Hydrol. Earth Syst. Sci., 19, 2505–2511, 2015
www.hydrol-earth-syst-sci.net/19/2505/2015/
doi:10.5194/hess-19-2505-2015

Hydrology and Earth System Sciences
Open Access

Variability of flow discharge in lateral inflow-dominated stream channels

C.-M. Chang and H.-D. Yeh

Institute of Environmental Engineering, National Chiao Tung University, Hsinchu, Taiwan

Correspondence to: H.-D. Yeh (hdyeh@mail.nctu.edu.tw)

Received: 17 December 2014 – Published in Hydrol. Earth Syst. Sci. Discuss.: 25 February 2015
Revised: 30 April 2015 – Accepted: 9 May 2015 – Published: 27 May 2015

Abstract. The influence of the temporal changes in lateral inflow rate on the discharge variability in stream channels is explored through the analysis of the diffusion wave equation (i.e. the linearized Saint-Venant equation). To account for variability and uncertainty, the lateral inflow rate is regarded as a temporal random function. On the basis of the spectral representation theory, analytical expressions for the covariance function and evolutionary power spectral density of the random discharge perturbation process are derived to quantify variability in stream flow discharge induced by the temporal changes in lateral inflow rate. The treatment of the discharge variance (square root of the variance) gives us a quantitative estimate of uncertainty in predictions from the deterministic model. It is found that the discharge variability of stream flow is very large in the downstream reach, indicating large uncertainty anticipated from the use of the deterministic model. A larger temporal correlation scale of inflow rate fluctuations, representing more temporal consistency of fluctuations in inflow rate around the mean, introduces a higher variability in stream flow discharge.

1 Introduction

Surface runoff originates from precipitation intensities exceeding the infiltration capacity of the surface (e.g. Duan et al., 1992; Sivakumar et al., 2000; Ruiz-Villanueva et al., 2012; Valipour, 2015). This process may result in lateral inflow to nearby stream channels. Significant lateral inflows may contribute to streams during storm-runoff periods when stream reaches are of large lateral watershed areas or upslope accumulated areas (Jencso et al., 2009). These lateral inflows may be not only a source of water to streams, but also a source of contaminants to surface water. Agricultural chemicals are frequently mixed into shallow soil layers and lateral inflows may cause the release and migration of them into streams (Govindaraju, 1996). The effect of the lateral inflow on the stream flow provides an important basis for analyzing contaminant transport in surface water. Understanding and quantification of the influence of inflow process on stream flow discharge is therefore essential for water resource planning and management.

Natural variability, such as significant variability of rainfall events on both temporal and spatial scales (e.g. Ogden and Julien, 1993; Redano and Lorente, 1993; Wheater et al., 2000; Zhang et al., 2001; De Michele and Bernardara, 2005; Haberlandt et al., 2008; Valipour, 2012; Bewket and Lal, 2014) and the great heterogeneity of soil types at the ground surface (e.g. Jencso et al., 2009; Fournier et al., 2013) and surface saturation (e.g. Schumann et al., 2009; Riley and Shen, 2014) over a watershed, creates a very complex runoff process on the land surface. Many practical problems of flood wave routing require predictions over relatively large time and space scales. The key issue is how one can realistically incorporate the effect of natural heterogeneity into models to predict flood wave behavior at large time and space scales. Due to a high degree of the natural heterogeneity of the surface runoff process, the use of deterministic analysis techniques in stream flow modeling is inevitably subject to large uncertainty. The theoretical understanding of variability in flood wave routing is far from complete. Motivated by that, this article focuses on quantification of the discharge variability in a lateral-inflow-dominated stream.

In the following, the response of the transient stream flow process to spatiotemporal lateral inflow in a diffusion wave model is analyzed stochastically by treating the fluctuations

Published by Copernicus Publications on behalf of the European Geosciences Union.

in lateral inflow rate as temporal stationary random processes. The non-stationary spectral techniques are employed to obtain closed-form solutions for quantifying the discharge variability in stream channels. These solutions provide variance relations for flow discharge, and thereby allow for assessing the impact of statistical properties of lateral inflow rate process on the discharge variability.

To the best of our knowledge, the issue on quantifying the effect of temporal variation of lateral inflow on the stream flow variability using non-stationary spectral techniques so far has not been addressed. The approach presented herein provides not only an analytical methodology but also a basic framework for understanding the response of transient stream flow process and quantifying the stream flow variability. It is hoped that the proposed approach and our findings obtained in this study are useful for further research in this area.

