbers) focused on Leicester*

GROWTH CURVE MODEL

The rainfall growth curve is modelled as a concatenation of l linear segments on the standardized variate versus Gumbel reduced-variate plot. The formulation for $l = 4$ is illustrated in Fig. 3 and defined by Eqn. 4.

$$xmodel = \begin{cases} 1 + a_1(y - y_1) & \text{for } y_0 \le y_1 \\ 1 + a_2(y - y_1) & \text{for } y_1 < y \le y_2 \\ 1 + a_2(y_2 - y_1) + a_3(y - y_3) & \text{for } y_2 < y \le y_3 \\ 1 + a_2(y_2 - y_1) + a_3(y_3 - y_2) + a_4(y - y_3) & \text{for } y > y_3 \end{cases} \quad (4)$$

Standardization by the median dictates that the growth curve takes the value 1.0 at the reduced variate corresponding to a return period of 2 years on the annual maximum scale. The median corresponds to $F = 0.5$ which, by Eqn. 3, yields $y = 0.3665$. The reduced variate axis is cut at this point, with one y-slice to the left and—depending principally on the number and record lengths of annual maximum series available for the particular duration—from four to eight y-slices to the right. Thus, y_1 is fixed at 0.3665. Because of the constraint to pass through (0.3665,1.0), derivation of the growth curve requires only that the gradients of the l segments be determined.

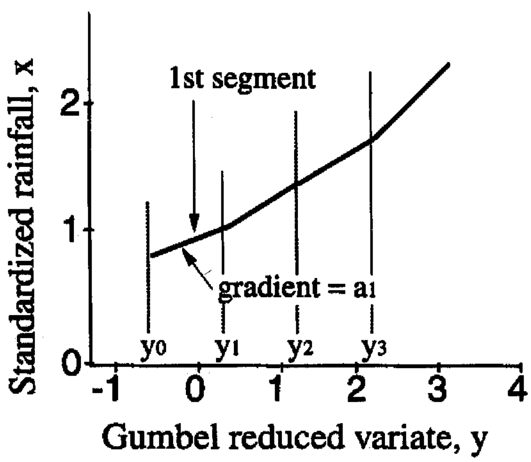

Fig. 3. *Definition sketch for rainfall growth model*

GROWTH CURVE DERIVATION

The growth curve is obtained as a linear least-squares fit moderated by a type of penalty function method (see for example Adby and Dempster, 1974). The latter is formulated to discourage large gradient changes between adjacent segments. It is added to the sum of squared differences (between observed and modelled standardized rainfall) to give the following objective function, which is minimized to determine the l unknown parameters:

$$\sum_{i=1}^{M}(x_i - xmodel_i)^2 + 0.05\frac{M}{l-1}\sum_{j=2}^{l}(a_j - a_{j-1})^2 \quad (5)$$

where M is the total number of data points and a_j is the gradient of the jth segment. Note that all points (both pooled and netmax) in Fig. 2 are accorded equal weight in the solution. The coefficient 0.05 in Function 5 was chosen experimentally; it provides a degree of smoothing whilst sustaining characteristic features of growth curves focused on particular sites.

Figure 4 presents the outcome of applying the above method to the Fig. 2 example: i.e. 1-day rainfall growth estimation for Leicester, England. The corresponding networks, eight in number, are indicated by the concentric circles in Fig. 1. At shorter return periods, the growth curve is substantially determined by pooled rainfall data from gauges close to Leicester. Yet the 1-day rainfall growth curve nevertheless extends to (and a little beyond) the 1000-year marker. The result is pleasing to the eye with, in this instance, a single inflection. Faulkner and Jones (1999) provide further examples of growth curves derived using the FORGEX procedure, and explore the construction of confidence intervals.

