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Mean velocities measured with the double pulse technique

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Abstract. It was recently observed that double-pulse measurements of the mean velocities of a wide asymmetric spectrum are a function of the time lag between the pulses (Uspensky et al., 2004[1]). Here we demonstrate that the observed relationship probably is influenced by the measurement technique in a way that is consistent with theoretical prediction. It is further shown that for small time lags the double pulse velocity is a good approximation to the mean Doppler velocity.

Key words. Ionosphere (auroral ionosphere; plasma waves and instabilities; ionospheric irregularities)

1 Introduction

Coherent radars have been widely used to study plasma processes in the Earth's E-region ionosphere (see, for example, Jackel, 2000; Nielsen, 1989; Villain et al., 1987). The $\boldsymbol{k}$-vector of a ground-based radar is pointed nearly perpendicular to the Earth's magnetic field at an altitude of $\sim$105 km, where the radar signal may be backscattered coherently from periodic plasma structures excited by instabilities in the plasma. The autocorrelation function (or spectrum) of the backscattered signal contains maximum information about the backscatter processes. However, often it is not the autocorrelation function or the spectrum that is directly used to study the ionosphere, but rather a single parameter derived to characterize these functions. Instead of working with the spectra themselves, it is often expedient to work with a parameter derived to characterize some aspects of the spectra, for example, the "mean Doppler frequency". The mean Doppler frequency is a measure of the "mean" frequency shift of the central radar frequency caused by the dynamic backscatter targets in the ionosphere. In the following "mean Doppler velocity" is the first moment of the power spectrum (see Eqs. 1 and 2 below). Other characteristic velocities derived for the spectrum are here referred to simply as "mean velocities". A mean velocity can be defined different ways. In the following various theoretical and experimental "mean velocities" are introduced and their relationship to each other examined. Observations with the STARE radar system are then used to illustrate these results. It is confirmed that the double pulse technique with a small separation between the pulses yields a useful approximation to the mean Doppler velocity, while the Doppler velocity associated with the peak in the backscatter power spectrum is better approximated using a large separation between double pulses.

2 Mean velocities

The "mean velocity", or the velocity one may use to characterize the spectrum with a single parameter, could, for example, be the frequency $f_{\max}$, at which most power is backscattered. To choose a parameter that is associated with the maximum power in the backscatter process would seem to be a good choice, since it is referenced to a well-defined physical property of the spectrum. However, this parameter has the draw-back in that it disregards the shape of the spectrum. It points only to the maximum but contains no information about the skewness of the spectrum. So even though $f_{\max}$ is the frequency of the spectral maximum, it does not inform us about the distribution of the received power relative to the spectral maximum; although $f_{\max}$ is the frequency of the spectral maximum, the distribution of the received power relative to the spectral maximum is unknown. When integrating over frequency, there may be actually more power received for frequencies other than those near the peak.

Another "mean velocity", $<f>$, is defined such that the total power received for frequencies less than $<f>$ is equal

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[1]Uspensky, M., Koustov, A., Sofieva, V., Amm, O., Kauristie, K., Schmidt, W., Nielsen, E., Pulkkinen, T., Pellinen, R., Milan, S., and Pirjola, R.: Double-pulse and multi-pulse velocities of STARE echoes, Ann. Geophysicae, submitted, 2004.

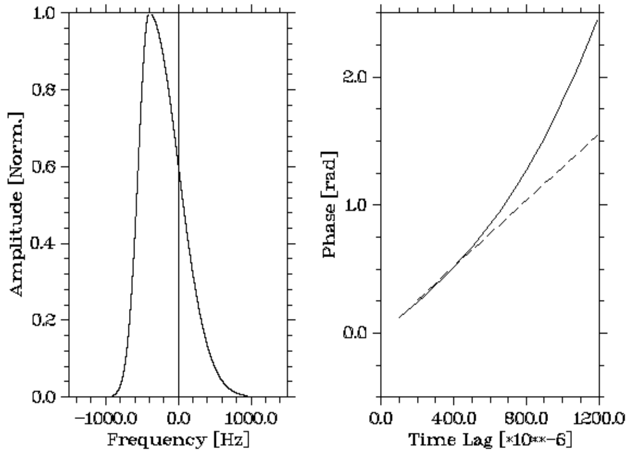

Fig. 1. (a) An asymmetric spectrum. Note, phase <0. **(b)** The simulated experimental phase shift (Eq. 7) between two pulses transmitted with a time lag of T is shown by the solid curve. The calculations were made for the asymmetric power in (a). The dashed line represents the hypothetical phase variation for a constant mean velocity equal to the mean Doppler velocity $<f>$ derived from Eq. (1) for the spectrum in (a). The slopes of the two curves are nearly identical for small T, which means that the mean Doppler velocity and the experimental velocity are compatible for small time lags, in this case close to $|220\,\mathrm{m/s}|$.