2 Description of the problem

This study considers the case of unsteady flow in open channels. The equations that describe the propagation of a flood wave with respect to distance along the channel and time in open channels are the so-called Saint-Venant equations, consisting of the continuity equation and the momentum equation. For most flood events, in most rivers the inertial terms appearing in the momentum equation of the Saint-Venant equations can be neglected as they are relatively smaller than the terms arising from gravity and resistance forces (Henderson, 1963; Dooge and Harley, 1967; Daluz Viera, 1983), leading to a simplified model of open channel flow. The diffusion wave equation is then expressed as (e.g. Moussa, 1996; Sivapalan et al., 1997)

$$\frac{\partial Q}{\partial t}+C_{\mathrm{d}}\left(Q,\frac{\partial Q}{\partial X}\right)\left[\frac{\partial Q}{\partial X}-q_{\mathrm{L}}\right]=\frac{1}{\sqrt{S_0}}\frac{\partial}{\partial X}\left[D_{\mathrm{h}}(Q)\left(\frac{\partial Q}{\partial X}-q_{\mathrm{L}}\right)\right], \tag{1}$$

where Q is the discharge, C_{d} and D_{h} are non-linear functions of discharge generally known as wave celerity and hydraulic diffusivity, respectively, S_0 is the bed slope, and $q_{\mathrm{L}}(X,t)$ represents the net lateral inflow distribution. The diffusion wave Eq. (1) is formulated by combining the continuity equations for both mass and momentum. The diffusion wave approximation is appropriate for simulations of the flood waves in rivers and on flood plains with milder slopes ranging between 0.001 and 0.0001 (Kazezyılmaz-Alhan, 2012). Most natural flood waves can then be described with the diffusion wave model. Some of the successful applications of the simplified channel flow models to flood routing are available in the literature (e.g. Ponce et al., 1978; Singh and Aravamuthan, 1995; Moramarco and Singh, 2002; Khasraghi et al., 2015).

Equation (1) is a nonlinear partial differential equation and has a complex behavior of the stream flow in general. No analytical solution of Eq. (1) is available in the literature. However, the problem can be solved analytically by some simplifications to Eq. (1), such as linearization for the case of an initially steady uniform flow. On the basis of expansion of the dependent variable and the nonlinear terms in Eq. (1) around the initial condition of steady uniform flow and limitation of the expansion to the first-order variation from the steady state, the resulting linearized Eq. (1) can be written as

$$\frac{\partial Q'}{\partial t}=D\frac{\partial^2 Q'}{\partial X^2}-C\frac{\partial Q'}{\partial X}+\left[Cq_{\mathrm{L}}-D\frac{\partial q_{\mathrm{L}}}{\partial X}\right]. \tag{2}$$

In Eq. (2), $Q'=Q-Q_0$ ($Q_0\gg Q'$, q_{L}), Q_0 is the initial uniform steady-state flow discharge, and C and D represent constant celerity and diffusivity, respectively, depending on the initially uniform flow (velocity and flow depth). The reader may be referred to Dooge and Napiorkowski (1987), Ponce (1990), Yen and Tsai (2001) or Tsai and Yen (2001) for the detailed development.

The problem of interest here is the stream flow response to the temporal changes in lateral inflow rate, which is governed by Eq. (2). The solution to Eq. (2) with associated initial and boundary conditions will serve as the starting point for conducting the following investigation of stream flow variability.

To derive the analytical solution of Eq. (2), one needs to specify the form of $q_{\mathrm{L}}(X,t)$. In the present work, the focus is placed on the case that the net lateral inflow is well-approximated by the following spatiotemporal distribution (e.g. Lane, 1982; Capsoni et al., 1987; Goodrich et al., 1997; Féral et al., 2003).

$$q_{\mathrm{L}}(X,t)=q_M(t)\exp\left(-\frac{X}{\eta}\right), \tag{3}$$

where q_M is the peak inflow rate, and η is the distance along the x axis for which the inflow rate decreases by a factor e^{-1} with respect to q_M. In particular, q_M is considered to be a temporally correlated stationary random field. It is apparent from Eq. (2) that the last two terms associated with the lateral inflow are introduced as the sources of fluctuations in stream flow discharge and treated here as temporally correlated stochastic processes. Equation (2) is then viewed as a stochastic differential equation with a stochastic output Q'. The solution of Eq. (2) will provide a rational basis for quantifying the flow variability through the representation theorem.

Consider that the flow domain is bounded within the range $0\le X\le L$. The associated initial and boundary conditions can be expressed as

$$Q'(X,0)=0, \tag{4a}$$

$$Q'(0,t)=0, \tag{4b}$$

$$\frac{\partial}{\partial X}Q'(L,t)=0. \tag{4c}$$

Equation (4) signifies that there is no perturbation from the reference discharge initially while Eq. (5) assumes no inflow at the upstream boundary at all times. The downstream boundary condition represented by Eq. (6) is under the condition of a zero-discharge gradient. Morris (1979) showed that this downstream boundary condition is applicable to a large class of problems.

3 General solutions via spectral theory

The approach followed is to develop the analytical solution of Eq. (2) in the Fourier frequency domain.

