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Eigenmode stability analysis of drift-mirror modes in nonuniform plasmas

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Abstract. Drift-mirror modes in a one-dimensional inhomogeneous model of the magnetosphere are studied by employing gyrokinetics, taking into account finite Larmor radius effects. A wave equation is derived which describes both the spatial structure of the modes, and its eigenvalue yields a growth rate of the mode. The finite Larmor radius effects are shown to raise the instability threshold especially for high-m waves, and lead to wave propagation across field lines.

Keywords. Magnetospheric physics (Magnetosheath; MHD waves and instabilities) – Space plasma physics (Kinetic and MHD theory)

1 Introduction

Generic properties of the circumterrestrial plasma are its inhomogeneity, high pressure (the plasma-to-magnetic pressure ratio $\beta{\sim}1$), and anisotropy. Under these circumstances, drift-mirror compressional waves can be collectively excited in the plasma (Hasegawa, 1969), provided that the mirror instability criterion is satisfied. From the observational point of view, drift mirror modes are often identified with compressional storm time Pc5 geomagnetic pulsations (Barfield and McPherron, 1978) and some kinds of magnetosheath modes (Narita and Glassmeier, 2005).

A realistic interpretation of drift-mirror modes must take into account finite Larmor radius (FLR) effects because these effects enter into wave equations through the combination $k_\perp \rho$ (here $k_\perp$ is a transverse to the magnetic field component of the wave vector and ρ is a particles' Larmor radius), and $k_\perp$ is proportional to the azimuthal wave number m, which is considered to be large, $m{\gg}1$. The compressional storm time Pc5 pulsations are characterized by high m values. Besides, these waves often have small parallel wavelengths (Takahashi et al., 1987), which implies that finite corrections of $k_\parallel / k_\perp$ should also be taken into account. Due to FLR effects, the wave frequency ω turns out to depend on $k_\perp$ (Hasegawa, 1969; Pokhotelov et al., 2004). But in inhomogeneous plasma $k_\perp$ should, in principle, be considered as an operator, and in the WKB approximation, as a function of coordinates. Thus, studies of instabilities of inhomogeneous plasma and mode spatial structure are intimately linked with each other. These were conducted by Vetoulis and Chen (1994, 1996) and Klimushkin (2000) for Alfvén waves, and by Crabtree and Chen (2004) for compressional waves due to trapped ions. For drift-mirror waves, such a nonlocal eigenmode stability analysis has not been carried out to our best knowledge. It is especially timely now, since the CLUSTER mission makes it possibile to study the small-scale structure of ULF waves. The study of this issue is the prime objective of the present paper.

2 The model and the main equations

In the model under consideration, the field lines of the ambient magnetic field B are supposed to have constant curvature (cylindrical model; e.g. Pokhotelov et al., 1986). All plasma parameters vary only across magnetic surfaces. The plasma-to-magnetic pressure ratio is $\beta{\sim}1$, but a cold electron population present in the plasma provides a shorting out of the parallel electric field ($E_\parallel{=}0$). The equilibrium distribution function F is assumed to be bi-Maxwellian, and the thermal velocities along and transverse to the magnetic field are $V_\parallel$ and $V_\perp$, respectively. Longitudinal and transverse pressures are $P_\parallel$ and $P_\perp$, and the particles concentration is n. Plasma temperature is taken to be uniform.

The perturbed quantities depend on space and time as $\exp\left[-i\omega t+ik_y y+i\int k_r(r)dr+ik_\parallel l_\parallel\right]$, where r is a radial coordinate (field lines' curvature radius), y and $l_\parallel$ are,

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consequently, the azimuthal and longitudinal coordinates, k_r is a wave vector radial component determined from the WKB ansatz.