to the power received, integrated over frequencies larger than $<f>$. This definition of the mean velocity yields information about the distribution of power relative to the mean velocity. This mean Doppler velocity, or mean Doppler frequency $<f>$, is mathematically defined as the first moment of the power spectrum, $S(f)$,

$$\int S(f)(f-<f>)df=0, \tag{1}$$

where $S(f)$ is normalized

$$\int S(f)df=1. \tag{2}$$

In practice, the characteristic Doppler velocity of a spectrum is often measured using a double pulse radar technique. The STARE coherent radar system is used to measure the autocorrelation functions of the E-region ionosphere irregularities or plasma waves. The measurement procedure is outlined in Nielsen and Rietveld (2003). Six pulses are transmitted in a certain time pattern, which allows double pulse pairs to be identified with a time lag of 200, 400, …, 1200 μs. Conceptually, the measurements may be regarded as being simultaneous for several different double-pulse pairs. For each pulse pair a mean velocity of the target can be calculated from the observed difference in phase shift ϕ, induced in the two pulses by the radar target (Jackel, 2000; Nielsen, 1989; Rummler, 1968).

This phase shift is a measure of how far the second pulse traveled relative to the first pulse. Equivalently, during the time interval between the pulses the second pulse increases or decreases its travel distance by twice the distance the target has moved since the first pulse. If the target in the interpulse time, T, has traveled half a wave length of the signal frequency, then the travel distance of the second pulse has decreased (or increased) by one wavelength, λ, and its phase ϕ, therefore, is changed by 2π relative to the first pulse. If the phase change is assumed to be proportional to the travel distance, we have for the mean velocity of the target,

$$V_D=\frac{\lambda/2}{T}\frac{\phi}{2\pi} = 1.66\times10^5\frac{\phi}{T}(\mathrm{m/s}), \tag{3}$$

where ϕ is measured in radians and T in μs. The radar wavelength (λ) is 2086 m. If ϕ is proportional to T, then the mean velocity is independent of T.

This experimental definition of the mean velocity is used in the STARE system: two pulses are transmitted with controlled phases, with a time interval of 200 μs, and the phase difference of the returned pulses is used to derive the mean Doppler velocity following Eq. (3). Similarly, in the multipulse mode, each point on the autocorrelation function is determined by pair wise correlating the signals backscattered from the transmitted pulses. Thus, the mean velocity can be determined as a function of time lag using the STARE multipulse observations.

In the following the relationship is examined between the experimental mean velocity and the mean Doppler velocity defined by Eq. (1). The autocorrelation function of the backscattered radar signal is given by the correlation (amplitude and phase) of the signals received in two pulses transmitted and then received with a given time interval, as a function of the time interval between the pulses. The Fourier transform of the autocorrelation function is the power spectrum of the received signal. Thus, the relationship between the autocorrelation function, $R(T)$, and the spectrum is

$$R(T)=\int S(f)e^{i\omega T}df. \tag{4}$$

To derive the equation which determines the "mean frequency", f_o, defined by the double pulse technique, the angle of the autocorrelation function is introduced as $\phi = \omega_o T$, where $\omega_o \pi f_o$ (The mean velocity is given by V_D=1.043× f_o.). Now $R(T)$ can be written as

$$R(T)=|R(T)|e^{i\omega_o T}. \tag{5}$$

Since the imaginary component of amplitude is zero, we have

$$\mathrm{Im}\left\{R(T)e^{-i\omega_o T}\right\}=0 \tag{6}$$

and combining Eqs. (4) and (6) yields

$$\int S(f)\sin\left[2\pi T(f-f_o)\right]df = 0. \tag{7}$$

The mean Doppler shift, f_o, is determined such that this integral is zero. Comparing this result with that in Eq. (1) it becomes apparent that the double-pulse velocity, f_o, is equal to $<f>$ for symmetric spectra (because the sine-function is uneven, an even spectrum results in a zero value integral).

