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Multiple triangulation analysis: application to determine the velocity of 2-D structures

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Abstract. In order to avoid the ambiguity of the application of the Triangulation Method (multi-spacecraft timing method) to two-dimensional structures, another version of this method, the Multiple Triangulation Analysis (MTA) is used, to calculate the velocities of these structures based on 4-point measurements. We describe the principle of MTA and apply this approach to a real event observed by the Cluster constellation on 2 October 2003. The resulting velocity of the 2-D structure agrees with the ones obtained by some other methods fairly well. So we believe that MTA is a reliable version of the Triangulation Method for 2-D structures, and thus provides us a new way to describe their motion.

Keywords. Magnetospheric physics (Magnetospheric configuration and dynamics; Solar wind-magnetosphere interactions; Instruments and techniques)

1 Introduction

In order to study the motion of various 2-D structures, several tools have been developed based on in-situ data from single or multiple satellites. Some of the most widely-used examples are: Spatio-temporal Difference, or STD method (Shi et al., 2006), DeHoffmann-Teller analysis (in detail, see Khrabrov and Sonnerup, 1998), and the Triangulation Method, also referred to as the "timing method" (Russell et al., 1983; Harvey, 1998).

The STD method assumes that the temporal change of the magnetic field, observed by the spacecraft, is only caused by the structure motion, which leads to a set of difference equations. By solving these equations, the velocities can be determined point by point. It should be noted that this method can be applied to 1-D, 2-D or 3-D structures.

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On the other hand, the DeHoffmann-Teller analysis was originally suggested to calculate the velocity of 1-D structures, such as current layers, by seeking a reference frame in which the mean square of the electric field is as small as possible. Khrabrov and Sonnerup (1998) further applied this analysis to 2-D cases, to calculate the frame velocity perpendicular to the invariant direction.

The Triangulation Method was also developed initially for 1-D structures, and makes use of the time differences between four spacecraft encountering the same planar structure, in order to calculate its normal direction along with the normal component of the velocity (Russell et al., 1983). It should be noted that due to the relative complexity of the 2-D structures, the application of this method to the 2-D structure is less successful, with difficulties in the determination of the time differences.

In many calculations for 2-D cases, one single characteristic contour plane (such as B_Z=0, et cetera) is selected as the signal to judge the time differences (e.g. Eastwood et al., 2005). However, the different signals selected often correspond to distinct timing consequences and thus lead to different velocities, and it is usually hard to tell why one should select a certain signal instead of another when performing the timing analysis. As a modified version, the time differences can be determined by maximizing the cross-correlation function between the magnetic profiles observed by different satellites (e.g. Fear et al., 2005). However, unlike 1-D cases, the cross-correlation peak is often less clear, and the maximum value is also much lower. In some of the cases, the time durations of the structure traversal for different satellites are shown to be different, which makes it even harder to precisely determine the time differences.

In this paper, we make use of the Multiple Triangulation Analysis (Zhou et al., 2006) as another version of the Triangulation Method, to calculate the velocity of the two-dimensional structures. As an example, this approach is also applied to Cluster data and the resulting velocity is compared

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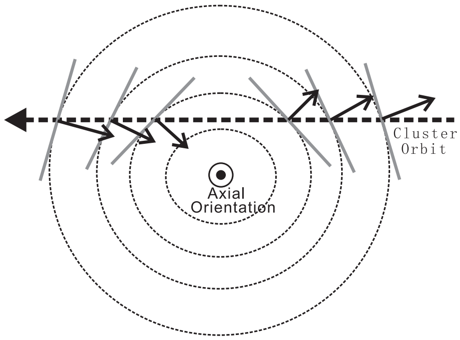

Fig. 1. Sketch of the Cluster constellation traversing a 2-D structure. The set of magnetic contour planes are represented by the dashed circles and the normal directions of these planes are the solid arrows. All of these arrows are shown to be perpendicular to the axial orientation of the 2-D structure.

with those obtained by other methods to test the accuracy of the MTA technique.