Temporal stationarity of the q_M perturbation process admits a spectral representation of the form (e.g. Priestley, 1965)

$$q_{\mathrm{L}} = q_M(t)\exp\left(-\frac{X}{\eta}\right) = \exp\left(-\frac{X}{\eta}\right)\int_{-\infty}^{\infty} e^{i\omega t}\,\mathrm{d}Z_q(\omega), \quad (5)$$

where ω is the frequency parameter, $Z_q(\omega)$ is an orthogonal process, and $\mathrm{d}Z_q$ is a zero-mean orthogonal increment process with

$$E\left[\mathrm{d}Z_q(\omega_1)\,\mathrm{d}Z_q^*(\omega_2)\right] = S_{qq}(\omega_1)\,\delta(\omega_1-\omega_2)\,\mathrm{d}\omega_1\mathrm{d}\omega_2, \quad (6)$$

in which E [–] denotes the ensemble average, the superscript asterisk stands for the complex-conjugation operator, and S_{qq} (–) is the power spectral density for the stationary random q_M perturbation process. On the other hand, without the restriction on the assumption of stationarity the random perturbed quantities Q' may be expressed in the form of the Fourier–Stieltjes integral representation as (e.g. Priestley, 1965; Li and McLaughlin, 1991)

$$Q'(X,t) = \int_{-\infty}^{\infty} \Theta_{Qq}(X,t,\omega)\,\mathrm{d}Z_q(\omega), \quad (7)$$

where Θ_{Qq} (–) is the transfer function depending on space, time, and frequency.

It follows from Eqs. (5) and (7) that Eq. (2) takes the form

$$\frac{\partial \Theta_{Qq}}{\partial t} = D\frac{\partial^2\Theta_{Qq}}{\partial X^2} - C\frac{\partial\Theta_{Qq}}{\partial X} + \exp\left(-\frac{X}{\eta}+i\omega t\right)\left(C+\frac{D}{\eta}\right), \quad (8)$$

subject to the following initial and boundary conditions

$$\Theta_{Qq}(X,0) = 0, \quad (9a)$$

$$\Theta_{Qq}(0,t) = 0, \quad (9b)$$

$$\frac{\partial\Theta_{Qq}}{\partial X}(L,t) = 0. \quad (9c)$$

The method of eigenfunction expansion is used to solve this inhomogeneous boundary value problem, and the solution of Eq. (8) with Eq. (9) is

$$\Theta_{Qq}(X,t,\omega) = 2\frac{D}{L}\left(\upsilon+\frac{1}{\mu}\right)\exp\left(\frac{\upsilon}{2}\xi\right)\sum_{n=0}^{\infty} \frac{a_n-\beta\exp(-\beta)\cos(n\pi)}{\beta^2+a_n^2}\sin(a_n\xi) \times \frac{\exp(-i\omega t)-\exp(-F_n t)}{F_n-i\omega}, \quad (10)$$

where $\upsilon = CL/D$, $\mu=\eta/L$, $a_n=\pi(2n+1)/2$, $\xi = X/L$, $\beta=(\upsilon/2)+1/\mu$, and $F_n = D[a_n^2+\upsilon^2/4]/L^2$. Rewriting Eq. (7), and using Eq. (10), yields the solution of Eq. (2) in the frequency domain as

$$Q'(X,t) = 2\frac{D}{L}\left(\upsilon+\frac{1}{\mu}\right)\exp\left(\frac{\upsilon}{2}\xi\right)\sum_{n=0}^{\infty} \frac{a_n-\beta\exp(-\beta)\cos(n\pi)}{\beta^2+a_n^2}\sin(a_n\xi)\times\int_{-\infty}^{\infty} \frac{\exp(-i\omega t)-\exp(-F_n t)}{F_n-i\omega}\mathrm{d}Z_q(\omega). \quad (11)$$

The covariance function of the flow discharge field, C_{QQ} (–), can be computed on the basis of the representation theorem for Q' by

$$\begin{aligned} C_{QQ}(X,t_1,t_2) &= E\left[Q'(X,t_1)\,Q'^*(X,t_2)\right] \\ &= \int_{-\infty}^{\infty}\Theta_{Qq}(X,t_1,\omega)\,\Theta_{Qq}^*(X,t_2,\omega)\,S_{qq}(\omega)\mathrm{d}\omega \\ &= 4\frac{D^2}{L^2}\left(\upsilon+\frac{1}{\mu}\right)^2\exp(\upsilon\xi)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{\sin(a_m\xi)\sin(a_n\xi)}{\left(\beta^2+a_m^2\right)\left(\beta^2+a_n^2\right)}\times\{a_m a_n \\ &\quad -\beta\exp(-\beta)\left[a_m(-1)^m+a_n(-1)^n\right] \\ &\quad +\beta^2(-1)^{m+n}\exp(-2\beta)\Big\}\times\int_{-\infty}^{\infty} \frac{\exp\left[i\omega\left(t_1-t_2\right)\right]-\exp\left(-F_m t_1-i\omega t_2\right)-\exp\left(i\omega t_1-F_n t_2\right)+\exp\left(F_m t_1+F n t_2\right)}{\left(F_m F_n+\omega^2\right)+i\frac{D}{L^2}\left(a_n^2-a_m^2\right)\omega} \\ &\quad S_{qq}(\omega)\mathrm{d}\omega, \end{aligned} \quad (12)$$

