



Challenges in modelling river flow and ice regime on the Ningxia–Inner Mongolia reach of the Yellow River, China

C. Fu¹, I. Popescu², C. Wang^{2,4}, A. E. Mynett^{2,3}, and F. Zhang⁵

¹NARI Group Corporation, water conservancy and hydropower technology branch company, Nanjing 211000, China

²UNESCO-IHE, Institute for Water Education, 2601DA, Delft, the Netherlands

³Delft University of Technology, Faculty of CiTG, 2600GA, Delft, the Netherlands

⁴Hydrology Bureau, Yellow River Conservancy Commission, Zhengzhou 450004, China

⁵Yellow River Institute of Hydraulic Research, Zhengzhou 450003, China

Correspondence to: I. Popescu (i.popescu@unesco-ihe.org)

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Abstract. During winter the Yellow River in China is frequently subjected to ice flood disasters. Possible dike breaking due to ice floods poses a serious threat to the part of the region located along the river, in particular the Ning–Meng reach (including Ningxia Hui and the Inner Mongolia autonomous regions). Due to its special geographical location and river flow direction, the ice dams and jams lead to dike breaking and overtopping on the embankment, which has resulted in huge casualties and property losses throughout history. Therefore, there is a growing need to develop capability in forecasting and analysing river ice floods. Research into ice floods along the river is taking place at the Yellow River Conservancy Commission (YRCC). A numerical model is one of the essential parts of the current research going on at the YRCC, which can be used to supplement the inadequacies in the field and lab studies which are being carried out to help understand the physical processes of river ice on the Yellow River. Based on the available data about the Ning–Meng reach of the Yellow River, the YRCC river ice dynamic model (YRIDM) has been tested for capabilities to conduct ice flood forecasting. The YRIDM can be applied to simulate water level, discharge, water temperature, and ice cover thickness under unsteady-state conditions. Different scenarios were designed to explore the model uncertainty for two bounds (5 and 95 %) and probability distribution. The YRIDM is an unsteady-state flow model that can show the basic regular pattern of ice floods; hence it can be used as an important tool to support decision making. The recommendation is that data and research should be continued in order to support the model and to measure improvements.

1 Introduction

River ice is a natural phenomenon that can be commonly seen in the cold regions of the world. River ice plays an important role in cold regions, being the main transportation means in wintertime: the ice roads and ice bridges caused by river ice in the northern regions of Canada, Russia, and the USA (Alaska), where the population is sparse, are a positive effect of the ice formation in cold regions (Petrow et al., 2007; Rojas et al., 2011). However, on the negative side, river ice can cause ice flooding, hamper hydropower generation, threaten hydraulic structures, hinder water supply and river navigation, and has many other important detrimental effects.

When it comes to floods caused by river ice, the Ning–Meng reach of the Yellow River, China, cannot be ignored. The ice flood of the Yellow River basin is one of the most dangerous situations of all the Chinese rivers. According to historical data, different ice flood events have been recorded in the past during the winter on the main stream and the tributaries of the Yellow River. These floods have caused huge casualties and property losses, especially on the Ning–Meng reach of the Yellow River. Since 1949 there have been 29 ice flood hazards, nine of them causing the levee to breach in the Ning–Meng reach.

Ice regime forecasting is an efficient method for ice flood control, regulation and decision making, and mathematical modelling is playing an essential part in the development of river ice research. Mathematical models can supplement the inadequacies of field and lab studies to help understand

the physical processes of river ice, and at the same time, they can also be a tool to help design and plan engineering projects. They have become valuable tools for exploring the research area of river ice, such as understanding the physical processes and simulating river ice phenomena, and even forecasting ice floods (Dahkle et al., 2012).

The development of the economy and society with the resulting climate change and human activities have changed the characteristics of ice regimes, especially ice disasters during the freezing or breaking-up periods, attracting more and more attention from water authorities and local governments. Hence, it is very important to know ice regime characteristics and use mathematical models to enable ice forecasting, ice flood prevention and ice flood alleviation.

At the YRCC a 1-D numerical model, named YRIDM has been developed to simulate ice regimes. It also contains data on observed data including hydrological, meteorological, and ice regime data, covering about fifty years of measurements. The available model and collected field data provide an opportunity to examine the possibility of ice flood forecasting. The choice of the 1-D model is given by the availability of data at YRCC. Currently there is no DEM available for the Ningxia–Inner Mongolia reach and the available collected data allowed for the development of a one-dimensional model. As the data collection continues a more complex model will be made available, by extending the existing model to a one-dimensional–two-dimensional (1-D2-D) one. It is important to use such a model in forecast conditions, in order to determine what are the possible times to have ice-formation and consequently flooding.

The purpose of this paper is to apply the YRIDM to the Ning–Meng reach of the Yellow River to examine the accuracy of the model.

This paper first describes the current state of the art in ice flood modelling, followed by a description of the case study. The modelling application is detailed next. Modelling of ice floods on the Ning–Meng reach is discussed with respect to model set-up, calibration, verification and uncertainty analysis. This paper ends with a conclusion section.

2 Ice flood modelling state of the art

2.1 Ice flood research

Ice floods in rivers are generated due to ice jams, ice dams, ice melt and snow melt (Liu et al., 2000). The ice flood process cannot be separated from the whole river ice process, although it mainly involves the ice-covered period and breaking-up period.

As summarized by Shen (2006), during the past fifty years engineering and environmental issues have largely driven the development of river ice research, and significant achievement has been made during this period. However, Beltaos (2008) thought that although there has been remark-

able progress in understanding and quantifying the complex river ice processes, many problems concerning river ice still remain today or are only partially solved.

Shen (2006) classifies the river ice research area into two parts: firstly, the energy budget methods and water temperature distribution calculations before and during the freezing-up period and secondly, the evolution of frazil ice, frazil floc, anchor ice and ice dams. Currently the water temperature distribution is well understood (e.g. Shen and Chiang, 1984), and the mechanisms of super cooling and frazil ice formation are also relatively well understood (Osterkemp, 1978; Daly, 1984). However, the evolution of frazil ice, frazil floc, anchor ice and ice dams needs to be studied further (Ye et al., 2004), the transitional conditions among different ice run regimes are not clearly understood (Hammar et al., 2002), and also knowledge on the mechanism of ice pans and ice floe formation is limited. In addition, a complete analytical formulation of the mechanical break-up should be developed.