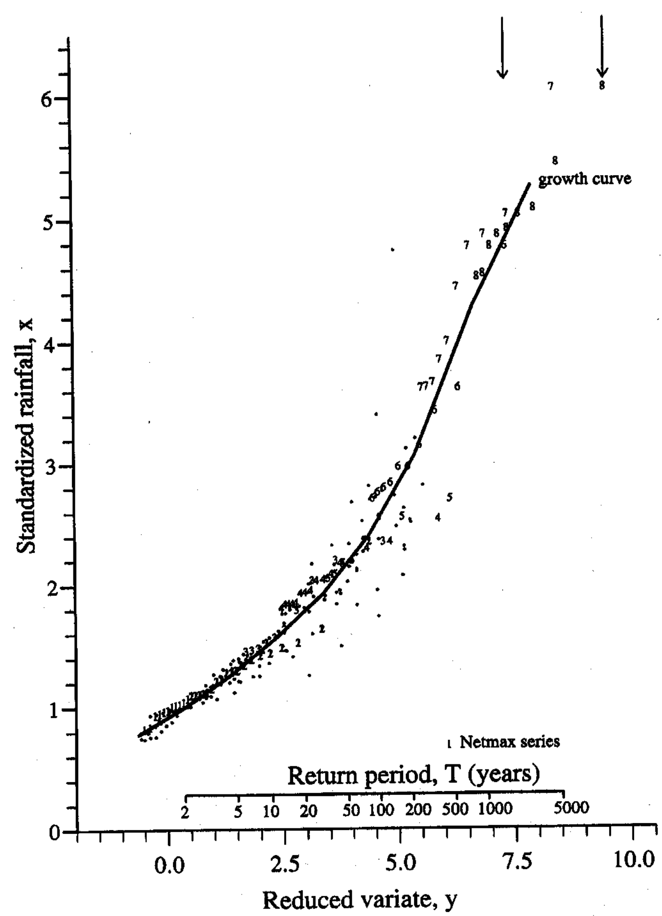

Fig. 4. *Growth curve for 1-day rainfall focused on Leicester; the arrows indicate the reduced-variate plotting positions of the highest shifted netmax points in the Pth and Lth networks*

To complete the description of the FORGEX method it remains to discuss the network maximum points included in Fig. 2: their definition, selection and plotting.

The network maximum series and its application

CONTEXT

Following Dales and Reed (1989), the *network maximum* (netmax) series is defined as the annual maximum series of the largest standardized value observed by the network. They show that the distribution of the network maximum from N independent and identically distributed GEVs lies exactly $\ln N$ to the left of the population growth curve on a variate versus reduced-variate plot. Reed and Stewart (1994) note that this result holds for any parent distribution. In practice, because of inter-site dependence in annual maxima, the netmax growth curve is found to lie a shorter distance to the left. Dales and Reed label this distance, $\ln N_e$, terming N_e the effective number of independent gauges. They refer to the underlying parent distribution as the *typical* distribution, being the distribution of the standardized variable at a typical site within the network.

SPATIAL DEPENDENCE MODEL

Dales and Reed present a spatial dependence model to estimate the offset distance, $\ln N_e$, for any UK raingauge network. The model is:

$$\ln N_e = \ln N(0.081 + 0.085 \ln AREA - 0.051 \ln N - 0.027 \ln D) \quad (6)$$

where N is the number of gauges and D the rainfall duration in days. $AREA$ is the nominal area spanned by the network, defined in the previous paper (Stewart *et al.*, 1999).

FORGE

In the FORGE method (Reed and Stewart, 1989), the spatial dependence model (Eqn. 6) is used to estimate the effective number of independent stations operating in each year of record. The number is aggregated over the entire period of record to estimate the effective number of independent station-years that the network supplies. This is then used to determine plotting positions for the very largest standardized events observed by the network, in a modification to the station-year method. Only the largest independent events are plotted, with the dates of annual maxima used to test for dependence. This procedure has two disadvantages. First, it is inconvenient to require date information, since sometimes only the month of the annual maximum is tabulated. Second, not all extreme rainfalls occurring on the same day (or adjacent days) are interdependent, particularly in large networks.