where $a_m=\pi(2m+1)/2$ and $F_m = D[a_m^2+\upsilon^2/4]/L^2$. The variance of flow discharge fluctuations is obtained by evaluating Eq. (12) at zero time lag as

$$\sigma_Q^2(X,t) = C_{QQ}(X,t,t) = 4\frac{D^2}{L^2}\left(\upsilon+\frac{1}{\mu}\right)^2 \exp(\upsilon\xi) \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\sin(a_m\xi)\sin(a_n\xi)}{\left(\beta^2+a_m^2\right)\left(\beta^2+a_n^2\right)} \times \{a_m a_n - \beta\exp(-\beta) \left[a_m(-1)^m + a_n(-1)^n\right] + \beta^2(-1)^{m+n}\exp(-2\beta)\} \times \int_{-\infty}^{\infty} \frac{1-\exp[-(F_m+i\omega)t] - \exp[(i\omega-F_n)t] + \exp[(F_m+F_n)t]}{\left(F_mF_n+\omega^2\right)+i\frac{D}{L^2}\left(a_n^2-a_m^2\right)\omega} S_{qq}(\omega)\mathrm{d}\omega. \quad (13)$$

In addition, following Priestley (1965), the variance of the Q' process may be written in the form of

$$\sigma_Q^2(X,t) = \int_{-\infty}^{\infty} |A_t(X,t,\omega)|^2 E\left[\mathrm{d}Z_q(\omega)\mathrm{d}Z_q^*(\omega)\right], \quad (14)$$

so that the evolutionary power spectral density of the non-stationary random process can be defined as

$$E\left[\mathrm{d}Z_Q(X,t,\omega)\mathrm{d}Z_Q^*(X,t,\omega)\right] = |A_t(X,t,\omega)|^2 E\left[\mathrm{d}Z_q(\omega)\mathrm{d}Z_q^*(\omega)\right], \quad (15)$$

where A_t (–) is referred to as the modulating function of the non-stationary process. The evolutionary spectrum has the same physical interpretation as the spectrum of a stationary process, namely, that it describes the distribution of mean square signal content (or fluctuations) of the random process at a given time t. Comparing Eq. (14) to Eq. (13) leads Eq. (15) to

$$S_{QQ}(X,t,\omega) = 4\frac{D^2}{L^2}\left(\upsilon+\frac{1}{\mu}\right)^2 \exp(\upsilon\xi)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{\sin(a_m\xi)\sin(a_n\xi)}{\left(\beta^2+a_m^2\right)\left(\beta^2+a_n^2\right)} \times \{a_m a_n - \beta\exp(-\beta) \left[a_m(-1)^m + a_n(-1)^n\right] + \beta^2(-1)^{m+n}\exp(-2\beta)\} \times \frac{1-\exp[-(F_m+i\omega)t] - \exp[(i\omega-F_n)t] + \exp[(F_m+F_n)t]}{\left(F_mF_n+\omega^2\right)+i\frac{D}{L^2}\left(a_n^2-a_m^2\right)\omega} S_{qq}(\omega), \quad (16)$$

where S_{QQ} (–) is the spectral density of the Q' perturbation process.

The infinite series in Eq. (10) converges rapidly when $\tau_c = Dt/L^2 \gg 1/\pi^2$. Accordingly, Eq. (10) can reduce to

$$\Theta_{Qq}(X,t,\omega) = \frac{D}{L}\left(\upsilon+\frac{1}{\mu}\right)\frac{\pi-2\beta\exp(-\beta)}{\beta^2+\frac{\pi^2}{4}}\exp\left(\frac{\pi}{2}\xi\right) \sin\left(\frac{\pi}{2}\xi\right)\frac{\exp(i\omega t)-\exp(-\tau)}{\rho+i\omega}, \quad (17)$$

where $\rho = D[\pi^2+\upsilon^2]/(4L^2)$ and $\tau = \rho t$. The timescale of the hydraulic system, τ_c, is referred to as the hydraulic response time (Gelhar, 1993). Here, it is interpreted as the characteristic time for a change in upstream discharge to reach the downstream end of the stream. For most practical applications it is much greater than unity, which is the main interest of this study.

