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An analytical solution for the estimation of the critical available soil water fraction for a single layer water balance model under growing crops

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Abstract

In the framework of simplified water balance models devoted to irrigation scheduling or crop modelling, the relative transpiration rate (the ratio of actual to maximal transpiration) is assumed to decrease linearly when the soil dries out below a critical available water value. This value is usually expressed as a fraction, *F*, of the maximal available soil water content. The present work aims to use the basic laws governing water transfer through the plants at a daily time step to compute *F* dynamically as the crop grows. It can be regarded as an expansion of Slabbers' (1980) approach to crop growing conditions. Starting from the mathematical representation given by single-root models (Gardner, 1960), an analytical expression for *F* is derived, using simplified hypotheses. This expression accounts for plant attributes such as the mean root radius, the critical leaf water potential for stomatal closure and the root length density profile growing with the crop. Environmental factors such as soil type and atmospheric demand also influence *F*. The stuctural influence of soil comes from the required introduction of the bulk soil hydraulic conductivity in the single-root model. The shape of the root length density profile is assumed to be sigmoïdal and a new profile is calculated at each value of the rooting depth. A sensitivity analysis of *F* to all those factors is presented. The first general result is that *F* decreases as the root system grows in depth. Differences in the shape of the root profile can be responsible for differential water stress sensitivity in the early stages of growth. Yet, low critical leaf water potential can compensate partially for a poor root profile. Conversely, *F* is relatively insensitive to the average root radius. *F* sensitivity to soil type seems somewhat artificial: given the bulk soil hydraulic conductivity formula, the soil sensitivity results from *F* being expressed as a fraction of the maximal available soil water content. The atmospheric demand together with the rooting depth appear as the most important factors. However, when assuming predictable climatic and crop evolution, compensation occurs between those two effects leading to a relative stability of *F* when the crop is fully developed. Though relying on well-known physical laws, the present approach remains in the framework of single layer models with the same limitations.

Introduction

Although mechanistic models of crop water dynamics exist (Molz, 1981), there is still a need for semi-empirical approaches requiring readily available input variables/parameters. Those semi-empirical water balance models are generally devoted to practical issues such as irrigation scheduling. They can also be embedded in crop growth simulators.

In a simplified mathematical representation of crop water dynamics, the relative evapotranspiration rate (ratio of actual to reference evapotranspiration) is dependent on soil water content. Van Bavel (1953) suggested a straight-line relationship allowing simple calculations of soil water balance. Subsequent studies have shown that the relationship was more likely to be curvilinear (Denmead and Shaw, 1962; Eagleman, 1971) or exponential (Baier, 1969). Nevertheless, a bilinear function may be a good representation of the experimental data (Burch *et al.*, 1978; Meyer and Green, 1981; Rosenthal *et al.*, 1985; Robertson and Fukai, 1994). Such a relationship assumes that a crop is able to take up soil water at a maximal rate to meet atmospheric demand until the soil water content falls below some threshold value. The bilinear function was adopted as the driving equation of many simple water balance models (see e.g. the FAO model from Doorenbos and Kassam, 1979 or the review by Leenhardt *et al.*, 1995).

Relying on this relationship, Hallaire (1964) divided the maximum available soil water into two fractions: the critical available water fraction (hereafter referred to as *F*

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expressed in percent of the maximal available soil water) below which the evapotranspiration rate decreases and the readily available soil water fraction (1–*F*) which is available to the crop without any reduction in the relative evapotranspiration rate. In many models, *F* is assumed to be a constant equal to 0.3, 0.4 or 0.5 (Hunt and Pararajasingham, 1995; Robertson and Fukai, 1994; Fisher and Elliott, 1996; Mailhol *et al.*, 1996). Yet *F* was shown to depend on atmospheric demand (Hallaire, 1964; Doorenbos and Kassam, 1979; Cordery and Graham, 1989), on species (Doorenbos and Kassam, 1979; Burch *et al.*, 1978; Novak, 1989; Gardner, 1991) and to vary during the crop cycle (Teixera *et al.*, 1996; Palacios and Quevedo, 1996).

In 1980, Slabbers proposed an operational formula to compute *F* from the atmospheric demand and the species. His approach relied on a simplification of the basic laws governing water transfer in the soil-plant atmosphere continuum. It was devoted to a fully developed crop. The present work uses a similar approach to compute *F* dynamically as the crop grows: this can be useful for crop growth models.

Framework and basic assumptions

SOIL WATER CHARACTERISTICS

In semi-empirical water balance models, soil water status is generally expressed as plant available water, defined as the amount of water held above the wilting point. The maximum plant available water refers to the amount of water between field capacity and wilting point. In this approach, the soil is assumed to be a single reservoir defined by an effective depth and by the water contents at field capacity (θ_{FC} in cm^3 cm^{-3}) and wilting point (θ_{WP} in cm^3 cm^{-3}). The question of whether it is better to use field rather than laboratory measured values for those soil characteristics has been debated extensively (see e.g. Ritchie, 1981). The standardised laboratory definitions: soil water contents at –33 or –10 kPa for θ_{FC} and –1500 kPa for θ_{WP}, usually available from soil survey data, are preferred.

