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A Regionalised Neyman-Scott Model of Rainfall with Convective and Stratiform Cells

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Abstract

A single-site Neyman-Scott Poisson cluster model of rainfall, with convective and stratiform cells, is fitted to data for 112 sites scattered throughout the UK using harmonic variables to account for seasonality. The model is regionalised by regressing the estimates of the harmonic variables on site dependent variables (e.g. altitude) to enable rainfall to be simulated at any ungauged site in the UK. An assessment of the residual errors indicates that the regression models can be used with reasonable confidence for urban sites. Furthermore, the regional variations of the model parameter estimates are found to be in agreement with meteorological knowledge and observation. Simulated 1 h extreme rainfalls are found to compare favourably with observed historical values, although some lack-of-fit is evident for higher aggregation levels.

Introduction

Observational studies on precipitation fields have shown that multiple types of rain cells exist within the same storm (e.g. areas of heavy convective rain and larger areas of lighter rain; Austin and Houze 1972). Furthermore, short-duration intense rain embedded within lighter stratiform rain can also be seen in plots of fine resolution data (e.g. see Fig. 1). Consequently, it is advantageous to generalize existing Poisson cluster models of rainfall by allowing the parameters of a rain cell to depend on the 'type' of cell (Cowpertwait, 1994). For example, in a generalized point process model convective rain could be represented by cells of short duration and high intensity whilst stratiform rain could be represented by cells of longer duration and lighter intensity.

Whilst the physical process can be used to guide the formulation of an appropriate Poisson cluster model, there is no guarantee that the fitted model will necessarily give insight into the underlying physical process. In particular, there is no guarantee that convective cells in the fitted model will correspond with those studied in meteorology. However, if the fitted model parameters compare favourably with some known physical properties, e.g. the expected lifetime of a convective cell obtained by radar or satellite, then this would suggest that a suitable stochastic model has been formulated.

Most applications of Poisson cluster models are based on the work of Rodriguez-Iturbe *et al.* (1987) and range from disaggregating daily data to hourly data (Glasby *et al.* 1995) to simulating rainfall scenarios for future climates resulting from the steady increase of greenhouse gases (Burlando and Rosso, 1991). Onof and Wheater (1993) and Khaliq and Cunnane (1996) have reported applications of the Bartlett-Lewis model to rainfall data from the UK and Ireland, while the single cell Neyman-Scott model has been regionalized over the UK for application in urban drainage studies (Cowpertwait *et al.*, 1996). The work in this last paper is further developed here to consider the regional variation of the parameters of stratiform and convective rain cells in the generalized Neyman-Scott model described by Cowpertwait (1994).

The paper is structured as follows. The generalised Neyman-Scott model is defined and its statistical properties given: a model with two cell types is fitted to data taken from 112 sites scattered throughout the UK and harmonic parameter estimates are regressed on site variables. The residual errors are assessed, regional variations in convective and stratiform cells across the UK are explored, and some comparisons made with observed extreme values. Finally some overall conclusions are given.