Beltaos (2008) showed the challenges and opportunities for research into river ice processes. The main challenge is to avoid or decrease the negative influence of river ice processes and to make sure that the positive influence is not affected by human activities (Beltaos, 2008). In order to meet this challenge, it is necessary that there is a good qualitative understanding of river ice processes. However, there still remain serious gaps, such as research into anchor ice, break-up, ice jamming, and the influence of climate change on the river-ice process. The main opportunities lie in how to use new technologies to understand the river ice processes. These new technologies include instrumentation, numerical modelling, mitigation and prediction of climate impact on river ice processes.

Research into ice cover has developed from static ice cover research, such as Beltaos and Wong (1986), into dynamic ice cover research due to the fact that the original research did not take ice dynamics into consideration. Dynamic ice cover research has been further divided into one-dimensional dynamic ice cover research (Shen et al., 1990) and two-dimensional dynamic ice cover research, such as the DynaRICE model (Shen et al., 2006), in order to take into account the frictional resistance of riverbanks and channel bottoms, irregular cross sections of river channels, and unsteady flow state in reality.

Research into ice transportation under the ice covers and ice jams started from a critical velocity criterion or Froude number criterion (Kivislid, 1959; Tesaker, 1975). However, this method could not provide a way to compute an accurate value of the critical velocity and Froude number. Hence, Shen and Wang (1995) developed a concept of ice transport capacity to simulate ice deposition under the ice cover, which was demonstrated by field observation data. Nevertheless Beltaos (2008) still thought that ice transportation under the ice cover was only partially understood, and further research should be continued in the future.

The process of the thermal growth and erosion of ice cover is relatively clearly understood. Firstly, research focused on the ice cover without layers of snow ice, snow slush or black ice (Shen and Chiang, 1984). Secondly, the layers of snow ice, snow slush and black ice were taken into consideration (Calkins, 1979; Shen and Lal, 1986). Finally, based on the previous research, a lot of models were produced to simulate the processes. The most classical one was the degree-day method proposed by Stefan (1889) that has been used to simulate ice cover growth for a long time. Shen and Yapa (1985) refined the classical degree-day method which they then developed into a modified degree-day method, which was demonstrated on the St. Lawrence River.

The break-up of ice cover can be classified as mechanical break-up or thermal break-up. A mechanical break-up can exert a negative influence on the hydraulic facilities and the safety of people along the river. However, the ability to simulate a mechanical break-up is still limited (Shen, 2006).

Only a few researchers have tried to simulate the propagation of ice-jam release waves. Most of them have used a one-dimensional model without considering the effect of ice (e.g. Blackburn and Hicks, 2003). Field investigations were conducted by Jasek (2003), who found out that release wave celerity seemed to change with different ice conditions. Based on this research output, Liu and Shen (2004) started to take into consideration the effect of ice on wave propagation. She and Hicks (2006) built a model, named River-1D, to look into the effect of ice on ice-jam release waves, which was tested by the release event on the Saint John River in 1993, on the Saint John River in 2002, and on the Athabasca River in 2002. At the same time, statistical methods and ANN are also used to forecast the break-up of river ice. The potential for using fuzzy expert systems to forecast the potential risk of ice jams was discussed (Mahabir et al., 2002), and the systems identified five years when high water levels could occur, including four years when the predicted high water level did occur. Based on this potential, more research has been carried out (Mahabir et al., 2006; Wang et al., 2008) which took more related parameters into consideration.

2.2 Ice flood modelling

Among all the river ice research, mathematical modelling is an essential part of the progress. Ice flooding models have developed from 1-D steady state to 1-D unsteady state and then into 2-D models, and data-driven models are also being applied to forecast ice floods.

The 1-D steady-state models are based on the static ice-jam theory, namely that the formulas to determine the final thickness of water surface ice can be deduced according to the static balance of internal and external forces on a floating ice block. The models based on the static ice-jam theory are HEC-2 (US Army, 1990), ICETHK (Tuthill et al., 1998), and HEC-RAS (US Army, 1998). The basic assumption is that the flow is steady, gradually varied, and one-dimensional,

and that river channels have small slopes (less than 1 : 10). The basic equation is the force and energy balance equations; a standard step method is used to solve the equation. However, the limitation of 1-D steady-state models is that they ignore ice dynamic conditions.

Due to the limitation of 1-D steady-state models, the development of 1-D unsteady-state models, such as RICE (Lal and Shen, 1991), RICEN (Shen et al., 1995), and the Comprehensive River Ice Simulation System (CRISSPID) model (Chen et al., 2006), was promoted. These models are based on the assumption of unsteady-state flow; the governing equations are 1-D Saint Venant equations (i.e. mass and momentum conservation equations with floating ice) and they can be used to simulate the entire ice process in rivers during the winter season. The basic assumptions of the RICE, RICEN, and CRISSPID models are that they ignore the following: (1) the effect on the water body mass balance due to the change in ice phase; (2) the river has a floating ice cover; (3) two-layer ice transportation theory; (4) suspension ice is full of the suspension ice layer; and (5) the thickness of the ice layer on the water surface is equal to the thickness of the ice block floating on the water surface. However, the limitation of these models is that they lack detailed consideration of complex flow patterns and river geometry (Shen, 2010).

The RICE and RICEN models have been used to simulate the ice regime for a few rivers; for example, the RICE model was used for the Upper St. Lawrence River near New York and the Peace River in northern British Columbia (Li et al., 2002) and the RICEN model was used for ice-jam simulation in 1995 on the Niagara River.

Based on the limitations of 1-D unsteady-state flow, 2-D models have also been developed, such as DynaRICE (Shen et al., 2000) and CRISSP2D (Liu and Shen, 2006) that can be used to simulate the ice regime. These models solve the 2-D depth-integrated hydrodynamic equations for shallow water flow. The two basic assumptions are that the movement of the surface ice layer is continuous, and ice is a kind of continuous medium. A finite element method is used to solve the equations. However, the limitation is that they lack detailed consideration of the third dimension.

DynaRICE has been applied to understand the ice-jam evolution, ice boom design and navigation structure design, on several rivers. DynaRICE was applied in the Niagara Power Project to study both the ice control and ice-period operation, on the Missouri–Mississippi River to study ice-jam formation, and on the Shokotsu River in Hokkaido, Japan, to study break-up jams. CRISSP2D has also been applied to several rivers and lakes to simulate the ice regime. It has been applied to the Nelson River to simulate freeze-up ice conditions (Malenchak et al., 2008); on the Red River near Netley Cut (Haresign and Clark, 2011) to model ice formation; and to test the potential for anchor ice growth after the Conawapa Power Generation Station was constructed and operational (Morris et al., 2008).