The use of Eq. (17), in turn, simplifies Eqs. (13) and (16), respectively, to

$$\sigma_Q^2(X,t) = \frac{D^2}{L^2}\left(\upsilon+\frac{1}{\mu}\right)^2\frac{[\pi-2\beta\exp(-\beta)]^2}{\left(\beta^2+\frac{\pi^2}{4}\right)^2}\exp(\upsilon\xi) \sin^2\left(\frac{\pi}{2}\xi\right)\times\int_{-\infty}^{\infty}\frac{1-2\exp(-\tau)\cos(\omega t)+\exp(-2\tau)}{\omega^2+\rho^2} S_{qq}(\omega)\mathrm{d}\omega, \quad (18)$$

$$S_{QQ}(X,t,\omega) = \frac{D^2}{L^2}\left(\upsilon+\frac{1}{\mu}\right)^2\frac{[\pi-2\beta\exp(-\beta)]^2}{\left(\beta^2+\frac{\pi^2}{4}\right)^2}\exp(\upsilon\xi) \sin^2\left(\frac{\pi}{2}\xi\right)\times\frac{1-2\exp(-\tau)\cos(\omega t)+\exp(-2\tau)}{\omega^2+\rho^2} S_{qq}(\omega). \quad (19)$$

Equation (19) states that the spectrum of the discharge is a result of a competitive relation between the signal frequency and the properties of the stream channel and inflow. Generally, it is very difficult to quantify the variability of inflow rate. Equation (19) thus provides information about the nature of inflow processes. For example, on the basis of an observed discharge perturbation time series with known hydraulic parameters, the nature of inflow processes may be determined from Eq. (19). After normalizing by the spectral density S_{qq} (–), the evolutionary power spectral density Eq. (19) as a function of dimensionless frequency for various time scales and locations are graphed in Fig. 1a and b, respectively. It shows that the spatial variation of spectral amplitude associated with a given frequency increases with the time and the distance from the upstream boundary as well. It reveals that the variability of flow discharge increases with time and distance.

4 Closed-form expressions for the variance and spectral density of discharge fluctuations

In this work, the spectrum of red noise is used to evaluate Eqs. (18) and (19) explicitly. The analysis of discharge variability in this section assumes an exponential form for the autocovariance function of the random fluctuations in the peak inflow rate (Jin and Duffy, 1994; Kumar and Duffy, 2009), namely,

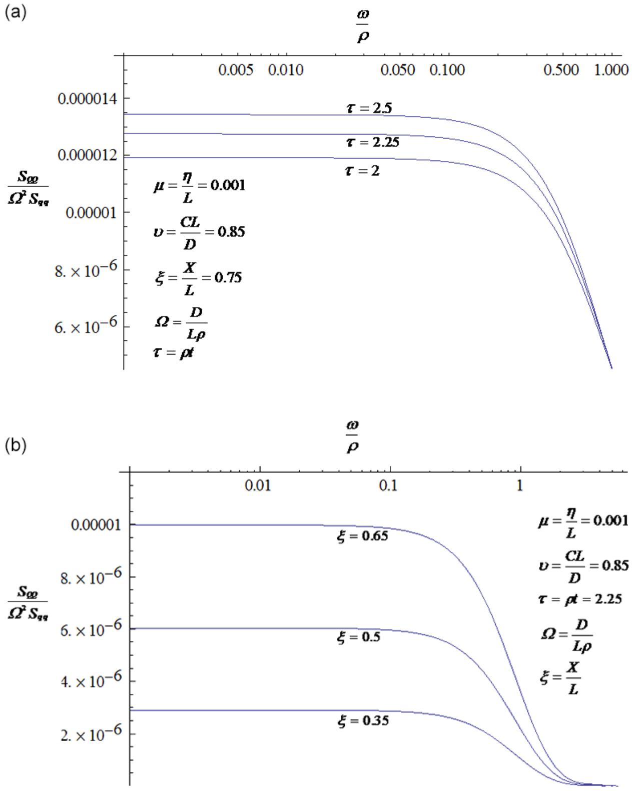

Figure 1. Dimensionless evolutionary power spectral density as a function of dimensionless frequency for various **(a)** time scales and **(b)** locations.

$$C_{qq}\left(\ell_{\mathrm{S}}\right)=\sigma_q^2 \exp\left(-\frac{|\ell_{\mathrm{S}}|}{\lambda}\right), \tag{20a}$$

which has the following spectral density function:

$$S_{qq}(\omega)=\frac{\sigma_q^2\lambda}{\pi\left(1+\lambda^2\omega^2\right)}, \tag{20b}$$

where ℓ_{S} is the time lag and σ_q^2 and λ are, respectively, the variance and temporal correlation scale of peak inflow rate fluctuations.

Upon substituting Eq. (20b) into Eq. (18) and integrating it over the frequency domain, one obtains the following expression for the variance of flow discharge fluctuations as

$$\sigma_Q^2(X,t)=16\sigma_q^2 L^2\frac{\left(\upsilon+\frac{1}{\mu}\right)^2}{\left(\pi^2+\upsilon^2\right)^2}\frac{\left[\pi-2\beta\exp(-\beta)\right]^2}{\left(\beta^2+\frac{\pi^2}{4}\right)^2}$$
$$\exp(\upsilon\xi)\sin^2\left(\frac{\pi}{2}\xi\right)\times\tau_R\left\{\frac{1+\exp(-2\tau)}{1+\tau_R}\right.$$
$$\left.-2\frac{\exp(-\tau)}{1-\tau_R^2}\left[\exp(-\tau)-\tau_R\exp\left(-\frac{\tau}{\tau_R}\right)\right]\right\}, \tag{21}$$

where $\tau_R=\rho\lambda$. Equation (21) indicates a linear relationship between the variances of fluctuations in the flow discharge and inflow rate, implying that the flow variability increases linearly with the heterogeneity of the inflow rate. With Eq. (20b), the resulting expression for the evolutionary power spectral density in Eq. (19) is given by