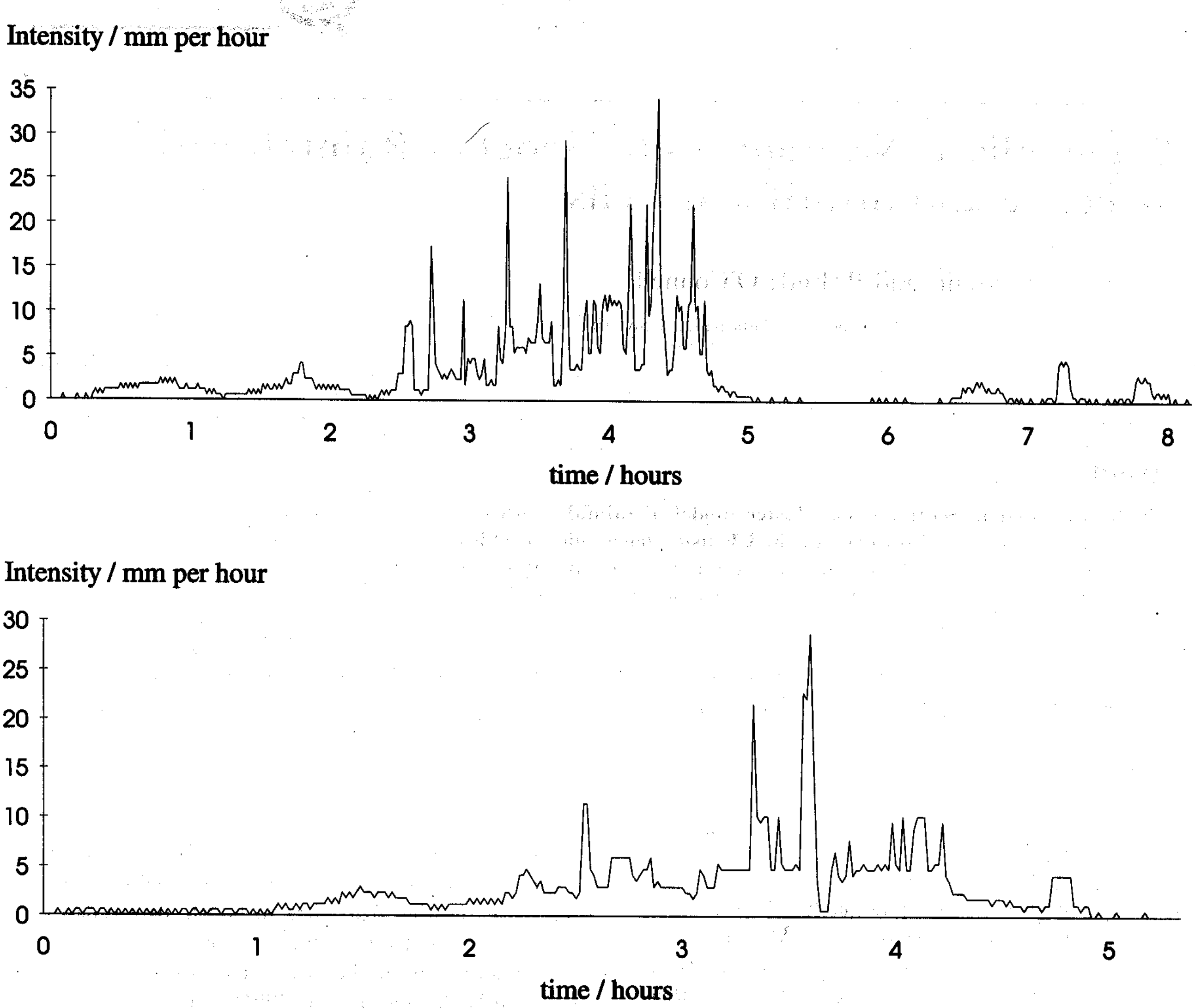


Fig. 1 *Plots of fine resolution data taken from sites in the South-West of Britain. The plots show intense short-duration rain (convective cells) embedded within lighter long-duration rain (stratiform cells).*

Generalized Neyman-Scott Model and Derived Properties

Following Cowpertwait (1994), let storm origins occur in a Poisson process of rate λ, each origin generating a random number C of cell origins. Let the waiting time for each cell origin after a storm origin be exponential with parameter β, no cell origin being located at the storm origin. Each cell origin is classified as one of n types, where α_i is the probability that a cell origin is of type $i(i = 1, \dots, n)$. A rectangular pulse (rain cell) is associated with each cell origin and depends on cell type (see Fig. 2); the duration of the pulse is an independent exponential random variable with parameter η_i for type i cell origins; the intensity of the pulse is an independent random variable X_i for type i cell origins. Let the mean number of cells per storm be denoted as μ_c and $E(X_i)$ as μ_i. The cell origins thus follow a Neyman-Scott point process, so that the model is a Generalised form of Neyman-Scott Rectangular Pulses (NSRP) model. This is abbrieviated to GNSRP(n) to denote a model with n cell types.

To fit the model to data, some statistical properties, e.g. second-order moments, are required. Let $Y(t)$ be the total rainfall intensity at time t given by the GNSRP(n) model, and let $X_{t-u}(u)$ denote the rainfall intensity at time t due to a cell with origin at $t - u$. Then,

$$X_{t-u}(u)\begin{cases} X_i \text{ with probability } \alpha_i e^{-\eta_i u} i, \dots, n; \\ 0 \text{ otherwise} \end{cases} \qquad (1)$$

The total rainfall intensity at time t, which is the summation of all cells active at time t, is given by:

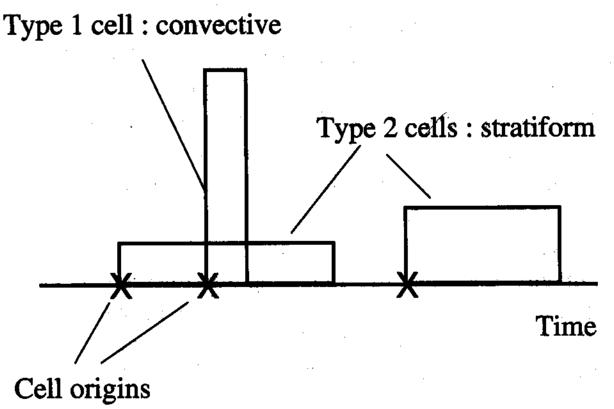


Fig. 2 *Schematic showing two cell types. Type 1 cells are heavy short-duration convective cells. Type 2 cells are lighter long-duration stratiform cells.*

$$Y(t) = \int_{u=0}^{\infty} X_{t-u}(u)dN(t-u), \qquad (2)$$

Now, let $S_k(h)$ be the aggregated rainfall depth in the kth time interval of length h, so that:

$$S_k(h) = \int_{(k-1)h}^{kh} Y(t)dt \qquad (3)$$

The second-order properties of $S_k(h)$ were derived by Cowpertwait (1994) and are given below:

$$E\{S_k(h)\} = h\lambda\mu_c \sum_{j=1}^{n} \alpha_j \mu_j / \eta_j \qquad (4)$$

$$Var\{S_k(h)\} = \sum_{j=1}^{n} \left\{ \frac{2C_j}{\eta_j^2}(h\eta_j + e^{-\eta_j h} - 1) \right\} + \frac{2C}{\beta^2}(h\beta + e^{-\beta h} - 1) \qquad (5)$$

and for $l \geq 1$,

$$Cov\{S_k(h), S_{k+1}(h)\} = \sum_{j=1}^{n} \left\{ \frac{C_j}{\eta_j^2} e^{-\eta_j(k-1)h}(1 - e^{\eta_j h})^2 \right. + \frac{C_\beta}{\beta^2} e^{-\beta(k-1)h}(1 - e^{\beta h})^2 \qquad (6)$$