2 Method

The Multiple Triangulation Analysis was initially developed to obtain the main orientation of 2-D structures, e.g. magnetic flux ropes (Zhou et al., 2006). Selecting a series of magnetic field magnitudes as the signals, the Triangulation Method can be applied to the 2-D structures for several times, each time with a resulting speed along a certain normal direction. Figure 1 sketches the orbit of the Cluster constellation (the dashed arrow) traversing a 2-D structure. During this traversal, the constellation penetrates a set of magnetic contour planes (the dashed circles). Although the shapes of these contour planes may be more complicated, their normal directions (solid arrows) should be basically contained within the cross-section plane and thus perpendicular to the axial orientation of the 2-D structure.

These normal directions $\boldsymbol{N}^{(m)}(m{=}1, M)$, along with the normal speeds $V^{(m)}$, are now put into the MTA calculation to obtain the the axial orientation $\boldsymbol{H}$, as the direction "most perpendicular" to these normals, by the minimization of

$$\sigma^2 = \frac{1}{M}\sum_{m=1}^{M} |\boldsymbol{N}^{(m)} \cdot \boldsymbol{H}|^2 \ ,$$

which leads to an eigenproblem of a symmetric matrix $\mathbf{L}$:

$$L_{\mu\nu} = \langle N_\mu^{(m)} N_\nu^{(m)} \rangle, \tag{1}$$

where the subscripts μ, ν=1,2,3 denote the Cartesian components of the vector $\boldsymbol{N}^{(m)}$ along the X, Y, Z directions, respectively. One of the three eigenvectors of $\mathbf{L}$, with the smallest eigenvalue, can be treated as the axial orientation of the 2-D structure (for a more detailed analysis and error estimation, see Zhou et al., 2006).

On the other hand, if the structure does not change its configuration significantly during the analyzed time interval, the set of normal speeds $V^{(m)}$ can be treated as the components of the structure velocity along their corresponding normal directions, i.e. $\boldsymbol{V}_S \cdot \boldsymbol{N}^{(m)} = V^{(m)}$. Here $\boldsymbol{V}_S$ denotes the structure velocity. In practice, this velocity can be obtained by the minimization of

$$D(\boldsymbol{V}_S) = \frac{1}{M}\sum_{m=1}^{M} |\boldsymbol{V}_S \cdot \boldsymbol{N}^{(m)} - V^{(m)}|^2. \tag{2}$$

A least-squares approach can be used to determine this velocity. After a straightforward analysis, the minimization problem leads to the linear equation

$$\mathbf{L} \cdot \boldsymbol{V}_S = \langle V^{(m)} \cdot \boldsymbol{N}^{(m)} \rangle, \tag{3}$$

where the brackets $\langle\ \rangle$ represent an averaging over the whole data set, and the symmetric matrix $\mathbf{L}$ is defined as (1). Therefore, the solution of Eq. (3), $\boldsymbol{V}_S = \mathbf{L}^{-1} \cdot \langle V^{(m)} \cdot \boldsymbol{N}^{(m)} \rangle$, would be the optimal estimation of the structure velocity.

Since we are discussing the velocity of a 2-D structure, it should be noted that the resulting structure velocity component along the axial orientation is less meaningful. Any arbitrary velocity component along $\boldsymbol{H}$ can be added to the resulting structure velocity with little change in the value of $\boldsymbol{V} \cdot \boldsymbol{N}^{(m)}$, due to the perpendicular properties between the axial orientation and each magnetic contour normal, by definition.

So it is natural to set the axial velocity equal to zero. The axial orientation (H_1, H_2, H_3), as was discussed before, is estimated as the eigenvector of the matrix $\mathbf{L}$ with the smallest eigenvalue. Then we can simply remove the axial component, thereby obtaining transversal velocity.

An alternative way to obtain the velocity is based on the following constraint:

$$H_1 V_{S1} + H_2 V_{S2} + H_3 V_{S3} = 0,$$

so that one of the velocity components, say V_{S3}, can be expressed as the function of the other two components. Thus, the minimization of (2) becomes the equation of:

$$\mathbf{K} \cdot \boldsymbol{V}_S^* = \langle V^{(m)} \cdot H_3 \cdot \boldsymbol{N}^{(m)*} \rangle \tag{4}$$

instead of Eq. (3). Here the asterisk denotes the first two components of the certain vector, and the 2×2 matrix $\mathbf{K}$ is defined as:

$$K_{\mu\nu} = N_\mu^{(m)} \cdot (N_\nu^{(m)} H_3 - N_3^{(m)} H_\nu),$$

where μ, ν=1, 2 corresponds to the Cartesian X and Y components. The solution of Eq. (4) would provide us the first two components of the structure velocity. Following the constraint that the velocity is perpendicular to the structure's axial orientation, the Z component of the velocity can also be obtained.

