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Toroidal and poloidal Alfvén waves with arbitrary azimuthal wave numbers in a finite pressure plasma in the Earth's magnetosphere

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Abstract. In this paper, in terms of an axisymmetric model of the magnetosphere, we formulate the criteria for which the Alfvén waves in the magnetosphere can be toroidally and poloidally polarized (the disturbed magnetic field vector oscillates azimuthally and radially, respectively). The obvious condition of equality of the wave frequency ω to the toroidal (poloidal) eigenfrequency Ω_{TN} (Ω_{PN}) is a necessary and sufficient one for the toroidal polarization of the mode and only a necessary one for the poloidal mode. In the latter case we must also add to it a significantly stronger condition $|\Omega_{TN}-\Omega_{PN}|/\Omega_{TN} \gg m^{-1}$, where m is the azimuthal wave number, and N is the longitudinal wave number. In cold plasma (the plasma to magnetic pressure ratio $\beta = 0$) the left-hand side of this inequality is too small for the routinely recorded (in the magnetosphere) second harmonic of radially polarized waves, therefore these waves must have non-realistically large values of m. By studying several models of the magnetosphere differing by the level of disturbance, we found that the left-hand part of the poloidality criterion can be satisfied by taking into account finite plasma pressure for the observed values of $m \sim 50-100$ (and in some cases, for even smaller values of the azimuthal wave numbers). When the poloidality condition is satisfied, the existence of two types of radially polarized Alfvén waves is possible. In magnetospheric regions, where the function Ω_{PN} is a monotonic one, the mode is poloidally polarized in a part of its region of localization. It propagates slowly across magnetic shells and changes its polarization from poloidal to toroidal. The other type of radially polarized waves can exist in those regions where this function reaches its extreme values (ring current, plasmapause). These waves are standing waves across magnetic shells, having a poloidal polarization throughout the region of its existence. Waves of this type are likely to be exemplified by giant pulsations. If the poloidality condition is not satisfied, then the mode is toroidally polarized throughout the region of its existence. Furthermore, it has a resonance peak near the magnetic shell, the toroidal eigenfrequency of which equals the frequency of the wave.

Key words. Magnetospheric physics (plasmasphere; MHD waves and instabilities) – Space plasma physics (kinetic and MHD theory)

1 Introduction

A great variety of Alfvén waves has been recorded in the magnetosphere to date. They are usually categorized into short-period (Pc 1-2 and Pi 1) and long-period (Pc 3-5 and Pi 2) oscillations. Of these waves, the former represent waves traveling along field lines, while the latter are standing waves similar to vibrations of guitar strings. Standing waves have small longitudinal wave numbers (i.e. the number of half-waves fitting along a field line between magnetically conjugate points of the ionosphere), $N \sim 1$, while traveling waves represent packets composed of harmonics with $N \gg 1$. Recently, it has been customary to categorize the long-period pulsations into azimuthally large-scale waves (the azimuthal wave number $m \sim 1$) and azimuthally small-scale waves ($m \gg 1$). A physical substantiation for such a categorization is the difference of the sources of these two wave modes: Alfvén oscillations with small m are generally thought of as being generated by a magnetoacoustic wave arriving from the outer boundary of the magnetosphere, and waves with large m by some source inside the magnetosphere (Glassmeier, 1995). Furthermore, long-period hydromagnetic waves in the magnetosphere are classed according to the predominant polarization (Anderson et al., 1990): azimuthally polarized, or toroidal if the magnetic field vector oscillates in an azimuthal direction, radially polarized, or poloidal if the magnetic field vector oscillates in a radial direction, and compressional if there is a significant disturbance of the magnetic field modulus (within the linear approximation, this signifies the presence of a longitudinal component of the wave's magnetic field). The question is how these categorizations are

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correlated, i.e. under what conditions the waves with particular values of m can have a particular polarization.