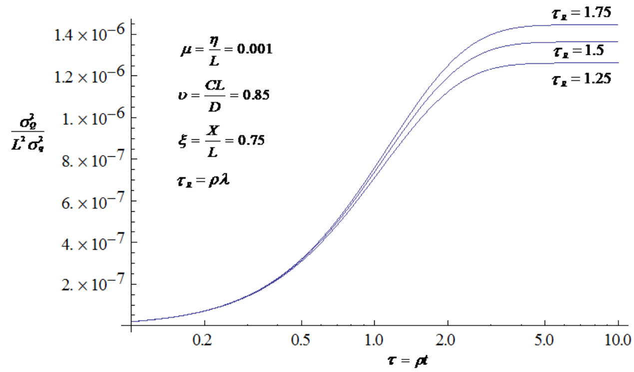

Figure 2. Dimensionless variance of discharge fluctuations as a function of dimensionless time for various dimensionless temporal correlation scales of inflow rate fluctuations.

$$S_{QQ}(X,t,\omega)=\frac{\sigma_q^2}{\pi}\frac{D^2}{L^2}\left(\upsilon+\frac{1}{\mu}\right)^2\frac{\left[\pi-2\beta\exp(-\beta)\right]^2}{\left(\beta^2+\frac{\pi^2}{4}\right)^2}$$
$$\exp(\upsilon\xi)\sin^2\left(\frac{\pi}{2}\xi\right)$$
$$\times\frac{1-2\exp(-\tau)\cos(\omega t)+\exp(-2\tau)}{\omega^2+\rho^2}$$
$$\frac{\lambda}{1+\lambda^2\omega^2}. \tag{22}$$

Figure 2 shows the plot of the dimensionless variance of discharge fluctuations in Eq. (21) as a function of dimensionless time for various dimensionless temporal correlation scales of inflow rate fluctuations. The figure indicates that the variability of flow discharge induced by the variation of inflow rate increases gradually with time toward its asymptotic value at large time. The correlation scale provides a measure of the strength of the persistence of fluctuations around the mean. It is anticipated that the stochastic processes will exhibit rather clear trends with relatively little noise (a smoother data profile) if the correlation scale is larger. In other words, the temporal fluctuations in inflow rate are either consistently above or below the profile of mean inflow rate in the case of a larger temporal correlation scale. Those larger inclusions in turn lead to larger deviations of flow discharge from the initially uniform steady-state flow discharge.

Variation of flow discharge with the distance from the upstream boundary is depicted in Fig. 3 according to Eq. (21). As noted in the figure, the variability of flow discharge grows monotonically with distance, implying that due to the naturally inherent variability of lateral inflow, uncertainty in the flow discharge calculations from a deterministic model increases with the distance from the upstream boundary. In

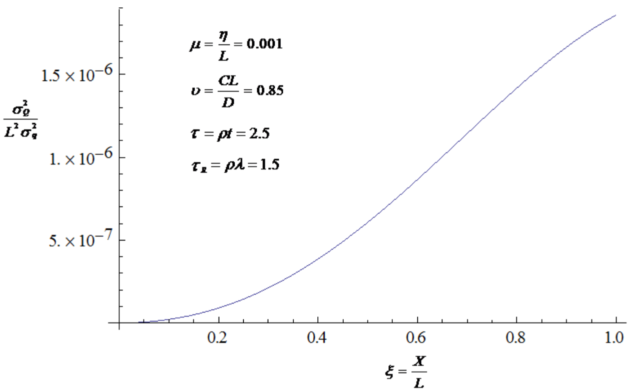

Figure 3. Dimensionless variance of discharge fluctuations as a function of dimensionless distance from the upstream boundary.

other words, the prediction of flow discharge distribution based on the deterministic simulation results is subject to the largest uncertainty in the downstream region. The downstream region is important in most real applications of modeling, and Eq. (21) provides a way of assessing the variation around the deterministic model prediction.

Many practical applications involving prediction over a large scale require measurement of uncertainty. Standard deviation is the best way to accomplish that. In this sense, the prediction results from a deterministic model are treated as the mean values. The mean value plus one standard deviation (square root of Eq. (21) provides a rational basis for extrapolating relatively small-scale field observations to these large space scales. Moreover, the likelihood of the flow discharge falling in the range of one standard deviation greater and smaller than the mean is about 68.27 %.

