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Reply to Stauning's comment on "The PC index: review of methods" by McCreadie and Menvielle (2010)

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1 Introduction

The underlying premise of our paper (McCreadie and Menvielle, 2010, hereafter referred to as MM2010) and the corrigendum (McCreadie and Menvielle, 2011, hereafter referred to as Corrigendum) was stated in the Abstract as "*What is in doubt is the methodology of the derivation of the index by different groups. The Polar Cap index (PC: PCN, northern; PCS, southern) described in Troshichev et al. (2006) and Stauning et al. (2006), both termed the "unified PC index", and the PCN index routinely derived at DMI are inspected using only available published literature.*". What we mean by "*only available published literature*" is peer reviewed papers, PhD theses and governmental scientific reports. We showed that the derivation procedures for all three PC indices are different. We argue that the term "unified" cannot be used because the value of the PC index depends upon the chosen derivation procedure. In order to avoid having too long a paper we decided not to reproduce developments whenever we found it not necessary in the line of the objective of the paper. We are aware that the choices we made in this respect may be challenged.

We should stress the fact that our objective was to make a critical analysis of the differences between the methods and of the points that are not fully described in the literature in order that these points will be addressed in future discourses. Our paper is not a critical analysis of the methods. We should also stress the fact that our main objective was not to provide an extensive description of the consequences of the errors that have occurred in the course of the PC index development, but to make clear the differences between the various derivative schemes that have been proposed in order to give the basis for the discussion on a "unified PC index". In this perspective, we don't think that a change of institute is an important development in the methodology of the index derivation.

The version of the PCN index which is computed by DTU is the DMI#2_2001 PCN index. Whilst the governing institute changed, the derivation of the index did not change. The DMI#2_2001 PCN index is available from many sources as it is widely used by the community and is commonly referred to as the PCN index; in particular, the World Data Centre for Geomagnetism, Copenhagen distributes the PCN index[1]. For the user of any magnetic index may we suggest you read the accompanying metadata to be sure you know which index you have. The metadata should state the official source of the index. If this is not the case, do not use it, and go directly to the places where reference values are available: the web page of the institute in charge of the derivation of the index, and in the case of IAGA endorsed geomagnetic indices the web page of the International Service of Geomagnetic Indices (ISGI, hosted by LATMOS, a UVSQ/CNRS laboratory)[2].

The nomenclature used in MM2010 reflects that used in most of the papers reviewed. We did this so the reader could easily compare our review with the literature.

For the sake of clarity, we use in this reply the same section titles as those used by Stauning (2011, hereafter referred to as S2011). The numbering of the sections given below

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[1]http://www.space.dtu.dk/English/Research/Scientific_data_and_models/World_Data_Center_for_Geomagnetism.aspx.
[2]http://isgi.cetp.ipsl.fr/

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is however different from the numbering of sections used in S2011.

1.1 Parameters of the basic expression for the PC index

Equations (1*), (2*) and (3*) in S2011 are equivalent to Eqs. (16), (17) and (7) in MM2010. Please note that V_{SW} is the magnitude of the solar wind velocity. The definition of GSM coordinates is: the origin lies at the center of the Earth; the x-axis is positive towards the Sun and is defined along the line connecting the center of the Earth to the center of the Sun; and the Z-axis is the projection of the Earth's magnetic dipole axis (positive North) on to the plane perpendicular to the x-axis.

1.2 Decomposition of the magnetic field in geographic and geomagnetic coordinate systems

C1. S2011 states that "*The vector $\boldsymbol{D}_H$ is misplaced. The quantity named D is really the geographic Y-component.*" The difficulty comes here from the fact that different names are used in the literature to denote the same geometrical quantity. It is in particular the case for the declination that is named D or D_E; D (D_E) is expressed in degrees, or in radians. We tried to express this in Fig. 5 of MM2010; in this figure D is definitely not the geographic Y-component. The quantity named $\boldsymbol{D}_H$ is the horizontal component in the easterly direction perpendicular to a reference Magnetic North direction $\boldsymbol{H}_0$; D_H is expressed in nT. Ideally, the $\boldsymbol{H}_0$ direction in the horizontal plane is chosen so that the instantaneous Geomagnetic North direction $\boldsymbol{H}(t)$ fluctuates around it; in geomagnetic observatory practice, $\boldsymbol{H}_0$ is chosen so that the angle between $\boldsymbol{H}(t)$ and $\boldsymbol{H}_0$ remains small enough for $\sin(\boldsymbol{H}(t),\boldsymbol{H}_0) \sim \tan(\boldsymbol{H}(t),\boldsymbol{H}_0) \sim (\boldsymbol{H}(t),\boldsymbol{H}_0)$, the angle $(\boldsymbol{H}(t),\boldsymbol{H}_0)$ being expressed in radians. The angle between the Geographic North and $\boldsymbol{H}_0$ is denoted D_0 (or D_{E0}): in other words, it is the declination corresponding to the reference Magnetic North direction and it is thus the reference value for the declination that is consistent with $\boldsymbol{H}_0$.

