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Non-linearity and spatial resolution in a cellular automaton model of a small upland basin

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Abstract

The continuing development of computational fluid dynamics is allowing the high resolution study of hydraulic and sediment transport processes but, due to computational complexities, these are rarely applied to areas larger than a reach. Existing approaches, based upon linked cross sections, can give a quasi two-dimensional view, effectively simulating sediment transport for a single river reach. However, a basin represents a whole discrete dynamic system within which channel, floodplain and slope processes operate over a wide range of space and time scales. Here, a cellular automaton (CA) approach has been used to overcome some of these difficulties, in which the landscape is represented as a series of fixed size cells. For every model iteration, each cell acts only in relation to the influence of its immediate neighbours in accordance with appropriate rules.

The model presented here takes approximations of existing flow and sediment transport equations, and integrates them, together with slope and floodplain approximations, within a cellular automaton framework. This method has been applied to the basin of Cam Gill Beck (4.2 km^2) above Starbotton, upper Wharfedale, a tributary of the River Wharfe, North Yorkshire, UK.

This approach provides, for the first time, a workable model of the whole basin at a 1 m resolution. Preliminary results show the evolution of bars, braids, terraces and alluvial fans which are similar to those observed in the field, and examples of large and small scale non-linear behaviour which may have considerable implications for future models.

Introduction

Fluvial action is often the most dynamic and important process in the evolution of a basin. For this and other reasons, fluvial models, operating at a variety of scales, have taken precedence in geomorphology. These range from the three dimensional modelling of circulation surrounding a confluence, detailed two dimensional finite element grids of water surface profiles (Nicholas and Walling, D.E., 1997; Bates *et al.*, 1997) and the more 'classic' one dimensional approach of calculating over cross sections, such as HEC II (Feldman, 1981). Many appear successful, but due to the complexity of solving the Navier-Stokes equations, are computationally restricted to operating in a confined area. Furthermore, they fail to account for processes outside this study reach, such as mass movement, hydrology and changes in upstream sediment supply.

Howard (1994, 1997); Polarski (1997); Tucker and Slingerland (1994); Willgoose *et al.* (1994) and Kirkby (1986) use a different approach, placing the emphasis on the slope processes. Howard (1994, 1997) and Tucker and Slingerland (1994) simplify channel operations to a subgrid cell process, with values for width and depth calculated using empirical relationships. This approach allows for aggradation and degradation of the channel, in the context of the whole basin, but does not allow the formation of terraces, a flood plain stratigraphy, or differing channel forms which geomorphologists use to interpret past environmental change.

Whilst both of these approaches are useful, the former, hydraulic approach trades basin scale realism for local flood plain accuracy, whereas the latter sacrifices channel accuracy for realism at the basin scale. Two reasons for this split can be identified. Firstly, numerical flow modelling comes mainly from a strongly engineering background, where the prime consideration is the channel. The second reason is scale.

A topic as complex as landscape evolution involves numerous processes interacting through a wide range of time and space scales. These vary from the entrainment of a grain in a split second, to the creep on a mountainside over thousands of years. The importance of a mass landslide in changing the landscape is obvious, but should the grains movement be ignored? If the landscape is assumed to be a chaotic system, highly sensitive to initial conditions, then the grains action is important, as is the butterfly effect to a climate modeller. Lane and Richards, K.S. (1997) seem to confirm this idea, suggesting that fluvial system behaviour is highly dependent upon its context.

This presents a major problem for a modeller in selecting an appropriate level of resolution. For example, in studying the Rhine Basin, what account should be taken of the turbulence generated by the movement of a 5 mm clast? In principle, the answer is not clear, as there are critical moments when it influences the outcome but, in practice, computational limits effectively exclude such a high level of detail.

Incorporating small scale processes in a basin model is troublesome, because of these scale ranges. The computationally intensive nature of finite element methods makes their use impracticable over the long timescale that slope influences require (>1000 years) and it is similarly impossible for them to provide models for the full spectrum of flood events. Furthermore, over the course of a flood, basins are spatially dynamic. Stream heads may extend, new tributaries and channels may form. For hydraulic modelling, this creates numerous problems, as changes in bed/floodplain topography and spatial changes in the network require a frequent re-definition of the mesh of nodes used, which is highly time consuming, especially if a curvilinear approach is used.