When crops are growing, the effective soil depth increases, which in turn increases the maximum available water. This increase may not be just proportional to the rooting depth because in the deeper soil layers the root density may not be sufficient to take up all of the available water. Hence, it seems better to avoid associating the soil and root components in the single notion of maximum available water. Since in the present approach, the root growth is accounted for separately, 'maximum available water content' (*MAWC* in cm^3 cm^{-3}) is defined as the difference in soil water contents at field capacity and wilting point (θ_{FC}–θ_{WP}) without any reference to the rooting depth. Similarly 'available water content' (*AWC* in cm^3 cm^{-3}) refers to the difference between the average current soil water content, (θ in cm^3 cm^{-3}) and θ_{WP}.

RELATIVE TRANSPIRATION RATE

The breakpoint between the maximum and reduced stages of relative evapotranspiration rate was identified as corresponding to the stomatal closure (Veihmeyer and Hendrickson, 1955; Hallaire, 1964). This physiological explanation sustains the use of a bilinear relationship, illustrated in Fig. 1, to estimate plant transpiration rather than total evapotranspiration. The processes involved in soil evaporation differ from those of plant transpiration (Ritchie, 1972; Brisson and Perrier, 1991). Despite this, it is generally accepted that, for crops covering the ground, using the bilinear relationship to assess crop evapotranspiration does not cause large errors because soil evaporation is negligible when compared to plant transpiration. Obviously, this is not the case in the earlier stages of crop growth, which is why the separation between evaporation and transpiration is an important feature in water balance of crop growth models (Leenhardt *et al.*, 1995).

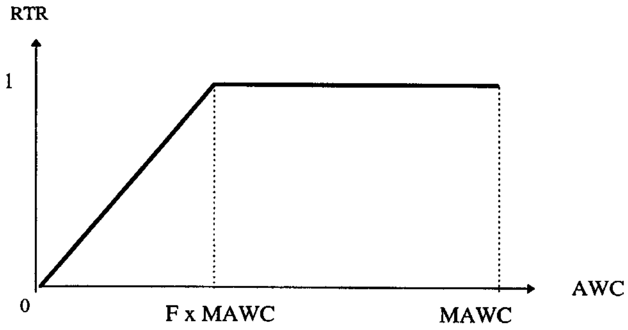

Fig. 1. *Definition of the critical available water fraction, F, as the MAWC proportion separating maximal RTR stage from decreasing RTR stage.*

For these reasons, the bilinear function is considered to relate the relative transpiration rate (RTR: the ratio of actual transpiration to atmospheric demand at plant level) and AWC. The critical (*F*) and the readily available (1 – *F*) soil water fractions are defined as ratios to MAWC as depicted in Fig. 1.

REFERENCE TO SLABBERS' WORK

Slabbers' approach (1980) linked the basic laws governing water transfers in the soil-plant-atmosphere continuum to the simplified approach of relative evapotranspiration rate reduction versus available soil water. From Slabbers, the four following basic ideas have been retained:

(1) root water uptake can be estimated using the well known Ohm's law analogue (Van den Honert, 1948) as the ratio of a soil-plant driving force, namely the difference in soil and plant water potentials, to a soil-plant resistance,

(2) when the reduced stage of (evapo)transpiration starts, the driving force is assumed to be equal to the critical leaf potential for stomatal closure, soil potential being negligible when compared to plant potential,

(3) at the start of the reduced stage of (evapo)transpiration, (evapo)transpiration is at its maximum value,

(4) at the daily timestep, the rate of (evapo)transpiration is assumed to be equal to root water uptake,

The rest of the work differs markedly from Slabbers' approach in that different formulations for soil-root resistances are used, relying a) on Gardner's model (1960), b) on the work by Taylor and Klepper (1975) and c) on the formulation proposed by Ritchie and Otter (1984) for the soil contribution to the resistance.

In addition, estimating F dynamically requires some modelling of the root growth.

As previously explained, the approach relies on a reference to atmospheric demand at plant level (maximal plant transpiration) rather than to standard atmospheric evapotranspiration. This needs, firstly, that a canopy variable such as the leaf area index or the proportion of covered soil could be available to derive the maximal plant transpiration. Secondly, the direct soil evaporation must be assessed separately to complete the water balance. These two last points are addressed in other papers (Brisson and Perrier, 1991; Brisson *et al.*, 1998a).

Mathematical formulations

EQUATIONS LEADING TO THE ANALYTICAL EXPRESSION FOR THE CRITICAL AVAILABLE WATER FRACTION, *F*

In the description of root water uptake by single-root models (Gardner, 1960; Cowan, 1965), the root is assumed to be an homogeneous cylinder of uniform water-absorbing properties. The cylinder is characterized by its inner (r in cm) and outer ($rcyl$ in cm) radii. If steady conditions are assumed, the daily rate of water uptake per centimetre of root (q in cm^3 water cm root^{-1} day^{-1}) can be calculated using the following equation (Taylor and Klepper, 1975):

$$q = \frac{\Delta\Psi}{\ln \frac{rcyl}{r} \Big/ 2\pi k} \qquad (1)$$

$rcyl$ is commonly computed as a function of root length density, l_v (cm root cm^{-3} soil) (Tardieu and Manichon, 1986):

$$rcyl = \frac{1}{\sqrt{\pi l_v}} \qquad (2)$$

The root radius, r, is often taken as 0.02 cm, yet Gallardo *et al.* (1996) found important differences between wheat (0.021 cm) and lupin (0.064 cm). $\Delta\psi$ (cm or cm^3 water root cm^{-2}) is the soil-plant driving force and k (cm day^{-1}) is the hydraulic conductivity at the soil-plant interface. k is often considered to be the hydraulic conductivity of the bulk soil.

It is assumed that equivalent values for k and l_v exist for the whole soil-root system, so that Eqn. (1) may be implemented globally. Those equivalent parameters are ke and l_{ev} respectively.