We start from the perpendicular plasma force balance equation (Pokhotelov et al., 2000)

$$\delta P_\perp + \frac{B\delta B_\parallel}{4\pi} = \frac{k_\parallel^2}{k_\perp^2}\left[\frac{\omega^2}{k_\parallel^2 v_A^2} - 1 - \frac{\beta_\perp - \beta_\parallel}{2}\right]\frac{B\delta B_\parallel}{4\pi},$$

where $k_\perp^2 = k_r^2(r) + k_y^2$. Later on, we will neglect the term $\omega^2/k_\parallel^2 v_A^2$ because we consider much lower frequencies. In also neglecting the coupling between the compressional and transverse Alfvén mode, this equation becomes

$$L_M b = 0. \tag{1}$$

Here, $b = \omega\delta B_\parallel/c$. The operator

$$L_M = k_\perp^2(\tau + a_M) + k_\parallel^2\left(1 + \frac{\beta_\perp - \beta_\parallel}{2}\right) \tag{2}$$

is a compressional (mirror) mode operator, where

$$\tau = 1 + \frac{4\pi}{c^2}2\pi\sum_{e,i}\frac{q^2}{m} \times \int dv_\parallel dv_\perp v_\perp^3 J_1^2\left(\frac{k_\perp v_\perp}{\omega_c}\right)\left(\frac{1}{V_\perp^2} - \frac{1}{V_\parallel^2}\right)F, \tag{3}$$

$$a_M = \frac{4\pi\omega^2}{c^2}\frac{1}{k_\parallel k_\perp^2}2\pi\sum_{e,i}\frac{q^2}{m}\int dv_\parallel dv_\perp v_\perp^3 J_1^2\left(\frac{k_\perp v_\perp}{\omega_c}\right) \times \left(v_\parallel - \frac{\omega - \omega_d}{k_\parallel}\right)^{-1}\left(\frac{\partial F}{\partial\varepsilon} + \frac{k_y F'}{\omega\omega_c}\right). \tag{4}$$

The prime means a differentiating with respect to the radius, r; ω_c is the gyrofrequency, and

$$\omega_d = \frac{k_y}{\omega_c}\left(\frac{B'}{2B}v_\perp^2 - \frac{v_\parallel^2}{r}\right)$$

is the drift frequency in the inhomogeneous magnetic field. The Eq. (1) could be readily derived using the linear gyrokinetic equations (Antonsen and Lane, 1980; Catto et al., 1981; Chen and Hasegawa, 1991).

First, we consider the "classical" drift-mirror mode, that is a mode in limits $k_\perp\rho \to 0$, $k_\parallel^2/k_\perp^2 \to 0$, and $|\omega - \omega_d| \ll k_\parallel V_\parallel$. We obtain from Eq. (1):

$$\tau_0 - \frac{8\pi n}{B^2}\sum_{e,i} m V_\perp^2 \frac{T_\perp}{T_\parallel} i\sqrt{\frac{\pi}{2}}\frac{\omega - \omega_*}{k_\parallel V_\parallel} = 0. \tag{5}$$

Here we have denoted

$$\tau_0 = 1 + \frac{8\pi n}{B^2}\sum_{e,i} m V_\perp^2\left(1 - \frac{T_\perp}{T_\parallel}\right), \tag{6}$$

$$\omega_* = k_y V_\parallel^2 \frac{(\log n)' + 3\alpha(\log B)'}{\omega_c}. \tag{7}$$

Later on, we will neglect the pressure of the electron component. We then readily cover the following well-known relations (Hasegawa, 1969):

$$\mathrm{Re}\,\omega = \omega_*, \tag{8}$$

$$\mathrm{Im}\,\omega = \gamma_M \equiv -\tau_0\sqrt{\frac{2}{\pi}}\frac{k_\parallel V_\parallel}{\beta_\perp}\frac{T_\parallel}{T_\perp}, \tag{9}$$

where $\beta_\perp = 8\pi n m_i V_\perp^2/B^2$. These expressions describe a wave with a drift frequency ω_* which grows if the mirror instability criterion $\tau_0 < 0$ is satisfied.