In this paper, a cellular automaton (CA) model, simple in concept yet complex in implementation, is applied to an entire small upland basin. This model aims to reconcile scale issues by dividing the basin into uniform 1 m^2 grid cells. This resolution is chosen as being small enough to allow representation of fluvial processes, yet large enough to encompass a whole basin. Furthermore, to resolve temporal scale problems, a variable time step is used which is dependent upon the erosion rates. This allows the representation of small scale processes such as fluvial erosion, yet incorporates the long term effects of vegetation change and soil creep. This model is being developed as part of on-going research to investigate the relative effects of climate change and human influence on the upland landscape over the Holocene (Coulthard *et al.*, 1996, 1998; Macklin and Lewin, 1993). In this paper, the authors wish to :

1. Focus on the model's unique application at this scale.
2. Investigate examples of non-linear behaviour in the relationships between processes.
3. Consider an appropriate choice of scale, for models of environmental change.

Method

The model is applied to the basin of Cam Gill beck, a tributary of the River Wharfe, above the hamlet of Starbotton, North Yorkshire, UK. The CA method used and details regarding its implementation are described in full by Coulthard *et al.* (1996, 1998) but summarised below.

The basin was digitised from 1:10,000 scale Ordnance Survey map contours. These data, with additional electronic distance meter (EDM) surveyed detail for the valley floor were combined using the TOPOGRID command

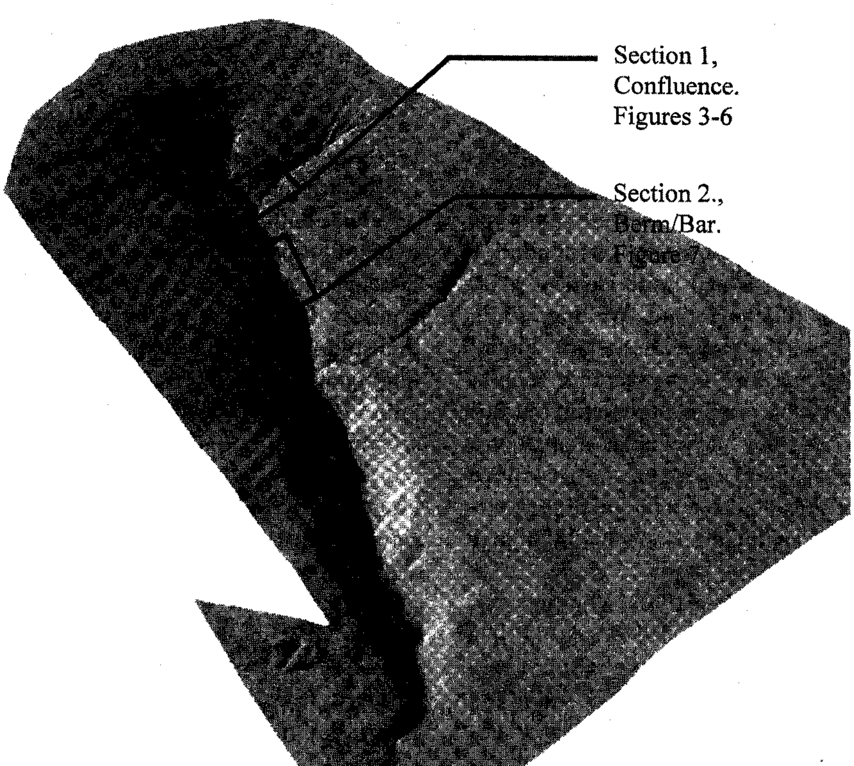

Fig. 1. *Draped image of Starbotton DEM. Scale 1600 by 2800 m.*

in ARC-INFO to create a 1 m^2 resolution digital elevation model (DEM), of 4.2 million cells (Fig. 1.). Within this topographic representation, each grid cell has properties of elevation, discharge, vegetation, water depth, and grainsize. For every model iteration, these values are altered in accordance only with their immediate neighbour and four sets of processes. The first component is a model of hill slope hydrology, using an adaptation of TOPMODEL (Beven and Kirkby, 1979) with an exponential soil moisture store. The second input is a hydraulic routing scheme, utilising bed slope and calculating depth with an adaptation of Manning's formula. Thirdly, fluvial erosion and deposition are assessed using the Einstein-Brown equation (Einstein, 1950). This is applied to nine grainsizes ranging from 0.004 to 1.025 metres in whole Phi classes incorporated with an eleven strata active layer system similar to that used by Parker (1990) and Hoey and Ferguson (1994). Finally, mass movement rates are calculated, incorporating a factor of safety which changes with soil saturation.