If T_p is the daily plant transpiration (mm day^{-1} or 10 cm^3 water cm^{-2} day^{-1}) and zr (cm) is the rooting depth, Eqn. 3 can be written according to the hypothesis (4):

$$T_p = q \times le_v \times z_r \times 10 \qquad (3)$$

Combining Eqns. 1, 2 and 3 yields:

$$T_p = -\frac{10 le_v z_r\, 2\pi\, ke\, \Delta\Psi}{\ln \dfrac{1}{r\sqrt{\pi\, le_v}}} \qquad (4)$$

When soil water content decreases to the critical available water content, leaf water potential reaches the critical potential at stomatal closure (Ψ_{sto}) whereas plant transpiration is still maximal (Tm_p) (hypothesis (2) and (3)). Under such conditions Eqn. 4 can be written as:

$$Tm_p = -\frac{10 le_v z_r\, 2\pi\, ke\, \Psi_{sto}}{\ln \dfrac{1}{r\sqrt{\pi\, le_v}}} \qquad (5)$$

Measuring hydraulic properties of the soil by laboratory or field procedures is difficult; in view of the large variability, some investigators have attempted to estimate them from available water content (Ritchie and Otter, 1984; Williams and Ahuja, 1993). In this study, for ke a similar formula as in Ritchie and Otter (1984) has been adopted:

$$ke = \alpha \exp(\beta\, AWC) \qquad (6)$$

The critical available water corresponds to:

$$AWC = F \times MAWC.$$

F can then be deduced from a combination of Eqns. 5 and 6:

$$F = \frac{1}{\beta\, MAWC} \ln\left[-\frac{Tm_p}{10\, \alpha\, z_r\, le_v\, 2\pi\, \Psi_{sto}} \times \ln\left(\frac{1}{r\sqrt{\pi\, le_v}}\right)\right] \qquad (7)$$

Eqn. 7 establishes that F depends on soil characteristics ($MAWC$), on plant attributes that can be considered as stable (Ψ_{sto} and r) and on rooting features (depth z_r and density le_v). In use, Eqn. 7 requires the estimation of the parameters α and β and a model of root growth delivering daily values of z_r and le_v.

BULK SOIL HYDRAULIC CONDUCTIVITY

Taylor and Klepper (1975) and Reid and Hutchison (1986) demonstrated that the bulk soil hydraulic conductivity affects root uptake only for very low levels of soil water content. In most cases, plants are responsible for limiting uptake (Reicosky and Ritchie, 1976). This process is generally accounted for by making absorption equal to transpiration (Eqn. 3) or by limiting uptake (e.g.: Ritchie and Otter, 1984 or Adiku *et al.*, 1996 used a threshold of 0.03 cm^3 water cm^{-1} root day^{-1}). Consequently, the parameterization of Eqn. 6 is particularly important in the range of low water contents. As in Taylor and Klepper (1975), the hydraulic conductivity of the whole soil-root system can be calculated as the minimum between the bulk soil hydraulic conductivity given by Eqn. 6 and the value obtained by rearranging Eqn. 5:

$$ksys = \min\left(\alpha \exp(\beta\, AWC);\; -\frac{Tm_p \ln \dfrac{1}{r\sqrt{\pi\, le_v}}}{10 le_v\, z_r\, 2\pi\, \Delta\psi} \right) \quad (8)$$

Parameters α and β have been identified by applying Eqn. 8 to the data of Taylor and Klepper (1975). The results are $\alpha = 10^{-8}$ and $\beta = 80$. Ritchie and Otter used values of $\alpha = 10^{-8}$ and $\beta = 64$ in combination with a constant leaf potential of −2.1 MPa. Both parameterizations give similar results in the range of low soil water contents when using leaf water potentials between −1 and −1.5 MPa. Figure 2 shows the hydraulic conductivity of the bulk soil (Eqn. 6) as a function of AWC, together with the hydraulic conductivity of the whole soil-root system, *ksys* (Eqn. 8). In the same figure were plotted the data from Taylor and Klepper (1975), using the minimum value of soil water content allowing plant uptake as wilting point to calculate AWC.

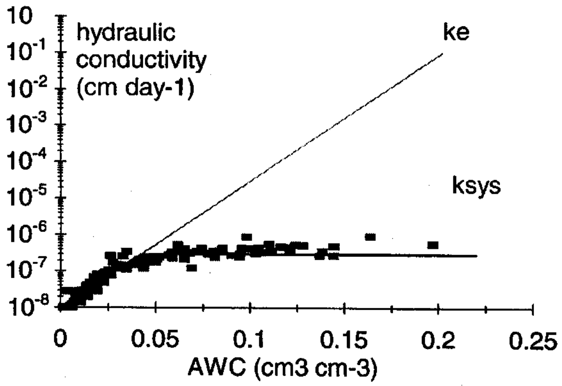

Fig. 2. *Comparison of bulk soil hydraulic conductivity (ke from Eqn. 6 with $\alpha = 10^{-8}$ and $\beta = 80$) and soil-root hydraulic conductivity (ksys from Eqn. 8) calculated in the same context as Taylor and Klepper's (1975). Points correspond to the experimental values of ksys calculated independently in Taylor and Klepper.*

If Tm_p and l_{ev} are varied widely, the plateau of *ksys* varies between 2×10^{-7} and 5×10^{-6} cm day^{-1}, which agrees with values reported in Taylor and Klepper for the cotton crop.