where C_j and C_β are given in (7) and (8) below:

$$C_j = \frac{\lambda\mu_c\alpha_j E(X_j^2)}{\eta_j} + \frac{\lambda\beta^2\mu_j\alpha_j E(C^2 - C)}{\beta^2 - \eta_j^2} \sum_{i=1}^{n} \frac{\mu_i\alpha_i}{\eta_i + \eta_j} \qquad (7)$$

$$C_\beta = -\tfrac{1}{2}\lambda\beta E(C^2 - C) \sum_{i=1}^{n} \sum_{j=1}^{n} \left\{ \frac{\mu_i\mu_j\alpha_i\alpha_j}{(\beta - \eta_j)(\beta + \eta_i)} \right. \qquad (8)$$

The probability that an arbitrary interval of length h is dry, $\phi(h) = \text{pr}\{S_k(h) = 0\}$, was also derived by Cowpertwait (1994), under the assumption that the number of cells per storm C is a geometric random variable with parameter ν;

$$\phi(h) = \exp[-\lambda \int_0^{\infty} \{1 - p_h(t)\}dt] \exp\left[-\lambda h + \frac{\lambda\nu}{\beta(1-\nu)} \ln\left\{ \frac{1}{\nu} + \left(1 - \frac{1}{\nu}\right) e^{-\beta h} \right\} \right] \qquad (9)$$

where $p_h(t)$ = pr{no rain in time interval $(t, t + h)$ due to a storm origin at time zero}

$$= \frac{\nu\{e^{-\beta(t+h)} + \omega(1 - e^{-\beta t})\}}{1 - (1-\nu)\{e^{-\beta(t+h)} + \omega(1 - e^{-\beta t})}$$

and,

$$\omega = \sum_{i=1}^{n} \alpha_i \omega_i \text{ where}$$

$$\omega_i = 1 - \frac{\beta(e^{-\beta t} - e^{-\eta_i t})}{(\eta_i - \beta)(1 - e^{-\beta t})} (i = 1, \ldots n).$$

The integral in (9) requires numerical evaluation.

The transition probabilities follow immediately from (9) as:

$$\phi_{dd}(h) = pr\{S_{k+1}(h) = 0 \mid S_k(h) = 0\} = \phi(2h) / \phi(h)\} \qquad (10)$$

$$\phi_{ww}(h) = pr\{S_{k+1}(h) > 0 \mid S_k(h) > 0\} = \{1 - 2\phi(h) + \phi(2h)\} / \{1 - \phi(h)\}. \qquad (11)$$

Regionalised GNSRP(2) Model

DATA

Hourly and daily rainfall data from 112 sites scattered throughout the UK were available for this study; details of the data base can be found in Cowpertwait *et al.*, (1996).

MODEL PARAMETERS

Guidance on the number of cell types to be included in the model can be obtained by considering the physical characteristics of the rainfall process and past observational studies of rainfall fields reported in the literature. These studies suggest that the rainfall field can be classified broadly into convective and stratiform rain. Consequently, the simplest Neyman-Scott model taking this information into account would be GNSRP(2), where type 1 cells are interpreted as 'heavy' short-duration convective cells and type 2 cells as 'light' long-duration stratiform cells. This single-site model has a total of 8 parameters (Table 1), assuming the intensities of the heavy and light rain cells (X_1 are X_2) are exponential random variables (with parameters ξ_1 and ξ_2), and C is geometric with $\mu_i = v^{-1}$ and $E(C^2 - C) = 2v^{-1}(v^{-1} - 1)$ in Equations (4)–(8).

Table 1. Parameters of GNSRP(2) Model

λ	the rate of storm origin arrival (per hour)
β	the mean waiting time for the raincells after the storm origin
v^{-1}	the mean number of raincells per storm
η_1^{-1}	the mean cell duration for heavy cells
η_2^{-1}	the mean cell duration for light cells
ξ_1^{-1}	the mean cell intensity for heavy cells
ξ_2^{-1}	the mean cell intensity for light cells
α	the proportion of heavy cells

To allow for seasonality, the model could be fitted to each calendar month of a rainfall record, which would result in 12 estimates per parameter, i.e. 96 estimates per site. However, to simplify the regionalisation, it is advantageous to reduce this number. An approach that also ensures a smooth seasonal variation in the parameter estimates is to assume that the parameters vary across the year according to harmonic functions, i.e. if ϕ_i is a parameter estimate of the GNSRP(2) model for the ith calendar month, so $\phi \in \{\lambda,\beta,\eta_1,\eta_2,v,\xi_1\xi_2,\alpha\}$, then:

$$\phi_i = m_\phi + \Lambda_\phi \sin\left(\frac{2\pi i}{12} + \theta_\phi\right)$$

where m_ϕ, Λ_ϕ, and θ_ϕ are harmonic parameters. These parameters can be estimated directly by minimising the following sum of squares:

$$SS = \sum_{i=1}^{12} \sum_{h \in H} \sum_{f_i \in F} \left\{1 - \frac{f_i(h)}{\hat{f}_i(h)}\right\}^2 \qquad (12)$$

(subject to: $m_\phi > 0$, $\Lambda_\phi \geq 0$, $2\pi \geq \theta_\phi \geq 0$, where F is a set of aggregated second-order properties for the GNSRP(2) model (e.g. given by Eqns. 3–11), $\hat{f}_i$ denotes the sample estimate of f_i for the ith calendar month, which is obtained by pooling all the available data for the month, and H is a set of aggregation levels (greater than or equal to 1 for hourly data).