Two main scenarios have been applied to the model. Firstly, fifteen floods of equal magnitude, equivalent to bankfull discharge have been simulated, to show cumulative changes in sediment discharge and morphology. Secondly, a larger flood approximating a 5 year flood event was simulated.

Results

Figure 2 shows the effects of simulating 16 floods of approximately bankfull discharge on the upper part of the basin. The initial conditions were set with every cell having the same grainsize distribution. Because of the basin size, initial conditions cannot feasibly be defined for every square metre; therefore, a 'spin up' time is required to allow the initial conditions to evolve. This meant that for the first few runs there was a large sediment discharge, until an armoured layer had developed on the channel bed. Subsequently, the basin displays a deterministic non-linear pattern of behaviour (Coulthard *et al.*, 1998), with irregular peaks in the sediment discharge. This may be attributed to the movement of 'slugs' (Nicholas *et al.*, 1995) of sediment downstream and the consequent remobilisation of these in later floods. Other observations show these peaks in activity can also be linked to the input of landslides, mass movement producing an input of hillslope fines to the armoured channel. Episodes of fluvial erosion and deposition correspond largely with the rising and falling limbs, respectively, of the hydrograph. There are, however, sporadic episodes of activity during periods of low flow. This results from the input of mass movement from the slopes, and demonstrates a partial de-coupling of fluvial and slope processes.

Figures 3 to 6 show valley floor evolution at the confluence of the two main upland channels. Figure 3 shows the initial conditions, where a small discharge has been applied to the basin, resulting in the formation of channels. Figure 4 shows the same region after the 16 floods and Fig. 5 shows the same area but after 1 large flood of approximately 5 year return interval. These three views show the development of several features. The series of flood events has led to the development of a fan at the mouth of the right hand tributary, formed of fines eroded from upland areas. This has caused the widening of the channel opposite and downstream. During the rising and falling limb of the hydrograph, a multiple channel forms as the large sediment influx causes the channel to diverge and converge. Figure 6 corroborates these observations, showing the grainsize distribution for the section after the 16 floods. This shows an 'armouring' down the centre of the multiple

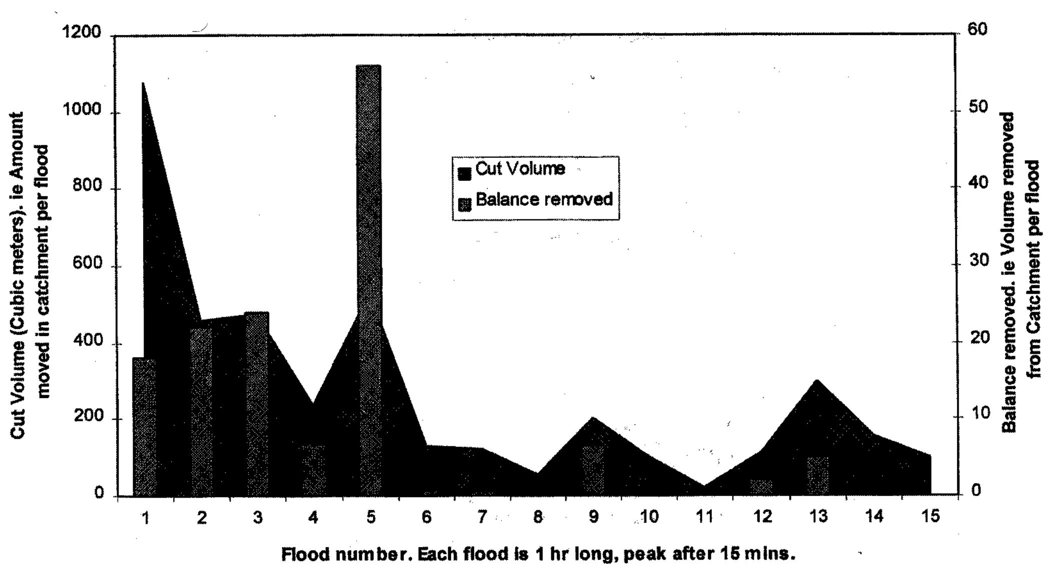

Fig. 2. *Graph showing volume of sediment moved and removed from the catchment for each flood.*

