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A new framework for resolving conflicts over transboundary rivers using bankruptcy methods

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Abstract. A novel bankruptcy approach is proposed for resolving transboundary river conflicts in which the total water demand or claim of the riparian parties is more than the available water. Bankruptcy solution methods can allocate the available water to the conflicting parties with respect to their claims. Four commonly used bankruptcy methods in the economic literature are used here to develop new river bankruptcy solution methods for allocating water to the riparian parties of river systems. Given the non-uniform spatial and temporal distribution of water across river basins, the proposed solution methods are formulated as non-linear network flow optimization models to allocate water with respect to time sensitivity of water deliveries at different locations in a river network during the planning horizon. Once allocation optimization solutions are developed, their acceptability and stability must be evaluated. Thus, a new bankruptcy allocation stability index (BASI) is developed for evaluating the acceptability of river bankruptcy solutions. To show how the proposed river bankruptcy framework can be helpful in practice, the suggested methods are applied to a real-world transboundary river system with eight riparians under various hydrologic regimes. Stability analysis based on the proposed stability evaluation method suggests that the acceptability of allocation rules is sensitive to hydrologic conditions and demand values. This finding has an important policy implication suggesting that fixed allocation rules and treaties may not be reliable for securing cooperation over transboundary water resources as they are vulnerable to changing socioeconomic and climatic conditions as well as hydrologic nonstationarity.

1 Introduction

Conflicts are integral to managing transboundary rivers due to the externalities associated with growing demand and development in riparian states. There are 148 riparian countries creating about 276 transboundary river basins in the world (De Stefano et al., 2012). These basins cover over 45 % of the Earth's land surface and provide about 60 % of the global river flows (Wolf et al., 2006). To facilitate cooperation over transboundary rivers, over 400 international agreements were signed in the 20th century (De Stefano et al., 2012), reflecting a great potential for cooperation over transboundary natural resources (Wolf et al., 2006). However, the stability of these agreements could be affected by socioeconomic and political changes in the riparian states as well as climatic and hydrologic variations. Dinar et al. (2010) reported 112 complaints about water deficit in transboundary river systems during droughts and floods in the 1950–2005 period, underlying the vulnerability of cooperation over transboundary water systems to abnormal hydrologic conditions.

Game theory – the mathematical study of competition and cooperation – is a useful method for studying transboundary river management problems. Both non-cooperative and cooperative game theory methods have been used in the past to study transboundary water conflicts (Parrachino et al., 2006; Madani, 2010).

Non-cooperative game theoretic methods are useful in studying the strategic behaviors of riparian parties, feasibility of cooperative solutions, and providing strategic insights into the conflicts (Madani and Hipel, 2011; Madani, 2013). Example transboundary river conflicts analyzed by

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non-cooperative game theory concepts include the conflict over flooding of Ganges and Brahmaputra rivers between India and Pakistan (Rogers, 1969), the lower Mekong river basin conflict between Cambodia, Laos, Thailand, and Vietnam (Dufournaud, 1982), the Jordan river conflict between Jordan, Israel, Lebanon, Palestine, and Syria (Madani and Hipel, 2007), and the Nile river conflict between Burundi, Congo, Egypt, Eritrea, Ethiopia, Kenya, Rwanda, Sudan, Tanzania, and Uganda (Elimam et al., 2008). These methods normally rely on qualitative information to find the likely outcomes of conflicts based on various stability definitions, which incorporate a range of decision makers' (players') characteristics such as risk attitude, foresight level, and information quality (Madani and Hipel, 2011; Madani, 2013). While these methods provide valuable insights into strategic conflicts and can help find possible resolutions to the conflict, their results are not necessarily quantitative and in most cases are only appropriate for studying as games with discrete solutions (strategies or actions).

Cooperative game theory solution methods normally seek to allocate the incremental benefits of cooperation (cost savings, added values, etc.) among the cooperating parties. In the context of transboundary river management, cooperative game theory concepts can be used to develop functional water allocation schemes. Example transboundary river conflicts analyzed by cooperative game theory include the Ganges river conflict between Bangladesh and India (Kilgoure and Dinar, 2001), the Euphrates and Tigris rivers conflict between Iraq, Syria, and Turkey (Kucukmehmetoglu and Guldmen, 2004), and the Syr Darya river basin conflict between Kyrgyzstan, Uzbekistan, and Kazakhstan (Teasley and McKinney, 2011). Cooperative game theory methods are appropriate for quantitative problems with a continuous solution domain.

Different resource allocation methods have been employed in the water resources literature to increase systems' efficiency and minimize conflicts. Social welfare maximization is perhaps the mostly commonly used approach for water allocation in the literature. Based on this approach, a social (central) planner seeks to maximize the system-wide benefits assuming there is perfect cooperation among the water users. The social planner approach pays minimal attention to individual gains and distribution of total benefits among the beneficiaries, making it less practical (Madani and Hooshyar, 2014).

Market mechanisms and cooperative game theory schemes are among the other water allocation methods that are used to make the social planner's solution practical. These methods focus on redistribution of the incremental benefits of cooperation and create win–win allocation solutions that make cooperation attractive to individual rational beneficiaries (Madani and Hooshyar, 2014). While these methods are promising, their application is limited to problems in which utility information is available for all parties and the incremental benefits of cooperation can be determined. Therefore, challenge remains in developing cooperative schemes for managing shared water systems for which utility information might not be readily available, agreeable, or reliable (common in transboundary river systems).

Operations research-based allocation methods (Madani et al., 2014a; Read et al., 2014) have also been employed in the water resources literature to allocate water. These methods are appropriate for problems with both discrete and continuous solutions and can be applied with and without utility information. However, as shown by Read et al. (2014), they seek distribution of dissatisfaction in an "optimal" way, disregarding the stability of allocation solutions. Therefore, their acceptability is questionable, making them less practical in real world.

Bankruptcy methods (O'Neil, 1982; Aumann and Maschler, 1985; Dagan and Volij, 1993) comprise another group of allocation methods used in the water resources literature for water allocation in the presence and absence of utility information (Sheikhmohammady and Madani, 2008; Ansink and Ruijs, 2008; Ansink and Weikard, 2012; Madani and Zarezadeh, 2012; Mianabadi et al., 2013; Madani and Dinar, 2013). These methods are promising due to their cooperative game theoretic nature and their attention to individual gains under different conditions (e.g., homogeneity of claims). However, as will be discussed in the following section, the previous applications of bankruptcy methods do not set an appropriate basis for using these methods for solving transboundary river allocation problems with temporal and spatial flow variability.

The objective of this paper is to bridge the gap of previous transboundary conflict resolution studies by developing a new bankruptcy-based water allocation mechanism that: (1) does not necessarily require the players' utility information (e.g., economic benefits of each beneficiary from the allocated water); (2) its application is not limited to problems in which cooperation must result in extra quantifiable benefits and (3) provides allocations solutions with respect to the temporal and spatial variability of water flows in transboundary river systems.

