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Climate of the Past

Statistical issues about solar–climate relations

P. Yiou[1,*], E. Bard[2], P. Dandin[3], B. Legras[4], P. Naveau[1], H. W. Rust[1], L. Terray[5], and M. Vrac[1]

[1]Laboratoire des Sciences du Climat et de l'Environnement, UMR CEA-CNRS-UVSQ 8212, IPSL, Gif-sur-Yvette, France
[2]Centre Européen de Recherche et d'Enseignement des Géosciences de l'Environnement, UMR 6635, Aix-en-Provence, France
[3]Météo-France, Division de la Climatologie, Toulouse, France
[4]Laboratoire de Météorologie Dynamique, UMR 8539, IPSL, Ecole Normale Supérieure, Paris, France
[5]Centre Européen de Recherche et Formation Avancées en Calcul Scientifique, Toulouse, France

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Abstract. The relationship between solar activity and temperature variation is a frequently discussed issue in climatology. This relationships is usually hypothesized on the basis of statistical analyses of temperature time series and time series related to solar activity. Recent studies (Le Mouël et al., 2008, 2009; Courtillot et al., 2010) focus on the variabilities of temperature and solar activity records to identify their relationships. We discuss the meaning of such analyses and propose a general framework to test the statistical significance for these variability-based analyses. This approach is illustrated using European temperature data sets and geomagnetic field variations. We show that tests for significant correlation between observed temperature variability and geomagnetic field variability is hindered by a low number of degrees of freedom introduced by excessively smoothing the variability-based statistics.

1 Introduction

The detection and attribution of climate change has been extensively investigated with various statistical methods (Hegerl et al., 2007). The role of solar activity on climate has been examined by many authors with climate models (Haigh, 1994; Shindell et al., 2001; Hansen et al., 2005; Meehl et al., 2009) and by comparing statistically solar and global or regional climate records (e.g. Lean and Rind, 2008, 2009; Benestad and Schmidt, 2009). Although weak in terms of energy balance, it seems to have played a significant role in various episode during the last millennium (Jansen et al., 2007).

Correspondence to: P. Yiou
(pascal.yiou@lsce.ipsl.fr)

There is an on-going debate on the exact role of solar activity on 20th century temperature variations. Siscoe (1978) pointed out flaws of the many papers debating on the subject, more than 30 years ago. More recently, Lockwood (2008) provided a critical assessment of the potential mechanisms linking solar variations to climate change, and showed that solar variations are not sufficient to explain the 20th century warming from elementary laws of physics and thermodynamics.

Three papers have recently reported statistical analyses of temperature and solar activity records (Le Mouël et al., 2008, 2009; Courtillot et al., 2010). Those papers introduce and discuss an unconventional method for time series analysis (at least in climate sciences). Those authors claim that this method (called "Mean Squared Deviation") gives information on the time evolution of the autocorrelation of a random process with slow heteroscedasticity. Our paper illustrates some statistical properties of such a transform, and provides tests of its significance from simple random processes. Those authors use European daily temperature records, and a record of geomagnetic intensity as a proxy for solar activity. Although this choice is debatable, we decided to be as close as possible as the framework of (Le Mouël et al., 2008, 2009).

The general message of the paper is that "data snooping" (White, 2000) can occur when relationships between data sets are not tested in an appropriate way. This implies that wrong conclusions can be derived out of statistical coincidences that are not robust to key (or mundane) parameters of the analyses. Data snooping is sometimes encountered in fields where the underlying mechanisms remain difficult to grasp and model. It consists of purposefully (or not) exploiting a feature from a data set that turns out to be a statistical artefact. This problem can be circumvented if clear null

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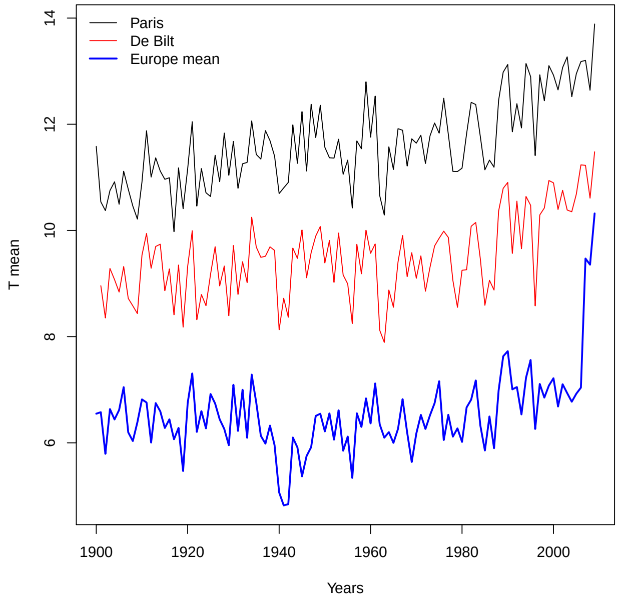

Fig. 1. Variations of the annual averages of daily mean temperature in Paris, De Bilt and a European mean, between 1900 and 2009. The data are taken from the ECA&D database (Klein-Tank et al., 2002).