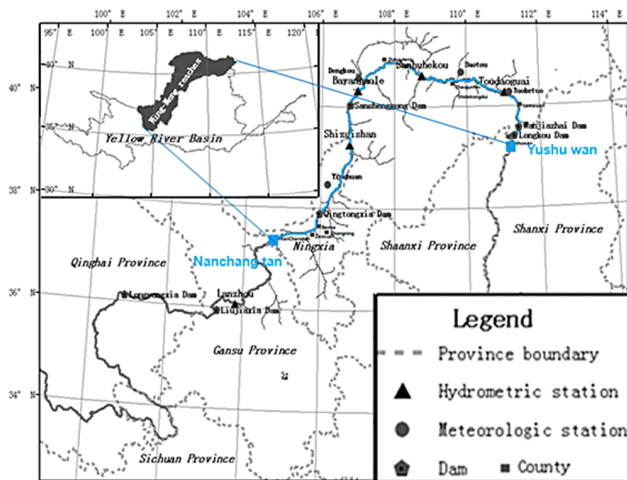


Fig. 1. Location of the Ning–Meng reach.

The potential for using fuzzy expert systems to forecast the potential risk of ice jam was discussed by Mahabir et al. (2002). Based on this research, more research has since been carried out (Mahabir et al., 2006; Wang et al., 2008). Data-driven modelling has been applied to the ice regime simulation several times and one example is the Yellow River (Chen et al., 2012). However, the limitation of using such methods is that ice processes cannot be understood by data-driven models.

3 Case study description

3.1 The Ning–Meng reach

The Yellow River is the second longest river in China and is also one of the most famous rivers in the world. Due to the fact that it is the cradle of the Chinese civilization, the river is also called the “Mother River of China”. The Yellow River originates from the Bayanhar Mountain in Qinghai Province and flows through the nine provinces of Qinghai, Sichuan, Gansu, Ningxia, Inner Mongolia, Shaanxi, Shanxi, Henan, and Shandong, and finally flows into the Bohai Sea in Shandong Province. It has a total length of 5464 km and a basin area of 752 443 km².

The Ning–Meng reach of the Yellow River (Fig. 1) is where the ice flood mainly happens. The length of the Ning–Meng reach is 1237 km and it consists of two consecutive parts, the Ningxia and Inner Mongolia reaches. The starting point for it is Nanchangtan, Zhongwei County in the Ningxia Hui autonomous region, and the end point is Yushuwan, Mazha Town, Zhungeerqi County in the Inner Mongolia autonomous region.

The Qingtongxia and Sanshenggong reservoirs are located on this reach. The total length of the Ningxia reach is 397 km and it flows from southwest to northeast. On this reach one

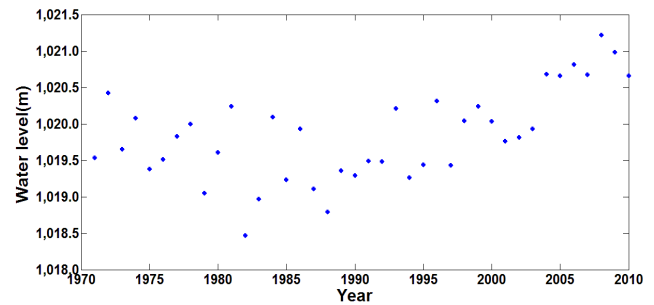


Fig. 2. Historical maximum recorded water levels.

part, from Nanchangtan to Zaoyuan, does not usually freeze because the bed slope is steep and the water velocity is large, and it only freezes in a very cold winter. The rest of the reach, from Zaoyuan to Mahuanggou, flows from south to north and it often freezes because the slope is gentle and the water velocity is low. The Inner Mongolia reach is located on the north of the Yellow River basin between 106°10' E–112°50' E, 37°35' N–41°50' N. The total length of the main stream is 830 km, and the total height difference is 162.5 m. The reach is wide with a gentle slope and meandering twists and turns. Although it is located in the middle and lower reaches, the slope of the reach that is under study is close to that at the Yellow River estuary.

3.2 Past ice floods

According to historical data for the Yellow River, ice disasters appeared frequently in the past and every year between 1855 and 1949, when dams were destroyed by ice floods 27 times. In the period between 1951 and 1955, the dams on the Lijing breach were also destroyed by ice flood. In addition, there were 28 ice flood seasons with ice disasters between 1951 and 2005 (Rao et al., 2012).

Especially on the Ning–Meng reach, ice jams, frazil jams, and other ice flood disasters have happened frequently. According to the statistics for ice flood disasters on the Ning–Meng reach, there were 13 ice flood events from 1901 to 1949, almost one event every 4 years. Even after the Liujiaxia Reservoir and the Qingtongxia Reservoir started operation in 1968 and 1960 respectively, ice flood disasters still occurred in 1993, 1996, 2003, and 2008 (Gao et al., 2012).

During the winter season 2007/2008, the water storage in the Inner Mongolia reach reached the largest value, 1835 million, 730 million more than during the normal flooding condition (1105 million). As a result, the water level at Sanhuhekou hydrological station reached a recorded level of 1021.2 m, 0.41 m higher than any recorded level at the station, which led to dike break at two sites, causing serious economic losses. During the break-up period in the winter 2005/2006, the water level at Sanhuhekou station reached 1020.81 m, for a maximum discharge of 772. Figure 2 show

the maximum recorded water levels, every decade between 1970 and 2009, at Sanhuhekou hydrological station.

The Yellow River basin is under the influence of cold air from the vicinity of Siberia and Mongolia; the mean air temperature during the whole winter is below 0°, and this lasts for 4 to 5 months. Due to the special geographical location and difference in latitude of the Ning–Meng reach on the Yellow River, together with the special river flow direction, the downstream part both freezes and breaks up earlier than the upstream part. Hence it is very easy for ice jams or backwater to be formed, which can result in ice flood disasters such as dams being destroyed and dike breaks. Based on the literature review on ice flood disasters, one can conclude that on the Yellow River the main problems regarding ice floods are caused by ice-dam floods and ice-jam floods which can result in dike breaks and overtopping on the embankment. This is the key problem to be solved on the Ning–Meng reach of the Yellow River.

4 Modelling application

4.1 Model set-up

Present research uses the YRIDM, which is a 1-D unsteady-state model. The YRIDM is based on the RICE and RICEN models, and designed taking river ice processes into account.