Clearly there are many choices available for F and H; here the 1 h mean, and the 1, 3, 6, 12, and 24 h variances, proportion of dry intervals, and wet spell transition probabilities are employed as in Cowpertwait *et al.* (1996).

REGRESSION ANALYSIS

A preliminary regression analysis showed that it was reasonable to treat three of the parameters, β, η_2, and ξ_1, as constants (i.e. these parameters appeared to have little or no dependence on season or location). Consequently, β, η_2, and ξ_1 were fixed at their mean values of 0.24 (h^{-1}), 0.53 (h^{-1}) and 0.26 (h mm^{-1}) respectively.

The parameters of the GNSRP(2) model were estimated for each of the 112 sites. For the sites having only daily data, a regression relationship was used to predict sample moments for aggregation levels less than 1 h (Cowpertwait *et al.* 1996).

For any UK site, the following variables could easily be found or estimated: the mean annual rainfall $\overline{AR}$ (in mm), the altitude A (in 10 m), the north grid reference N (in 100km), and the east grid reference E (in 100km).

A 'Stepwise' regression was carried out for each of the estimated harmonic variables, where interactive terms were allowed to enter the different regression models being considered. The best fitting regression models are given below:

$$\hat{m}_\lambda = 7.52 \times 10^{-6} \times \overline{AR} + 0.00168 \times N - 2.19 \times 10^{-5} \times N^3 + 0.0140 \, (R^2 = 0.45,\ SE = 0.0025\text{h}^{-1}); \quad (13)$$

$$\hat{m}_v = -2.47 \times 10^{-4} \times \overline{AR} + 0.489 \quad (R^2 = 0.40,\ SE = 0.067); \quad (14)$$

$$\hat{m}_{\xi_2} = 9.42 \times 10^{-4} \times \overline{AR} + 1.53 \quad (R^2 = 0.11,\ SE = 0.60\text{h mm}^{-1}); \quad (15)$$

$$\hat{m}_\alpha = -0.0154 \times N - 1.07 \times 10^{-4}\overline{\text{AR}} + 4.64 \times 10^{-6} \times A^3 - 7.31 \times 10^{-4} \times A \times E + 0.434 \quad (R^2 = 0.41,\ SE = 0.058); \quad (16)$$

$$\hat{\Lambda}_\lambda = -0.00125 \times E + 0.0109 \quad (R^2 = 0.18,\ SE = 0.0029\ \text{h}^{-1}); \quad (17)$$

$$\hat{\Lambda}_{\xi_2} = 0.46 \ (SE = 0.034\ \text{hr mm}^{-1}); \quad (18)$$

$$\hat{\Lambda}_\alpha = -1.37 \times 10^{-4} \times \overline{AR} + 0.356 \quad (R^2 = 0.11,\ SE = 0.088); \quad (19)$$

$$\hat{\theta}_\lambda = 0.0146 \times N^2 + 1.37 \quad (R^2 = 0.13,\ SE = 0.66\ \text{rads}); \quad (20)$$

$$\hat{\theta}_{\xi_2} = -0.729 \times E + 5.20 \quad (R^2 = 0.24,\ SE = 1.4\ \text{rads}); \quad (21)$$