There are two error sources in the MTA velocity calculation: the systematic parts and the random ones. The systematic errors are mostly produced by the nonplanar properties

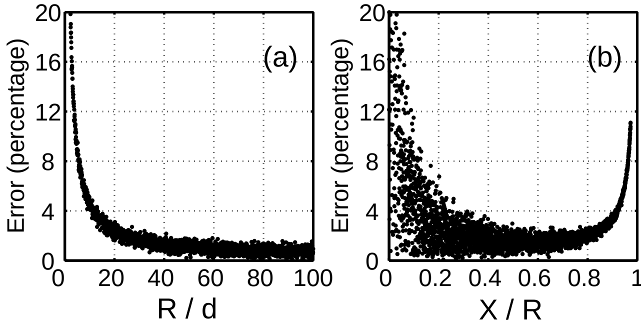

Fig. 2. The error range obtained by the set of MTA tests, selecting 10 contour planes within the structure as the signals. The bootstrap procedure is further applied in each of the MTA tests to display the effect of quantization errors. The difference between the modeled "true" velocity and the MTA velocity, divided by the "true" velocity, can be defined as the error. **(a)** The error versus flux rope radius R (normalized to d=400 km) in a certain type of Cluster trajectory with X (the constellation's closest distance to the axis) of 0.707 R. **(b)** The error dependency on X, which is normalized to the structure radius R), when R is fixed to be 30 times greater than the satellite separation d.

of the magnetic contour planes within the structure, while the random ones may occur because of the quantization effect in applying the timing method.

In order to estimate the error range of the MTA velocity, we simulate a number of 2-D structures with different sizes and make the Cluster constellation cross it along different paths. The quantization errors in the determination of traversal times (randomized for each satellite, with the maximum error of 0.08 s) are also taken into account in each simulation, using the bootstrap procedure (e.g. Kawano and Higuchi, 1995; Sonnerup and Scheible, 1998).

For simplicity, the cross-section of the 2-D structure is set to be circular in the xy-plane, and the z-direction-orientated structure is moving with the velocity of 200 km·s^{-1} along the x-direction. The formation of Cluster is set to be a regular tetrahedron, with the separation of 400 km between each satellite pair. Selecting 10 contour planes as the signals, the MTA technique can be applied on each simulated crossing, and the difference between the resulting velocity and the "true" one can be treated as the error in the simulation. The error shows strong dependencies on both the structure radius R and the constellation crossing path (characterized by the closest distance X from the constellation to the structure axis), as is displayed in Fig. 2.

Figure 2a shows the error dependency on the radius of the 2-D structure, with a fixed constellation trajectory of X/R=0.707. For the cases with the spacecraft separation to be comparative to the scale of the structure, the error is large because the magnetic contour planes are no longer planar, which violates the basic assumption of the Triangulation Method.

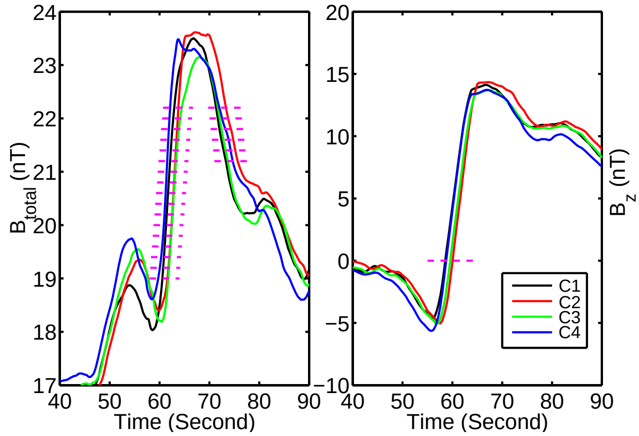

Fig. 3. A 2-D structure observed by FGM/Cluster on 2 October 2003, 00:46:40 UT to 00:47:30 UT. (Left panel) Magnetic strength (Right panel) B_z components, as the function of time (both 6-s sliding averages at 0.2-s resolution).