The YRIDM model is simulating the ice dynamics, by describing the skim ice formation and frazil ice as it is in the RICE model. Skim ice formation is based on the empirical equation, developed by Matousek (1984); frazil ice along the river channel is described by the mathematical model defined by Shen and Chiang (1984) and the ice dynamics is described by the static border of the ice formation, which is defined by Svensson et al. (1989) in form of a critical value. The other elements of the ice dynamics are the growth of the border ice, the undercover accumulation, erosion of the ice, and erosion of the ice cover due to thermal growth. Though YRIDM is based on the RICE model, there is a major difference in how the break-up of the ice is checked in the two models. In the case of RICE, the critical value of discharge for which the break-up appears is a constant value that has to be determined before using the model, by experiments on site, while in the YRIDM model the critical discharge is determined by an empirical equation.

The YRIDM model was designed to simulate the ice flood for unsteady-state flow, which uses the governing Eqs. (1)–(5) below:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0, \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} = -gA \left(\frac{\partial Z}{\partial x} + S_f \right), \tag{2}$$

$$S_f = n_c^2 \frac{|u|u}{R^{4/3}}, \tag{3}$$

$$n_c = \left[(n_i^{3/2} + n_b^{3/2})/2 \right]^{2/3}, \tag{4}$$

$$n_i = n_{i,i} + (n_{i,i} - n_{i,e})e^{-\alpha_n T}, \tag{5}$$

where Q = discharge ($\text{m}^3 \text{s}^{-1}$); A = net flow cross-sectional area (m^2); x = distance (m); t = time (s); g = gravitational acceleration (m s^{-2}); S_f = friction slope; n_c = equivalent roughness; n_i = roughness of ice cover; n_b = roughness of river bed; u = average velocity (m s^{-1}); R = river hydraulic radius (m); $n_{i,i}$ = initial roughness of ice cover; $n_{i,e}$ = final roughness of ice cover at the end of an ice-covered period (0.008 ~ 0.012); T = freeze-up time (day); and α_n = decay coefficient.

The YRIDM equations for ice transportation model are different from the ones in RICE, that is, YRIDM uses convection–diffusion equation between ice run and floating ice, while RICE uses a two-layer mode for the surface ice transportation. The critical value of discharge for which the break-up appears is determined by the empirical Eq. (6):

$$Q \geq \frac{9.5 * a * h_i}{\sum T_{a+} + 1}, \tag{6}$$

where Q is discharge ($\text{m}^3 \text{s}^{-1}$); h_i is the ice cover thickness when air temperature varies from positive values to negative ones (m); a is the break-up coefficient ($a = 4$ for mechanical break-up and $a = 22$ for thermal break-up); and T_{a+} is the average daily accumulated positive air temperature, measured from the day when air temperature turns positive.

The roughness of ice cover is usually computed based on the end roughness of ice cover and a decay constant; however, the YRIDM model is developed with a constant initial roughness of ice cover, which has been determined by YRCC through several measuring campaigns.

The logical flowchart of the YRIDM model is presented in Fig. 3. It consists of three main components, the river hydrodynamics, thermodynamics, and ice dynamic modules. The advantage of this model is that it can be subdivided even further into the following modules: river hydraulics, heat exchange, water temperature and ice concentration distributions, ice cover formation, ice transport and cover progression, undercover deposition and erosion, and thermal growth and decay of ice covers.

Based on the available data analysis for the Ning–Meng reach of the Yellow River, and literature review on ice models, the YRIDM has been selected to conduct the modelling for ice flood middle-term prediction (10–15 days ahead). The ice regime on the Ning–Meng reach is mainly determined by thermal factors, dynamic factors, channel course conditions and human activities. During the whole research period between 2001 and 2011, the most serious ice flood occurred during two winters, namely the winter from 2007 to 2008 and the winter from 2008 and 2009. The winter from 2008 to 2009 was chosen as the simulation winter to calibrate the

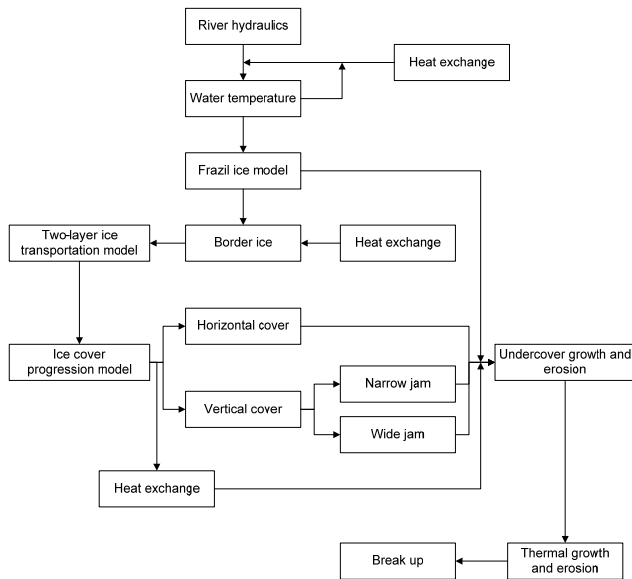


Fig. 3. YRCC river ice dynamic model flowchart.

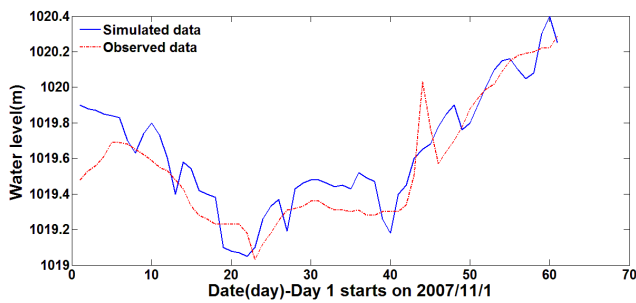


Fig. 4. Verification of the model: water levels.

model, and the winter from 2007 to 2008 was used to verify the calibrated model.

The reach between Bayangaole station and Toudaoguai station was chosen to be modelled due to the fact that ice flood has happened frequently here and data was available on the reach. The simulation reach has a total length of 475.6 km. A hydrological station named Sanhuhekou is positioned in the middle of the reach (i.e. 205.6 km away from Bayangaole station).