ROOT GROWTH MODEL

The depth of the tap root, *zr*, also corresponding to the extraction front, can be calculated using various models. The simplest one is a linear function of time since sowing (Subbaiah and Rao, 1993; Robertson and Fukai, 1994; Mailhol *et al.*, 1996). The average rate of growth is generally between 3 and 4 cm day^{-1}. Robertson *et al.* (1993) demonstrated that the root penetration rate of sorghum was relatively insensitive to soil and crop management conditions; Meinke *et al.* (1993) found similar conclusions for sunflower. Another possibility is to make the depth of the tap root depend on thermal time using constant penetration rates ranging between 0.11 to 0.23 cm degree day^{-1} depending on species (Stone *et al.*, 1975; Brisson *et al.*, 1992; Plauborg *et al.*, 1996). In dry tropical climates, crops are sown at the beginning of the rainy season, so that the rooting depth may be assumed to follow the soil wetting front (Bonachela, 1996; Franquin and Forest, 1977).

It is not possible to implement Eqn. 7 using a constant value for the mean root length density, l_{ev}. When the tap root grows, the root length density profile is likely to be modified in a non-conservative way so that l_{ev} is modified as well. To compute the relationship between z_r and l_{ev}, it is necessary to assume a root length density profile. The conventional approach, first proposed by Gewitz and Page (1974), relies on an exponential description of the root distribution with depth. Sigmoidal description is preferred here; it allows the water uptake efficency of root density to be limited in the superficial soil layers. There is an optimal threshold of root density allowing a maximal uptake of available water (Cowan, 1965; Tardieu and Manichon, 1986; Gardner, 1991; Kage and Ehlers, 1996). For Tardieu and Manichon (1986), the distance between roots corresponding to this threshold depends on the spatial arrangement of roots (Tardieu *et al.*, 1992). In the calculation by Aura (1996), the maximal depletion distance from the root axis (*rcyl*) ranges between 0.5 and 1 cm. Bonachela (1996) showed that above a root length density of 0.5–0.7 cm cm^{-3} all the available water is extracted. Kage and Ehlers (1996) and Robertson *et al.* (1993) suggested lower values of 0.1 and 0.25 cm cm^{-3} respectively. Based on the results of Aura (1996) and Bonachela (1996), a value of $lopt_v = 0.5$ *cm.cm*$^{-3}$ was adopted. This value corresponds to *rcyl=0.8cm* according to Eqn. 2.

A reference effective root length density profile is assumed as the profile achieved at full development. The sigmoidal shape is defined by three depth parameters (Fig. 3). The ploughed layer (z_{pl}) is considered as the zone where roots do not limit plant water uptake, namely $l_v(z) \cong lopt_v$. At z_{sl}, the effective root length density is reduced by half and z_{lim} is the depth of the tap root. The equation of the reference profile (Fig. 3) is:

$$l_v(z) = \frac{lopt_v}{1 + \exp[-S(z - z_{sl})]} \quad \text{and} \quad S = \frac{\varepsilon_s}{z_{pl} - z_{sl}} \quad (9)$$

The value of ε_s was chosen arbitrarily so that the proliferation of roots in the plough zone is maximal ($\varepsilon_s = -4.6$) for the reference profile.

At each rooting depth corresponding to a certain stage of growth, a new profile is recalculated using the same shape parameters as in the reference profile so that the root

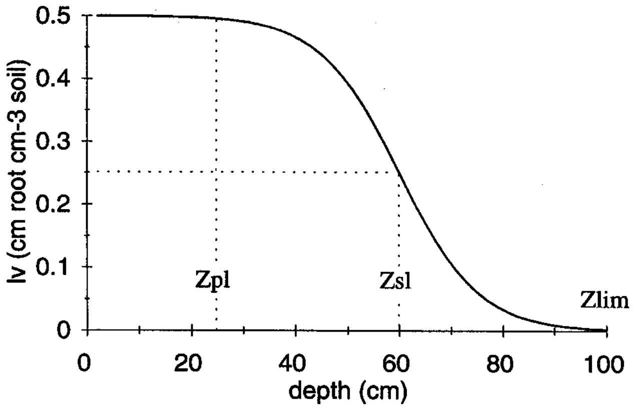

Fig. 3. *The reference profile of efficient root length density.*

system is assumed to grow in an homothetical way (the shape is respected). At this precise time, the rooting depth is z_r and the level where the length density is reduced by half is $z_{0.5}$.

$$l_v(z) = \frac{lopt_v}{1 + \exp[-S(z - z_{0.5})]} \quad \text{and}$$

$$z_{0.5} = \max\left(z_r - z_{\lim} + z_{sl};\ \frac{1.4}{S} \right) \qquad (10)$$

The minimal value $z_{0.5} = 1.4/S$ was arbitrarily chosen; it allows a minimal water uptake of about 20% of the maximal available water content (over the rooting depth) at the very beginning of the crop cycle. le_v is derived from the analytical integration of Eqn. 10.

$$le_v = \frac{1}{z_r} \int_{z=0}^{z} \frac{lopt_v}{1 + \exp\left[-S(z - z_{0.5})\right]} = -\frac{lopt_v}{S z_r} \ln \frac{1 + \exp(-S\, z_{0.5})}{1 + \exp\left[S(z_r - z_{0.5})\right]} \qquad (11)$$