It can be clearly seen from Fig. 2b that the MTA velocity error is highly dependent on the path of the Cluster constellation, which is typically less than 4% in the region of 0.2 $R<X<0.9$ R. For the $X>0.9$ R cases, the relatively larger systematic errors are caused by the skimming motion over the 2-D structure. On the other hand, for the $X<0.2$ R cases, for which the path is very close to the structure axis, the errors are mainly random ones, which should be produced by the ambiguity in the determination of the axial orientation (see the error estimation part of Zhou et al., 2006). However, because of the approximate parallelism of the set of normal directions in these cases, the resulting minimum eigenvalue λ_3 should be very close to the intermediate one λ_2. So the relatively smaller value of λ_2/λ_3 can provide us with a warning of the invalidation of the MTA technique on the estimation of both axial orientation and traversal velocity.

3 Applications

On 2 October 2003, the Cluster constellation was in the magnetotail observing a 2-D structure. This event was originally discovered by Eastwood et al. (2005), as an example of a flux rope-type structure moving earthward. The authors assumed that the surface B_z=0 is planar on the scale of the spacecraft tetrahedron and by applying the timing method, found a speed of 140±13 km·s^{-1} along the direction of (0.778, 0.595, 0.158) GSM.

Now we apply the Multiple Triangulation procedure to this data set as a comparison, using magnetic field data (FGM data) (Balogh et al., 1997). Figure 3 shows the events in the time interval of 00:46:40 UT–00:47:30 UT: $|B|$ in the left panel, and B_z in the right panel. Note that the high-frequency fluctuations were removed with a sliding average procedure

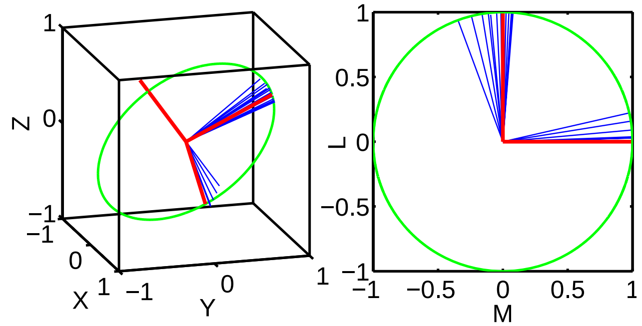

Fig. 4. The MTA analysis on the event during 00:46:40 UT to 00:47:30 UT. The blue lines show the normal directions obtained by the Triangulation method as the first step of MTA, and the red lines are the three eigenvectors of the corresponding matrix. The green circle represents the cross-section plane of the flux rope, basically containing all of the blue lines, while the red line perpendicular to the green circle suggests the flux rope orientation. The left box is in GSM, while the right one is in the LMN coordinate system defined by three eigenvectors.

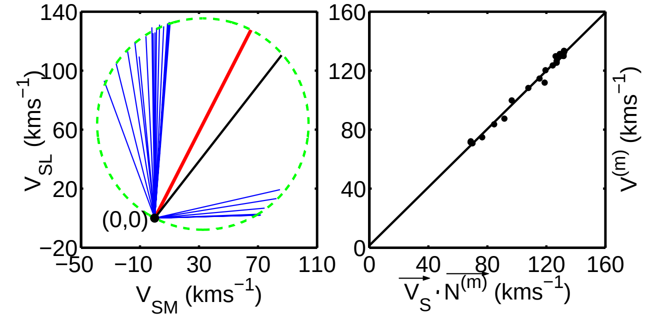

Fig. 5. (Left panel) The set of velocities $V^{(m)} \cdot \boldsymbol{N}^{(m)}$ as blue lines and the structure velocity $\boldsymbol{V}_S$ as the red line in the LMN coordinate system defined by three eigenvectors. (Right panel) Scatter plot of the speed $V^{(m)}$ as the function of $\boldsymbol{V}_S \cdot \boldsymbol{N}^{(m)}$. The correlation coefficient is 0.995.