FORGEX

The FORGEX method supplies extreme data points for use in extending the growth curve to long return periods by an approach that more nearly parallels the derivation of the spatial dependence model. For each network in turn, the netmax annual maximum series of standardized rainfalls is constructed and the values ranked and plotted as described below. The netmax points are shifted to the right by $\ln N_e$, where N_e is the effective number of independent stations and $\ln N_e$ is estimated by Eqn. 6.

The procedure is complicated by the fact that the network gauges supplying valid annual maxima change year-by-year, as annual maximum records for individual gauges commence, are interrupted, resume or cease. It is therefore necessary to shift each netmax point individually. Appropriate plotting positions for the shifted netmax values are determined jointly, using a maximum likelihood procedure specially devised by Jones (1997).

Expanding the network

The main rule for expanding the gauge network was defined earlier: the network is expanded progressively until there are at least 20 pooled data points in the designated y-slice of interval 1.0. This rule defines the first P networks. For the illustrated example of 1-day rainfall growth at Leicester, there are six such networks (i.e. $P = 6$).

The Lth network (in general, L is greater than P) is determined by *distmax*, the limit set for the maximum range over which rainfall data are used. Exceptionally, if the highest shifted netmax point in the Lth network is less than 0.2 units to the right of the highest point in the Pth network, only P networks are used. Otherwise, the Lth network is always used.

An auxiliary rule is required to define any intermediate network between the Pth and the Lth. The procedure followed is to note the plotting positions of the highest shifted netmax points in the Pth and Lth networks. As an aid to the description, these are marked in Fig. 4 by downwards-pointing arrows. If these plotting positions are more than 2.0 units apart, one or more intermediate networks are inserted. In the example, the highest netmax points in the 6th and Lth networks are 2.2 units apart on the reduced variate axis, and there is room for one intermediate network (the 7th). The largest network is thus determined to be the 8th (i.e. $L = 8$).

In choosing how many intermediate networks can be accommodated, the rule is followed that the highest shifted netmax points in consecutive networks should have plotting positions separated by no less than 1.0 and no more than 2.0 units.

The spatial extent of any intermediate network is determined by progressively increasing the defining radius to accommodate additional gauges until the plotting position of the highest shifted netmax value is the required proportion of the distance between the plotting positions of the highest shifted netmax values for the *P*th and *L*th networks. When using a maximum pooling distance of 200 km, there will rarely be more than one intermediate network in typical UK applications.

Defining the growth curve segmentation by reference to y-slices

For the lower part of the growth curve, where there are sufficient pooled points, the y-slices are defined to be of width 1.0. In the Fig. 4 example, the first six segments of the growth curve, extending to $y = 5.3665$, have unit width. For the upper part of the growth curve, somewhat coarser y-slices are used. In order to avoid excessive reliance on the very highest netmax points, the growth curve is fully drawn only as far as the plotting position of the third highest netmax point in the largest network, although the highest netmax points are included in the least-squares summation (Function 5). The upper part of the growth curve is then divided into segments of equal width, the width being no less than 1.0 nor greater than 2.0 units.

For the Fig. 4 example, the procedure defines eight segments, so that $l = 8$. Thus, the eight hierarchical networks support a growth curve with eight segments. While it is typical to have equal numbers of networks and y-slices, this need not always be the case. It is for this reason that the different notations L and l have been adopted.

Performance check

It is customary to assess the overall performance of a rainfall frequency generalization by deriving growth curves for many gauged sites and counting the number of exceedances of rainfall depths with various rarities in the gauged record at each site (e.g. Buishand, 1991). Trials with FORGEX indicate a small bias, with 10 to 20% fewer exceedances observed in sets of 500 UK raingauge records selected at random, than expected according to FORGEX. An overestimation of 15% in frequency corresponds to an underestimation of about 0.15 in y. This feeds through to a typical overestimation of rainfall growth factors of about 4%.