Consider now the variations of the horizontal geomagnetic components with respect to D_0 (or D_{E0}) and $\boldsymbol{H}_0$. If the magnetic variations in the horizontal plane are referred to the geographic frame, the horizontal components are expressed as $X = X_0 + dX$ and $Y = Y_0 + dY$ (X, X_0, dX, Y, Y_0, dY are expressed in nT); if they are referred to the frame where the vector unit for the "x"-axis is $\boldsymbol{H}_0/H_0$, the horizontal components are expressed as $H = H_0 + dH$ and $D_H = dD_H = H_0 \tan(D_E - D_{E,0})$ [or $H_0 \tan(D - D_0)$ since D and D_E denote the same quantity]. H_0 is the modulus of $\boldsymbol{H}_0$, and $\boldsymbol{D}_H$ is the horizontal component in the easterly direction perpendicular to $\boldsymbol{H}_0$; H_0 and $D_H = dD_H$ are expressed in nT, D (or D_E) and D_0 (or $D_{E,0}$) are expressed in degrees, or in radians. A further difficulty arises from the fact that D_E and D_H are often referred to as D(deg) and D(nT), respectively, and it then comes $dD(nT) = H_0 \tan[D(\mathrm{deg}) - D_0(\mathrm{deg})]$: dD then denotes the same quantity as dD_H and, although they are expressed in nT, dD_H and dD are also called "variations of the declination".

C2. S2011 also states that, "*If $\boldsymbol{F}$ is the total magnetic field vector then its horizontal component, $\boldsymbol{H}$, is not necessarily situated in the magnetic meridian in the northerly direction. The same error is repeated in the Corrigendum.*". The correctness of this sentence depends on the definition of the magnetic meridian, and of that of the magnetic field vector. If the magnetic meridian is defined as the direction of the reference field $\boldsymbol{H}_0$, this sentence is correct provided $\boldsymbol{H}$ (resp. $\boldsymbol{F}$) refers to the instantaneous field $\boldsymbol{H}(t)$ (resp. $\boldsymbol{F}(t)$); if on the contrary the magnetic meridian is defined as the direction of the instantaneous field $\boldsymbol{H}(t)$, this sentence is incorrect provided $\boldsymbol{H}$ (resp. $\boldsymbol{F}$) refers to the reference field $\boldsymbol{H}_0$ (resp. $\boldsymbol{F}_0$). This illustrates the importance of clearly denoting whether the considered field is the reference or the instantaneous one; this is for instance achieved by denoting the reference field with a 0 subscript (e.g. $\boldsymbol{F}_0$ and $\boldsymbol{H}_0$), as we did in the Corrigendum. As stated in the Corrigendum, the variation with time of the geomagnetic field implies that at time t the direction of the reference field $\boldsymbol{H}_0$ generally does not correspond to the current direction of the horizontal vector. In other words, the direction of the "reference meridian" (i.e. that of $\boldsymbol{H}_0$) and that of the "instantaneous meridian" (i.e. that of $\boldsymbol{H}$ at time t) are different.

It is worth noting here that in Fig. 1* of S2011, the magnetic meridian (denoted as magn. N) is defined with respect to the reference field having a declination denoted as $D_{E,0}$, the instantaneous field is denoted as **H**, and the projection of **H** on the magn. N direction is denoted as H. With the notations used in the Corrigendum and in the present reply, the magnetic meridian in Fig. 1* is the "reference" meridian defined with respect to the reference field denoted as $\boldsymbol{H}_0$ with a declination denoted as $D_{E,0}$, the instantaneous field is denoted as $\boldsymbol{H}$ ($\boldsymbol{H}(t)$ in the present reply), and the projection of $\boldsymbol{H}$ on the "reference" meridian is denoted as H.

There is no difference between the formulas that we give in the Corrigendum and those given in S2011: we compute D_H using the reference field $\boldsymbol{H}_0$ (notations used in the Corrigendum) while S2011 computes D_H using the instantaneous field $\boldsymbol{H}$ (notations used in the Corrigendum).

1.3 The projection angle

C3. In MM2010, Eq. (7), p. 1892 is correct but there is a typo error in the explanations for this equation. This error is corrected in the Corrigendum (point 6).