SUMMARY OF THE OPERATIONAL ESTIMATION OF THE CRITICAL AVAILABLE WATER FRACTION, *F*

Finally F is estimated using the following set of equations:

$$F = \frac{1}{80\, MAWC} \ln\left[-\frac{Tm_p}{10^{-7}\, z_r\, le_v\, 2\pi\, \psi_{sto}} \times \ln\left(\frac{1}{r\sqrt{\pi\, le_v}} \right) \right] \qquad (12)$$

$$le_v = -\frac{0.5}{S z_r} \ln \frac{1 + \exp(-S z_{0.5})}{1 + \exp\left[S(z_r - z_{0.5})\right]} \qquad (13)$$

$$S = \frac{-4.6}{z_{pl} - z_{sl}} \qquad (14)$$

$$z_{0.5} = \max\left(z_r - z_{\lim} + z_{sl};\ \frac{1.4}{S} \right) \qquad (15)$$

According to Eqns. (12) to (15), F is affected by both time-dependent (variables) and time-independent (parameters) characteristics of the soil-plant-atmosphere system. The influence of the soil type is merged in the parameter *MAWC*. The plant attributes are the parameters r, Ψ_{sto} and the triplet (z_{pl}, z_{sl} *and* z_{lim}) giving the shape of the root profile. The two variables driving F during the crop cycle are the atmospheric demand at the plant level, Tm_p, and the depth of the tap root, z_r. Tm_p is not a purely atmospheric reference, because it also depends on the amount of leaves interacting with the atmosphere. Tm_p is generally calculated as a simple combination of the reference atmospheric evapotranspiration and the crop leaf area index (LAI), using an analogy to the Beer's law (Ritchie, 1985a; Allen, 1990; Brisson *et al.*, 1992; Villalobos and Fereres, 1990; Van Keulen and Seligman, 1987; Ragab *et al.*, 1990; Binh *et al.*, 1994). Hence, indirectly the crop LAI also operates on F.

Application of the derived equations

To implement the previous set of equations within a water balance model, it is essential to test the sensitivity of F to the various parameters/variables. Therefore, in this section a sensitivity analysis considers the parameters/variables as independent and then F is calculated dynamically assuming actual atmospheric conditions and predictable crop evolution.

F SENSITIVITY ANALYSIS

Each parameter/variable is varied in a range reported in Table 1, while the other ones are given the default values. The variable z_r is used as a driving variable in that analysis so that the results are given as a function of both z_r and the tested parameter/variable.

Three reference profiles of effective root length density were assumed. These profiles vary only in the parameter *zsl* (Fig. 4). The profile B can be the case of a deep tap root (a pivot) with the main part of the roots exploring the superficial layers; at the opposite profile C can be the case of roots exploring the deeper layers very early.

At each rooting depth, z_r, the shape defined by the reference profile (Fig. 4) is applied to derive l_{ev} (Eqns. 13 to 15).

In any root reference profile depicted in Fig. 4, F decreases as the root system grows in depth (Fig. 5). Yet the shape of the root profile influences F significantly: for the profile B, F remains at the maximal value for a long time whereas for the profile C, F displays a smoother evolution between 0.45 and 0.25.

The roles of the average root radius, r, and of the critical potential, Ψ_{sto}, are displayed in Figs. 6 and 7, showing that F can be delayed by a low critical potential or, to a lesser extent, by thick roots.

The environmental factors: soil type and atmospheric

Table 1. Default values and ranges of variation for each parameter/variable (*: 10 000 cm = 1MPa).

Parameter/variable	Definition	Range of variation tested	Default value
z_{pl}, z_{sl}, z_{lim}	reference root profile	[B] to [C] see Fig. 4	[A] see Fig. 4
r	root radius	0.005 to 0.07 cm	0.02 cm
ψ_{sto}	critical leaf potential	–0.5 to –2.5 MPa*	–1.5 MPa*
$MAWC$	maximal available water content	0.10 to 0.20 $cm^3\ cm^{-3}$	0.15 $cm^3\ cm^{-3}$
Tm_p	maximal plant transpiration	1 to 9 mm day^{-1}	4 mm day^{-1}

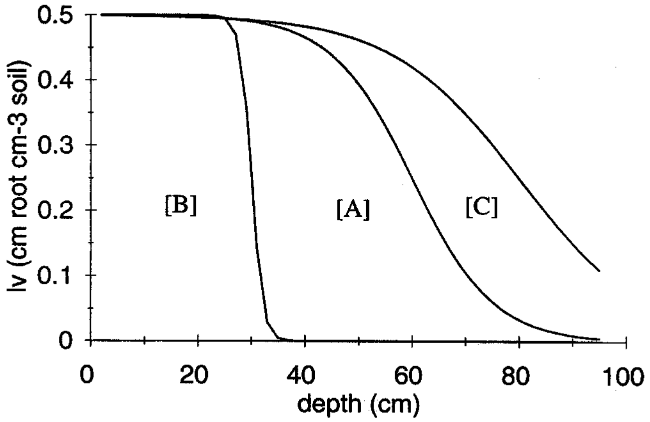

Fig. 4. *Shape of the three reference profiles assumed for the sensitivity analysis. The depths used are: profile A (z_{pl} = 25 cm; z_{sl} = 60 cm; z_{lim} = 100 cm), profile B(z_{pl} = 25 cm; z_{sl} = 30 cm; z_{lim} = 100 cm) and profile C (z_{pl} = 25 cm; z_{sl} = 80 cm; z_{lim} = 100 cm).*