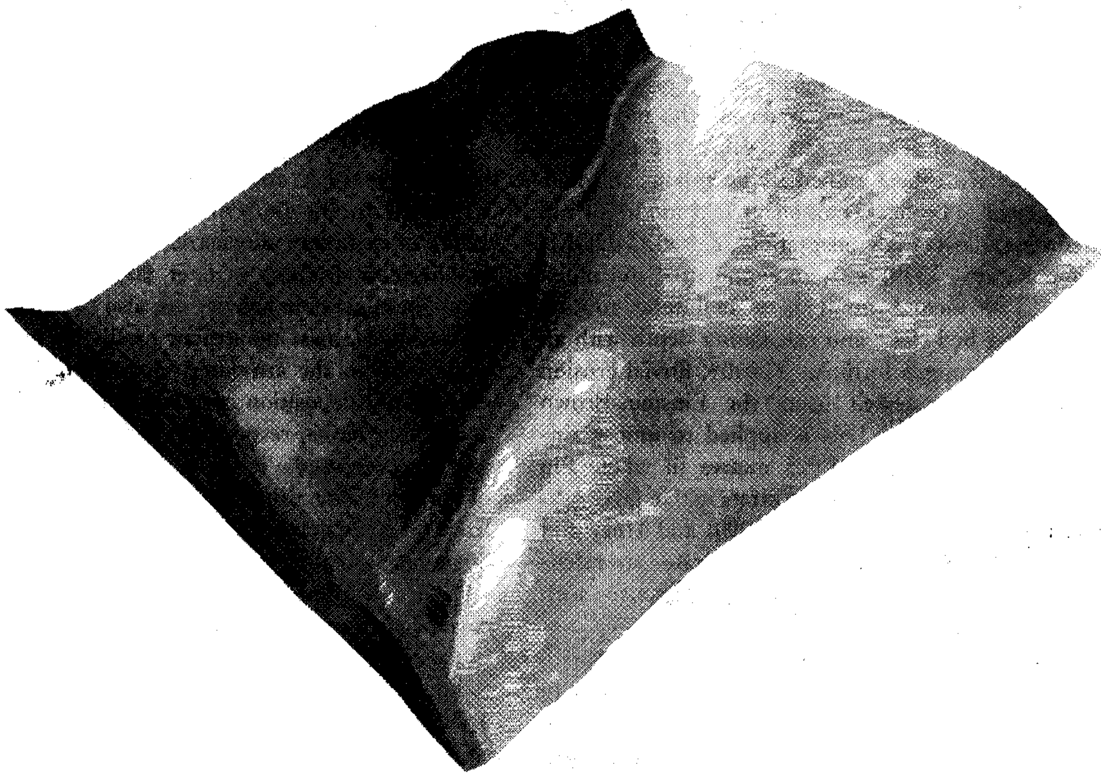

Fig. 3. *Confluence section before flood series.*

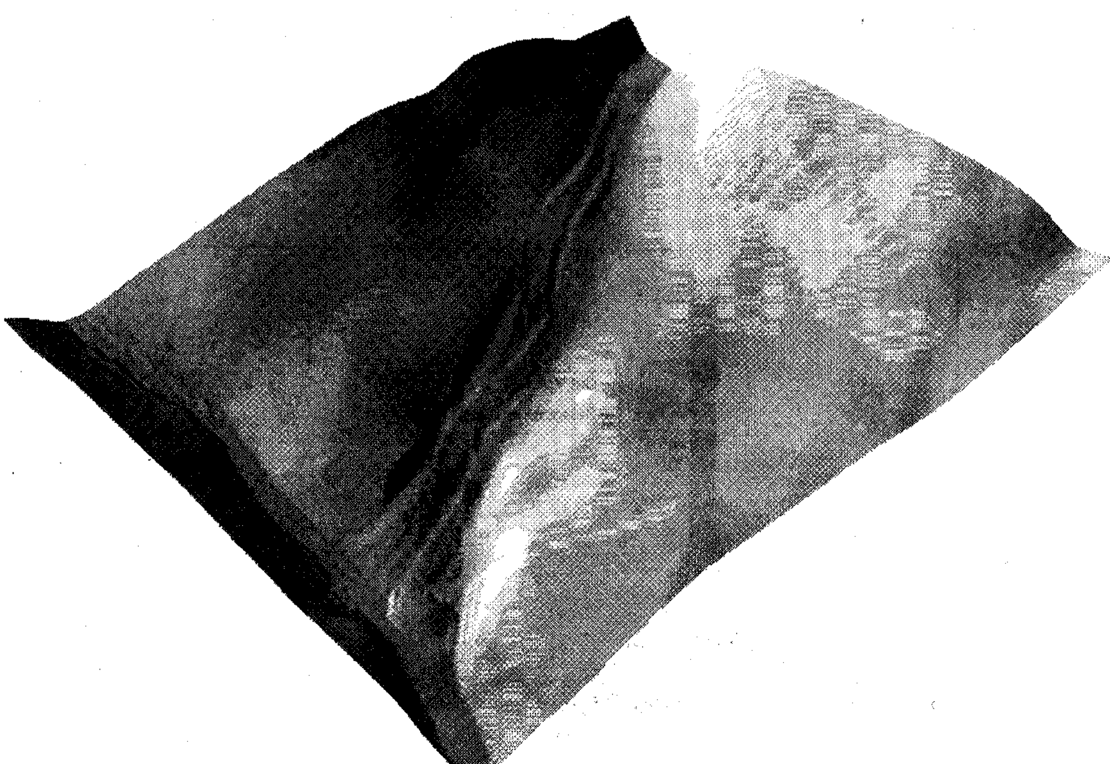

Fig. 4. *After 16 floods of bankfull discharge.*

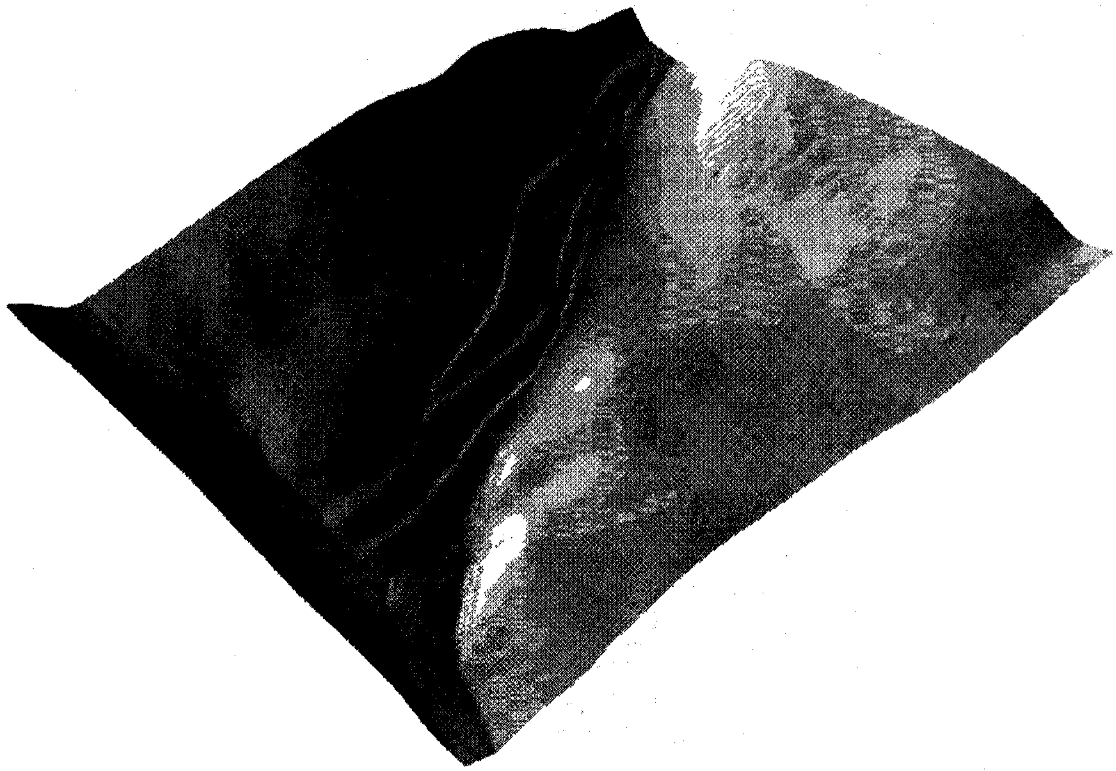

Fig. 5. *After a '5 year' flood event.*

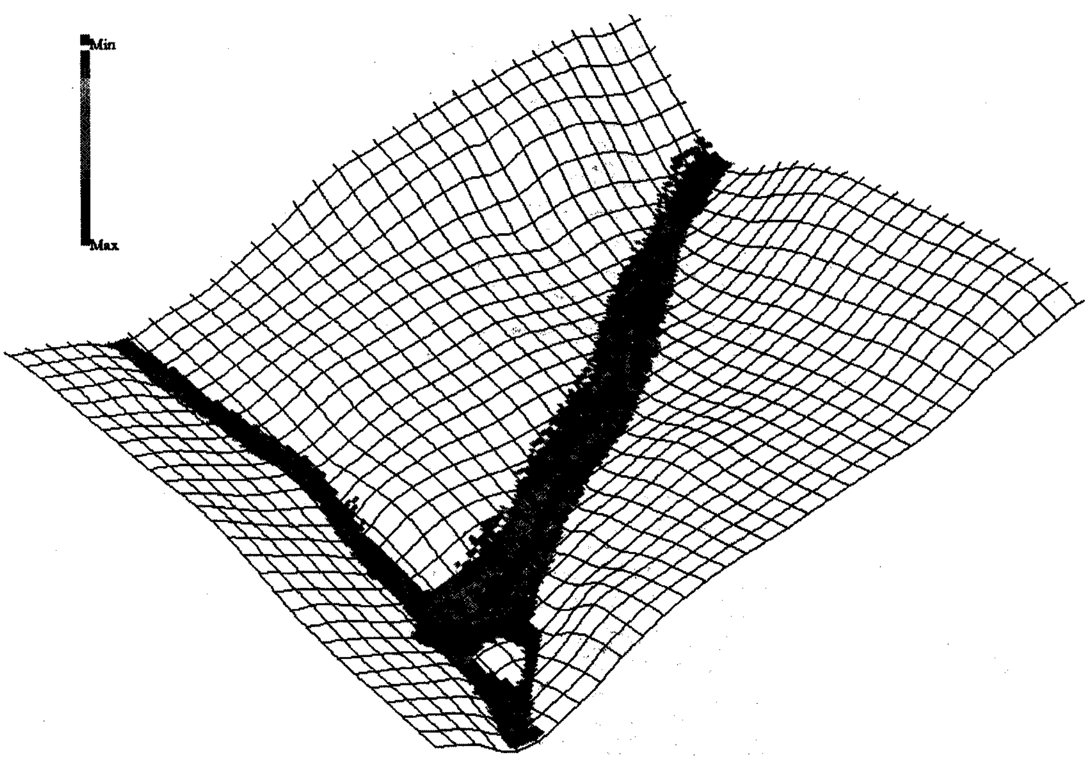

Fig. 6. *Grainsize composition of confluence section.*

channels and fine material deposited at the base of the fan.

Figure 7 details the section outlined in Fig. 1. Figure 7a shows a shaded plan view, Fig. 7b the grainsize and Fig. 7c four cross sections. Here flow (from top to bottom) emerges from a narrow section into a wider section of the valley floor resulting in deposition and the formation of a coarse deposit on the right bank of the channel. 30 m downstream, the tail of this deposit is being eroded as the valley floor narrows and steepens, forming a deposit of fines on the left bank. These features are similar to a boulder berm and side bar/terrace in plan form, elevation and grainsize. Although this is only a preliminary simulation, cross sections from the model (Fig. 7c) compare favourably with those from a field survey (Fig. 7d).