Now we consider small but finite values of $k_\perp\rho$, where ρ is ion Larmor radius. Also, we are going to retain the $k_\parallel^2/k_\perp^2$ corrections. Let us introduce the designations

$$\alpha = \frac{3}{2}\beta_\perp\left(\frac{T_\perp}{T_\parallel} - 1\right), \quad \delta = \frac{k_\parallel^2}{k_y^2}\left(1 + \frac{\beta_\perp - \beta_\parallel}{2}\right),$$

$$\omega_t = \sqrt{\frac{2}{\pi}}\frac{k_\parallel V_\parallel}{\beta_\perp}\frac{T_\parallel}{T_\perp}, \quad \tilde\tau = \tau_0 + \alpha k_y^2\rho^2, \quad h = i\frac{\omega - \omega_*(r)}{\omega_t(r)}.$$

Then, instead of Eq. (5), we obtain the following drift-mirror mode dispersion relation:

$$k_\perp^2[\tilde\tau(r) + \alpha k_r^2\rho^2 - h(\omega, r)] + k_y^2\delta = 0. \tag{10}$$

This expression can be obtained from Eq. (35) of Hasegawa (1969), if we let $k_\perp\rho \ll 1$ there.

3 Drift-mirror eigenmode stability analysis

We are going to consider modes trapped across the magnetic shells. This means that the radial mode width depends on the macroscopic scale length and, thus $|k_r| \ll |k_y|$. Then Eq. (10) further reduces to

$$k_r^2\alpha\rho^2 + [\tilde\tau(r) - h(\omega, r) + \delta] = 0. \tag{11}$$

This equation can be written in the form

$$\alpha(r)k_r^2\rho^2 - i\frac{\omega - \omega_0(r)}{\omega_t(r)} = 0, \tag{12}$$

where ω_0 is the solution in the $k_r\rho = 0$ case determined by the equality

$$\tilde\tau(r) - h(\omega_0, r) + \delta = 0.$$

The solution of this equation is $\omega_0 = \omega_*(r) + i\gamma_M(r)$, where

$$\gamma_M = -(\tilde\tau + \delta)\omega_t.$$

Because all of the quantities in Eq. (12) are functions of the radial coordinate r, the radial wave vector k_r must also depend on r. Hence, Eq. (12) should be considered as an equation in the WKB approximation which determines the function $k_r{=}k_r(\omega, r)$. Thus, the drift-mirror instability constitutes an eigenmode stability analysis. To determine the eigenvalue ω, we need to derive the corresponding eigenmode equation and impose the appropriate boundary conditions. The simplest approach is replacing k_r by the differential operator $-id/dr$; and we obtain

$$\rho^2\alpha(r)\frac{d^2 f}{dr^2} + i\frac{\omega-\omega_0(r)}{\omega_t} f(r) = 0. \tag{13}$$

Here $f(r)$ is the wave function. Certainly, any equation which differs from this one with the term like df/dr yields the same leading order WKB approximation, Eq. (12), as Eq. (13) does. But these terms do not affect the eigenvalues ω in the large k_y asymptotic limit, so it is sufficient to restrict the consideration to Eq. (13). A natural boundary condition to this equation in the coordinate r is the condition that $|f|$ decays away from the localization region.

Then, we will consider the mode localized near the surface r_m, where the function $\omega_0(r)$ takes its maximum value. Near this point we can use an expansion

$$\omega_0(r) = \omega_{0m} + (1/2)\omega_0'' x^2,$$

where $x{=}r{-}r_m$, and the m-index means a function value in the point r_m. Besides, we will consider the case $|\gamma|\ll|\omega_*|$. Then,

$$\omega_0'' \simeq -\frac{|\omega_{*m}|}{l^2},$$

where l is a characteristic scale of the variation of the drift frequency across magnetic shells. Denoting

$$E = \frac{\omega-\omega_{0m}}{\omega_{tm}\rho_m^2\alpha_m},$$

Equation (13) becomes

$$\frac{d^2 f}{dx^2} + \left[iE - \frac{i}{2}\frac{|\omega_*|}{\omega_t\alpha(\rho l)^2}x^2\right] f = 0. \tag{14}$$

The boundary condition, meanwhile, is

$$|f(x\to\pm\infty)| \to 0, \tag{15}$$

or $|f|$ vanishes as $|x|{\to}\infty$. From Eq. (14) and below, the subscript "m" is omitted, assuming all plasma equilibrium parameters are evaluated at the point r_m.