The available data includes river hydraulics, meteorological, ice regime, and cross sectional data at the four hydrometric stations, namely Shizuishan, Bayangaole, Sanhuhekou, Toudaoguai stations, and it covers the period of ten winters, from 2001 to 2011. River hydraulic data includes water level and discharge with daily measuring frequency. Meteorological data includes air and water temperature with a daily measurement of the water temperature and twice a day measurement of air temperature (the daily highest and lowest temperature per day). Ice regime data includes ice run data, freeze-up date, break-up date, and ice cover thickness. The measured frequency is per winter except that the measured

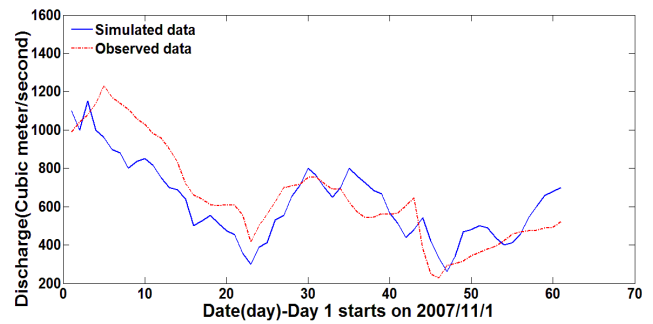


Fig. 5. Verification of the model: discharge.

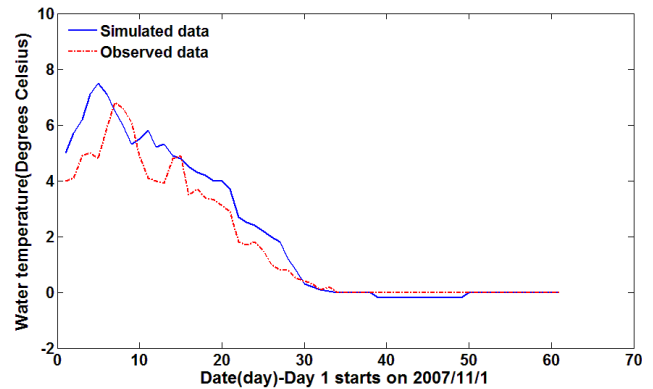


Fig. 6. Verification of the model: water temperature.

frequency of ice cover thickness is every 5 days. Data for four cross sections for 2009 is available.

During the winter of 2008/2009 the discharge and water temperature at Bayangaole station, the water level at Toudaoguai station, air temperature at Sanhuhekou and Toudaoguai stations, and cross sections and bed elevation were used to build the model.

4.2 Model calibration and verification

During the winter of 2008/2009, daily mean discharge at the Bayangaole station is taken as the upstream boundary condition to the model, and the daily mean water level at the Toudaoguai station was set as the downstream boundary condition. The air temperatures at the Sanhuhekou and Toudaoguai stations were input into the model, and the air temperature at the Sanhuhekou station was used to cover the reach from the Bayangaole station to the Sanhuhekou station, and at the same time, the air temperature at the Toudaoguai station was used to cover the reach from the Sanhuhekou station to the Toudaoguai station. The daily lowest and highest temperatures were interpolated by sine function. Daily mean water temperature at the Bayangaole station was input into the model as the upstream boundary condition of water temperature. The initial conditions were water temperature, water level, and discharge.

Table 1. Results of the sensitivity analysis of the parameters of the YRCC river ice dynamic model.

Sensitivity parameters	Physical meaning	Reference range	Unit	Sensitive object
Nb	Bed roughness	0.019 ~ 0.045	–	Water level
$n_{i,e}$	End-ice roughness	0.008 ~ 0.035	–	Water level
α_n	Decay constant	0.005 ~ 0.4	1/day	Water level
Coe_Cw	Heat exchange coefficient between water and ice	15 ~ 18	–	–
Coe_Hia	Heat exchange coefficient between ice and air	6 ~ 12	–	Ice cover thickness
Coe_Co	Heat exchange coefficient between water and air	15 ~ 25	–	Ice cover thickness

Table 2. Calibrated parameters values.

Parameters	Physical meaning	Value	Unit
<i>N1</i>	Bed roughness (cross1–cross26)	0.011	–
<i>N2</i>	Bed roughness (cross27–cross51)	0.017	–
<i>N3</i>	Bed roughness (cross52–cross87)	0.045	–
<i>N4</i>	Bed roughness (cross88–cross121)	0.004	–
α_n	Decay constant	0.001	1/day
$n_{i,e}$	End-ice roughness	0.01	–
Coe_Cw	Heat exchange coefficient between water and ice	16.62	–
Coe_Hia	Heat exchange coefficient between ice and air	12	–
Coe_Co	Heat exchange coefficient between water and air	19	–

Based on the literature review and the reference book of the numerical model from YRCC, the sensitivity parameters are listed in Table 1. The one-at-a-time sensitivity measure method was used to conduct the sensitivity analysis, which means the value of one parameter is changed from the minimum value to the maximum value while at the same time other parameters are kept constant at the mean value, and then the variation of the model output is checked.

After the sensitivity analysis, scenarios of the model calibration can be designed based on the sensitivity analysis results (Table 1). If the model is sensitive to a certain parameter, then the parameter needs to be calibrated carefully, otherwise the default value is used. In this case, bed roughness, end-ice roughness, and decay constant are sensitive to the water level at the Sanhuhekou station; heat exchange coefficient between ice and air, and heat exchange coefficient between water and air are sensitive to ice cover thickness at the Sanhuhekou station. These five parameters should be calibrated carefully; the heat exchange coefficient between water and ice is not sensitive to the output, hence the default value can be set. The model can be calibrated under different ice regime conditions, which means that some parameters can be calibrated under open water conditions, such as the Manning coefficient of river beds and the heat exchange coefficient between water and air. Some parameters can be calibrated under ice conditions, such as the Manning coefficient under ice cover, end-ice roughness, and decay constant. Hence the model calibration procedure is divided into a model calibration under open water conditions and a model

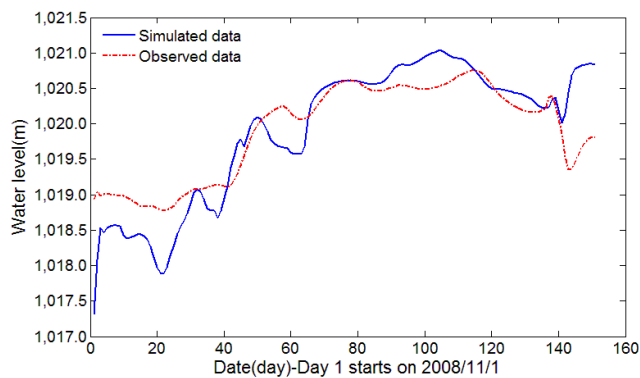
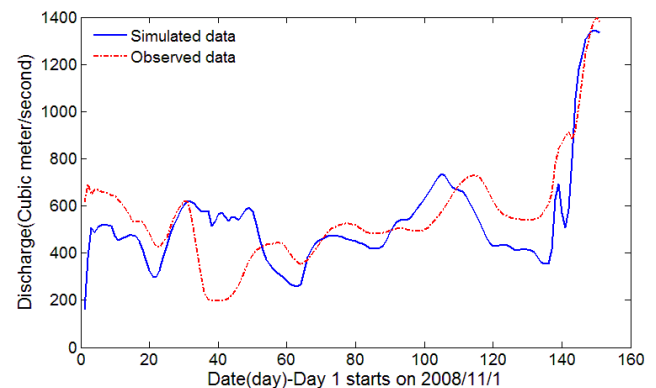
calibration under ice conditions. In the calibration procedure, RMSE (root mean square error) is used as the criterion to check the model performance.