2 River bankruptcy problem

2.1 General description

Water conflicts can develop when the yield of a water system is not sufficient to fully satisfy the demands of all beneficiaries. Such a situation is similar to a bankruptcy state in which the total assets of an individual/entity are not enough to fully satisfy their/its debts. In other words, in a bankruptcy problem the total value of the claims of the beneficiaries is more than the value of the available resource. Such a similarity between water allocation problems and bankruptcy problems has been the main motivation for using bankruptcy methods, rooted in the economics and mathematics literature

(O'Neil, 1982; Aumann and Maschler, 1985; Dagan and Volij, 1993) to solve water resources bankruptcy problems (Sheikhmohammady and Madani, 2008; Ansink and Ruijs, 2008; Sheikhmohammady et al., 2010; Grundel et al., 2011; Ansink and Weikard, 2012; Madani and Zarezadeh, 2012; Mianabadi et al., 2013; Madani and Dinar, 2013).

Bankruptcy methods can be categorized as cooperative game theory solutions (Sheikhmohammady and Madani, 2008). Nevertheless, these methods are different in principle from the commonly used cooperative game theory methods such as Nash–Harsanyi bargaining solution (Harsanyi, 1959), Shapley value (Shapley, 1953), and the nucleolus method (Schmeidler, 1969), among others. While the bankruptcy methods focus on allocation of the total deficit (the difference between the total claim and the value of the available resource) among the parties, commonly used cooperative game theoretic solution methods have been primarily developed for allocation of the incremental benefits of cooperation among the cooperating parties. Therefore, they are not readily applicable to the bankruptcy situations with no incremental benefit from cooperation, or to cases in which the available information about the utilities of the parties from their resource shares are missing or are not reliable.

In some river basins, developing agreeable utility functions to estimate the utility (e.g., economic gain) of each riparian from its water use is very challenging due to the lack of trust and information as well as the absence of cooperative tendencies in the region. Therefore, river sharing games are often played as zero-sum games in which parties are mainly bargaining about their volumetric shares from the river, while in reality, due to the difference in the non-linear utility functions of the parties these games are not zero-sum (Madani, 2011; Madani and Lund, 2012). In fact, the zero-sum perception of the riparian parties is one of the main reasons for competition rather than cooperation, which makes economically efficient cooperative game theoretic institutions or other mechanisms such as water trading and markets impractical and unacceptable. To address these issues, bankruptcy methods can be applied for developing water allocation solutions. Although bankruptcy methods provide solutions which are economically less efficient than those provided by common cooperative game theory methods, they are potentially more publicly acceptable and practical. Most bankruptcy methods are based on common sense and are relatively easy to understand by the general public, unfamiliar with the economic principles and fairness rationales of the mathematically sophisticated cooperative game theory methods. This advantage has been the main reason for the success of some of the bankruptcy methods in practice since ancient times in different eras and locations. The proportional cutback principle is an example of one of the oldest bankruptcy methods that has been widely used for water resources management during droughts in different areas of the world (e.g., qanat water allocation in the Persian Empire and groundwater allocation in California.)

2.2 Essential elements

The two essential elements of a bankruptcy problem include (1) the amount of resource available and (2) the values of beneficiaries' claims. In most water resources bankruptcy problems, the first element simply equals the available water to be allocated to the beneficiaries in a given location at a specific time. Also, finding the claim values is straightforward in some water systems (e.g., claims of groundwater users in the case of groundwater bankruptcy are determined based on their groundwater rights in a regulated system). Therefore, bankruptcy solutions have been already used in the literature for solving water allocation problems with known (predetermined) claims and/or without temporal and spatial variability in resource availability/access (Sheikhmohammady and Madani, 2008; Ansink and Ruijs, 2008; Sheikhmohammady et al., 2010; Grundel et al., 2011; Ansink and Weikard, 2012; Mianabadi et al., 2013; Madani and Dinar, 2013). Nevertheless, solving transboundary river bankruptcy problems with original bankruptcy methods can be challenging for two reasons: (1) the lack of an acceptable method by all parties to estimate the credible claims of the beneficiaries and (2) the temporal and spatial change of the flow along the river basin.

2.3 Claims

Determination of the beneficiaries' claims in transboundary water systems is challenging and highly controversial due to the lack of information, unreliability of parties' claims and narratives, and absence of a globally acceptable framework for determining the credible claims of riparian parties. Thus, in Sect. 4 this paper suggests three different possible claim estimation methods for transboundary river bankruptcy problems with potential applications in real-world water conflicts.

2.4 Physical constraints: spatial and temporal variability

Classical bankruptcy methods assume homogenous resource accessibility and are appropriate for one-shot allocation problems. Therefore, they are not necessarily applicable to problems with temporal and spatial heterogeneity in resource availability. Due to the change of the flow over time and space, especially in river systems with multiple tributaries, water availability might be limited at a given location at a specific time. Therefore, original bankruptcy methods may produce infeasible allocation solutions for river systems.

While temporal resource variability has not been considered in previous bankruptcy studies, few studies have tried to address the spatial variability of resource in bankruptcy problems. Ilkılıç and Kayı (2012) formulated a network (graph) model for bankruptcy allocation with respect to the possible geographical and infrastructural constraints in distributing the resource among beneficiaries. While their method satisfies the fairness principle, i.e., "equal treatment of the equals", it considers "no restrictions on the possible networks between sources and agents" (Ilkılıç and Kayı, 2012). Therefore, their model is not generally applicable to river sharing problems in which the physical characteristics of the resource (river) system impose restrictions on networks between sources (river sections) and agents (riparian parties). In another study, Ansink and Weikard (2012) proposed a sequential allocation approach for solving river bankruptcy problems using classical bankruptcy methods. However, their method is only applicable to linear river systems with a single tributary, and is based on some assumptions which might limit its applicability to complex multi-tributary transboundary river systems involving equally powerful parians seeking equal treatment of the equals (Mianabadi et al., 2013; Zarezadeh et al., 2014).

This paper proposes a new approach for solving transboundary river bankruptcy problems with consideration of the constraints imposed by the temporal and spatial variability of water flows within river networks. The general characteristics of the proposed method are discussed in the next section. While transboundary water allocation motivates this work, the proposed method is applicable to other bankruptcy problems with temporal and spatial resource availability constraints. Suggested examples of such problems by Ilkılıç and Kayı (2012) include aid relief during and after disasters, utility (gas, electricity, water) distribution in supply shocks, and common property fisheries.

2.5 Acceptability

Given that the developed bankruptcy allocation solutions have no practical value unless they are acceptable and considered to be fair by the beneficiaries, evaluating the acceptability of the developed solutions is essential. Ex post analysis of the stability of allocation solutions is common in the water resources literature (e.g., Dinar and Howitt, 1997; Teasley and McKinney, 2011; Madani and Dinar, 2012; Read et al., 2014; Madani and Hooshyar, 2014). Once different allocation solutions are developed, the stability of solutions is normally evaluated using quantitative measures to determine the solutions with higher potential acceptability. Alternatively, an axiomatic (ex ante) approach can be adopted for allocation stability evaluation. Based on this approach, which is more common in the economics literature, the attractiveness of allocation rules is evaluated based on their properties such as monotonicity or independence properties, with respect to various possible perturbations of the problem at hand (Herrero and Villar, 2001; Thomson, 2003).