The bias is an intrinsic part of the FORGEX method, which arises from the use of netmax points. In cases where no extreme event has been recorded locally, FORGEX provides a netmax point—representing an extreme value observed in some wider network—which inevitably pulls the rainfall growth curve upwards. Other possible sources of bias include the netmax plotting positions and the model for N_e (Eqn. 6). The bias disappears when the segmented growth curve is re-fitted to the pooled data points alone.

This minor bias in FORGEX is considered beneficial. It recognizes the likelihood that, as additional data become available, rainfall growth estimates are more likely to increase than decrease. This conclusion is consistent with the typically asymmetric confidence intervals derived by Faulkner and Jones (1999).

Reed and Stewart (1991) present an example in which a regional flood frequency analysis is re-visited as each new water-year of record becomes available. The example shows that, prior to 1979/80, a conventional pooled analysis supports use of a Gumbel distribution for flood growth in South Wales. However, the regional frequency analysis reacts strongly to the occurrence of a widespread extreme event in December 1979 and, thereafter, the growth curve is always notably upwards-curving. The sensitivity is seen as a weakness of ignoring inter-site dependence, particularly when record lengths are relatively short.

It is suggested that the use of netmax points within FORGEX introduces a small but valuable element of pro-action to the regional analysis. The method allows the most extreme events in the regional or national dataset to influence estimates at sites in districts where no unusually large rainfall has yet been gauged.

Concluding remarks

The Focused Rainfall Growth Extension (FORGEX) method has been described, and an example given of its application to estimate 1-day rainfall growth rates for a site in eastern England. Faulkner and Jones (1999) present further examples and explore the construction of confidence intervals for the rainfall growth factors. A special attribute of the method is that estimates of rainfall growth are obtained for return periods as long as 1000 years.

Once derived, the rainfall growth curve must be combined with estimates of the index variable, *RMED*, to yield design rainfall estimates. The geostatistical technique of kriging can be used to map *RMED* directly from observed data in densely gauged regions (Stewart and Reed, 1989; Stewart *et al.*, 1995). The mapping of *RMED* in sparsely gauged mountainous areas is considered by Prudhomme and Reed (1998).

The FORGEX method derives rainfall growth curves for a given duration such as one clock-hour or one measurement-day. When combining to estimate rainfall frequency for intermediate durations, it is necessary to correct for discretization effects and to ensure that the resultant description of rainfall depth-duration-frequency is internally consistent. Dwyer and Reed (1995) give guidance on the former. Faulkner (1999) presents a method for constructing a family of depth-duration-frequency curves.

The FORGEX method exploits an existing model of spatial dependence in UK rainfall extremes. This model was derived and calibrated by reference to the same kind of data, i.e. gauged annual maximum rainfalls. To facilitate application to other environmental variables (e.g. wind speed, air temperature), or to rainfall growth estimation in climates to which the Dales and Reed model may not apply, it would be helpful if calibration of the spatial dependence model could be focused on the subject site. The analyses might then form a single unified procedure.

It can be valuable to explore the properties of estimation procedures through large-scale simulation studies. These experiments use extensive datasets generated by random sampling from an assumed distribution function that is considered realistic of the variable (or variables) concerned. It is suggested that such simulation studies will be of value in proving the performance properties of FORGEX when it becomes possible to generate annual maximum rainfalls that exhibit a spatial dependence structure that is realistic of real-world data.

Acknowledgements

The FORGEX method has been developed in a succession of research projects for the Department of the Environment, South West Water Authority, the Ministry of Agriculture Fisheries and Food, and the National Rivers Authority. Research to develop rainfall frequency estimation procedures for the Flood Estimation Handbook was funded by the Environment Agency, the Department of Agriculture Northern Ireland, and the Scottish Office. The co-operation of the Met. Office in making rainfall records available under Memoranda of Understanding is gratefully acknowledged. All the above are UK organizations. We are grateful for suggestions provided by two referees.

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