The calibrated bed roughness for different cross sections was varied for a large range of values (i.e. from 0.004 to 0.017). These values are provided by YRCC and reflects the four different types of channel beds of the modelled river reach. The Ningxia River reach, which is 397 km long, starts at Nanchangtan and ends at Mahuanggou, in Shizuishan city. From Nanchangtan to Zaoyuan (ca. 135 km) there is an uncommon freezing state of the river reach, because the bed slope is steep and water velocity is high. This state allows the river reach to freeze up only during very cold winters. On the reach from Zaoyuan to Mahuanggou (ca. 262 km), on the other hand, there is a common freezing state of the river reach because the slope is gentle and water velocity is small. The Inner Mongolia reach is located on the north of Yellow River basin and has a total length of 830 km. The reach is wide with gentle slopes, meandering twists and turns. Although it is located in the middle and lower Yellow River, its slope is close to that of the Yellow River estuary. A summary of the slopes, width and roughness of these four types of considered reaches is given in the Table 3.

After calibration of YRIDM during the winter of 2008/2009, the discharge and water temperature at the Bayangaole station, the water level at the Toudaoguai station, air temperature at the Sanhuhekou and Toudaoguai stations, and cross sections and bed elevation during the winter of 2007/2008 were used to build the model. The simulated

Table 3. Channel characteristics of Ning–Meng reach.

Autonomous region	Section	Channel type	Channel length (km)	Average channel width (m)	Main channel width (m)	Channel slope (%)	Roughness
Ningxia	Nanchangtan–Zaoyuan	valley type	135	200–300		0.8–1.0	0.005–0.014
	Zaoyuan–Mahuanggou	transition type	262	500–1000		0.1–0.2	0.005–0.014
Inner Mongolia	Mahuanggou–Wuda bridge	valley type	69.0	400	400	0.56	0.011–0.020
	Wuda bridge–Sanshenggong	transition type	106.6	1800	600	0.15	0.011–0.020
	Sanshenggong–Sanhuhekou	wandering type	205.6	3500	750	0.17	0.009–0.018
	Sanhuhekou–Zhaojunfen	transition type	126.2	4000	710	0.12	0.009–0.018
	Zhaojunfen–Lamawan	bend type	214.1	3000–2000	600	0.10	0.002–0.010
	Lamawan–Yushuwan	valley type	118.5				0.002–0.010
Total	Nanchangtan–Yushuwan		1237				

**Fig. 7.** Water levels at Sanhuhekou station.**Fig. 8.** Discharge at Sanhuhekou station.

water level, discharge, water temperature, and ice cover thickness at the Sanhuhekou station during the winter of 2007/2008 were used to compare with the observed values to verify the calibrated model.

The simulation period used for verification of the model was done for the available data from 1 November 2007 to 31 December 2007. Results of the verification are presented in Figs. 4, 5 and 6 for water level, discharge and temperature, respectively. Though there was just little data measured and available for verification, results shows that the model could capture the trend of water level, discharge, and water temperature. It is worth to be noted that in 2008/2009, the water level did not exceed the embankments.

4.3 Model uncertainty analysis

Based on the sensitivity analysis results, the water level at the Sanhuhekou station is sensitive to the Manning coefficient of river bed, decay constant, and end-ice roughness; and the ice cover thickness at the Sanhuhekou station is sensitive to the heat exchange coefficient between ice and air, and heat exchange coefficient between water and air. Hence, the uncertainty analysis is divided into an uncertainty analysis of the water level at the Sanhuhekou station and the uncertainty analysis of ice cover thickness at the Sanhuhekou station, and

the Monte Carlo simulation is used to conduct the parametric uncertainty analysis (Moya Gomez et al., 2013).

Due to the limitation of time, it was impossible to run the model several times. When conducting the uncertainty analysis of the water level at the Sanhuhekou station, the related four parameters were the Manning coefficients of the river bed at the upstream and downstream of the Sanhuhekou station, decay constant, and end-ice roughness. The scenarios of the uncertainty analysis of the water level at the Sanhuhekou station were designed based on the calibrated parameters, the range was calculated by increasing and decreasing the calibrated value by 20 %, the sample generation was uniformly random, and the number of simulation was 500. When conducting the uncertainty analysis of the ice cover thickness at the Sanhuhekou station, the two related parameters were the heat exchange coefficient between ice and air, and the heat exchange coefficient between water and air. The scenarios of the uncertainty analysis about ice cover thickness at the Sanhuhekou station were designed based on the calibrated parameters, the range was calculated through increasing and decreasing the calibrated value by 20 %, the sample generation was uniformly random, and the number of simulations was 400.

Based on the above case designs, the parameters were input into the model, the model was run and the result were

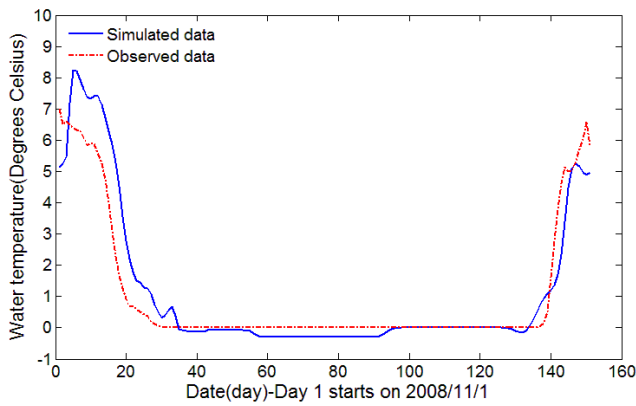


Fig. 9. Water temperature at Sanhuhekou station.