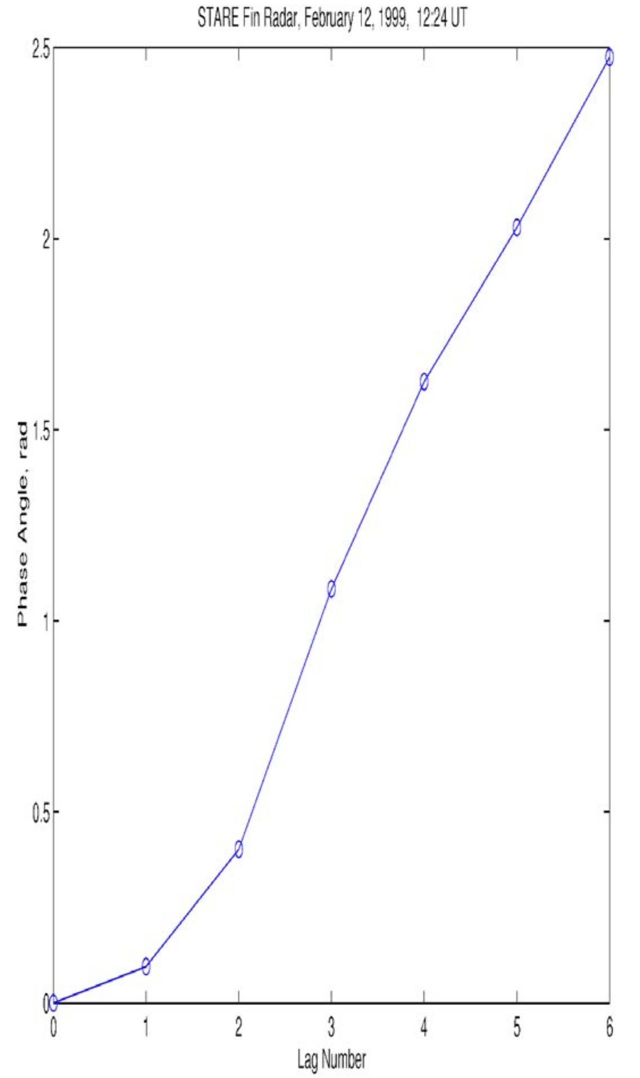

Fig. 2. Phase, ϕ, vs. time lag (=Lag Number * 200 μs) for STARE multi pulse experiment (after Fig. 5b in Uspensky et al., 2004[1]).

To further examine how f_o depends on T, the sine function in Eq. (7) has been expanded in a Taylor series,

$$\int S(f)\left[(f-f_o)-\frac{1}{3!}(2\pi T)^2(f-f_o)^3+\frac{1}{5!}(2\pi T)^4(f-f_o)^5-...\right]df=0 \quad (8)$$

$$f_o=<f>-\frac{1}{6}(2\pi T)^2\int(f-f_o)^3 S(f)\,df+\frac{1}{120}(2\pi T)^4\int(f-f_0)^5 S(f)df-... \quad (9)$$

Defining the nth moment of the normalized power spectrum

$$M_n=\int(f-f_o)^n S(f)df\,, \quad (10)$$

we find

$$f_o=<f>-\frac{(2\pi)^2}{6}T^2M_3+\frac{(2\pi)^4}{120}T^4M_5-... \quad (11)$$

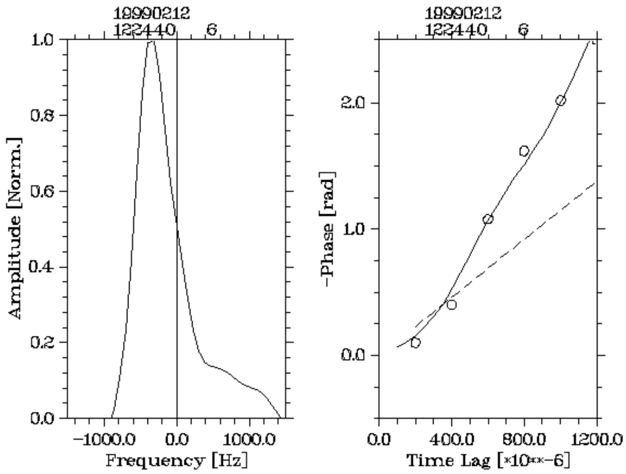

Fig. 3. **(a)** The backscattered power spectrum observed with STARE averaged between 12:23 and 12:25 UT on 12 February 1999. This spectrum, as shown, was used in the phase calculations. **(b)** The calculated variation of the phase change between two double pulses as a function of time lag for the observed spectrum in (a) (solid curve). The "circles" represents the observed data points taken from Fig. 2. The dashed line represents the mean Doppler velocity (Eq. 1) calculated using the spectrum in (a), and is |195 m/s|.