(Haaland et al., 2004). Here we use a sliding window of 6 s, with a time resolution of 0.2 s, and apply linear interpolation between each consecutive measurements. Then we select a set of magnetic field values as the signals, shown as magenta dashed lines in the left panel of Fig. 3, to obtain 23 normal directions and speeds using the timing method. The 23 directions, displayed as blue lines in Fig. 4, thus lead us to the calculation of the matrix **L** and its three eigenvalues and eigenvectors (shown as red lines). The eigenvector (0.289, −0.570, 0.769) GSM with the smallest eigenvalue (λ_2/λ_3=66.6) should be, in principle, perpendicular to all of the 23 normals, as the estimated axial orientation of the 2-D structure. The large separation between eigenvalues further suggests that the statistical error of the estimated axial orientation is as small as 1.5 degree (see Zhou et al., 2006). As can be clearly seen in Fig. 4, all of the 23 normals are very well confined in the plane (green circle) composed by the other two eigenvectors.

The estimated axial orientation can now be used to calculate the matrix **K**. Solving Eq. (4) thus leads to the calculation of a structure velocity of around (95.19, 100.03, 38.28), i.e. the speed of 143.3±2.4 km·s^{-1} along the direction of GSM (0.664, 0.698, 0.267), with an angular error of 2.0 degree.

All of the 23 velocity vectors are plotted together (as blue lines) in the LMN coordinate system, defined by the **L** eigenvectors, in the left panel of Fig. 5. As was discussed before, the LM plane of this coordinate system can be treated as the cross-section plane of the certain 2-D structure. The structure velocity, obtained by the least-squares procedure, is also shown in the figure, as the red line. Since the 23 velocity vectors are believed to be the components of the structure velocity, all of the vector-end points should be theoretically located in the circle with the structure velocity as the diameter, as is also very clearly seen in the figure.

To give an impression of the precision of the obtained velocity, the two speeds, $\boldsymbol{V}_S \cdot \boldsymbol{N}^{(m)}$ and $V^{(m)}$, are plotted against each other, in the right panel of Fig. 5. The correlation between these two speeds is high, with the correlation coefficient of 0.995. Then we may go back to the result obtained by Eastwood et al. (2005), with the speed of 140±13 km·s^{-1} along the direction of GSM (0.778, 0.595, 0.158). However, in their timing analysis, only one signal is selected (the magenta dashed line in the right panel of Fig. 3). Although their resulting velocity seems to be similar with the MTA velocity, it should be noted that this velocity cannot be treated as the structure velocity based on our discussion. Instead, the velocity is one component of the structure velocity along the normal direction of the B_z=0 plane. Therefore, the velocity (shown in the left panel of Fig. 5 as the black line) should play the same role as the 23 normal velocities discussed before. The vector-end point of this black line is found to be precisely located in the green circle, similar to the 23 vector-end points. Actually, the component of $\overrightarrow{V_S}$ in this direction is 139.6±2.3 km·s^{-1}, which agrees with their result extremely well.

As another comparison, the STD method is also applied in this case. During this time period, the STD velocity is rather stable, with the mean speed of 152.8 km·s^{-1} along the direction of GSM (0.706, 0.675, 0.213). The estimated speed is 9.5 km·s^{-1} larger than the MTA result and the angular deviation between them is only 4.2 degrees. Their similarities, in some sense, confirm the reliability and consistency of both the MTA and STD methods.

DeHoffmann-Teller analysis, as a classical technique to obtain the velocity of structures, should be also applied to examine the precision of MTA approach. However, in this 2-D structure crossing, the time interval is fairly short, which invalidates the V_{HT} result.

4 Summary

In order to avoid the ambiguity in the application of the Triangulation Method to 2-D structures, the approach of the Multiple Triangulation Analysis is presented as a new version of Triangulation Method. While the traditional Triangulation Method can only provide one single component of the structure velocity, the MTA approach makes use of the set of velocity components obtained by the traditional method to calculate both the axial orientation and the optimal structure velocity. Real Cluster data sets are used and comparisons with other methods are also made to prove the precision of this approach. In addition, the MTA approach can have the ability to provide more accurate results for future missions with more than 4 satellites.

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