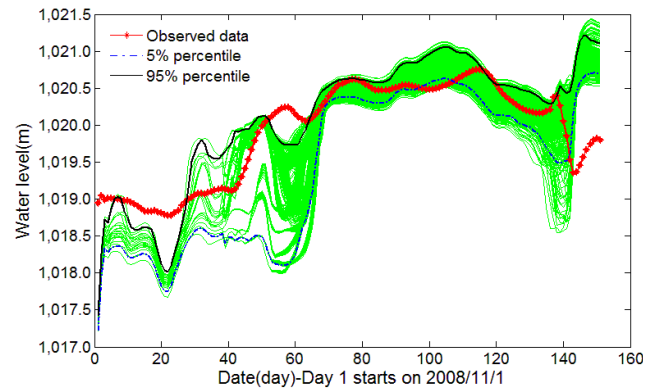


Fig. 11. Water levels at Sanhuhekou station.

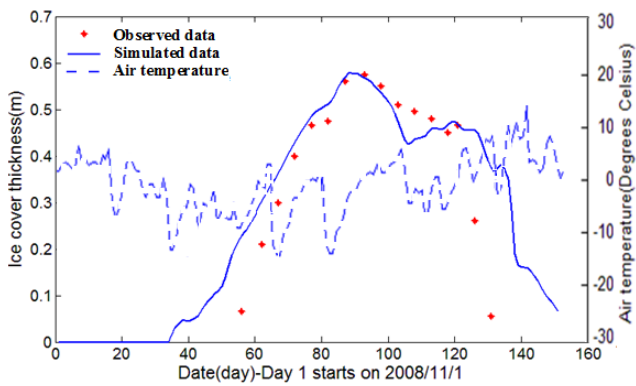


Fig. 10. Ice cover thickness at Sanhuhekou station.

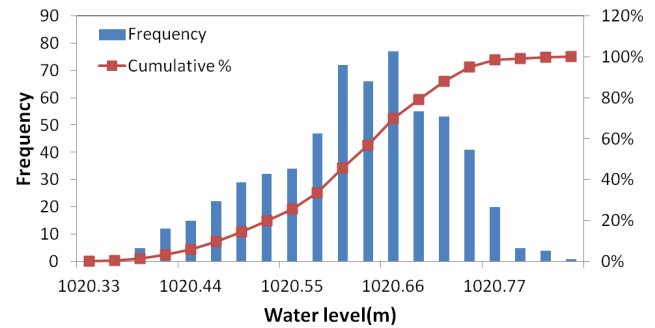


Fig. 12. Probability distribution of the water level on Day 90.

stored, the distribution and quartile of the output were analysed, namely the pdf (probability density function) at one time step and two bounds (5 and 95 %).

5 Results and discussion

The results of the sensitivity analysis show that bed roughness, end-ice roughness, and the decay constant are sensitive to the water level at the Sanhuhekou station, and that the heat exchange coefficient between ice and air, and the heat exchange coefficient between water and air are sensitive to ice cover thickness at the Sanhuhekou station.

Based on the results of the sensitivity analysis, the model was calibrated, and the values of calibrated parameters can be seen in Table 2.

Figure 7 shows the water level comparison at the Sanhuhekou station during the winter of 2008/2009 between the simulated data and observed data. The resulting RMSE is 0.464. The simulated result is acceptable and reasonable. From day 1 to day 40 the simulated result is not good, but the trend in the simulated results is consistent with the observed results, which is because in the beginning, the simulated water level is affected by the initial conditions. From

day 125 to the end, the simulated result is not accurate and the trend is even adverse, which is because the model cannot simulate an ice-jam break-up during the break-up period. The data observed during the period clearly shows that the water level decreases suddenly, which means that the ice jam at the downstream of the Sanhuhekou station collapses, but the model cannot simulate this phenomenon, which is why the simulated results are not so accurate. During the rest of the period, the results are acceptable.

Figure 8 shows a comparison between the simulated discharge data and observed discharge data at the Sanhuhekou station during the winter of 2008/2009. The RMSE is 160.5 compared with the peak discharge of $1400 \text{ m}^3 \text{ s}^{-1}$, so the error rate is 11.2 %, and hence the simulated result is acceptable and reasonable. From day 1 to day 25, the simulated results are not accurate, but the trend in the simulated results is consistent with the observed trend, which is because in the beginning, the simulated water level is affected by the initial conditions. From day 25 to day 50, the simulated results are also not accurate. The discharge decreases suddenly; this is because during the freeze-up period, the channel storage capacity increases when the water has been changed into ice, then the amount of water decreases, which can result in a sudden decrease in discharge, but the discharge can increase again when the channel storage capacity of the river continues to be stable. However, there is an assumption that the

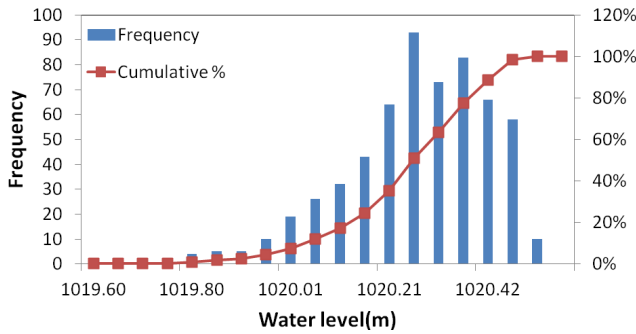


Fig. 13. Probability distribution of water level on Day 130.

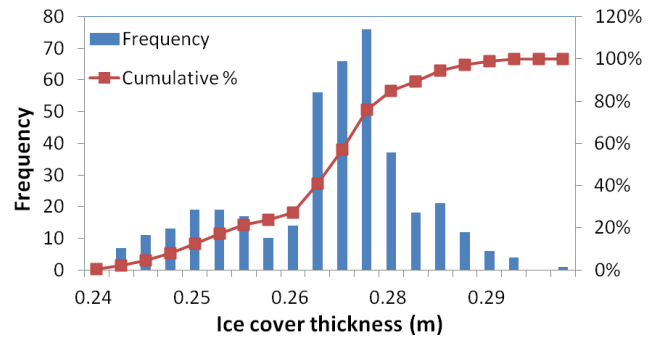


Fig. 15. Probability distribution of ice cover thickness on Day 60.

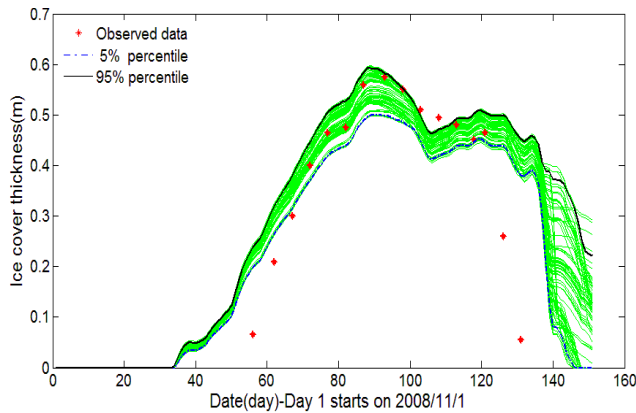


Fig. 14. Ice cover thickness at Sanhuhekou station.