5 Conclusions

The problem of fluctuations in flow discharge in open channels in response to temporal changes in lateral inflow rate is investigated stochastically for a finite flow domain. In this study, the inflow perturbation field is modeled as a temporally stationary random process. For a complete stochastic description of flow discharge variability, expressions for the covariance function and evolutionary power spectral density of the random flow discharge perturbation process are developed. These expressions are obtained using a spectral representation theory. The variance relation developed here provides a rational basis for quantifying the uncertainty in applying the deterministic model.

This work represents an initial step in stochastic study of the effect of temporal variation of lateral inflow on the stream flow discharge variability. To take the advantage of a closed-form solution, the linearized diffusion wave equation (Eq. 2) is therefore used as the starting point for this research. It is important to recognize that the results developed in this work are valid only for the case of small variations in flow discharge around an initially uniform flow regime.

It is found from our closed-form expressions that the discharge variability in stream channels induced by the temporal changes in lateral inflow rate increases gradually with time toward its asymptotic value at large time. A larger temporal correlation scale of inflow rate fluctuations, which is of a more persistence of inflow perturbation process, will introduce more variability of the flow discharge. The increase of discharge variability with the distance from the upstream boundary suggests that prediction of flow discharge distribution in channels using a deterministic model is subject to large uncertainty at the downstream reach of the stream.

Acknowledgements. This research work is supported by the Taiwan Ministry of Science Technology under the grants NSC 101-2221-E-009-105-MY2, 102-2221-E-009-072-MY2 and NSC 102-2218-E-009-013-MY3. We are grateful to the editor Zhongbo Yu and anonymous referees for constructive comments that improved the quality of the work.

Edited by: Z. Yu

References

Bewket, W. and Lal, R.: Recent spatiotemporal temperature and rainfall variability and trends over the Upper Blue Nile River Basin, Ethiopia, Int. J. Climatol., 34, 2278–2292, 2014.

Capsoni, C., Fedi, F., Magistroni, C., Paraboni, A., and Pawlina, A.: Data and theory for a new model of the horizontal structure of rain cells for propagation applications, Radio Sci., 22, 395–404, 1987.

Daluz Viera, J. H.: Conditions governing the use of approximations for the Saint-Venant equations for shallow water flow, J. Hydrol., 60, 43–58, 1983.

De Michele, C. and Bernardara, P.: Spectral analysis and modeling of space-time rainfall fields, Atmos. Res., 77, 124–136, 2005.

Dooge, J. C. I. and Harley, B. M.: Linear routing in uniform channels, Proc. Int. Hydrol. Symp., 1, 57–63, 1967.

Dooge, J. C. I. and Napiorkowski, J. J.: The effect of the downstream boundary condition in the linearized St. Venant equations, Q. J. Mech. Appl. Math., 40, 245–256, 1987.

Duan, Q., Sorooshian, S., and Gupta, V.: Effective and efficient global optimization for conceptual rainfallrunoff models, Water Resour. Res., 28, 1015–1031, 1992.

Féral, L., Sauvageot, H., Castanet, L., and Lemorton, J.: A new hybrid model of the rain horizontal distribution for propagation studies: 1. Modeling of the rain cell, Radio Sci., 38, 1056, doi:10.1029/2002RS002802, 2003.

Fournier, B., Guenat, C., Bullinger-Weber, G., and Mitchell, E. A. D.: Spatio-temporal heterogeneity of riparian soil morphology in a restored floodplain, Hydrol. Earth Syst. Sci., 17, 4031–4042, doi:10.5194/hess-17-4031-2013, 2013.

Gelhar, L. W.: Stochastic Subsurface Hydrology, Englewood Cliffs, Prentice Hall, New Jersey, 1993.