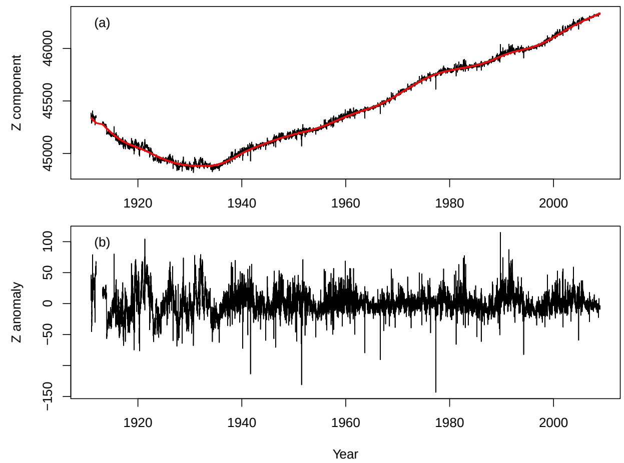

Fig. 2. **(a)** Daily variations of the Z component of the geomagnetic field measure at Eskdalemuir between 1911 and 2008. The red line represents the smoothed data with a spline function with 20 degrees of freedom. **(b)** Anomaly of the Z component with respect to the spline smoothing function.

hypotheses are formulated and proper statistical tests are performed. When mathematical transforms (such as the mean squared deviation) are involved, such tests should explore the sensitivity of the results to the parameters of the transforms.

Section 2 describes the data sets used in the paper. The methodology is detailed in Sect. 3. Three types of results are provided in Sect. 4.

2 Data

We used the daily mean temperature from the ECA&D database in Europe (Klein-Tank et al., 2002). In particular, we randomly focused on the Paris-Montsouris (France) and De Bilt (Netherlands) stations between 1900 and 2009.

We also used a set of European temperatures starting before 1920 and ending after 2000, and yielding less than 10% of missing or doubtful data. We computed the spatial average for each day of this data set, although its spatial distribution tends to give a lot of weight to Central Europe. The annual means of the three data sets are shown in Fig. 1. They all exhibit a warming over the 20th century. The exceptional warming between 2004 and 2009 in the European mean is due to the fact that a large proportion of "cold" stations (from central to eastern Europe) have systematic missing values after 2004. Thus, when the arithmetic mean of the ensemble is computed after 2004, it tends to be pulled toward higher temperatures. This feature should not be interpreted as an exceptional warming, but is an effect of missing data in ECA&D stations after 2004. We point out that the problem of missing data also appears before 1940. Such a problem could be circumvented by removing a seasonal cycle from temperature data. Hence all time series are centered (i.e. with zero mean), as well as their average. Thus, there is no artificial drift of the mean due to missing observations. We computed a mean seasonal cycle from the daily temperature data for the 1960–1990 period, and we removed it from the raw time series to obtain daily anomalies of temperature. We hence used daily temperature anomalies in computations of this paper. For brevity, the term "temperature" now refers to "anomalies of temperature with respect to a seasonal cycle", unless otherwise specified.

Homogeneity problems have been detected on daily time steps, and to our knowledge they have not been corrected in the ECA&D database (O. Mestre, personal communication, 2009). The lack of homogeneity (due to changes in instruments, orientation or simply reporting errors) can generate artificial non-climatic discontinuities. In this paper, we do not question or evaluate the quality of the temperature data, although most stations are indicated as "suspect" on the ECA&D database. This evaluation is done in a further study (Legras et al., 2010).

We compare those temperature datasets with records of solar activity. Before the period of precise irradiance measurements by satellites, there are several ways of approximating solar activity by using sunspots observed with telescopes (Hoyt and Schatten, 1998; Solanki, 2002), geomagnetic activity (Mayaud, 1972; Cliver et al., 1998; Lockwood et al., 1999), the frequency of aurora (review by Silverman, 1992) and cosmogenic isotopes such as ^{10}Be and ^{14}C (e.g. Beer et al., 1988; Bard et al., 1997). The variations of the geomagnetic field are measured in many stations around the earth. The intensity of the geomagnetic field is dominated by dynamo processes of the interior of the earth, varying on secular time scales. The fast variations, on time scales of minutes to years, are mainly influenced by solar activity and galactic cosmic rays, as discovered by Sabine, Wolf, Gautier and Lamont in the 19th century. It is then reasonable to assume that the fast variations recorded by stations over the Earth mostly reflect the fluctuations of solar activity (see Bartels, 1932, for a review). The intensity of the geomagnetic field is measured in three directions. It appears that the fast variability of the horizontal and vertical components are very similar (Le Mouël et al., 2009). Thus we focus on the vertical component variations (Z) of the geomagnetic field.

In this study, we followed the choice of Le Mouël et al. (2009) and used the geomagnetic data from Eskdalemuir (UK) as a proxy for solar activity. The data was obtained from the World Data Center for Geomagnetism (www.wdc.bgs.ac.uk/catalog/master.html). We computed daily averages from the hourly data, from 1911 to 2008. The time series shows a steady increase since 1938 (Fig. 2). This trend reflects secular changes in the geomagnetic field. We thus consider the small variations around this trend.

Mean daily temperature and geomagnetic times series have no mutual correlation (see Table 1, first column). Hence, the investigation of a potential relationship between the two variables motivates a focus on other statistical diagnostics. The next section gives an example of such analysis.