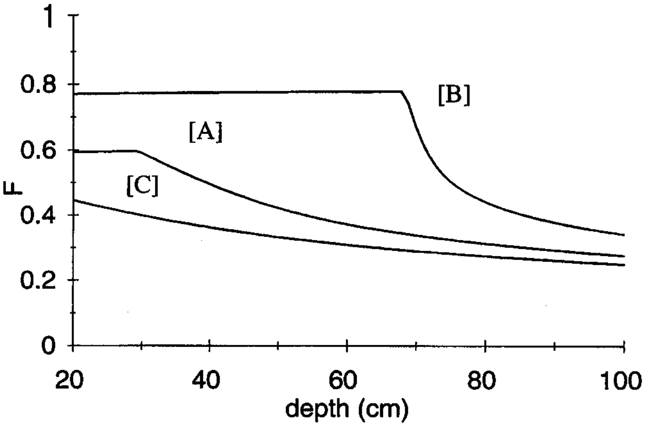

Fig. 5. *Influence of the root profile (Fig. 4) on F.*

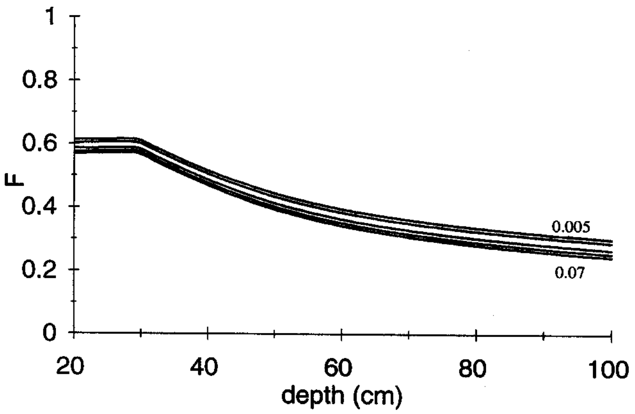

Fig. 6. *Influence of mean root radius on F. The five curves correspond to 0.005, 0.01, 0.03, 0.05 and 0.07 cm root radius.*

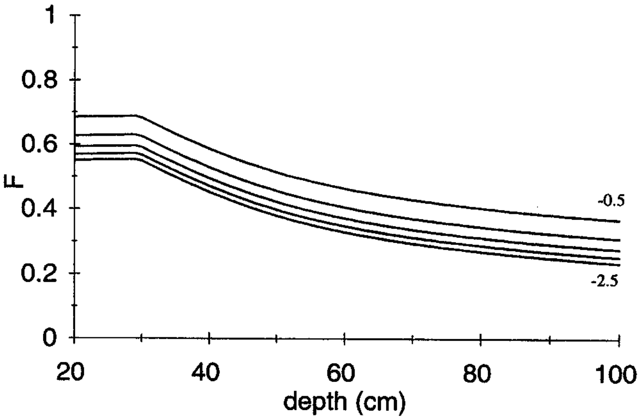

Fig. 7. *Influence of the critical leaf potential on F. The five curves correspond to –0.5, –1.0, –1.5, –2.0, –2.5 MPa.*

demand, influence the value of F throughout the crop cycle (Figs. 8 and 9). In general, the more sandy the soil (low MAWC) and the higher the atmospheric demand, the higher is the value of F.

In temperate climates, irrigation often starts when roots are fully developed. Hence, it is interesting to focus on the sensitivity of F at full root growth (Table 2). At the final stage of root growth, the importance of soil type and reference root profile on F are proportionally less than during growth, although the effects of critical leaf potential and atmospheric demand remain. The sensitivity to the root radius is even lower while the sensitivity to the soil type is in the same proportion as the range of variation of the parameter MAWC.

Table 2. Sensitivity of F resulting from the variation of each parameter (as described in Table 1) once the root system is installed.

Parameter	Sensitivity of F at full root growth (100 cm)
Reference root profile	0.25–0.34
Root radius	0.24–0.30
Critical leaf potential	0.23–0.37
Maximal available water content	0.21–0.42
Maximal plant transpiration	0.16–0.35

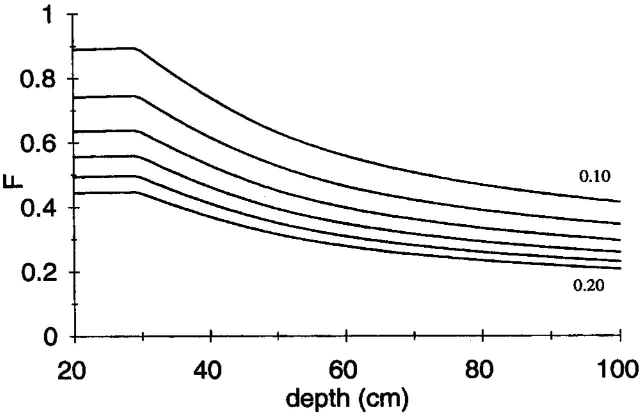

Fig. 8. *Influence of the soil type characterized by the maximal available water content on F. The five curves correspond to 0.1, 0.12, 0.14, 0.16, 0.18 and 0.2 cm³ cm⁻³.*

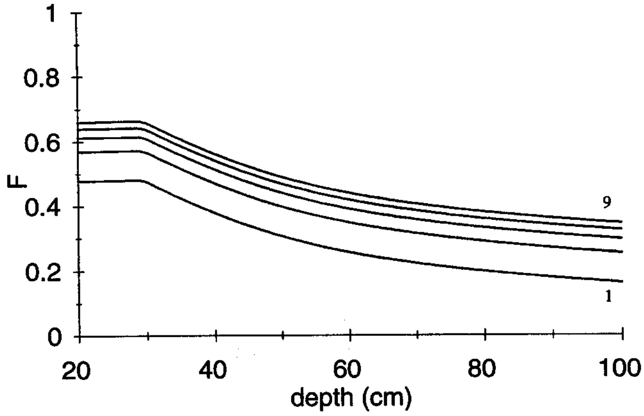

Fig. 9. *Influence of the atmospheric demand at the plant level on F. The five curves correspond to 1, 3, 5, 7 and 9 mm day⁻¹.*