The mean velocity is a function of the mean Doppler velocity, the double pulses' time lag and the uneven moments of the backscattered power spectrum.

If the argument in the sine function in Eq. (7) is so small that it approximates its sine value, $\theta\sim\sin\theta$, then $f_o\sim<f>$, independent of whether the spectrum is symmetric or asymmetric. This is equivalent to require that the spectral moments with $n\geq3$ on the right-hand side of Eq. (11) are small compared to the term with the 1st moment. This situation occurs if the spectrum is narrow enough and if the backscattered power only exceeds the noise in a narrow interval around f_o, and $<f>$ is $\gg0$.

Since $\theta=\pi T(f-f_o)$ the approximation of the sine function by its argument is better satisfied the smaller T is. If, say, the spectrum exceeds the noise level for $|f-f_o|<500$ Hz, and $T=200\,\mu$s; then θ is a good approximation of $\sin\theta$ over the whole frequency interval (the worst approximation is $\theta\sim0.62$ and $\sin\theta\sim0.59$ for $|f-f_o|=500$ Hz). In this realistic case the 200 μs double pulse measurement will yield a good approximation of the mean Doppler velocity (Eq. 1). If $T=400\,\mu$s, then $\theta\#\sin\theta$ ($\theta\sim1.25$ and $\sin\theta\sim0.95$ for $|f-f_o|=500$ Hz), and consequently, f_o is not a good approximation of $<f>$. For larger T the approximation further deteriorates.

In conclusion, for small values of the double pulse time lag, T, of, say, 200 μs, the experimental mean velocity is a good approximation of the mathematically defined mean Doppler velocity, Eq. (1). For a larger time lag the values of f_o and $<f>$ remain compatible for symmetric spectra and for narrow spectra, but for wide asymmetric spectra the mean velocity, f_o, becomes a function of the time lag, T.

Table 1. Observed and estimated (numerical) velocities.

	Simple model (Fig. 1) m/s	Observations (Fig. 2) m/s	Derivations from spectrum (Fig. 3) m/s
Mean Doppler velocity (Eqs. 1 and 2)	220		195
Mean velocity for:			
small-T ($\sim$200 μs)	220	180	175
large-T ($\sim$1200 μs)	350	345	345
(MP) curve fit	360	380	385

As an illustration the "ϕ vs. T" curve implied by Eq. (3) has been calculated for a wide asymmetric backscattered power spectrum. This simple spectrum is chosen in order to demonstrate the basic variation of the phase as a function of time lag in a double pulse measurement. The spectrum is shown in Fig. 1a, and the associated phase versus time lag in Fig. 1b.

The mean velocity is related to the ratio of the phase and time lag (Eq. 3). In Fig. 1b one notices the absence of proportionality between phase and time lag for the asymmetric spectrum: the slope of the simulated experimental curve (solid) is actually increasing with time lag. This implies that the mean velocity increases with time lag. The (numerical) Doppler velocity increases from 220 m/s at 200 μs to 350 m/s at 1200 μs, and is 360 m/s for a linear fit to the whole curve. For the dashed curve ϕ/T is constant and equal to the mean Doppler velocity. The difference between the solid and dashed curves arise owing to the asymmetry of the spectrum. The slope of the dashed line is a good approximation to the slope of the "experimental" (solid) curve at small time lags, indicating, as expected, that the experimental velocity for small T is a good approximation of $<f>$.

Figure 1 is only used to illustrate the theoretical results. It shows that the detailed behavior of phase with time lag depends on the spectrum associated with the observed data. The next step is therefore to introduce an observed phase vs. time lag, together with the associated spectrum, and then recalculate the phase vs. time lag for that spectrum and compare with the observed variation.

3 Observations

Using the STARE system Uspensky et al. (2004)[1] observed the phase ϕ as a function of the double pulse time lag. An example of their results is shown in Fig. 2. The mean velocity is for a given double-pulse time lag determined by the ratio between phase and time lag.