3 Method

Auto-regressive processes of order 1 (AR(1)) are often used in climate studies to define a null hypothesis to describe variations of temperature time series (Allen and Smith, 1994; Ghil et al., 2002; Maraun et al., 2004). By definition, an AR(1) process $R(t)$ is:

$$R(t+1) = aR(t) + b(t), \tag{1}$$

where $0 \le a < 1$ and $b(t)$ is a centered white noise with unknown but finite variance σ_b^2. The parameter a gives information on the memory of the process from one time step to the next. We use this simple random process as a pedagogical benchmark for the study of variability properties of time series, because they can be explicitly derived for various kinds of quadratic transforms. This process is commonly denoted "red noise" because its power spectrum decreases with frequency (Priestley, 1981).

Such a model can be refined to include slow time variations of a and σ. Such variations alter the probability distribution of $R(t)$ and an exaustive list of the properties of such processes lies beyond the scope of this paper (e.g. Embrechts et al., 2000).

For a given centered time series $X(t)$ (of observations, for example), one wants to find an AR(1) process that fits "best" the statistical characteristics of $X(t)$, like its variance and auto-covariance. The classical maximum likelihood estimate (MLE) $\hat{a}$ for a gives (Priestley, 1981):

$$\hat{a} = \frac{C_X(1)}{C_X(0)}, \tag{2}$$

where $C_X(0)$ is the sample variance of X and $C_X(\tau)$ is its sample auto-covariance at lag τ:

$$C_X(\tau) = \frac{1}{N-\tau} \sum_{t=1}^{N-\tau} X(t+\tau)X(t).$$

For the AR(1) process R of Eq. (1), the auto-covariance is (Priestley, 1981):

$$C_R(\tau) = \frac{\sigma_b^2 a^{|\tau|}}{1-a^2}. \tag{3}$$

An alternative approach, suggested by Le Mouël et al. (2009), is to define the mean squared daily variation: $\zeta_\Theta(t)$ for a given time window Θ:

$$\zeta_\Theta(t) = \frac{1}{\Theta} \sum_{\tau=t-\Theta/2}^{t+\Theta/2} (X(\tau+1) - X(\tau))^2. \tag{4}$$

For an AR(1) process ($R(t)$ in Eq. (1)), the expected value of $\zeta_\Theta^R(t)$ converges to:

$$E[\zeta_\Theta^R] = \frac{2\sigma_b^2}{1+a},$$

for all Θ. This can be verified by expanding Eq. (4) and using Eq. (3).

The mean interannual squared variation $Q_\Theta(t)$ is defined as:

$$Q_\Theta(t) = \frac{1}{\Theta} \sum_{\tau=t-\Theta/2}^{t+\Theta/2} (X(\tau+365) - X(\tau))^2. \tag{5}$$

For an AR(1) process $R(t)$, the expected value of $Q^R(t)$ converges to:

$$E[Q_\Theta^R] = \frac{2\sigma_b^2}{1-a^2},$$

for all Θ. We note that both $E[\zeta_\Theta^R(t)]$ and $E[Q_\Theta^R(t)]$ depend on a and σ_b for an AR(1) process, but not on Θ.

The "lifetime" function is defined by Le Mouël et al. (2009) as the normalization of $Q_\Theta(t)$ by $\zeta_\Theta(t)$:

$$L_\Theta(t) = Q_\Theta(t)/\zeta_\Theta(t). \tag{6}$$

This denomination is not connected with the lifetime notion in statistical survival theory (e.g. Lawless, 2003). For an AR(1) process, it follows that the expected value of $L_\Theta^R(t)$ is:

$$E[L_\Theta^R(t)] = \frac{1}{1-a} \equiv \lambda. \tag{7}$$

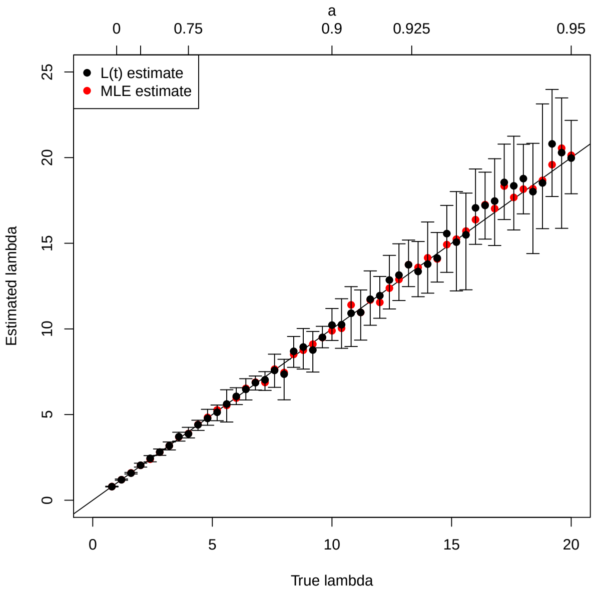

Fig. 3. Bias estimates of λ from an MLE computation $\hat{\lambda}$ (red circles) and mean "lifetime" computation $\bar{\lambda}$ (black circles). The bars around the $\bar{\lambda}$ estimates indicate the 5th and 95th quantiles of $L_\Theta(t)$ variations of AR(1) realizations. See text for formulas. The first diagonal is indicated for reference. The upper axis indicates variations of a (as in Eq. (1)).