Ideally, the results of both approaches should coincide and this needs to be verified by future research. Nevertheless, the water resources literature tends to commonly use the ex post approach, finding it more attractive from the practical standpoint. In practice, developing a compromise over an ex ante approach, before clarifying to the beneficiaries what their actual gains would be, more challenging unless decision is made by an authorized intervener (e.g., social planner, government, or regulator), whose decision is enforceable and acceptable by all parties. In the ex post approach, on the other hand, multiple allocation solutions with transparent utility information (i.e., volumetric gains in case of river bankruptcy) can proposed to the negotiators. This expands the feasible solution set and creates more opportunities for cooperation through providing "substance" to negotiations (Bruce and Madani, 2014).

What makes the application of an ex ante approach even more complex in of river bankruptcy problems is the asymmetric accessibility of the resource (water) at a given time and location. Therefore, even if the solution properties are reasonable, the actual solutions (volumetric allocations in this case) will be affected by the physical limitations of the system. Thus, the same solution principles might not yield similar results in two river bankruptcy problems with similar claims and total water availability as the physical aspects of the system can make the actual allocations different, i.e., water might not be available at a given location at a given time, as desired based on the allocation rule. This makes the axiomatic approach less practical as it does not guarantee feasible allocation solutions based on a given allocation rule, failing to provide clear information to the parties about their actual gains. Therefore, an ex post stability evaluation method is proposed and used in this study.

While various methods have been used in the literature to evaluate the stability and acceptability of water allocation solutions (Dinar and Howitt, 1997; Madani and Dinar, 2012; Read et al., 2014), these methods cannot be readily used to evaluate the acceptability of bankruptcy solutions. Therefore, a new quantitative stability evaluation method is developed in this study to evaluate the potential acceptability of the proposed bankruptcy solutions.

2.6 Robustness

Normally, in water allocation negotiations, the amount of available water in a given time-step (e.g., month) is determined based on the average historical flow in that time-step. Given that water flows are different in dry, wet, and normal years, water allocation agreements can vary depending on the hydrologic conditions. Water allocation based on historical flows might make allocation agreements vulnerable to hydrologic variability, uncertainty, and non-stationarity. Therefore, instead of relying on fixed water shares, riparian parties can try to agree over a flexible allocation framework that adjusts allocation solutions considering the changing conditions of the system. This study seeks to propose a robust bankruptcy solution framework that can provide water allocation solutions that are not vulnerable to changing conditions and can update allocations accordingly. Therefore, stability of the allocation solution is examined under different hydrological conditions to determine if allocation rules and solutions must be changed under different hydrologies or if a

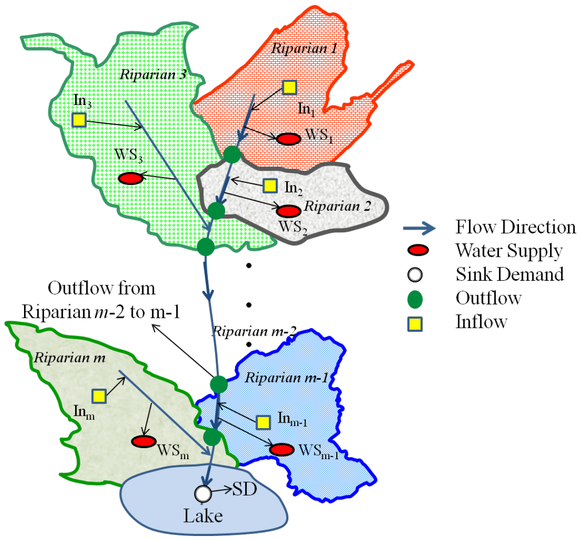

Figure 1. Schematic map of a transboundary river network.

fixed allocation rule can provide acceptable allocations that are insensitive to hydrologic variability.

3 River bankruptcy allocation models

Figure 1 shows a schematic of simple transboundary river system with multiple tributaries, a lake (sink) at the outlet and m riparians ($i = 1, 2, \ldots, m$), each having different types of water demand (e.g., domestic, agricultural, and environmental). Water bankruptcy occurs when the total demand of the riparians exceeds the stock of water. Bankruptcy rules can be applied to allocate the available water to the riparians with respect to their water demands (claims). In river systems, however, the classic bankruptcy rules might produce infeasible allocation results due to spatial and temporal flow variability and water access heterogeneity. Thus, bankruptcy rules must be modified to account for the physical characteristics of the river network. We propose developing non-linear network flow optimization models that facilitate application of four commonly used bankruptcy methods, namely proportional (P), adjusted proportional (AP), constrained equal award (CEA), and constrained equal loss (CEL) rules to river bankruptcy problems, with respect to the water availability constraints. These models have the following general characteristics:

1. Each bankruptcy optimization model is based on a specific bankruptcy rule.

2. An allocation solution developed by a river bankruptcy optimization model is always unique.

3. The allocation solution set developed by a bankruptcy optimization model is always feasible, considering the temporal and spatial flow variability in the river network.

4. The allocation solutions developed by a river bankruptcy optimization model are expected to satisfy the composition properties suggested by Ansink and Weikard (2013) for river bankruptcy problems. Nonetheless, composition properties have not been used in this study to derive the bankruptcy solutions.

5. The bankruptcy optimization models seek "equal treatment of the equals" to the extent possible. This means that the bankruptcy optimization models minimize hydro-hegemony (Zeitoun and Warner, 2006) in the system by equal treatment of all riparians regardless of their location in the system (upstream vs. downstream) and level of contribution to the total flows. This characteristic makes the proposed framework different in essence from the methods proposed by Ansink and Weikard (2012) and Mianabadi et al. (2013).

6. In the absence of water availability restrictions, a bankruptcy optimization model based on a given bankruptcy rule produces allocation results that are identical to allocation solutions based on that bankruptcy rule. In other words, when allocation based on a given bankruptcy rule is feasible, original bankruptcy solutions match the solutions produced by the corresponding bankruptcy optimization model.

7. In presence of water availability restrictions (when the application of original bankruptcy solutions methods infeasible allocation solutions), bankruptcy optimization models minimize the difference between their allocation solutions and the solutions based on the corresponding original bankruptcy rules.

8. Water allocations to riparians with equal claims are not necessarily equal in river bankruptcy problems, due to the uneven access to water along the river system.

9. A bankruptcy optimization model can be applied to any river network, irrespective of its physical characteristics (shape, number of tributaries, number of riparians, etc.).