As we see, Eq. (14) has the same form as the Schrödinger equation for the harmonic oscillator, and value E plays the role of energy. The solution of this equation is written as

$$f(x) = H_n\left(\frac{x}{\lambda}\right)\exp\left(-\frac{x^2}{2\lambda^2}\right), \tag{16}$$

where H_n are Hermitian polynomials, $n{=}0, 1, 2, ...$, and λ is a characteristic radial wavelength determined by the condition

$$\frac{i}{2}\lambda^4\frac{|\omega_*|}{\omega_t\alpha(\rho l)^2} = 1.$$

To determine the proper root of λ^4, we impose the boundary condition (15). Hence, it follows that $\mathrm{Re}\,\lambda^{-2}{>}0$, that is

$$\lambda^2 = e^{-i\pi/4}\left|\frac{2\omega_t\alpha}{\omega_*}\right|^{1/2}\rho l. \tag{17}$$

In terms of order of magnitude, $\lambda{\sim}O(\sqrt{\rho l})$.

The quantization condition on the eigenvalue is $iE\lambda^2{=}2n{+}1$, hence

$$\mathrm{Re}\,\omega_n = \omega_* + \left(n+\frac{1}{2}\right)\frac{\rho}{l}\sqrt{\alpha\,\omega_t\,|\omega_*|}, \tag{18}$$

$$\gamma_n = \mathrm{Im}\,\omega_n = \gamma_M - \left(n+\frac{1}{2}\right)\frac{\rho}{l}\sqrt{\alpha\,\omega_t\,|\omega_*|}. \tag{19}$$

Following Hasegawa (1969), let us introduce the parameter

$$\Delta = \frac{3}{4}\left[\beta_\perp\left(\frac{T_\perp}{T_\parallel}-1\right)-1\right],$$

representing the measure of the overshooting of the instability condition. The growth rate takes a maximum value when $\partial\gamma_M/\partial k_\parallel{=}0$, $\partial\gamma_M/\partial k_y{=}0$. Thus, γ peaks at

$$k_y^* = \frac{1}{\rho}\frac{\sqrt{2}}{3}\left(\frac{\Delta}{\frac{4\Delta}{3}+1}\right)^{1/2}, \quad \delta^* = \frac{\Delta}{3},$$

$$k_\parallel^* = \sqrt{\frac{2}{27}}\frac{\Delta\Upsilon}{\rho}, \tag{20}$$

where

$$\Upsilon = \left(1+\frac{\beta_\perp-\beta_\parallel}{2}\right)^{-1/2}\left(\frac{4\Delta}{3}+1\right)^{-1/2}.$$

Both k_y^* and $k_\parallel^*$ are real when $\Delta{>}0$. Taking maximizing values of k_y, δ, we find

$$\gamma_M^* = \frac{2}{\sqrt{27\pi}}\frac{V_\parallel T_\parallel}{\beta_\perp T_\perp}\frac{2}{3\rho}|\Delta|\Delta\Upsilon.$$

We see that the condition for γ to be positive is still $\Delta{>}0$, as in the case when FLR and $k_\parallel/k_y$ terms are neglected. In order of magnitude,

$$\gamma_M^* \sim \Delta^2\frac{V_\perp}{\rho}.$$

So, as the Larmor radius increases, the growth rate decreases, but remains positive. However, according to Eq. (19), when the radial mode number n becomes large, the instability is stabilized. The critical value is

$$n_c \sim \left(\frac{L}{\rho}\right)^{3/2}\Delta^{5/4},$$

i.e. FLR effects are favorable for lowering the stabilizing n number.