The interesting point of this quotient is that it no longer depends on σ_b for an AR(1) process. Thus for a given Θ, we obtain an estimator $\bar{\lambda}$ for λ:

$$\bar{\lambda} = \text{mean}(L_\Theta(t)),$$

where mean(.) is the sample mean over t. Thus, in principle, the $L_\Theta(t)$ transform can be used to estimate a for an AR(1) process ($\bar{a} = 1 - 1/\bar{\lambda}$). However, the properties (bias, variance) are not a priori known but can be assessed by simulation (see next section). As a consequence of this convergence, it is hoped that if a varies slowly in time (over a scale larger than Θ) in Eq. (1), then it should be possible to estimate the variation rate from Eq. (6). This motivates the $L_\Theta(t)$ transform.

The MLE estimate of a in Eq. (2) also provides an estimate of λ:

$$\hat{\lambda} = \frac{1}{1-\hat{a}}.$$

The second goal of the paper is to obtain the significance of correlations between the transformations $L_\Theta(t)$ and $Q_\Theta(t)$ of two time series X and Y.

In general, the non nullity of a correlation r_{XY} between two time series X and Y can be tested with a Student t-test (von Storch and Zwiers, 2001):

$$t = |r_{XY}| \left(\frac{n-2}{1 - r_{XY}{}^2} \right)^{1/2}, \qquad (8)$$

where n is the number of dregrees of freedom in X and Y. From the value of t, one can derive a p-value that is the risk of wrongly rejecting the null hypothesis $r_{XY} = 0$ (von Storch and Zwiers, 2001). The value of n is a priori lower than the number of data points. When transforms such as $Q_\Theta(t)$ or $L_\Theta(t)$ are applied, the upper bound for n can be estimated by $n \approx N/\Theta$, where N is the number of data points. If the time series cover one century on daily time steps and $\Theta = 11 \times 365$, then the number of degrees of freedom n for $Q_\Theta(t)$ and $L_\Theta(t)$ is $n \approx 10$.

We remind that when a significant correlation is found between two time series there is no proof (or even a suggestion) of causality between the variables because correlation is a symmetric operator. A fortiori, finding a correlation between the L_Θ, Q_Θ or ζ_Θ quadratic transforms of two time series X and Y does not provide any causality relation between X and Y.

4 Results

4.1 Bias

The first step of this study is to verify that the $L_\Theta(t)$ transform is indeed a practical estimator of λ, i.e. with satisfactory properties of convergence and bias. We designed an ensemble of numerical experiments with AR(1) processes, by sampling the $(0, 20)$ interval of λ values with increments of 0.4. From those samples of λ, we take a $(= 1 - 1/\lambda)$, and simulate AR(1) processes, with a unit variance σ_b^2 for $b(t)$ (in Eq. (1)). The experiments are carried on $N = 30000$ increments. For each realization, we computed $L_\Theta(t)$ with $\Theta = 11 \times 365$ and its average $\bar{\lambda}$. We also determined the variance and autocovariance, to estimate $\hat{a}$ in a direct way from Eq. (2). We then plotted the values of $\hat{\lambda}$ for both estimates, as functions of the "real" λ from which the processes were constructed. The choice of $\Theta = 11 \times 365$ is somewhat arbitrary. Its heuristic justification is that it covers a sunspot cycle of $\approx$11 years. The exact length of Θ should not alter the L_Θ estimates if an interpretation in term of memory needs to be done.

The results (Fig. 3) show that the MLE estimate $\hat{\lambda}$ generally performs well. The "lifetime" estimate $\bar{\lambda}$ does not yield an apparent bias. The confidence intervals for $\bar{\lambda}$ seem to increase with a, indicating a higher variability of $L_\Theta(t)$ when a (or λ) increases. This is explained by the singularity at $a = 1$.

This exercise can be performed on the Paris or De Bilt temperature data. We obtain respectively $\hat{a} \approx 0.8$ and $\bar{a} \approx 0.7$. If the same computation is done on daily mean European temperature, we obtain $\hat{a} > 0.93$. Thus, assuming temporarily that local temperatures can be represented by AR(1) processes, this shows that their memories are rather high ($a > 0.8$). However, we point out that the average of independent AR(1) processes is *almost never* an AR(1) process (Embrechts et al., 2000). This implies that the interpretation