The observed variation of phase with time lag has features similar to the general predicted behavior (Fig. 1b). For small lags the phase shift is reduced in magnitude compared to the shift associated with larger lags. Figure 2 shows that ϕ and T are not proportional, but rather that the ratio between the phase and lag time tends to increase with increasing lag time. Thus, the mean frequency is observed to be an increasing function of the time lag. From Fig. 2 we find a variation of the Doppler velocity from 180 m/s at 200 μs to 345 m/s at 1200 μs, and 380 m/s for a linear fit to all the observations.

The observed variation of "ϕ vs. T" can be accounted for by the observed asymmetric backscatter power spectrum. The average backscattered power spectrum observed with STARE over a 2-min time interval centered on 12:24 UT is shown in Fig. 3a. It has an asymmetric main peak, as the simple spectrum in Fig. 1a, and in addition, a tail extending towards negative frequencies. Using this observed spectrum and Eq. (7), the phase variation with time lag was determined, Fig. 3b.

The agreement with the observation data in Fig. 2 is quite striking. The "curved" behavior below 600 μs, as well as the nearly linear behavior above 600 μs, is reasonably well reproduced. The (numerical) mean velocity increases from 175 m/s at 200 to 400 μs, to 345 m/s at 1200 μs, and to 385 m/s for a linear fit to all the observations. The measurements, f_o, for small T are a good approximation to the calculated mean Doppler velocity, $<f>$ (the slopes of the solid and dashed curves are compatible for small time lags). One may argue that the fit between the solid and dashed curves is not as perfect as is the case for the corresponding curves in Fig. 1b. Indeed, Uspensky et al. (2004)[1] suggest that in particular this "poor" fit is a result of cross-talk between different backscatter ranges (see Sect. 4). However, in this work the key point is to note that spectral asymmetry in itself causes a nonlinear variation of the phase versus lag between double pulses, which is quite similar to the observed variation. A higher order effect as cross-talk is being treated separately. The parameter values derived from Figs. 1, 2 and 3 are listed in Table 1.

4 Discussion

It is not straightforward to determine which is the most appropriate and useful definition of the mean velocity or mean frequency of the spectrum. It could be the frequency associated with maximum backscattered power, or the mathematically defined mean frequency of the spectrum. The mean Doppler velocity is defined as the first moment of the power spectrum; it is a defined parameter and it is therefore an important parameter to measure.

In practice, the mean frequency is often determined using a double-pulse measurement technique. In this report it is demonstrated that the mean velocity obtained with a double-pulse measurement technique is a good approximation of the mean Doppler velocity when the measurement is made at a small time lag. The experimental and theoretical mean Doppler velocity is compatible for symmetric spectra and for narrow (symmetric or asymmetric) spectra, independent of time lag. But for wide asymmetric spectra the mean Doppler velocity and experimental defined mean velocities have different values for large time lags. Typically, coherent spectra are wide and asymmetric for large flow angles and high electron drift velocities. It has been shown with an example of STARE radar observations, that the experimental mean velocities measured using double-pulses, are related to the backscatter spectrum, consistent with theoretical prediction.

Uspensky et al. (2004)[1] reported that on 12 February 1999, around 12:24 UT, the mean velocities observed with a double-pulse technique were increasing for increasing time delay between the pulses. The authors suggested that the observations could be accounted for by assuming some correlation between the signals backscattered from the two pulses at different ranges. It has generally been assumed that there is only correlation between signals backscattered from the same range. The pulse length used in the experiment corresponds to a range resolution of 15 km, and the inter-pulse time is >30 km. Thus, the new assumption is that there may be a correlation between signals backscattered from widely separated regions, >30 km. On the other hand, this report suggests that these observations can be (at least partly) accounted for by the nature, or technique, of the double pulse measurements. It has been demonstrated, that for the observed asymmetric spectrum the calculated double pulse phase variation with time delay (Eq. 7) is in agreement with the observed phase variations. The observed phase variation (and therefore also mean velocities) at small, as well as at large time delays is consistent with the values predicted by calculations using the observed spectrum (Fig. 3). This does not exclude the possibility of some correlation of signals from widely separated regions, but it does indicate that before a final conclusion is made the observations ought to be corrected for the influence of the double pulse technique, and then the correlation-analysis repeated on the corrected data.

The observed non-proportionality of phase and time lag is a consequence of the wide asymmetric spectrum. Since not all spectra observed with STARE are asymmetric, this also accounts for observations of linear phase change, which implies a mean velocity independent of time lag (see Uspensky et al., 2004[1]). The observations during morning hours in the westward electrojet (reported by these authors) may be associated with spectra less skewed or asymmetric than those associated with the observations in the eastward electrojet. This would explain that the observed non-proportionality between phase and time lag would be more pronounced for eastward electrojet observations.