C4. The word that should be used to name the angle γ can be discussed at length; our aim in MM2010 was to make clear its geometrical definition, and accordingly how the horizontal direction used to compute the magnetic quantity F_k

used in the PC index derivation is defined. We went to length to describe the rotation of the axis so the equations could be understood, especially the choice of addition and subtraction. There are errors in the literature and we wanted to check which equations were the correct ones. Once we had gone from first principles of rotating coordinate axis we found it such a useful exercise that we thought first time users would appreciate this insight into the derivation of the F_k.

The vectors are rotated about an axis as shown in Figs. 2 and 3 MM2010. This enabled Eqs. (13) and (15) to be understood. In order to project (or visualise) a vector in another coordinate system the original coordinate system must be rotated and translated into the new coordinate system. The angle γ is termed the projection plane angle (Please see paragraph under Eq. 13). The rotation angle described in Eq. (10) is the projection plane angle described in Eqs. (13), (14) and (15).

C5. The comment in p. 1895 MM2010 stating: "*...(reader please note, Eq. (18) is not the linear correlation coefficient, see Aitken, 1947)...*" is quite correct. In Aitken, fifth edition (1947) p. 86, one finds that the mean product is given by;

$$r = \frac{1}{N}\sum\left(x - m'_{10}\right)\left(y - m'_{01}\right)/S_1 S_2$$

"*We shall call r the Pearson coefficient or product-moment coefficient, of correlation of x and y in the frequency distribution*". N is the number of individuals in a population, m'_{10} and m'_{01} are the means, and $S_1^2$and S_2^2 are the variances of x and y, respectively.

The variance of a frequency distribution is defined in Aitken, fifth edition (1947) on p. 35 as

$$S^2 = \sum\left(x - m'_1\right)^2 f(x)$$

where m'_1 is the mean of the population, and $f(x)$is the relative frequency of each sample in the population. The relative frequency is 1 because we address members of the population individually within the summation, thus

$$S^2 = \sum\left(x - m'_1\right)^2$$

Substituting this into r we find

$$r = \frac{1}{N}\frac{\sum\left(x - m'_{10}\right)\left(y - m'_{01}\right)}{\sqrt{\sum\left(x - m'_{10}\right)^2 \sum\left(y - m'_{01}\right)^2}}$$

This is not the equation given by Stauning et al. (2006) which is replicated in Eq. (18). There is a factor of $1/N$ missing in their equation. Whilst the equation given by Stauning et al. (2006) is not the correlation coefficient it is a relative number which they use to define a set of data.

1.4 Regression methods

C6. We used the terms linear and orthogonal in the table because we required a way of distinguishing the two methods. Yes both are linear regression methods. The ordinary least square (OLS) estimate (of Y on X) will minimize the vertical distance from the points to the regression line. This is what we have termed "Linear Regression Coefficients". Likewise, the OLS estimate (of X on Y) would minimize the horizontal distance to the regression line. The orthogonal regression takes the middle ground by minimizing the orthogonal distance to the regression line. This is what we have termed "Orthogonal Regression Coefficients".

C7. Thank you for the clarification.

C8. We did not wish to analyse the issue in greater detail because we wanted only to compare the derivation procedure and note that here was an item where the PCN and PCS indices differed.

The coefficients from regression analyses are dimensionless as the only concern is the distance from the regression line.

For us, it is clear from Eq. (5) that the x-term is electric and the y-term is magnetic as convention dictates. The values of the coefficients will be different depending on the method chosen. The method of orthogonal regression is well defined and the reasons for using it are given in Vennerstrom (1991). Whether those reasons are correct or not are a matter of scientific debate and should be addressed in another forum. Our purpose here was only to note the differences in derivation procedure.

1.5 Derivation of the quiet reference level at AARI

C9. We stand by our brief description. It is only when there are no data that the longer method is employed. This is described well by Janzhura and Troshichev (2008). As recalled in the introduction of the present paper, we chose to consider only available published literature in our review, so we cannot comment on the basis of information that is not available in this corpus, such as that referred to in S2011 point (iv).

1.6 Derivation of the quiet reference level at AARI

C10. It would be useful to have the method of the DMI qwnl baseline procedure outlined in a publication.

C12. To our best knowledge, the actual weights cannot be found in the available published literature. An example of a calculation would be nice so a comparison of methods could be done. As it stands an independent person cannot compute the coefficients given the explanations found in the literature.

1.7 PC index sampling less frequent than data sampling

C14. Equation (8) is incorrect in MM2010, and in the Corrigendum. We deeply apologise about the Corrigendum error.