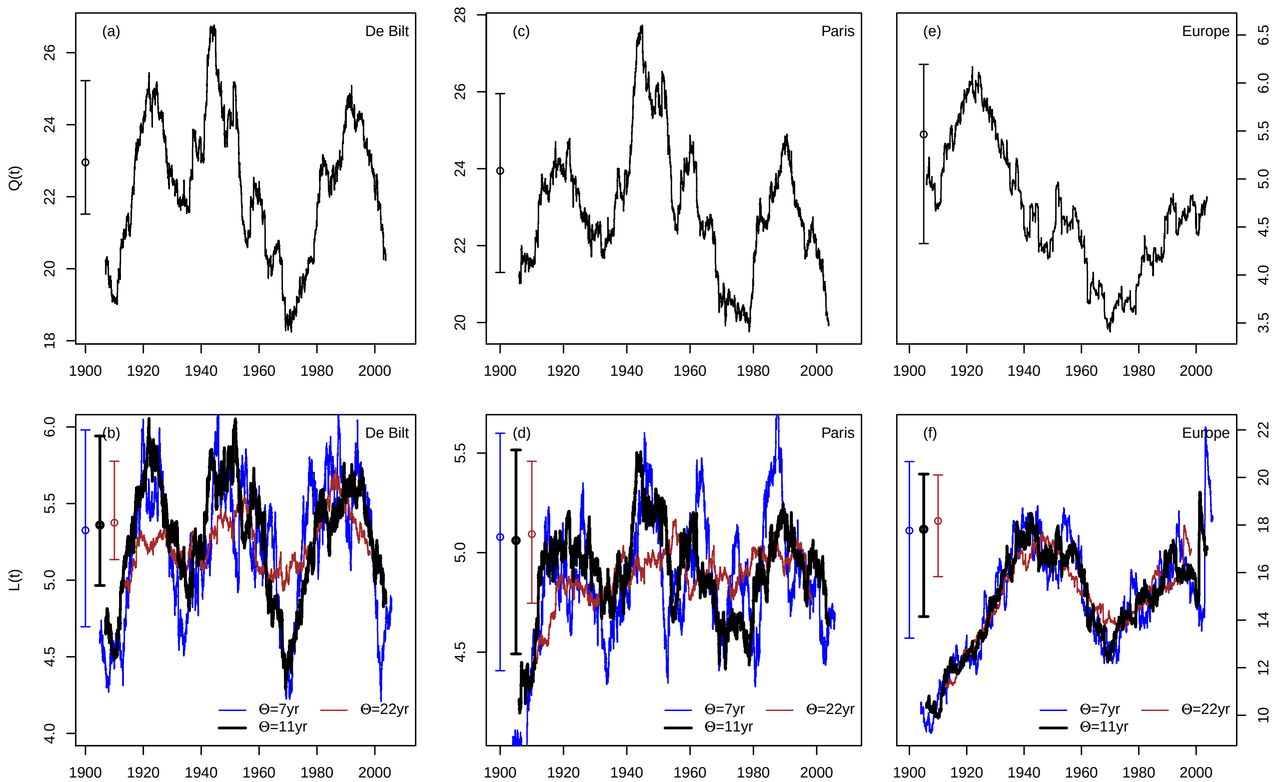

Fig. 4. **(a)** Variations of daily $Q_\Theta(t)$ for De Bilt temperature between 1900 and 2008, with $\Theta = 11$ years. **(b)** Same convention as (a) with $L_\Theta(t)$. The colored continous lines show the variations of $L_\Theta(t)$ when the parameter Θ is varied from 7 years to 22 years. **(c)** Same as (a) for the mean daily Paris temperature. **(d)** same as (b) for the mean daily Paris temperature. **(e)** Same as (a) for the mean daily European temperature. **(f)** same as (b) for the mean daily European temperature. The vertical confidence intervals indicate the median, 5th and 95th quantiles for the variations of $Q_\Theta(t)$ (or $L_\Theta(t)$) of an AR(1) process with the same variance and auto-covariance.

of $\bar{\lambda}$ of the European mean temperature in term of process memory is a priori not possible.

4.2 Variability of $Q_\Theta(t)$ and $L_\Theta(t)$ for temperature

The second step of the study is to investigate the range of variations of $Q_\Theta(t)$ (the mean squared daily variations) and $L_\Theta(t)$ for a random process (AR(1)). From Eq. (2), we determined $\hat{a}$ and $\hat{\sigma}$ for the De Bilt and Paris temperature anomalies. This allows us to simulate AR(1) processes having the same variance and autocovariance (and length) as those temperature series. For each random realization and the temperature data, we computed $Q_\Theta(t)$, $\zeta_\Theta(t)$ and $L_\Theta(t)$. We determined the 5th, 50th and 95th quantiles of $Q_\Theta(t)$ and $L_\Theta(t)$.

The variations of $Q_\Theta(t)$ for De Bilt and Paris temperature anomalies and the 90% bounds for red noise are shown in Fig. 4a, c. Overall, the large deviations of $Q_\Theta(t)$ are significant, with respect to an AR(1) process. The variations of $L_\Theta(t)$ for temperature anomalies (Fig. 4b, d) show generally similar shapes as those of $Q_\Theta(t)$ for De Bilt and Paris and they are also significant with respect to an AR(1) process. We note that when the window Θ increases from 11 to 22 years, some of the maxima of $L_\Theta(t)$ lose their significance with respect to an AR(1) process (Fig. 4b, d). Curiously, the variations of $Q_\Theta(t)$ and $L_\Theta(t)$ are different for the mean European temperature. This can be explained by the fact that the statistical properties of the mean of time series are not the same as individual series, in particular in term of persistence.

We note that the $L_\Theta(t)$ transform for raw temperature (i.e. without removing the seasonal cycle) has the same general behavior and order of magnitude as for the temperature anomalies. This feature motivates the use of $L_\Theta(t)$ to investigate the variability of time series with periodic components that are potentially complex.