10. Each river bankruptcy optimization model satisfies the following initial, mass balance (flow continuity), and non-negativity constraints:

$$T_{i,t} = I_{i,t} + O_{i-1,t} \quad \forall i, \tag{1}$$

$$O_{i,t} = T_{i,t} - S_{i,t} \quad \forall i, \tag{2}$$

$$O_{0,t} = 0, \tag{3}$$

$$O_{m,t} = D_t, \tag{4}$$

$$S_{i,t} \geq 0 \quad \forall i, \tag{5}$$

$$S_{i,t} \leq T_{i,t} \quad \forall i, \tag{6}$$

$$\sum_{i=1}^{m} S_{i,t} \leq E_t, \tag{7}$$

$$E_t = \sum_{i=1}^{m} I_{i,t} - D_t, \tag{8}$$

where for $i = 1, 2, \ldots, m$ in a given time step t: $T_{i,t}$ is the total available water in riparian i's territory (decision variable); $I_{i,t}$ is the riparian i's contribution to the river system through the tributaries originating in its territory (known variable); $O_{i,t}$ is the total outflow from riparian i to the downstream riparian state $(i+1)$ (decision variable); $S_{i,t}$ is the allocated water supply to riparian i in each month (decision variable); D_t is the sink demand at the system's outlet (known variable); and E_t is the total asset water (available water) to be shared in the bankruptcy problem.

While the sink node at the system's outlet can be treated as a riparian, here we assume that the environmental need of the sink has a high priority. Therefore, Eq. (4) ensures that the lake's environmental demand is fully met.

11. A bankruptcy optimization model is only appropriate for a bankrupt river system in which the total demand exceeds the total available water. This makes $E_t \leq \sum_{i=1}^{m} C_{i,t}$ a necessary condition for validity of the optimization model, where $C_{i,t}$ is the claim (demand) of riparian i in time step t (known variable).

12. The amount of water allocated to a riparian party i (by a river bankruptcy optimization model) never exceeds its claim/demand. Thus, all bankruptcy optimization models satisfy the following constraint:

$$S_{i,t} \leq C_{i,t} \quad \forall i, \tag{9}$$

In the case of downstream excess flow, i.e., when water availability is more than the claim of the downstream riparian, this riparian must be excluded from the river bankruptcy problem, as technically this riparian is not expected to be subject to any conflict of water allocation.

Sections 3.1–3.4 present the mathematical formulations for each of the proposed river network bankruptcy optimization models. All river bankruptcy optimization models are subject to Eqs. (1)–(9). One advantage of using a network bankruptcy model is that it can be applied to any river network with any shape. Thus, the river network should not necessarily match the natural river system and it can include water diversion/transfer infrastructure (already developed or under consideration during allocation negotiations). It must be noted that while the proposed models have been developed for river networks, they are generally applicable to any network bankruptcy problem in which the agents' accessibility to the resource can be determined based on the network structure.

Given the time sensitivity of water deliveries, water solutions must be found using an appropriate time-step to prevent disruption in water deliveries to any riparian party. In systems without enough storage capacity to regulate and carry over water, smaller time-steps (e.g., months) can be used as the basis of allocations. In this case, allocations are time specific and can be done for each time-step independently, i.e., the bankruptcy rule is applied to a given time-step (e.g., 1 month) during the planning horizon (e.g., 1 years), based on the water availability and claims in that month only, regardless of the allocations in other months. In regulated systems, operations are more flexible. Thus, bankruptcy rules can be applied to the whole planning horizon

Bankruptcy optimization models in this case is that specific concerns of the riparian parties during the planning horizon as optimization constraints. Examples of such constraints would be minimum acceptable water supply, minimum environmental flow, minimum reservoir storage/energy head for hydroelectricity generation, maximum/minimum temperature, minimum acceptable reliability/resiliency of water supply, and maximum acceptable vulnerability of water supply at particular points within the river network in a given time (e.g., day, week, month) during the planning horizon (e.g., year).

In this study, the proposed modeling framework is applied to an unregulated system for which bankruptcy (cutback) allocation decisions in each time-step are independent from other time-steps.

3.1 Proportional (*P*) rule

The P rule satisfies an equal proportion of the creditors' claims. Based on this ancient bankruptcy method, the equal portion ($\lambda_{P_{i,t}}$) is calculated by dividing the total available resource by total demand. The P rule's water allocation optimization model for river systems is proposed in the following mathematical form:

$$\text{Minimize } \lambda_{P_t} - \prod_{i=1}^{m} \lambda_{P_{i,t}}, \tag{10}$$

subject to:

$$\lambda_{P_{i,t}} = \frac{S_{i,t}}{C_{i,t}} \quad \forall i, \tag{11}$$

$$\lambda_{P_{i,t}} \leq \lambda_{P_t} \quad \forall i, \tag{12}$$

where for $i = 1, 2, \ldots, m$ in a given time step t: $\lambda_{P_{i,t}}$ is the riparian i's proportional allocation coefficient (decision variable), and λ_{P_t} is the maximum proportional allocation coefficient (decision variable). This optimization model tries to minimize the latter variable. The second term in the objective function ensures that the model has a unique solution and the agents' allocation coefficients ($\lambda_{P_{i,t}}$) are as close to each other as possible. The minimum objective value for a given problem is achieved when the riparians' allocation coefficients are equal. In that case, $\lambda_{P_t} = \lambda_{P_{i,t}}$ and the proportional allocation coefficients match the proportional cutback rate under the original bankruptcy rule, which is equal to $\frac{E_t}{\sum_{i=1}^{m} C_{i,t}}$. This case occurs when the flow variability does not make the original bankruptcy rule infeasible.

3.2 Adjusted proportional rule (AP)

Based on this rule (Curiel et al., 1987), what remains for beneficiary i once all other creditors are satisfied is the basis for allocation. To determine the initial amount of allocation to creditor i, the summation of claims of all other beneficiaries is compared with the available stock of water. In the case of surplus, the initial allocation to riparian i is equal to the remaining water stock once the demands of all other riparians are satisfied. Otherwise, the initial allocation to i is set to 0. It is assumed that the initial allocation calculated through this procedure is agreeable by all beneficiaries. Once initial allocations are determined, claims are revised. The revised claim of a given beneficiary is set equal to the minimum of the remaining water stock and the difference between the beneficiary's initial claim and its initial allocation. The P rule is then applied to the remaining water stock and the revised claims.