$$\hat{\theta}_\alpha = 3.60 \ (SE = 0.041\ \text{rads}); \quad (22)$$

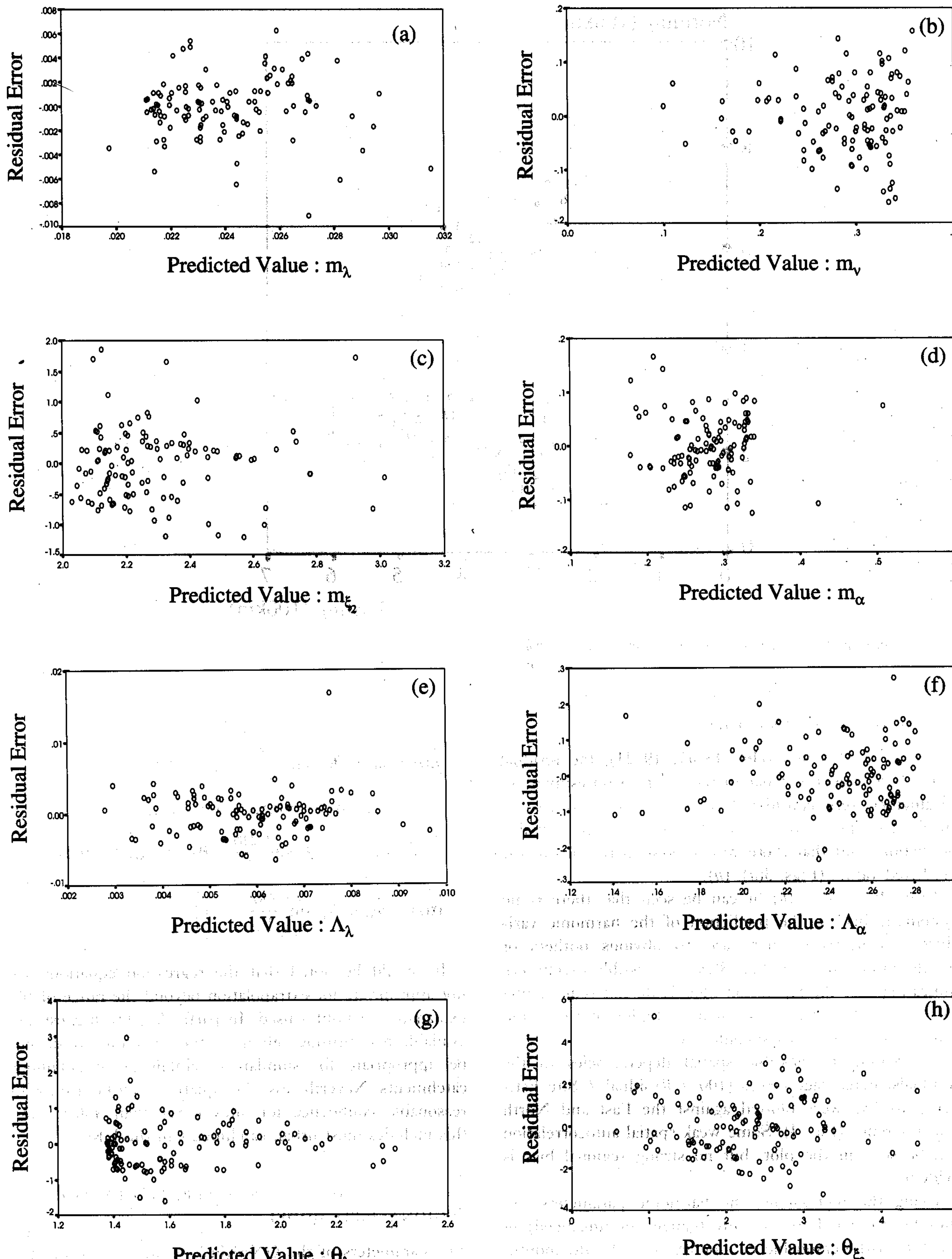


Fig. 3 *Plots of residual errors against predicted values for each of the dependent variables: (a), (b), (c), (d), (e), (f), (g), and (h).*

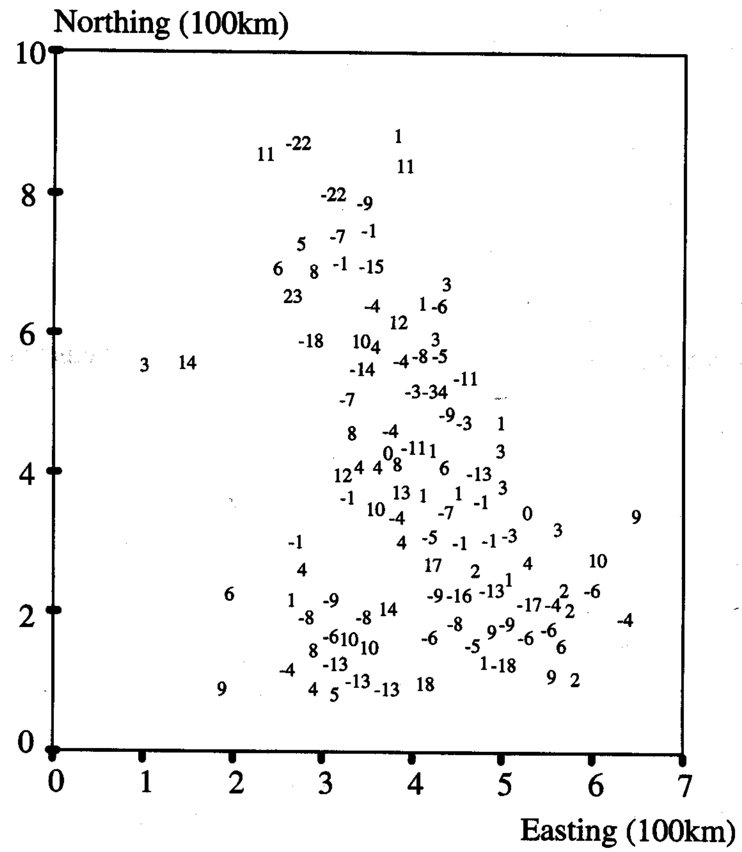


Fig. 4 *Spatial plot of percentage errors in the prediction of (which also shows the approximate locations of the sites).*

RESIDUAL ERROR ANALYSIS

For each regression model (13–17, 19–21), the residual errors (site estimate of parameter – regression estimate of parameter) were considered in a validation exercise to check that the regression models were giving sensible predictions and that there was no systematic bias in the predicted values (Figs. 3(*a*)–(*h*)).

From Figs. 3(*a*)–(*h*), it can be seen that there is no systematic bias in the prediction of the harmonic variables. Furthermore, there are no obvious outliers or trends (although Λ_λ in Fig. 3(*e*) is a possible exception), indicating that the regression models do not require further explanatory variables, such as higher powers, and can be applied with some confidence

To search for possible spatial dependencies in the residuals, percentage errors (100 × Residual / Site Estimate) for m_λ were plotted against the East and North grid reference (Fig. 4). Some weak spatial autocorrelation can be seen in the plot, but no strong regional bias is evident.