F AS A FUNCTION OF TIME USING ACTUAL ATMOSPHERIC CONDITIONS

In temperate and Mediterranean climates a compensation can occur between the root growth effect (which tends to decrease F) and the maximal transpiration effect (which tends to increase F) as the weather becomes warmer and drier and as leaves are growing. To estimate the possible importance of such a compensation, F has been calculated by using actual weather data from the meteorological station of the Institut National de la Recherche Agronomique at Avignon-France (43°54' N, 4°48'E), 1992. Maximal transpiration is derived from the Penman reference evapotranspiration (RET) and the leaf area index (LAI) of an hypothetical crop. The Beer's law analogue (Brisson *et al.*, 1992) is used to derive the maximal transpiration from the reference evapotranspiration using an extinction coefficient of 0.4 (Eqn. 16).

$$Tm_p = RET\,\exp(-0.4LAI) \qquad (16)$$

A very simple evolution of the LAI in three stages has been assumed for summer crops maize or soyabean (Fig. 10). Default parameters (Table 1) for plant attributes have been used as well as two contrasting soil conditions (MAWC of 0.1 and 0.2 $cm^3\ cm^{-3}$). The growth of the tap-root is calculated using a constant rate of 0.15 cm root per degree-day from sowing to the date of maximal LAI.

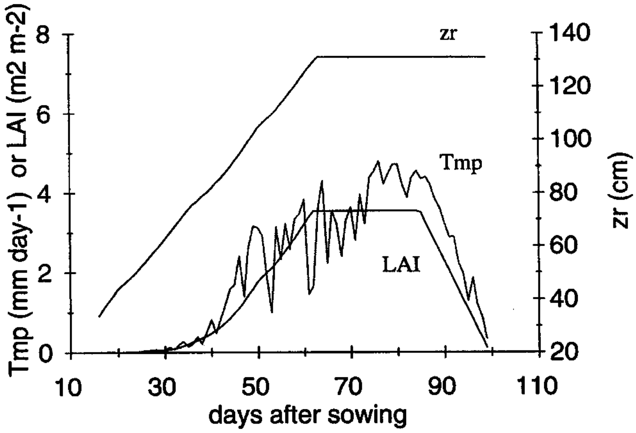

Fig. 10. *Evolution of the climatic demand and the rooting depth of an hypothetical summer crop in Avignon 1992. The LAI is assumed to vary in three stages.*

Apart from the first 45 days after sowing and the very end of the cycle, the evolution of F (Fig. 11) does not display any decreasing or increasing trend. Fluctuations due to the atmospheric demand can be seen and the soil type gives the level of the average F value.

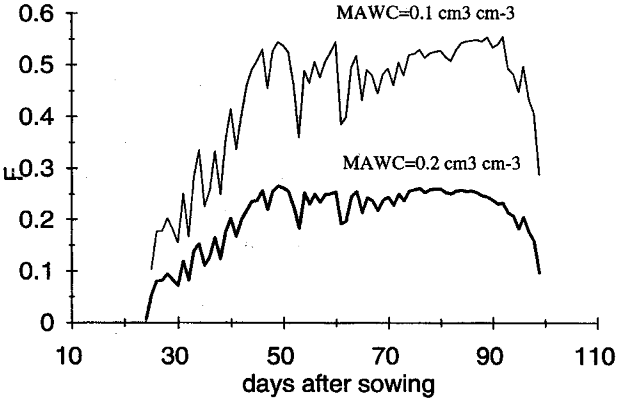

Fig. 11. *F evolution throughout the crop cycle for the soil of high water retention in bold line (0.2 cm^3 cm^{-3} maximal available water content) and for the soil of low water retention in normal line (0.1 cm^3 cm^{-3} maximal available water content.*

Discussion

ON THE ROLE OF PLANT ATTRIBUTES

The influence of the root profile shape on the estimate of *F* is much more important at the beginning than at the end of the growth period (Fig. 5). The more superficial the roots (for example profile B), the higher the value of *F*. This indicates that water stress sensitivity is highest during the early stages of crop growth for poor root systems. Burch *et al.* (1978) noticed a significant difference in the root profiles in sorghum and soyabean on water uptake. Once roots have completed growth, the *F* value seems to be controlled only by the tap-root depth whatever the shape of the profile, which agrees with the results of Ehlers *et al.* (1991).

The critical leaf potential also plays a significant role in controlling *F* (Fig. 7). A low critical leaf potential results from the inherent ability of the plant to delay stomatal closure and tolerate dehydration. It is considered a major drought resistance feature (Turner, 1979; Laffray and Louget, 1990). Slabbers (1980) selected values of critical leaf potentials for the main agricultural crops from the literature: between −1.3 and −1.7MPa for maize, sorghum, wheat, barley and alfalfa, between −0.8 and −1 MPa for peas, soybeans and tomatoes, between −0.35 and −0.75 for sunflowers, sugarbeet and potatoes. Laffray and Louget (1990) give similar values.