Goodrich, D. C., Lane, L. J., Shillito, R. M., Miller, S. N., Syed, K. H., and Woolhiser, D. A.: Linearity of basin response as a function of scale in a semiarid watershed, Water Resour. Res., 33, 2951–2965, 1997.
Govindaraju, R. S.: Modeling overland flow contamination by chemicals mixed in shallow soil horizons under variable source area hydrology, Water Resour. Res., 32, 753–758, 1996.
Haberlandt, U., Ebner von Eschenbach, A.-D., and Buchwald, I.: A space-time hybrid hourly rainfall model for derived flood frequency analysis, Hydrol. Earth Syst. Sci., 12, 1353–1367, doi:10.5194/hess-12-1353-2008, 2008.
Henderson, F. M.: Flood waves in prismatic channels, ASCE J. Hydr. Div., 89, 39–67, 1963.
Jencso, K. G., McGlynn, B. L., Gooseff, M. N., Wondzell, S. M., Bencala, K. E., and Marshall, L. A.: Hydrologic connectivity between landscapes and streams: Transferring reach- and plot-scale understanding to the catchment scale, Water Resour. Res., 45, W04428, doi:10.1029/2008WR007225, 2009.
Jin, M. and Duffy, C. J.: Spectral and bispectral analysis for single and multiple input nonlinear phreatic aquifer systems, Water Resour. Res., 30, 2073–2095, 1994.
Kazezyılmaz-Alhan, C. M.: An improved solution for diffusion waves to overland flow, Appl. Math. Model., 36, 4165–4172, 2012.
Khasraghi, M. M., Sefidkouhi, M. A. G., and Valipour, M.: Simulation of open- and closed-end border irrigation systems using SIRMOD, Arch. Agron. Soil Sci., 61, 929–941, 2015.
Kumar, M. and Duffy, C. J.: Detecting hydroclimatic change using spatio-temporal analysis of time series in Colorado River Basin, J. Hydrol., 374, 1–15, 2009.
Lane, L. J.: Distributed model for small semiaridw atershedsJ, Hydraul. Div. Am. Soc. Civ. Eng., 108, 1114–1131, 1982.
Li, S.-G. and McLaughlin, D.: A nonstationary spectral method for solving stochastic groundwater problems: Unconditional analysis, Water Resour. Res., 27, 1589–1605, 1991.
Moramarco, T. and Singh, V. P.: Accuracy of kinematic wave and diffusion wave for spatial-varying rainfall excess over a plane, Hydrol. Process., 16, 3419–3435, 2002.
Morris, E. M.: The effect of small slope approximation and lower boundary conditions on solution of Saint Venant equations, J. Hydrol., 40, 31–47, 1979.
Moussa, R.: Analytical Hayami solution for the diffusive wave flood routing problem with lateral inflow, Hydrol. Process., 10, 1209–1227, 1996.
Ogden, F. L. and Julien, P. Y.: Runoff sensitivity to temporal and spatial rainfall variability at runoff plane and small basin scales, Water Resour. Res., 29, 2589–2597, 1993.
Ponce, V. M.: Generalized diffusion wave equation with inertial effects, Water Resour. Res., 26, 1099–1101, 1990.
Ponce, V. M., Li, R. M., and Simons, D. B.: Applicability of kinematic and diffusion models, J. Hydraul. Div.-ASCE, 104, 353–360, 1978.
Priestley, M. B.: Evolutionary spectra and non-stationary processes, J. Roy. Stat. Soc. Ser. B, 27, 204–237, 1965.
Redano, A. and Lorente, J.: Modelling the spatial and temporal distribution of rainfall intensity at local scale, Theor. Appl. Climatol., 47, 25–32, 1993.
Riley, W. J. and Shen, C.: Characterizing coarse-resolution watershed soil moisture heterogeneity using fine-scale simulations, Earth Syst. Sci., 18, 2463–2483, 2014.
Ruiz-Villanueva, V., Borga, M., Zoccatelli, D., Marchi, L., Gaume, E., and Ehret, U.: Extreme flood response to short-duration convective rainfall in South-West Germany, Hydrol. Earth Syst. Sci., 16, 1543–1559, doi:10.5194/hess-16-1543-2012, 2012.
Schumann, G., Lunt, D. J., Valdes, P. J., de Jeu, R. A. M., Scipal, K., and Bates, P. D.: Assessment of soil moisture fields from imperfect climate models with uncertain satellite observations, Earth Syst. Sci., 13, 1545–1553, 2009.
Singh, V. P. and Aravamuthan, V.: Accuracy of kinematic wave and diffusion wave approximations for time-independent flows, Hydrol. Process., 9, 755–782, 1995.
Sivakumar, B., Berndtsson, R., Olsson, J., Jinno, K., and Kawamura, A.: Dynamics of monthly rainfall-runoff process at the Gota basin: A search for chaos, Hydrol. Earth Syst. Sci., 4, 407–417, doi:10.5194/hess-4-407-2000, 2000.
Sivapalan, M., Bates, B. C., and Larsen, J. E.: A generalized, nonlinear, diffusion wave equation: theoretical development and application, J. Hydrol., 192, 1–16, 1997.
Tsai, C. W.-S. and Yen, B. C.: Linear analysis of shallow water wave propagation in open channels, J. Eng. Mech., 127, 459–472, 2001.
Valipour, M.: Critical areas of Iran for agriculture water management according to the annual rainfall, Eur. J. Sci. Res., 84, 600–608, 2012.
Valipour, M.: Long-term runoff study using SARIMA and ARIMA models in the United States, Meteorol. Appl., doi:10.1002/met.1491, in press, 2015.
Wheater, H. S., Isham, V. S., Cox, D. R., Chandler, R. E., Kakou, A., Northrop, P. J., Oh, L., Onof, C., and Rodriguez-Iturbe, I.: Spatial-temporal rainfall fields: modelling and statistical aspects, Hydrol. Earth Syst. Sci., 4, 581–601, doi:10.5194/hess-4-581-2000, 2000.
Yen, B. C. and Tsai, C. W.-S.: On noninertia wave versus diffusion wave in flood routing, J. Hydrol., 244, 97–104, 2001.
Zhang, X., Hogg, W. D., and Mekis, E.: Spatial and temporal characteristics of heavy precipitation events over Canada, J. Climate, 14, 1923–1936, 2001.