The mathematical formulation of the AP rule's river bankruptcy optimization model is proposed as follows:

$$\text{Minimize } \lambda_{\text{AP}_t} - \prod_{i=1}^{m} \lambda_{\text{AP}_{i,t}}, \tag{13}$$

subject to:

$$R_{i,t} = \sum_{j \neq i} C_{j,t} \quad \forall i, \tag{14}$$

$$\upsilon_{i,t} = \frac{E_t - R_{i,t} + |E_t - R_{i,t}|}{2} \quad \forall i, \tag{15}$$

$$C^*_{i,t} = \text{Min}(C_{i,t}, E_t) \quad \forall i, \tag{16}$$

$$\lambda_{\text{AP}_{i,t}} = \frac{S_{i,t} - \upsilon_{i,t}}{C^*_{i,t} - \upsilon_{i,t}} \quad \forall i, \tag{17}$$

$$\lambda_{\text{AP}_{i,t}} \leq \lambda_{\text{AP}_t} \quad \forall i, \tag{18}$$

$$\lambda_{\text{AP}_t} \leq \frac{E_t - \Sigma_{i=1}^{m} \upsilon_{i,t}}{\Sigma_{j=1, j \neq i}^{m} (C^*_{j,t} - \upsilon_{j,t})} \quad \forall i, \tag{19}$$

where for $i = 1, 2, \ldots, m$ in a given time step t: $R_{i,t}$ is the summation of all riparians' claims excluding riparian i; $\upsilon_{i,t}$ is the initial allocation to riparian i (amount of water conceded to riparian i by all other riparians); $\lambda_{\text{AP}_{i,t}}$ is the riparian i's AP allocation coefficient (decision variable), and λ_{AP_t} is the maximum AP allocation coefficient (decision variable). Similar to the P rule's optimization model, the minimum objective value is achieved when the original bankruptcy solution is feasible and the optimized allocations match the allocations under application of the original bankruptcy rules. The second term in the objective function ensures having a unique solution and minimizes the differences between the allocation coefficients of the riparians.

3.3 Constrained equal award rule (CEA)

This ancient rule, adopted by rabbinical legislators (Dagan and Volij, 1993) allocates the minimum of λ_{CEA_t} and $C_{i,t}$ to all beneficiaries, provided that the sum of allocations equals the total available resource. CEA tries satisfying the lower claims to the extent possible in order to minimize the number of unsatisfied creditors. This rule is supposed to favor the lower claims, normally belonging to weaker beneficiaries who can be more affected by losses (Madani and Dinar, 2013). Based on CEA, the initial allocation to all beneficiaries is equal to the lowest claim, provided that the sum of initial allocations does not exceed the demand. The fully satisfied creditor is then excluded and the process continues with the remaining creditors after updating their unsatisfied claims as well as the remaining resource value. At any stage (including the initial stage) when allocating an amount equal to the lowest claim to all remaining creditors is not feasible (due to insufficiency of remaining resource) the remaining resource is distributed equally among all remaining creditors.

The mathematical formulation of the CEA rule's river bankruptcy optimization model is proposed as follows:

$$\text{Minimize } \lambda_{\text{CEA}_t} - \frac{\prod_{i=1}^{m} \lambda_{\text{CEA}_{i,t}}}{(\lambda_{\text{CEA}_t})^{m-1}}, \tag{20}$$

subject to:

$$\lambda_{\text{CEA}_{i,t}} = S_{i,t} \quad \forall i, \tag{21}$$

$$\lambda_{\text{CEA}_{i,t}} \leq \lambda_{\text{CEA}_t} \quad \forall i, \tag{22}$$

where for $i = 1, 2, \ldots, m$ in a given time step t: $\lambda_{\text{CEA}_{i,t}}$ is the feasible allocation to the riparian i (decision variable), and λ_{CEA_t} is the highest feasible allocation to the creditors (decision variable). The second term in the objective function is to enforce a unique solution and to minimize the difference between the allocations which would be equal in the absence of resource accessibility limitations. Given that λ_{CEA_t} can be more than 1 (different from the λ_{P_t} and λ_{AP_t} which are always less than or equal 1) the second term is divided by a positive number of comparable magnitude to ensure that the second term is always smaller than or equal to the first term.

3.4 Constrained equal loss rule (CEL)

This rule can be viewed as the opposite of CEA, as it gives priority to satisfying the highest claims (more powerful creditors) first. Once the highest claim is satisfied, the process is repeated with the remaining resource and creditors. The process stops at any stage (including the first stage) if the available resource is not sufficient to satisfy the highest claim of the remaining creditors. At this stage, the remaining resource is split equally among the remaining creditors. By doing this, CEL allocates $C_i - \lambda_{\mathrm{CEL}_i}$ to all beneficiaries whose claims are bigger than λ_{CEL_t}, allocating 0 to those who do not fall in this category. Thus, the final allocation to each beneficiary is equal to max $\left\{0, C_i - \lambda_{\mathrm{CEL}_i}\right\}$.

The CEL rule's river bankruptcy optimization model is proposed as follows:

$$\text{Minimize } \lambda_{\mathrm{CEL}_t} - \frac{\prod_{i=1}^{m} \lambda_{\mathrm{CEL}_{i,t}}}{(\lambda_{\mathrm{CEL}_t})^{m-1}}, \tag{23}$$

subject to:

$$\lambda_{\mathrm{CEL}_{i,t}} = C_{i,t} - S_{i,t} \quad \forall i, \tag{24}$$

$$\lambda_{\mathrm{CEL}_{i,t}} \leq \lambda_{\mathrm{CEL}_t} \quad \forall i, \tag{25}$$

where for $i = 1, 2, \ldots, m$ in a given time step t: $\lambda_{\mathrm{CEL}_{i,t}}$ is the unmet claim of the riparian i (decision variable), and λ_{CEL_t} is the maximum unmet claim of all riparians (decision variable).

4 Example: the Qezelozan-Sefidrud river system bankruptcy problem

The proposed framework is applied to develop bankruptcy-based water allocation schemes for resolving a real-world transboundary river conflict in Iran. The Qezelozan-Sefidrud river basin (Fig. 2) is located at the intersection of the Iran's Alborz and Zagros mountain ranges, with an area about 59 400 km^2, making it the largest basin of the nation. The basin overlaps with eight provinces (Kurdistan, Hamadan, Zanjan, Eastern Azerbaijan, Ardebil, Tehran, Qazvin, and Gilan) and the river provides the basis for important economic activities in these provinces. The river eventually flows into the Caspian Sea in north of Iran, which is the largest enclosed body of water in the world and the source of more than 90 % of the world's caviar supply (Madani et al., 2014b). Supplying the required environmental flows of the Caspian Sea is essential to the health of its valuable ecosystem.

The study basin is not international. Nevertheless, inter-provincial or interstate basins are effectively equivalent to international basins as long as their boundaries do not match political boundaries and they are managed by more than one authority. The Qezelozan-Sefidrud river system is an example of a transboundary river basin, in which serious conflict has arisen as a result of recent socioeconomic (i.e., population increase and development), political (i.e., changes in the water resources management structure), and hydrologic/climatic (i.e., frequent droughts) changes. As a result of political changes in the country, the Qezelozan-Sefidrud river system, which was historically shared by six Iranian provinces and managed by only one water management authority, is now shared by eight provinces and managed by eight water authorities. As a result of population increase and development in the region, each province is trying to increase its share from the river and minimize the outgoing flow, resulting in significant reduction of water flowing into downstream provinces. To increase their uses from the river, the upstream provinces have aggressive water resources development plans. These development plans include construction of multiple new reservoirs, which are currently under construction or in the study phase. Complete implementation of these plans will negatively impact the downstream provinces, which historically have had more access to the river system due to their stronger political and economic power as well as higher populations. Therefore, the political tension has increased in the basin, making the Qezelozan-Sefidrud river system the subject of one of the most intractable conflicts over water resources in Iran. To show the utility of the proposed model in solving transboundary water allocation conflicts, the proposed framework is applied to derive new water allocation schemes for the Qezelozan-Sefidrud river system.