Discussion

Observations of basin dynamics show many examples of non-linear behaviour, from the hydrograph output to sediment discharges (Lane and Richards, 1997; Evans, 1996). This model exhibits similar behaviour, with an unpredictable sediment discharge, showing a partial decoupling between the hydrograph and sediment transport processes. This reflects the supply-limited nature of these temperate basins; there cannot be much sediment transport without a flood, but a flood does not always lead to sediment transport. The initial runs of the model, as described above, show the formation of landslides, berms, bars, braids, terraces and alluvial fans, of similar magnitude and form to those observed in the study area. These features have evolved from featureless valley floors, with equal initial conditions and distributions of sediment. The behaviour and formation of these features are all examples of non-linear behaviour. The grainsize distribution in Fig. 6 is a good example, with fines in areas of lower slopes where sediment has collected and there is armouring in the channels. Throughout the 16 runs of the model, there is a constant interaction between the channel and these stores, which are re-mobilised and dispersed by some floods, yet left by others. The braided pattern observed in Figs. 4 and 5 is a result of these non-linearities. The planform is constantly shifting, with channels growing in one area, yet declining in another.

The model shows chaotic tendencies with its sensitivity to initial conditions. When the elevation data is saved to file, the values are truncated to 6 decimal places. When the data are re-loaded and the model run, results differ from those obtained when the values are retained in the computer memory at their full length.

Are these complex responses simply a condition of the model's design? What happens to this response if more processes are integrated, such as a better hydrological model, or slope representation? Initial sensitivity testing hints that whilst altering the laws used gives different results, they are very similar. For example with Fig. 7, if this is run with a different sediment transport law, the exact dimensions of the berm/terrace sections differ, but their form and location are the same. Computational insta-

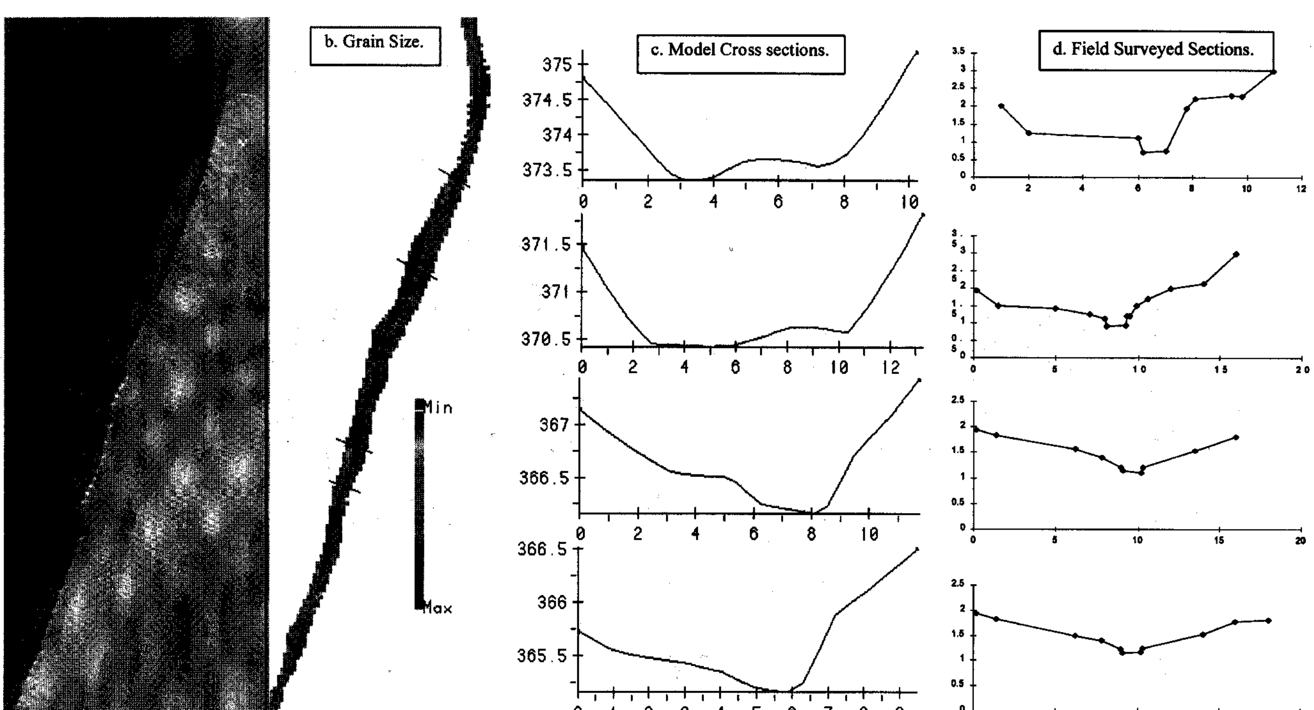

Fig. 7. *Section 2. View a and b are 120 by 30 m. All units in metres.*

bilities could explain non-linear outputs, but to maintain stability, the amount eroded or deposited between each cell is limited to within a few percent of the local slope. If the grid cell area is doubled to 2 m^2 (1.4 by 1.4 m) or even quadrupled to 4 m^2 (2 by 2 m), a virtually identical sediment discharge results, although detail is lost in the form of the features.