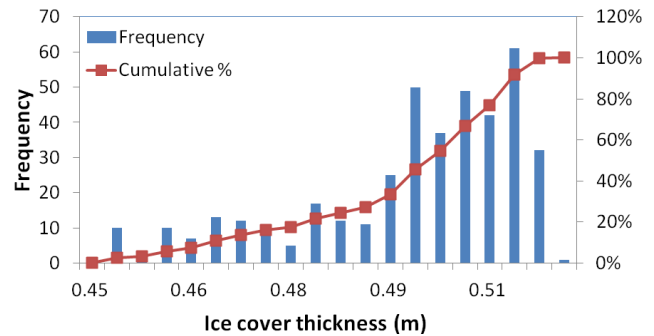


Fig. 16. Probability distribution of ice cover thickness on Day 120.

model ignores the effect on the water body mass balance due to changes in the ice phase; this is the reason why the results are not good enough. During the other period, the result is acceptable.

Figure 9 shows the comparison between the simulated data and observed data for the water temperature at the Sanhuhekou station during the winter of 2008/2009; the RMSE is 0.854. The simulated results are acceptable and reasonable. From day 1 to day 15, the simulated results are not good, which is because in the beginning, the simulated water temperature is affected by the initial water temperature. From day 55 to day 95, the water temperature is below 0°; this is the super cooling phenomenon, which means before the formation of ice cover the water body loses heat very quickly, which can result in the negative value for the water temperature. However, once the ice cover forms, it can prevent heat exchange between the water body and the air; this is the reason why the water temperature keeps steady at 0° between day 100 and day 130. During the rest of the period, the results are acceptable.

Figure 10 shows the comparison between the simulated data and observed data for ice cover thickness at the Sanhuhekou station during the winter of 2008/2009. Because the ice cover thickness data was measured per five days, the observed ice cover thickness data is not continuous and

the RMSE between the simulated data and observed data is 0.109 m. The simulated results can describe the variation trend for ice cover thickness, and the simulated maximum value of the ice cover thickness, 0.58 m on 28 January 2009, has the same value as the measured one

The prepared data during the winter of 2007/2008 were input into the model to verify the calibrated model. In the winter of 2007/2008, the water level exceeded the height of embankment at Duguitakuisu county, and resulted in a dike breach, which could not be captured by the model because as per the reference guide for the model, it cannot work when the water level exceeds the height of the embankment. In this respect YRIDM should be improved so that it has the ability to deal with such kind of problems.

After running the model, the uncertainty analysis results can be summarized in Fig. 11. Figure 11 shows the observed data, 5 % percentile bound, 95 % percentile bound, and the results of 500 cases. Day 90 (Fig. 12) and Day 130 (Fig. 13) are chosen to show the probability distribution. When it comes to the probability distribution of the uncertainty analysis results, if the distribution looks like a normal distribution, the uncertainty analysis results are good. According to Figs. 12 and 13, the probability distribution of the water level on day 90 looks good, which is because it looks like the normal distribution. Although the probability distribution of the water level on day 130 is a skewed normal distribution, at

least it shows the basic shape of normal distribution, and hence the uncertainty analysis results are reasonable.

After running the model, the uncertainty analysis results can be summarized in Fig. 14. Figure 10 shows the observed data, 5 % percentile bound, 95 % percentile bound, and the results of 400 cases. Day 60 (Fig. 15) and Day 120 (Fig. 16) are chosen to show the probability distribution. According to Figs. 15 and 16, the uncertainty analysis results are not good, due to the fact that the probability distribution of the ice cover thickness on these two days is not a normal distribution. This result is due to the fact that the number of cases designed for the uncertainty analysis might not be sufficient to show the characteristics of a model uncertainty.

Results show the sensitivity of bed roughness, end-ice roughness, and decay constant to water level at the Sanhuhekou station; and that both the heat exchange coefficient between ice and air and the heat exchange coefficient between water and air are sensitive to ice cover thickness at the same station. The model however cannot work when the water level exceeds the height of embankments, nevertheless reasonable results about uncertainty analysis can be achieved. Based on the obtained results it can be concluded that the model can be applied to the Ning–Meng reach to simulate its ice regime, and once calibrated, it can be used to forecast the ice regime to support decision making, such as on artificial ice-breaking and reservoir regulation

6 Conclusions

Tracking ice formation from observations and combining with numerical model predictions for advanced warning requires proper understanding of all scientific issues that play a role. In the case of the Yellow River, ice floods impose a threat every year, which is why the Yellow River Commission is putting considerable effort in verifying theoretical formulations with actual field measurements in order to better understand the scientific mechanisms that play a role. Transforming this knowledge into an early warning system that can help save lives is a scientific issue that requires attention (Almoradie et al., 2014; Jonoski et al., 2014; Popescu et al., 2012).

Based on the obtained results with the YRIDM model it can be concluded that it is applicable to the Ning–Meng reach for simulating ice regimes, and once calibrated, it can be used to forecast the ice regime to support decision making, such as on artificial ice-breaking and reservoir regulation. There is however a limitation of the YRIDM model that it cannot simulate a mechanical break-up during the break-up period because the effect of ice phase change on the water body mass balance is ignored.

The used model has the advantage that for 10–15 days forecasts of the meteorological data that are used as hydrological and hydraulic input into the model, the ice regime can be predicted and support decision making.

As the data collection continues the base to determine the possible times of ice formation and consequently flooding is enlarged and improved (Debolskaya, 2009). Based on ice-formation predictions decision makers can take appropriate measures to reduce the risk of flooding. Flooding during cold season is very important, therefore determination of the moments of ice formation that could possibly eliminate flooding, due to the decisions taken is also an important task in modelling. For ice formation and based on data availability a 1-D model is sufficient to be used; however, for the determination of the flood extent and time of flood occurrence, a more complex model, such as a 1D-2D, needs to be made available (Gichamo et al., 2013; Shen et al., 2008).

Though the present research focussed on ice formation rather than floods it can be generally concluded that the measured elements and frequency should be increased, and as recommendation if floods need to be captured and simulated then the one-dimensional models should be extended to two-dimensional models.

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