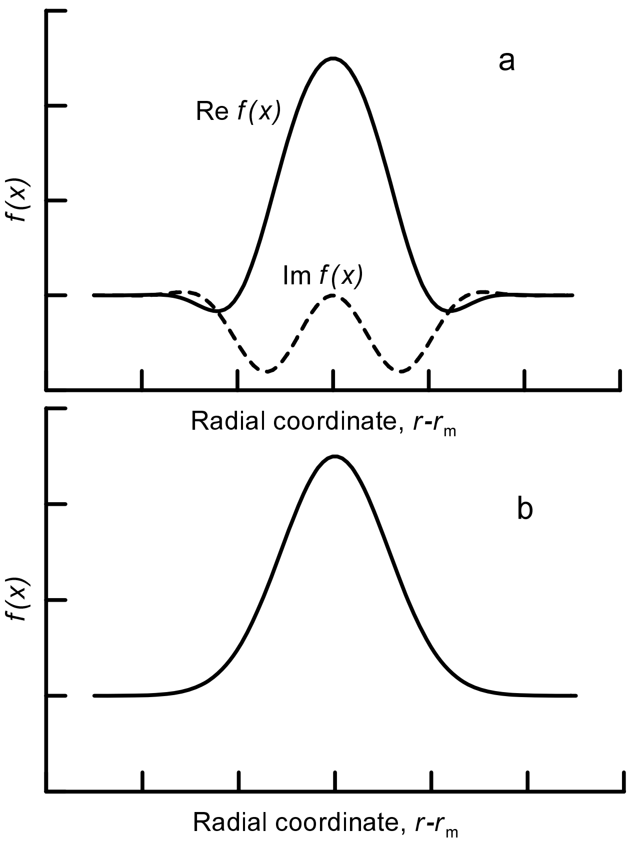

Fig. 1. The mode structure across magnetic shells: **(a)** when $|\gamma|\ll|\omega_*|$, **(b)** when $|\gamma|\gg|\omega_*|$.

Considering the marginal instability case when $0<\Delta\ll 1$ and $k_\parallel=k_\parallel^*$, $k_y=k_y^*$, the characteristic radial wavelength scales are

$$\lambda^* \sim L\left(\frac{\rho}{L}\right)^{1/4}\Delta^{1/8},$$

where L is the inhomogeneity scale. We see that λ^* only weakly depends on both ρ/L and Δ.

4 Discussions

In this paper, we have analyzed the growth rate and the spatial structure of the drift-mirror modes as a nonlocal eigenmode problem. The following picture is beginning to emerge. At the first sight the finite Larmor radius favors the instability stabilization because the condition for the inequality becomes

$$\tau_0+\alpha(k_y\rho)^2+(k_\parallel/k_\perp)^2[1+\frac{1}{2}(\beta_\perp-\beta_\parallel)]<0.$$

However, at the maximizing values of k_y and $k_\parallel$, the instability criterion remains as in the $k_y\rho=0$ case: $\Delta>0$. FLR effects influence the instability in another way, via lowering the threshold value of the radial harmonic number n. This value n_c decreases rapidly with increasing ρ, and the harmonics with $n>n_c$ are stable.

The radial structure of the mode is described by Eq. (16). The presence of the imaginary part leads to the propagation of the wave across the magnetic shells. The real part of the radial wave vector is $\mathrm{Re}\,k_r=-x|\lambda^{-2}|\sin(\pi/4)$, i.e. the phase velocity is directed toward the left when $r>r_m$ and toward the right when $r<r_m$. As Eq. (17) indicates, k_r does not depend on ω, i.e. energy is not transmitted across magnetic shells (like in an Alfvén wave in a homogeneous plasma). Notice that the radial structure has the oscillatory character even at $n=0$ (Fig. 1a). The wave is modulated by the Gaussian function. Surprisingly, the value of the characteristic radial wavelength has a rather weak dependence on the ratio ρ/L and on the measure of the overshooting of the instability condition Δ.

The situation is entirely analogous in the regions where the function $\omega_*(r)$ reaches minima, except that the sign of the radial phase velocity is opposite.

The conclusions about the instability remain valid when $|\gamma|\gg|\omega_*|$. But in this case $\mathrm{Im}\,\lambda^2=0$, i.e. the mode is standing across magnetic surfaces (Fig. 1b).

The spatial structures predicted in this paper could be compared with the CLUSTER satellite observations in the magnetosphere and in the magnetosheath.

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