Using the regressions, the harmonic parameters can thus be estimated for any site (gauged or ungauged) in the UK with reasonable confidence. For the *i*th month, the estimates of the GNSRP(2) model parameters are then given by:

$$\hat{\lambda}(i) = \hat{m}_\lambda + \hat{\Lambda}_\lambda \sin\left(\frac{2\pi i}{12} + \hat{\theta}_\lambda\right); \quad \hat{\beta}(i) = 0.24;$$
$$\eta_i(i) = 3; \quad \hat{\eta}_2(i) = 0.53; \quad \hat{v}(i) = \hat{m}_v;$$
$$\hat{\xi}_2(i) = \hat{m}_{\xi_2} + 0.46 \sin\left(\frac{2\pi i}{12} + \hat{\theta}_{\xi_2}\right); \quad \hat{\xi}_i(i) = 0.24;$$
$$\hat{\alpha}(i) = \hat{m}_\alpha + \hat{\Lambda}_\alpha \sin\left(\frac{2\pi i}{12} + 3.6\right). \tag{23}$$

It should be noted that the regression equations are not appropriate for extrapolation beyond the range of the explanatory variables used. In particular, the highest site available had altitude 380 m, so the equations would not be appropriate for simulating rainfall in mountainous catchments. Nevertheless, the equations can be used with reasonable confidence for sites lower than 400 m, and this includes most urban catchments in the UK.

SOME EXTREME VALUES GENERATED BY THE REGIONALISED MODEL

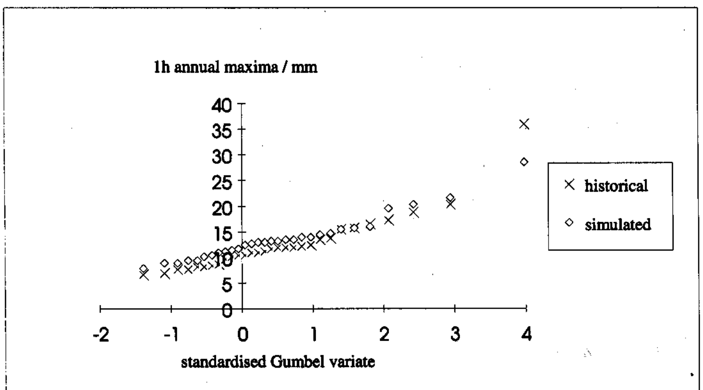
The parameters of the GNSRP(2) model were estimated using site variables for a 30-year record of hourly data (Farnborough: Altitude: 69 m; North Grid reference:

1544 (0.1 km); East Grid reference: 4867 (0.1 km); Mean annual rainfall: 750 mm). A synthetic time series of thirty years of rainfall data was then generated for Farnborough using the GNSRP(2) model with parameters estimated via Equations (23), and maximum annual totals extracted and plotted and compared with the observed historical values (Fig. 5).

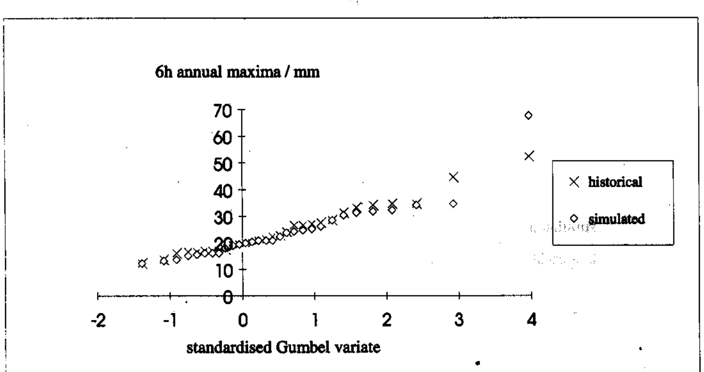
Good agreement was obtained for the 1 hr and 6 hr annual maxima (Fig. 5(*a*) and 5(*b*) respectively). However, there was some evidence of underestimation of the daily maxima for lower return periods (Fig. 5(*c*))

As a further validation exercise, the parameters of the regression models were re-estimated with the data from a 30-year record taken from Hampstead (the second longest record of hourly data after Farnborough) removed from the regression analysis. The GNSRP(2)

(a)



(b)



(c)

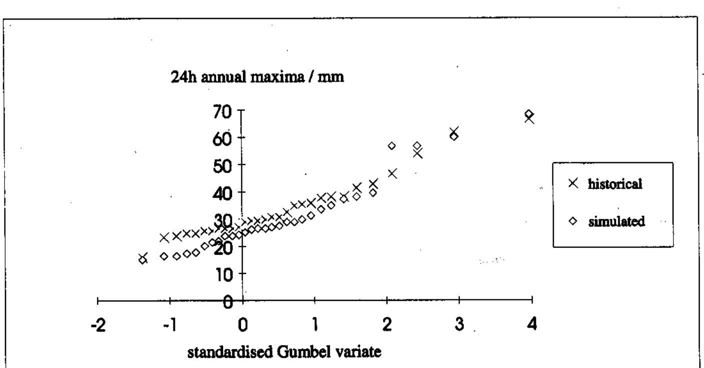


Fig. 5 *Annual maximum rainfall totals for Farnborough plotted against the standardised Gumbel variate: (a) hourly maxima, (b) 6-hourly maxima, and (c) daily maxima.*

model parameters for Hampstead were then estimated from equations (22) by inputting site variables into the revised regression equations. A time series of thirty years of hourly rainfall data was then generated using the GNSRP(2) model. The maximum hourly, six-hourly and daily values for each year were plotted against the standardised Gumbel variate, together with the equivalent values extracted from the historical record (Fig. 6).