We checked the dependence of the analysis on Θ in the $L_\Theta(t)$ estimate. Indeed, the interpretation of a single outstanding "event" in $L_\Theta(t)$ should be robust to the choice of the window Θ in Eq. (6). We hence computed $L_\Theta(t)$ with $\Theta = 7, 11$ and 22 years (Fig. 4b) for the De Bilt and Paris daily temperature. This illustrates that an alteration in the window size changes important details of the transform $L_\Theta(t)$. This provides an additional (albeit more heuristic) way of verifying the robustness of the oscillations of $L_\Theta(t)$.

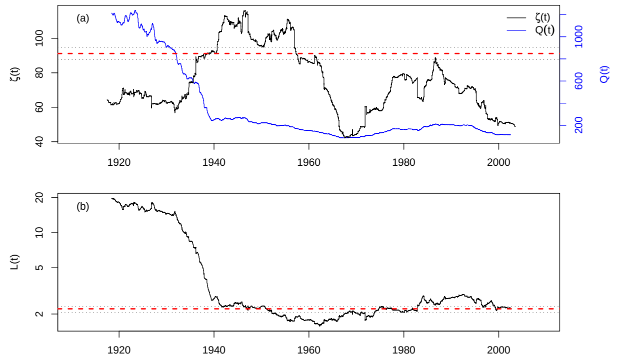

Fig. 5. **(a)** $Q_\Theta(t)$ of the geomagnetic data (blue line). Mean squared daily deviation ($\zeta_\Theta(t)$) of Z (black line). The confidence intervals are computed for the mean squared daily deviation $\zeta_\Theta(t)$ between 1940 and 2008. **(b)** $L_\Theta(t)$ of the geomagnetic data with $\Theta = 11$ years. The confidence intervals are determined for this parameter between 1940 and 2008 (vertical line).

If De Bilt or Paris temperature series are considered, the major peaks or troughs of $Q_\Theta(t)$ and $L_\Theta(t)$ found for $\Theta = 11$ years seem unstable to this parameter, and it is not reasonable to load them with a physical interpretation.

We have also applied the analysis to the ensemble mean of temperature from the ECA&D database. The autocorrelation function $C_X(1)$ of the ensemble daily mean is larger than for each station. The time variation for the ensemble mean are coherent with the De Bilt or Paris data, but with a higher baseline (Fig. 4e, f). The conclusions of the significance of the variations of $L_\Theta(t)$ remain for ensemble averages of temperature.

4.3 Significance of correlations

Should one persist in using $L_\Theta(t)$ as an estimator of variability for a time series, the next question that arises is the significance of correlations between two transformed data sets. The $L_\Theta(t)$ transform intrinsically reduces the number of degrees of freedom of a time series: if N is the number of data samples, N_{dof} the number of degrees of freedom of the data, then the number of degrees of freedom of $L_\Theta(t)$ is approximated by $\min(N_{\mathrm{dof}}, N/\Theta)$. In other words, the number of degrees of freedom of $L_\Theta(t)$ is bounded by the number of independent windows in the sum in Eq. (6). Further filters, like moving averages or splines, also contribute to the reduction of the number of degrees of freedom in a time series.

The goal here is to compare the time variations of the $Q_\Theta(t)$ and $L_\Theta(t)$ functions of two time series X and Y through their correlation. The idea of this approach is to compare the variability of X and Y, although both time series might be uncorrelated. In any case, we repeat that the correlation between $L_\Theta(t)$ transforms does not allow for an inference of a mutual relation between the original time series, unless a specific model of covariation is provided.

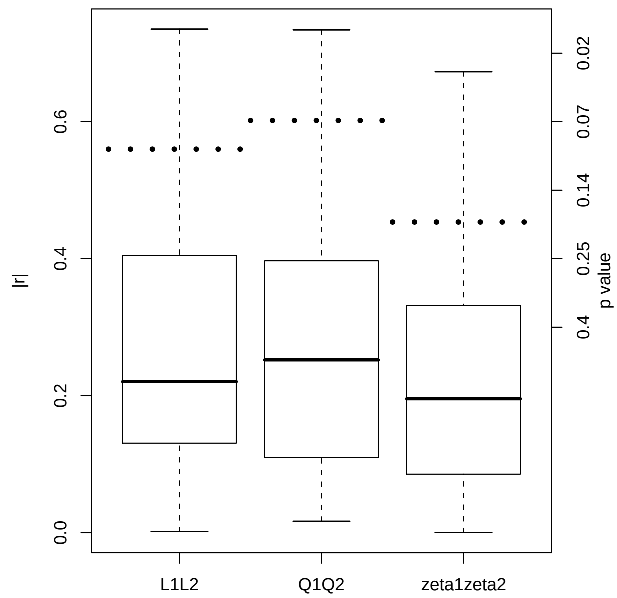

Fig. 6. Distribution of absolute values of correlation $|r|$ between $L_\Theta(t)$, $Q_\Theta(t)$ and $\zeta_\Theta(t)$ transforms of two random AR(1) processes with the respective variance and autocovariance of ECA&D European temperatures and geomagnetic field from Eskdalemuir. The box and whisker plots indicate the 25, 50 and 75th quantiles q (boxes). The upper whisker corresponds to: $\min(\max(|r|), q_{50} + 1.5 \times (q_{75} - q_{25}))$. The right hand vertical axis shows the p values for $r = 0.3$ to 0.7, with increments of 0.1, corresponding to a null hypothesis of no correlation and $n = 10$ degrees of freedom. The horizontal thick dotted lines indicate the 90th quantile of $|r|$ values.