Past analysis of experimental data showed that $<f>$ is typically 15% larger than f_o (private communication, J. A. Waldock). But both $<f>$ and f_o are different from $f_{\max}$ (except for symmetric spectra, where they are all of equal magnitude). So which of these "Doppler velocities" is the velocity relevant as a typical characterization of the spectrum? If we want to characterize the spectrum by a single "Doppler velocity", which one should be used? Fundamentally, this is a question to be answered by the theories of the backscatter process. One may therefore suppose the mean velocity, $<f>$, to be the relevant parameter. There is no reason to expect that just because a parameter value is large, that it is also "better". Increasing the double-pulse time separation increases the mean velocity, but it does not necessarily lead to a more relevant mean velocity. Using a double pulse time lag of 200 μs ensures that the observed mean velocities are useful approximations to the mean Doppler velocity, the first moment of the spectrum. This is therefore a relevant parameter with a well-defined meaning.

There is a further important argument for this choice of time lag. The larger the time lag, the smaller the velocity that can be measured without ambiguity. If the radial motion of the radar target is so fast that there is more than 2π variation in the phase of the second pulse relative to the first, then the phase shift during the time lag becomes ambiguous, and the mean velocity can no longer be determined with accuracy. Since flow velocities in the auroral ionosphere are known to exceed 1500 m/s, a small time lag in the double-pulse measurements is required for accurate observations. In the STARE system we earlier used T=300 μs, which corresponds to an unambiguous maximum observable Doppler velocity of 1667 m/s. Later, software and hardware changes did not allow that time delay. Instead, a value T=200 μs was used. This corresponds to an unambiguous maximum velocity of 2500 m/s.

The deviation from proportionality between phase and time lag increases with asymmetry and width of the spectrum. Wide and narrow spectra can be observed simultaneously from different directions in the same backscatter volume. The spectral width tends to increase with increasing flow angle. The asymmetry tends to increase with increasing electron drift velocity in the plasma. In the eastward electrojet (Nielsen et al., 1984) narrow spectra could be observed for all flow angles, but wide spectra were observed only for large flow angles (>70°). The width of spectra increased rapidly with increasing flow angle and increasing electron drift velocity. The spectrum tends to be asymmetric when the line-of-sight increases above ∼400 m/s. In the westward electrojet (Haldoupis et al., 1984) the average spectral properties are similar to those reported for the eastward electrojet. Thus, wide asymmetric spectra are expected in both the eastward and westward electrojets at large flow angles for large electron drift velocities.

The STARE system operates at ∼140 MHz. But coherent echoes have been observed over a wide frequency range, from 10 (Milan and Lester, 2001) to 3000 MHz (Leadabrand et al., 1967).

A spectrum may be asymmetric, owing to a particular physical process working in the backscatter region. The mean velocities, including the mean Doppler velocity, have then relevance to that physical process. However, an asymmetric spectrum may also be considered a superposition of one or more line spectra, with each line spectrum the result of a particular process or a separate backscatter region (see, for example, Whitehead et al., 1983). If the observed spectrum actually is a superposition of several (two or more) separate peaks, one can of course still derive the mean Doppler velocity, as well as mean velocities for the total spectrum. However, the interpretation becomes even more complicated. In that case it would seem better to deconvolve the spectra and apply an analysis to each separate spectrum. Which approach to use would depend on the actual spectral observations.

It has been outlined in theory how the mean velocity of a coherent radar spectrum is a function of the double-pulse time lag and of the spectrum's uneven moments. For one observed asymmetric spectrum (Fig. 3a) it was demonstrated that the observed phases of the autocorrelation as a function of the time lag were fairly well reproduced by the theory (Fig. 3b). The mean velocity for $T \rightarrow 0$ approaches the mean Doppler velocity, the lower limit of the observed mean velocities (Fig. 3b). In this framework all the mean velocities measured for different time lags are correct, and described analytically by Eq. (11). Since all mean velocities are correct, they can all be used. Which one to use may depend on the problem to which the data are being applied. To discuss that is beyond the scope of this paper.

We have demonstrated that the observed phase versus time lag (Fig. 2) can at least partly be accounted for by the double-pulse technique used in the measurements. One should note that the discussed relation between mean velocities and time lag is a property of the double-pulse technique; it is not bound to any particular radar system, but is valid for any coherent radar using the double-pulse technique.

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