Fig. 6(*a*) suggests that the regionalised model shows a good fit to the observed annual 1 h maxima. However, a tendency to under-estimate the largest annual maxima at the 6 and 24 h levels of aggregation is evident (Figs. 6 (*b* and *c*)). In general, the regionalized model shows a good fit to historical annual maxima at the 1 h level of aggregation, but has a tendency to under-estimate the largest annual maxima at the 24 h levels of aggregation. To allow for the sampling error of the maxima, parametric tests on fitted generalised extreme value distributions

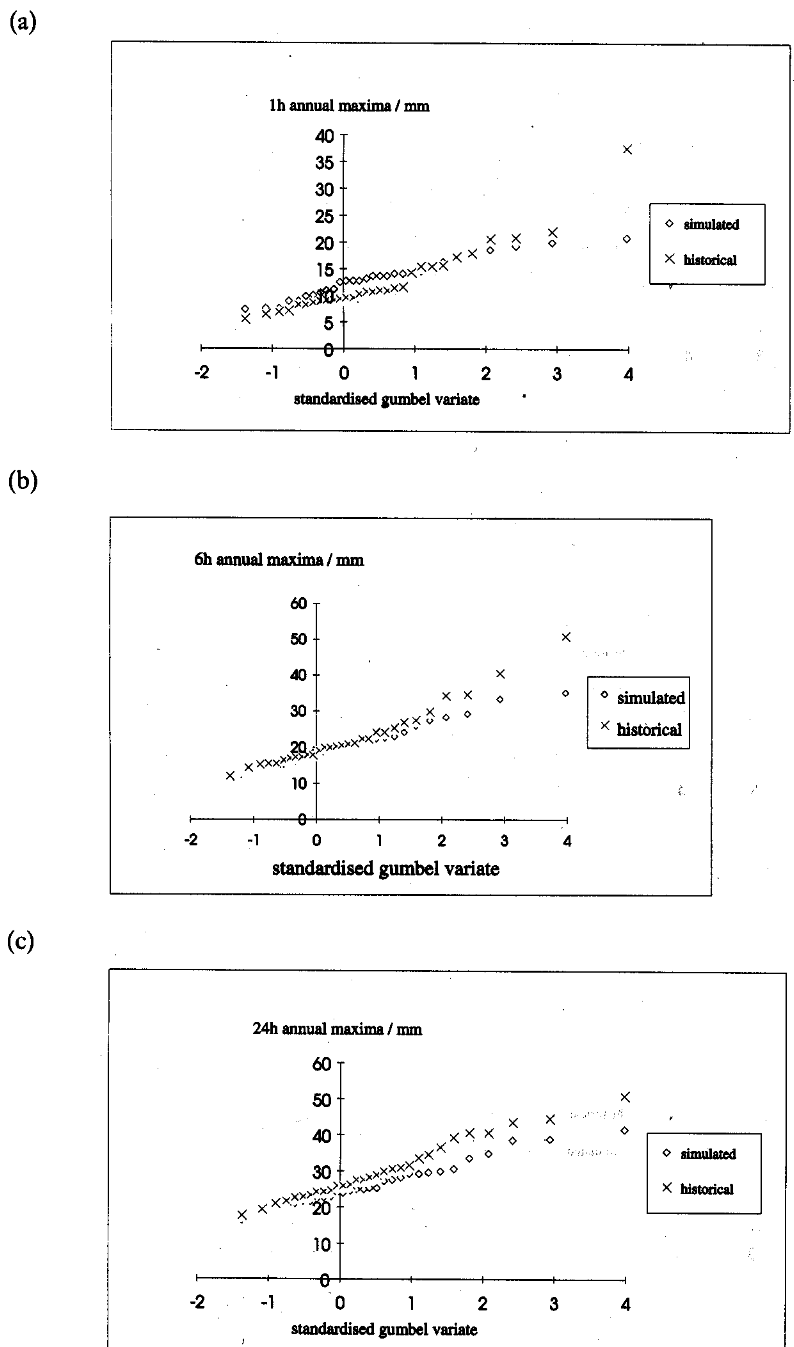


Fig. 6 *Annual maximum rainfall totals for Hampstead: (a) hourly maxima, (b) 6-hourly maxima, and (c) daily maxima.*

could be carried out, but this was beyond the scope of the present paper.

More complex regionalisation procedures may give better representations of the extreme rainfall events. For example, the country could be divided into a number of non-overlapping regions, such as those used by Dales and Reed (1989), and separate regression models fitted within each region. However, to obtain reliable regression equations, numerous sites per region would be needed and these were not available for this study.