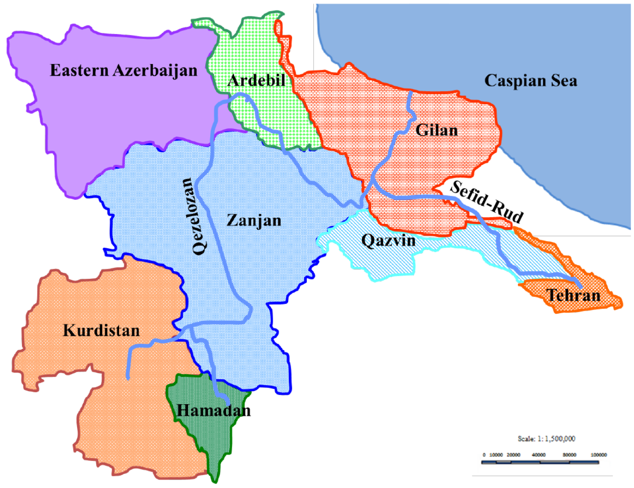

Figure 2. Qezelozan-Sefidrud river system and its eight riparian provinces.

The first step in solving river bankruptcy problems is determining the legitimate claims of the riparian parties. This step is challenging in unregulated systems without established water rights. We propose three alternatives for determining the claims of the riparian parties in the example case. These alternatives, which help setting the upper and lower boundaries of the claims include:

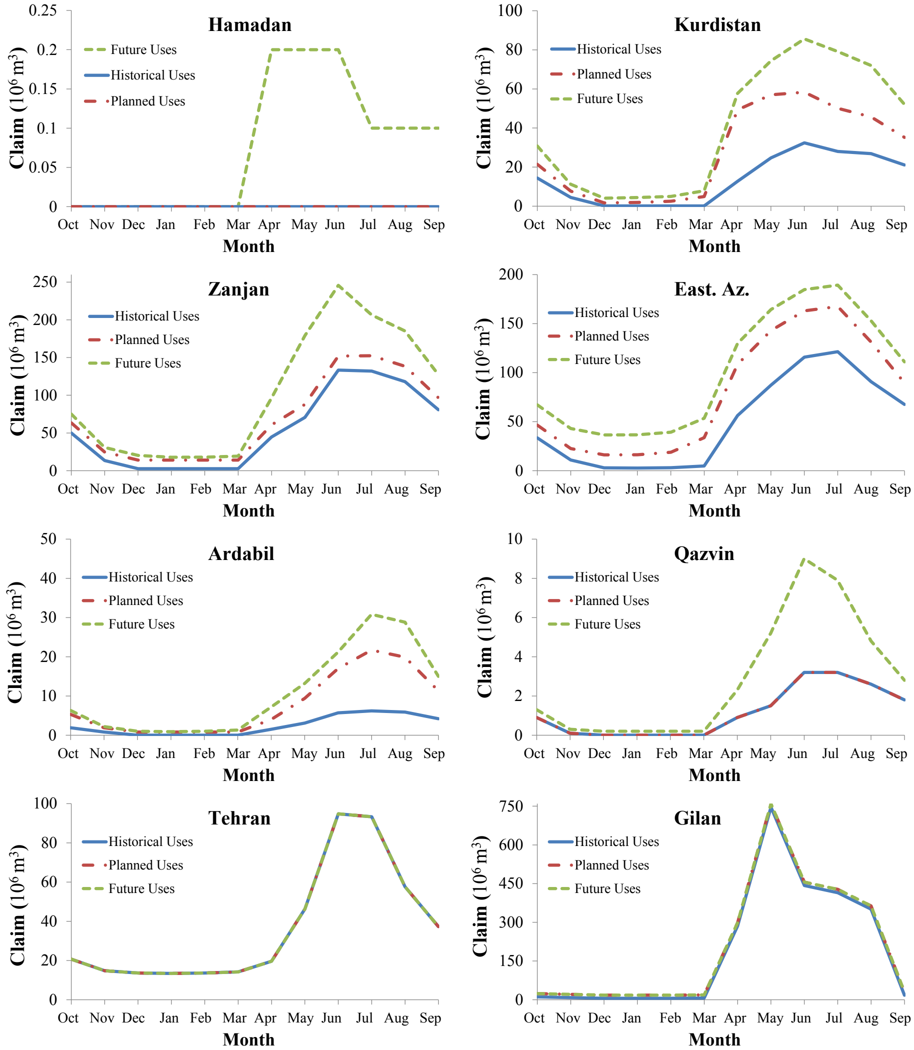

Figure 3. Estimated monthly claims of the riparian provinces based on the three proposed claim calculation methods.

1. Historical uses: based on this alternative, historical uses of the river system, revealed by the historical water use data are set as the claims of the riparians. The water use values are calculated based on the difference between the recorded inflows and outflows of each province at the hydrometric stations. This alternative sets the lower claim boundary for each riparian.

2. Planned uses: Iran is currently the world's third dam builder with respect to the dams it has under construction (Madani, 2014). Several water storage projects are under development at different locations in the riparian states of the Qezelozan-Sefidrud river system. These projects have been approved by the central government, and are receiving financial support from the central and provincial governments. Each project has an associated estimation of sectoral water demands (i.e., domestic, agricultural, industrial, and environmental) used for the calculation of the required storage capacity. Based on this alternative, the total claim of each riparian is set equal to the total documented water demands of different river system-related reservoirs within the watershed boundaries, which are already in operation or under development.

3. Future uses: beside projects under construction, each riparian state has plans for getting approval for constructing additional water storage infrastructure to meet its increasing water demand as a result of development. Based on this claim estimation alternative, water demands of these additional facilities will be added to the water claims calculated based on alternative 2 only if the construction plans of these facilities have been publicly announced. This alternative sets the upper claim boundary for each riparian.

Figure 3 indicates the estimated monthly water claims of the riparian states of the Qezelozan-Sefidrud river system based

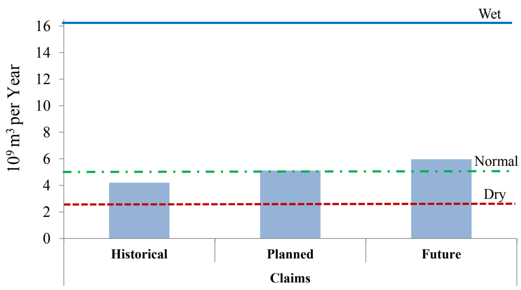

Figure 4. Total annual claims (including the Caspian Sea's water demand) based on three different claim estimation methods and total annual water yield under three different hydrologic scenarios.