The effect of the root radius appears to be negligible (Fig. 6), which is in agreement with Kage and Ehlers' calculations (1996) and the commonly accepted idea that radial hydraulic conductivity is not a limiting factor for water uptake (Habib *et al.*, 1991). Nevertheless, Gallardo *et al.* (1996) showed that low critical leaf potential combined with thick roots enhancing axial hydraulic conductance can compensate for a poor root profile.

In reality, plant attributes cannot be regarded as independent of each other. It is better to consider them as a whole, varying together, in order to analyze the global plant strategy to water availability. Also, the three plant parameters (average root radius, potential of stomatal closure and shape of the root profile) are assumed not to vary with time. This is a fairly general assumption in crop water models (Ritchie, 1985b; Jones *et al.*, 1987) because the possible variation in these parameters is far smaller than the time variation due to root growth and atmospheric demand.

ON THE ROLE OF ENVIRONMENTAL FACTORS

The high sensitivity of *F* to soil type is apparently in contradiction with the statement of limited effect of soil hydraulic conductivity on water uptake. In fact, the critical available water *fraction* (*F*) depends on soil type only because it is expressed as a fraction of MAWC. Eqn. 13 shows that the critical available water *content* (*F* × MAWC) does not depend on soil. Obviously, this conclusion relies on the formulation adopted for the bulk soil hydraulic conductivity. Since the soil-dependency of *F* seems somewhat artificial, it is better to avoid it by using the critical available water content.

The result confirms that *F* is substantially sensitive to atmospheric demand. In 1971, Eagleman established that atmospheric demand was sufficient to explain the variability in the relationship between relative evapotranspiration rate and soil water content. In the present parameterization, the atmospheric demand is considered at the plant level. It is, thus, not only a matter of climate but also the amount of leaves. At the beginning of the crop cycle, water stress sensitivity is low as a result of a low leaf area index, which compensates for a superficial root system.

ON THE COMPENSATORY EFFECTS RESULTING FROM TIME-DEPENDENT VARIABLES

When *F* is calculated dynamically in a realistic framework of crop and climate evolution, steady-state conditions tend to develop more or less rapidly according to the local atmospheric conditions. They result from the opposing effects of root growth and atmospheric demand. In the example in Fig. 11, the period of stability starts 45 days after sowing and lasts until 10 days before physiological maturity. Similarly, Palacios and Quevedo (1996) proposed three values for *F* throughout the crop cycle, with a maximum during the reproductive stage.

Those results explain why approaches using constant *F* values work quite well when applied to the most sensitive crop phases as recommended for irrigation scheduling.

ON THE LIMITS OF THE APPROACH

Obviously, some limitations exist in view of the crude assumptions of the approach.

Firstly, steady-state daily conditions, which allow transpiration and uptake to be equal, cannot be assumed in the case of severe drought.

Secondly, assuming constant water-absorbing properties throughout the root profile, as well as constant hydraulic properties throughout the soil profile is not realistic (Aura, 1996; Bruckler *et al.*, 1991). Nevertheless, those assumptions are commonly used in water uptake models working at macroscopic levels (Molz, 1981; Habib *et al.*, 1991). In a precise experimental case, Kerr *et al.* (1993) observed no advantage in calculating the soil water balance by dividing the soil into numerous layers rather than treating it as a single layer.

In the literature, values for the optimal root length density ranged between 0.1 and 0.7 cm cm^{-3}. In spite of that, the parameter *lvopt* is considered constant, which assumes a certain pattern of root arrangement, which may not always be consistent with reality.

The present approach assumes that roots grow in density following the same strategy throughout crop cycle; this does not account for an eventual adaptative reaction of the plant root system to the environment. Moreover, the root profile is supposed not to be affected by soil whereas the soil is known to modify root dynamics in particular through its fertility.

Consequently, this work must be regarded as a simple way to adapt the convenient reservoir water balance model to various and evolving conditions without forgetting the limitations inherent of the single layer approach.

Conclusion

This study relates two approaches to root water uptake: a mechanistic one relying on Gardner's single-root description (1960) and an empirical one based on the linear relationship between relative transpiration rate and available soil water. With some simplifying assumptions, it is possible to derive an analytical formula for the critical available water as a function of plant attributes and atmospheric evaporative demand throughout the crop cycle. Information on soil type is required only for calculating the critical available water fraction, namely the ratio of the critical available water content to the maximum defined as the difference between field capacity and wilting point water contents.

Concerning the plant attributes, the average root radius has only a minor influence on the estimate of the critical available water content. The influence of the root profile is particularly important at the beginning of the cycle. After root growth is completed, the critical leaf potential and the atmospheric demand play the main role.

Actual plant attributes are likely to be interdependent and may contribute to a crop behaving uniformly regarding water uptake. The analytical formula (Eqn. 12), applied to realistic crop and climate conditions, indicates that crop growth can compensate for increasing atmospheric demand, so that considering F as a constant proportion of MAWC once the crop is fully developed can be accepted from a pragmatic point of view of irrigation management. Nevertheless, the chosen value for F is soil and crop dependent. The present work can help to obtain this value. To avoid the soil-dependency, it is better to consider the critical available water content rather than the critical available water fraction.

The interest in this approach lies in its simplicity in using the classical analogue of soil reservoir while avoiding the shortcomings of a purely empirical approach in accounting for the effects of plant attributes, growth and environment. However, the same limitations as for all single layer models remain.

In the framework of crop modelling, this analytical estimate of the critical available water content can be useful in calculating plant transpiration and water stress factors without using complex water transfer models. It is used in the model STICS (Brisson *et al.*, 1997, Brisson *et al.*, 1998b).

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