VARIATION IN MODEL PARAMETERS ACROSS THE UK

Some further confidence in the regionalisation can be gained by noting that the predicted GNSRP(2) model

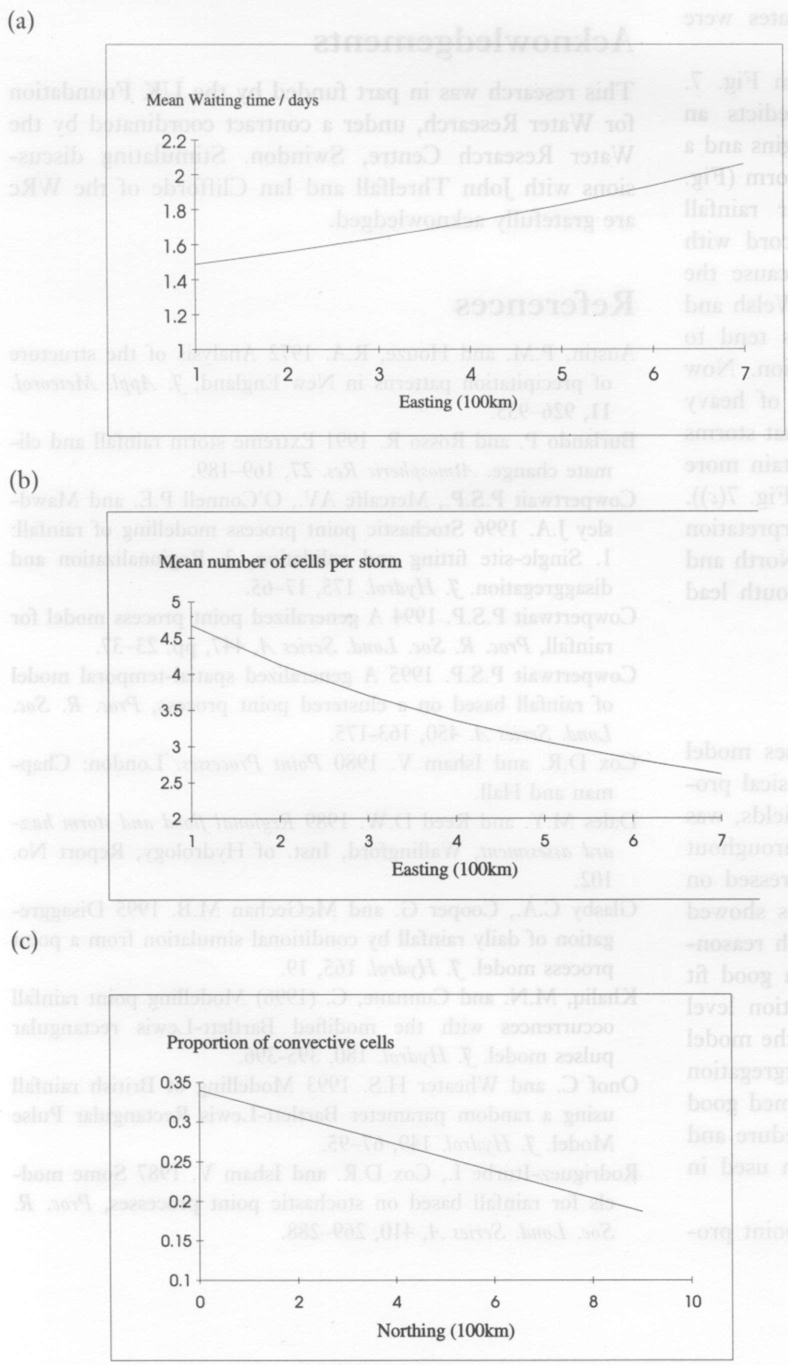


Fig. 7 *Regional variation in the parameters: (a) mean waiting time between storm origins, (b) mean number of raincells per storm, and (c) proportion of heavy convective raincells.*

parameters vary across the UK in agreement with meteorological knowledge and observations. To amplify interpretation of the regional variations in the parameters, a simple regression model was fitted to each of the harmonic variables, where the explanatory variables were non-interactive site variables only. Regional trends were then investigated by considering changes in m_{ϕ}, where ϕ was a parameter of the GNSRP(2) model, ignoring seasonal variations. The most significant regional trends were then plotted against North and East grid references, where the reciprocals of the parameter estimates were taken for ease of interpretation (Fig. 7).

The following deductions can be made from Fig. 7. Moving from West to East, the model predicts an increase in the waiting times between storm origins and a decrease in the mean number of raincells per storm (Fig. 7 (*a* and *b*)), i.e. the model predicts fewer rainfall 'events' in the East of Britain. This is in accord with meteorological knowledge and observations because the East of Britain is in a rain shadow due to the Welsh and Pennine mountain ranges, and frontal systems tend to move across the country in an easterly direction. Now moving from South to North, the proportion of heavy raincells decreases, so that the model predicts that storms in the North of Britain will be longer and contain more light stratiform rain than storms in the South (Fig. 7(*c*)). This variation is supported by a physical interpretation based on temperature differences between the North and South of Britain: higher temperatures in the South lead to more convective rainfall.

Conclusions

A Generalised Neyman-Scott Rectangular Pulses model of rainfall, taking into account some of the physical processes observed and measured in precipitation fields, was fitted to data taken from 112 sites scattered throughout the UK and harmonic parameter estimates regressed on site variables. An analysis of the residual errors showed that the regression equations could be used with reasonable confidence for urban sites. Furthermore, a good fit to observed extreme values at the 1 h aggregation level was found. However, there was a tendency for the model to under-estimate extreme values at higher aggregation levels. Overall the fit to the extreme values seemed good given the simplicity of the regionalization procedure and that properties of extreme values had not been used in the fitting procedure.

An interesting area of research is extending point process models into the spatial domain, to allow realistic series of multi-site rainfall to be simulated. Recent research (Cowpertwait 1995) has provided properties (e.g. the cross-correlation function) which enable the model to be fitted to multi-site data. The model thus has the potential to provide a complete description of space-time rainfall. Further work is needed to assess the performance of the spatial model against multi-variate extreme values.

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