The model highlights the importance of mass movement and slope processes in the evolution of a small basin. Disabling the landslide module resulted in a partial reduction in the non-linearity. This suggests that the input from channel banks and heads is important, both as a sediment input and trigger for erosion/deposition episodes. Analysis of cut-fill sequences, shows stream heads to be major contributing areas, as producers of sediment. This re-enforces research claims by Kirkby (1994) regarding the importance of the stream head in a network's evolution. There is still some non-linear sediment response even when the mass movement section is removed; this demonstrates that re-mobilisation and dispersal of sediment throughout the basin are important aspects of system behaviour. For example, the deposition of a clast may result in the lateral migration of the channel towards a pre-existing deposit, re-mobilising fresh material. In contrast to these positive feedbacks, several negative ones control or pacify the model's operation. For example, at the base of Fig. 7, where the channel has cut a terrace, incision is resulting in a stable channel pattern.

The implications of a model generating such non-linear responses are considerable; a simple regression-based model is inappropriate, because the response of the system is complex. The spatially distributed nature of the system means that processes throughout the basin have to be taken into account. It is not the 'random' input from weather systems that is solely responsible for the non-linear behaviour of the fluvial systems; there is an inherent chaotic instability within the whole system. This is further demonstrated by the model's sensitivity to initial conditions. Unfortunately, most fluvial modelling schemes fail to account for non-linear behaviour in any form.

If basin behaviour is unstable and sensitive to small perturbations in initial conditions, how can changes, so small as to appear inconsequential, be incorporated? Subsequently, they may prove to be important. Paola (1996) treats a 'whole' braided river system as a stochastic one, and finds the addition of a random element contributes to the accuracy of estimates of total flow and sediment flux. However, a chaotic system which appears to give a stochastic response is in fact deterministic. The LAB (Bridge and Leeder, 1979) model of alluvial architecture is driven by an avulsion frequency, derived from a probability distribution around an observed mean. Whilst there are many other limitations to their approach (Heller and Paola, 1996), similar approximations may represent one answer. Another approach may take the form of an artificial intelligence (AI) answer, such as a fuzzy logic application or 'training' a neural net to incorporate this chaotic element. However, a true deterministic answer may never be obtained; reliance may have to be placed upon an average of model runs, as climate modellers do.

By choosing the 1 m^2 scale, the effects of basin scale processes such as hydrology and slope processes can be studied and smaller scale basin dynamics such as the in-channel storage and re-mobilisation of sediment may be incorporated. This provides a clear advantage over previous models in which separate slope and channel modules are coupled. With these schemes, different spatial and time scales have to be resolved and feedbacks have to be defined explicitly. Further advances over previous work include the inclusion of units, the use of a real landscape, representation of nine grainsizes and the development of an alluvial architecture and stratigraphy. Furthermore, by selecting a 1 m^2 scale, this model demonstrates synergistic behaviour and shows that the overall basin behaviour cannot be simulated simply from the sum of its individual component processes. The authors believe that this technique shows great potential to demonstrate the relative effects of climate change, human influence and catastrophism (Thorn and Welford, 1994) on an upland basin.

Conclusions

Non-linearities in basins are crucially important at all scales and it will never be possible to account fully for all of them. It is not practical for large basin scale models to simulate three dimensional flow around clasts, yet the broader impact of such small scales must be incorporated. Similarly, three dimensional coupled flow and sediment transport models will have to account for irregularities in the time and space distribution of the arrival of sediment from upstream. Ultimately, the accurate incorporation of such factors will determine the power of the next generation of geomorphological models. Given the increases in computer power and advances in modelling techniques, it may prove that these 'chaotic' terms are the most important.

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