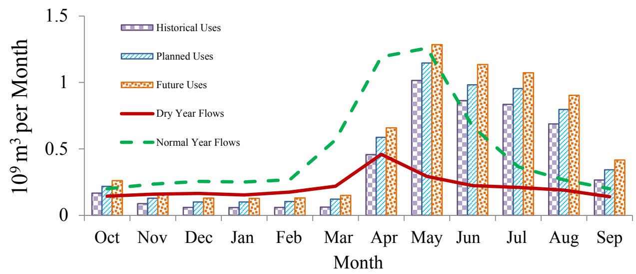

Figure 5. Total monthly claims (including the Caspian Sea's water demand) based on three different claim estimation methods and total annual water yield under normal and dry hydrologies.

on the three proposed methods. Detailed calculations of the water claims based on proposed claim determination alternatives can be found in Zarezadeh (2011).

Given that allocation solutions can be sensitive to climatic/hydrologic conditions, three different water availability scenarios, representing three distinct hydrologic conditions, normal (average), dry, and wet, were initially considered for solving the example river bankruptcy problem. In the normal scenario, river flows are based on the average monthly river discharges during the 1956–2006 period. Dry scenario flows match the average monthly river discharges during the major drought of 1998–2001 in the region. The wet scenario flows are based on the monthly flows during the 1968–1969 period. The annual river discharge under the wet scenario will be sufficient to meet the historical, planned, and future claims of the riparian states as well as the Caspian Sea's (sink) water demand (Fig. 4). Therefore, river bankruptcy problem is solved only for the normal and dry cases.

Due to the unclear status of water rights/claims and the future status of reservoir networks in the Qezelozan-Sefidrud river system, this system is considered to be unregulated, disregarding the possible benefits resulting from coordination of reservoir operation strategies in the basin. The problem is solved using a monthly time-step and allocations are determined in each time-step separately. Under this approach, the total allocation to each riparian over the whole planning horizon (e.g., one year) is the summation of bankruptcy allocations in the existing time-steps within the planning horizon (e.g., twelve months).

Figure 5 indicates the monthly water yield of the Qezelozan-Sefidrud river system under the normal and dry conditions as well as the total monthly claims of the riparian parties (including the Caspian Sea's water demand). This figure clearly shows the water bankruptcy status of the example river system for almost half of the year, especially in warmer months with higher agricultural water demands.

The four proposed bankruptcy optimization models in Section 3 were run under two hydrologic scenarios to calculate bankruptcy allocations under normal and dry conditions. The models were first run on a monthly basis to calculate the monthly allocations. The summation of 12 monthly allocations based on each model with a given set of claims under a given hydrology determines the corresponding annual allocation of each province. The annual bankruptcy allocations based on different bankruptcy models, claims, and hydrologies are presented in Fig. 6.

As expected based on the definition, the CEL method favors the creditor with the highest claim (in this case, the downstream Province of Gilan). The opposite is true for the CEA method which gives priority to satisfy the claims of the creditors with lower claims (in this case, the provinces upstream of Gilan). The AP and P methods can be considered as moderate allocation methods which result in allocations that are between the high and low allocations estimated by the other two methods. In comparison with P, the AP method allocates a higher share to the parties with lower claims and a lower share to the parties with higher claims, trying to address the bias toward higher claims in the P method. The difference between the allocation values for different claims and hydrologies underline the sensitivity of bankruptcy allocation schemes to the difference in claim values and hydrologic conditions.

5 Stability evaluation

The suggested bankruptcy optimization models provide different allocation solutions, based on various notions of fairness. Therefore, their acceptability is always questionable, given that there is always at least one beneficiary who finds one of the given alternatives unfair because they can gain more under another rule (Madani and Lund, 2011). As one of the most commonly used social choice (voting) methods (Sheikhmohammady and Madani, 2008; Madani et al., 2014c), the plurality index can be considered as an indicator of potential acceptability of a decision rule in multi-participant decision-making problems. Based on this index, the number of stakeholders who prefer one method to the others is simply an indicator of the degree of acceptance of that method (Dinar and Howitt, 1997).

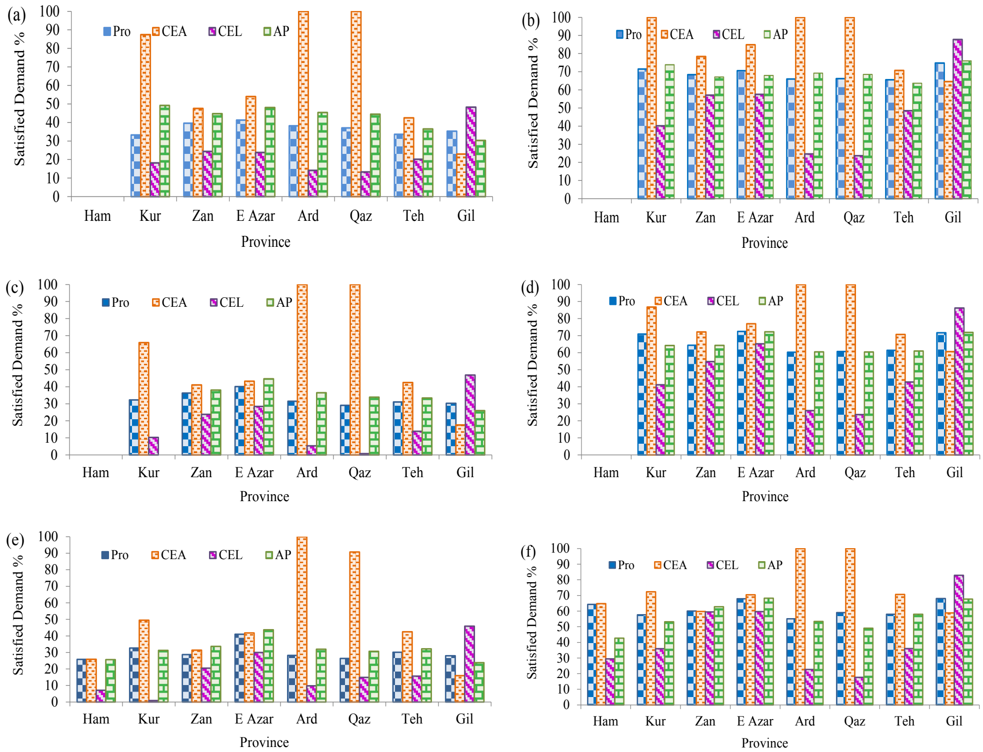

Figure 6. Satisfied annual water claim (%) of the riparian provinces based on different bankruptcy solution methods for different claims and hydrologies: **(a)** historical claim in a normal year, **(b)** historical claim in a dry year, **(c)** planned claim in a normal year, **(d)** planned claim in a dry year, **(e)** future claim in a normal year, and **(f)** future claim in a dry year.

Table 1. Plurality index of different bankruptcy solutions for different claims and hydrologies.