It should be noted that correlating ζ_Θ variations of one variable with L_Θ variations of another variable is very difficult to justify. Indeed, ζ_Θ measures mean fluctuations of one day to the next, and L_Θ measures fluctuations of days separated by one year. Nevertheless, in order to have an extensive view of the correlations among transforms of temperature and geomagnetic data, we compute the correlations between the $L(t)$ transforms of temperature data sets, and $\zeta(t)$, $Q(t)$ and $L(t)$ transforms for geomagnetic data.

From the behavior of the geomagnetic data (Fig. 5a), it is reasonable to subtract the low frequency part of the time series, which is connected to slow internal dynamo processes, and not relevant to solar activity. We hence smoothed the data with a spline function with 20 degrees of freedom (Green and Silverman, 1994), and retained the anomalies with respect to this spline function. We verified that those anomalies do not have a trend. The $L_\Theta(t)$ and $Q_\Theta(t)$ transforms of the geomagnetic data are actually insensitive to the removal of the trend. This removal is done to estimate the variance and auto-covariance of the solar influence on the geomagnetic field, and hence simulate an AR(1) with the same parameters for Monte Carlo tests.

The first step is to estimate the probability of spurious correlation of $L_\Theta(t)$, $Q_\Theta(t)$ and $\zeta_\Theta(t)$ for two independent AR(1) processes X and Y. For the sake of the exercise, we computed the variance and autocovariance of mean daily temperature over Europe and the geomagnetic field intensity anomalies. We then generated 100 independent realizations of each AR(1) process over 30000 time steps (approximating the number of days during the 20th century), and computed $L_\Theta(t)$, $\zeta_\Theta(t)$ and $Q_\Theta(t)$ for each realization. We took $\Theta = 11$ years for those experiments.

The correlation distributions show that it is relatively frequent (with probability $p > 0.1$) that the correlations between $Q_\Theta(t)$ of two independent processes exceed 0.5 in absolute value (Fig. 6). The p-values of correlations for $n \approx 10$ degrees of freedom are below 0.07 only when $r > 0.6$, see Fig. 6 (right axis). This implies that $L_\Theta(t)$ or $Q_\Theta(t)$ transforms of two independent AR(1) processes with the same auto-covariance features (variance and lag-1 autocovariance) as temperature and geomagnetic field are likely to yield large absolute values of correlation. Conversely, this means that large absolute values of correlation for $L_\Theta(t)$ or $Q_\Theta(t)$ transforms do not necessarily imply statistical significance. This emphasizes the importance of correlation testing, especially when the number of degrees of freedom is low.

In a second step, we computed $\zeta_\Theta(t)$, $Q_\Theta(t)$ and $L_\Theta(t)$ for the geomagnetic daily anomaly data anomalies, with the same parameters as Le Mouël et al. (2009).

From the geomagnetic anomalies, we determined the AR(1) process with the same variance and covariance, to estimate confidence intervals. The $\zeta_\Theta(t)$ and $Q_\Theta(t)$ variations are overall significant with respect to an AR(1). The variations of $L_\Theta(t)$ are generally not significant after 1940 because their amplitude is smaller than for an AR(1) process. The $L_\Theta(t)$ and $Q_\Theta(t)$ variations present a major decrease near 1940, and $L_\Theta(t)$ yields relatively weak variations after this date. The reasons for this change are undocumented, but could come from changes of instrument, or measurement frequency.

The (linear) correlations between the $L_\Theta(t)$ of daily mean temperature (resp. Paris, De Bilt and European average) and $L_\Theta(t)$ of Z are generally weak and not significant, as sumarized in Table 1. The correlation even changes signs when European mean temperature is considered. The correlations between $L_\Theta(t)$ of temperatures and $Q_\Theta(t)$ of Z are also weak and not significant. The correlations between $L_\Theta(t)$ of temperatures and $\zeta_\Theta(t)$ of Z is higher, and can reach $r = 0.61$. But the p-value of the correlation is $p > 0.06$, which still make it unsignificant by usual standards, because of the low number of degrees of freedom.

Overall, this shows that no conclusion about a covariation diagnostic can be derived from the comparison of the $L_\Theta(t)$, $Q_\Theta(t)$ or $\zeta_\Theta(t)$ transforms of temperature and geomagnetic activity. We also point out that higher correlation values (although not significant) are obtained with a special choice of transforms, which have no a priori justification. Thus, choosing different kinds of data transforms (i.e. $L_\Theta(t)$ vs. $\zeta_\Theta(t)$) to compare two data sets, in order to maximize a correlation can potentially lead to "data snooping".

Table 1. Correlations between ζ_Θ, Q_Θ and L_Θ transforms of daily temperature series and the Z component of the geomagnetic field, between 1940 and 2009 ($\Theta = 11 \times 365$). p-values for $n = 10$ degrees of freedom are indicated in parentheses when the correlation coefficient exceeds 0.4. Correlations under 0.4 are not considered significant.