Equation (8*) defined in S2011 is also incorrect. Equation (8) is in fact

$$F_k = \sum_{i=(k-1)d}^{j+i} \delta M_{j+i} \sin\gamma_{j+i} \mp \delta N_{j+i} \cos\gamma_{j+i}$$
$$\{j=1,...,d\}\{k=1,...,k_T\} \text{ and } d=\frac{\kappa}{\tau}$$

2 Further comments

C15. The series of events described in S2011 were not available before in published literature.

However, we were trying to summarize Sect. 4.1 of Papitashvili et al. (2001). They discuss the daily variation of the PCN index after it was corrected (DMI#2_2001). They found that if the PCN index was averaged over eight years then no recognisable daily variation can be found. However, if the index is split into amplitude dependent sections a recognisable daily variation exists. Their reasoning is "*…different physical mechanisms cause the standard (positive PCN) and reverse (negative PCN) currents over the station in the polar ionosphere (e.g., Troshichev et al., 2000). The positive PCN index is a measure of the dawn-dusk ionospheric electric fields, related to the southward IMF conditions; the negative PCN values are mainly recorded during northward IMF conditions.*" Papitashvili et al. (2001) showed in Fig. 5 that the average UT curves can exhibit a variation of 0.4 in a day; in Fig. 6 the seasonal variation of PCN is shown to be 0.3 in a year; Fig. 7 shows the solar cycle variation where the average solar cycle variation of PCN is 0.7. Thus the daily variation, the seasonal variation and the solar cycle variation are comparable in magnitude.

C16. We think that the original developments in the field of PC indices are worth being mentioned, in case an historian wanted the full picture of idea synthesis. We thank Dr. Stauning for the precision he gives in his comment, but we want to make clear that we do not fail to state clearly the meaning of the PC_L and MAGPC indices.

(i) The original idea for the PC index came from previous works which included the PC_L and $\mathrm{PC}(B_z)$. We included them in case an historian wanted the full picture of idea synthesis. The fact that PC_L is a range index makes no difference in the development of the PC indices.

(ii and iv) We clearly state in Eq. (3) what the definition of MAGPC (Troshichev and Andrezen, 1985) is. It is the same definition as $\mathrm{PC}(B_z)$ (Troshichev et al., 1979). As we state on p. 1890: "*The term MAGPC index was coined in Troshichev and Andrezen (1985). However the method of deriving the index was first defined in Troshichev et al. (1979).*" From the introduction of Troshichev and Andrezen (1985) we find, "*The procedure of calculation for such characteristic was given by Troshichev et al. (1979a). Further this characteristic will be termed the MAGPC index*".

(iii) We state this in the paragraph above Eq. (3): "*In Troshichev and Andrezen (1985) the projection plane was changed to 03:00–15:00 LT.*".

C17. See Corrigendum point (3). Please note that Tables 2 and 3 in MM2010 give the characteristics of the PCN and PCS indices respectively, and attach the publication to the label given by us.

C18. To our best knowledge, the information given in point (i) cannot be found in the available published literature. As for point (ii), we refer the reader to point 2 of the Corrigendum.

C19. We were in fact trying to find similarities in the procedures, and we ended up with a confusing statement. We should not have done this. We should have pointed out that not including secular variation may in fact harm the PC index calculation.

In our opinion, the DMI#4_2006 procedure does not avoid the secular variation problem. The DP2 current system lies on the magnetic field. If the geometry of the geomagnetic field lines with respect to the geographic axis changes with time (secular variation) then the orientation of the DP2 current system should change with time. Therefore the secular variation will impact on the value of the angle ϕ. Perhaps this should be investigated.

C20. Thank you for the clarification.

C21. This point is already answered in the Corrigendum, at point (10).

C22. The point we wished to make here was that any magnetic station situated within the polar cap can be used to determine the PC index, although stations located close to the centre of the DP2 transpolar current are likely to give better PC determination.

C23. We are concerned in our paper with the derivation procedures and how they differ. A major difference between the derivations of the two indices is in the quiet level determination.

3 Conclusion

The PC index is a complicated issue, in particular because it is a rather new topic in the field of geomagnetic indices, as compared to the planetary indices that inherit the knowledge regarding the C index, or to ring current characterization that began in the nineteen-fifties! Much literature has been published, with nomenclatures and methods that evolve with time so as to improve the index and get the most reliable possible information on the convection in the Polar ionosphere. We aimed at clarifying the situation in MM2010. The Corrigendum and S2011 strikingly illustrate the need of the task we undertook, and of its difficulty.

It is clear that new contributions, considering the usage of the PC indices, and clarification of the PC indices would be of great help to pave the way towards a definition of the PC index that gains consensus within the scientific community. We welcome any new contributions to the clarification elucidation of the determination, meaning, and usage of the PC indices.

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