Claim	Hydrology									
	Normal					Dry				
	P	AP	CEL	CEA	Winner	*P*	AP	CEL	CEA	Winner
Historical	0	0	1	*6*	CEA	0	0	1	*6*	CEA
Planned	0	0	1	*6*	CEA	0	1	1	*5*	CEA
Future	0	1	1	*6*	CEA	0	2	1	*5*	CEA

The higher the allocation to a riparian state, the more preferred the allocation rule (bankruptcy method) by that state. Table 1 shows the plurality index (number of votes received) of each bankruptcy solution method for different claim values under different hydrologies. The assumption is that each party selects the allocation scheme that gives it the highest share. Given that the Hamadan Province has no historical or planned claim, its vote is only counted when future claims are considered. Based on the plurality index, the CEA method, which highly satisfies the riparians with lower claims (majority in this case) in both normal and dry conditions, is the winner. However, given the absolute objection of the most powerful province, i.e., Gilan, to this method, which allocates low shares to this province, this solution is not practical without strong intervention of the central government or providing strong cooperation incentives to Gilan.

The majority does not necessarily win in multi-participant decision-making problems with asymmetric powers, especially when the minority group is powerful. Therefore, the plurality index might not be appropriate for identifying the feasible solution when there is a power imbalance among the beneficiaries. Other methods can be used to quantify the potential acceptability of allocation solutions (Read et al., 2014). Loehman et al. (1979) used the following power index (α_i), originally developed by Shapley and Shubik (1954), to evaluate the power of players in cooperative game theory problems in which players seek the best method for allocating the incremental benefits of cooperation

Table 2. Bankruptcy allocation stability index (BASI) values of different bankruptcy solutions for different claims and hydrologies.

Scenario	BASI							
	Normal				Dry			
	P	AP	CEL	CEA	*P*	AP	CEL	CEA
Historical	0.11	0.10	*0.07*	0.15	*0.90*	1.00	1.33	1.27
Planned	0.31	0.31	*0.21*	0.43	1.11	1.01	1.68	*0.77*
Future	6.57	6.49	*3.48*	9.65	1.01	0.95	1.54	*0.73*

to coalition members:

$$\alpha_i = \frac{x_i - x_i'}{\sum\limits_{j \epsilon N} (x_j - x_j')} \quad i \epsilon N, \sum_{i \epsilon N} \alpha_i = 1, \tag{26}$$

where x_i is the allocated cooperative benefit share to player i, $x_i^{'}$ is the status quo (non-cooperative) gain of player i, and N is the set of all players.

A high power index value reflects less power or a higher willingness to cooperate. A stable allocation solution can be achieved when power is distributed more or less equally among the players (Dinar and Howitt, 1997). Therefore, the coefficient of variation of powers, also known as the stability index (S_α) is used as an indicator of the stability of allocation solutions:

$$S_\alpha = \frac{\sigma_\alpha}{\overline{\alpha}}, \tag{27}$$

where σ_α is the standard deviation of powers and $\overline{\alpha}$ is the mean power. The lower the index, the more stable the allocation solution.

Given that cooperation in bankruptcy problems does not have incremental benefits and parties' gains are zero in the status quo, the power index is not readily quantifiable in bankruptcy problems. Therefore, we propose a modified power index (BPI) for bankruptcy problems as follows:

$$\mathrm{BPI}_i = \frac{S_i - \upsilon_i}{\sum\limits_{j \epsilon N} (S_j - \upsilon_j)} \quad i \epsilon N, \quad \sum_{i \epsilon N} \mathrm{BPI}_i = 1, \tag{28}$$

where:

$$S_i = \sum_{t=1}^{n} S_{i,t}, \tag{29}$$

$$\upsilon_i = \sum_{t=1}^{n} \upsilon_{i,t}, \tag{30}$$

BPI_i is the bankruptcy power index (BPI) of riparian i, υ_i is the sum of the conceded water to riparian i by all other riparians in all time-steps in the overall planning horizon, N is the set of riparians, and n is the number of time-steps in the planning horizon ($n = 12$ months in the study example).

The bankruptcy allocation stability index (BASI) is then equal to the coefficient of variation of BPIs, which can be used to evaluate the potential acceptability of a bankruptcy solution:

$$\mathrm{BASI} = \frac{\sigma_{\mathrm{BPI}}}{\overline{\mathrm{BPI}}}, \tag{31}$$

where σ_{BPI} is the standard deviation of riparian powers and $\overline{\mathrm{BPI}}$ is the mean power. The higher the index, the less stable the allocation solution.

Table 2 shows the BASI value for each bankruptcy solution under a given hydrology for a unique claim set. Based on this table, the CEL method is the most stable method under the normal hydrology even though this method is not the most popular method (based on the popularity index). Given that stability (feasibility) is more important than popularity (social optimality) in conflict resolution (Read et al., 2014) we can conclude that CEL is the best mechanism for water allocation in this bankruptcy example. Nevertheless, the stability of CEL is sensitive to hydrological conditions and this method becomes the least stable allocation method under the dry conditions. Under the dry hydrology, the P rule is the most stable with lower demands. As demands increase, the CEA method (the most popular method) becomes more stable. The changes in stability of allocation rule, with changes in demand and hydrology show that the stability of allocation mechanisms is sensitive to both the hydrologic conditions (water availability) and the claim set characteristics. Future studies can focus on understanding the correlations of the BASI of allocation rules with the claim set characteristics (magnitude of claims, heterogeneity of claims, etc.) and the resource availability conditions.

6 Conclusions

This work formed the basis and set practical guidelines for developing allocation schemes for resolving transboundary water allocation conflicts based using bankruptcy methods. Although the suggested approach does not necessarily maximize the total welfare in the basin and might result in suboptimal allocations from an economic standpoint, it can be used to develop practical solutions when side payments are not feasible, parties are not highly cooperative (or not interested in implementing solutions based on conventional cooperative game theory solutions), and utility information is not available or reliable.

Considering the non-uniform spatial and temporal variability of water flows, resulting in unequal access to water in river systems, non-linear optimization models were proposed for solving river bankruptcy problems. Four river bankruptcy network flow optimization models were developed based on four conventional bankruptcy rules, i.e., proportional (P), adjusted proportional (AP), constrained equal award (CEA), and constrained equal loss (CEL), for transboundary water allocation. The models can be applied to any river network (or bankruptcy network) problem, irrespective of its shape and resource variability/accessibility conditions.

Acknowledging the difference in the notion of fairness and the possibility of the rejection of suggested allocations by the beneficiaries, who find certain allocation rules unfair, there is a need for evaluating the acceptability of different bankruptcy solutions. While popularity of each solution is a simple indicator of its potential acceptability, it was argued that in the case of asymmetric powers, the majority cannot necessary determine the feasible solution, especially when powerful parties do not support the most popular solution. Therefore, a new index (bankruptcy allocation stability index (BASI)) was formulated for evaluating the potential acceptability/stability of allocation solutions with respect to the distribution of claims and dissatisfaction among the beneficiaries.

The evaluation of the stability of different bankruptcy allocation solutions for different water demand and hydrologic scenarios in the example case suggested that acceptability is sensitive to both water demand (claim) and water availability. This finding has a significant policy implication for transboundary water management, suggesting that inflexible water allocation agreements and treaties that have been developed based on stationary assumptions are not resilient, especially in face of expected socioeconomic and climatic changes.

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