Variables	Z	ζ^Z	Q^Z	L^Z
T^{Paris}	0.07			
L^{Paris}		0.52 (0.12)	0.14	0.08
T^{DeBilt}	0.06			
L^{DeBilt}		0.53 (0.11)	0.33	0.26
T^{Europe}	0.04			
L^{Europe}		0.61 (0.06)	0.30	−0.43 (0.21)

4.4 Identification of causality in $L_\Theta(t)$

Although none of the transforms outlined above provide tools to assess any causal link between two variables, one may *assume* that such a relationship exists, and determine the amplitude of the link. For example, consider two generalized AR(1) variables X and Y such that the memory parameter a^Y is controled by a function $F^X(t)$, i.e. $a^Y(t) = F^X(t)$. The question we want to address is whether $L^Y(t)$ is correlated to the function $F^X(t)$.

In general, the answer to such a question is negative because Eq. (7) is not true when a varies continuously (even slowly) with time. We illustrate this point by creating a generalized AR(1) process:

$$R(t+1) = a(t)R(t) + b(t), \tag{9}$$

where $0 \le a(t) < 1$ slowly with time and $b(t)$ is a white noise with zero mean. In this example, we set X to be the mean squared daily variation of the Z component of the geomagnetic data after 1940 and Y is a mean daily temperature in Paris modeled by the generalized AR(1) process of Eq. (9).

We generate a function $F^X(t)$ by scaling $\zeta_\Theta^{(Z)}(t)$ (the Z component of the geomagnetic data) to vary between the bounds of $L_\Theta^{(\text{Paris})}(t)$ (the mean daily Paris temperature). This arbitrary choice is motivated by Le Mouël et al. (2009) who computed the correlation between $\zeta_\Theta^{(Z)}(t)$ and $L_\Theta^{(\text{Paris})}(t)$ of temperature. We then determine $a^Y(t)$ from $F^X(t)$. Hence, $a^Y(t)$ is directly controled by $\zeta_\Theta^{(Z)}(t)$. We simulated 100 such generalized AR(1) processes and computed the correlation between $F^X(t) \equiv \zeta_\Theta^{(Z)}(t)$ and $L_\Theta^Y(t)$, with $\Theta = 11$ years. In those realizations, the variance of $b(t)$ is the same as the sample variance of temperature anomalies in Paris.

We find that the correlation between $L_{\Theta}^{Y}(t)$ and $\zeta_{\Theta}^{(Z)}(t)$ is generally positive (in 90% of cases), although the median correlation is $r = 0.53$ and is hence not significant because of the low number of degrees of freedom ($n = 7$). This synthetic example shows that when the causality between memory parameters $a(t)$ is assumed, then the $L(t)$ transforms is barely sufficient to retrieve the original forcing function. This poor score, even in an idealized case, is due to the fact that the variance of $b(t)$ plays a role in $L(t)$ when $a(t)$ varies with time in Eq. (9), although it does not appear in Eq. (7). This further illustrates that an interpretation of $L(t)$ for a generalized AR(1) process with varying $a(t)$ cannot be obtained in a satisfactory manner. In general, $L(t)$ does not allow for an estimate of the time variations of the parameters of a generalized AR(1) process, unless those variations follow stepwise constant functions (as outlined by Le Mouël et al. (2009)). If the underlying process is more complicated than an AR(1) (for example if the variance of $b(t)$ also varies with time), then the interpretation of $L(t)$ in terms of process memory is impossible.

5 Conclusions

We analyzed the potential of $L_{\Theta}(t)$, $\zeta_{\Theta}(t)$ and $Q_{\Theta}(t)$ transforms of time series to characterize the variability of a time series. Using elementary statistical techniques of hypothesis testing, based on Monte Carlo simulations, we have provided a framework to check the statistical significance of such transforms, and hence help with their physical interpretation.

Following Le Mouël et al. (2009), we applied those procedures to temperature and geomagnetic activity time series. We found that the $\zeta_{\Theta}(t)$, $Q_{\Theta}(t)$ and $L_{\Theta}(t)$ variations of those variables are generally significant with respect to an AR(1) process. Moreover, a rigorous test between both variables shows that no significant correlation exists between them. Of course, we do not exclude that such transforms would not give significant results with other data sets.

This study can be extended to other transforms and statistical diagnostics. We emphasize the importance of testing statistical estimates with respect to reasonably chosen null hypotheses in order to avoid data snooping. In the case of $L_{\Theta}(t)$, the most reasonable and simple null hypothesis is an AR(1) process, for which $L_{\Theta}(t)$ has a direct interpretation. But temperature variations generally cannot be modelled by an elementary AR(1) process (Huong Hoang et al., 2009; Yiou et al., 2009). If time variations in a and σ_b are introduced in Eq. (1), their contribution to the $\zeta_{\Theta}(t)$ and $Q_{\Theta}(t)$ transforms cannot be separated as simply as for an AR(1). This strongly limits the use and interpretation of those transforms to diagnose the variability of a time series.

Finally, we point out that all the results presented in this paper are based on second order statistics of time series. The L_{Θ} diagnostic omits variations of first order, i.e. variations of the mean. From a quick inspection of Fig. 1, it is clear that the mean temperature evolves with time on multi-annual time scales. None of the presented analyses allow to explain such a temperature variation with a geomagnetic series. The increase of temperature (or temperature anomalies) after 1940 is still unexplained by the variations of the geomagnetic field anomalies.

Supplementary material related to this article is available online at:
http://www.clim-past.net/6/565/2010/
cp-6-565-2010-supplement.zip.

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