Usually, this question is given a very simple answer: when $m \sim 1$ the Alfvén wave is predominantly toroidally polarized, and when $m \gg 1$ its polarization is predominantly poloidal. This conclusion is in general agreement with experimental data. A theoretical substantiation for this conclusion is the solution of the MHD equation in dipole geometry in two limiting cases: when $m = 0$ the mode is purely toroidal, and when $m = \infty$, it is purely poloidal (Dungey, 1967; Radoski, 1967). Nevertheless, the large value of the azimuthal wave number cannot be recognized as a sufficient condition for the poloidal polarization of the Alfvén wave. Krylov et al. (1981) showed that both toroidal and poloidal modes can have both low and large m values. For example, at any m in a plasma that is inhomogeneous across magnetic shells, at a certain frequency of the wave there is a surface on which the wave field has a singularity accompanied by the toroidal polarization of the mode (Krylov and Lifshitz, 1984; Wright and Thompson, 1994).

Leonovich and Mazur (1990) noticed one paradox which called into question the very existence of poloidal modes. The paradox is as follows. The eigenfrequency of poloidal oscillations varies across magnetic shells. In order for the mode to be poloidally polarized, it is necessary that the wave frequency ω equals the eigenfrequency of poloidal oscillations. This means that the poloidal mode is concentrated only on the magnetic shell where these frequencies are equal. In this case, however, the radial component of the wave vector must be equal to infinity, as well as the azimuthal component, the role of which is played by the number m. On the other hand, in order for the Alfvén wave to be poloidally polarized, it is necessary that the radial wavelength exceeds significantly the azimuthal one.

To resolve this paradox, Leonovich and Mazur (1990) investigated the wave field structure by assuming that the wave frequency differs little from the poloidal eigenfrequency. They showed that the wave's transverse structure is described by the Airy equation, the solution of which has the form of a wave outside of the poloidal surface. The mode is poloidally polarized if the radial wavelength far exceeds the azimuthal wavelength. This condition is satisfied at sufficiently large values of the azimuthal wave number m. Hence, the poloidal mode does exist, but it is not localized near the only one magnetic shell but is more-or-less widely distributed in space.

This example shows that studying the polarization of the mode necessarily leads to the study of its global structure. Of course, such an investigation is important per se, especially now that the system of four CLUSTER satellites holds much promise for the separation of the spatial and temporal structure of the mode (Glassmeier et al., 2001). When studying the structure of the toroidal and poloidal modes, it is appropriate to take into account the plasma inhomogeneity not only across magnetic shells, but also in the direction along the external magnetic field, and, in addition, the field line curvature and finite plasma pressure, because all of these factors affect the difference between the frequencies of poloidal and toroidal oscillations (Krylov et al., 1981; Walker, 1987). A study of the global structure of the wave was carried out by Leonovich and Mazur (1993), Klimushkin et al. (1995), Kouznetsov and Lotko (1995), Vetoulis and Chen (1996), and Klimushkin (1998a, b). However, the question still is: What are the conditions and magnetospheric regions where Alfvén waves can have particular polarization properties? This question is addressed in the present paper.

This study is based on using an axisymmetric model of the magnetosphere, taking into account all of the above-mentioned factors. Plasma pressure is considered small but finite. The presence of the plasmapause and ring current is taken into account. Our treatment is based on the equations of ideal magnetohydrodynamics, which leads us to exclude storm-time compressional Pc 5 waves from our consideration, as there are grounds to believe that they are mirror modes (Woch et al., 1988), an understanding of which requires to leave the ideal MHD.

This paper is organized as follows. Section 2 provides a system of equations describing MHD waves in plasma of finite but low pressure. In Sect. 3, the frequencies of toroidal and poloidal oscillations are studied analytically and numerically. It is also established in this section that the longitudinal structure of these modes for $N \sim 1$ differs little from each other. Based on this fact, in Section 4 we derive an ordinary differential equation describing the structure of the wave across magnetic shells. This equation is solved in Sect. 5. In Sect. 6, we summarize our knowledge of the conditions of the toroidal and poloidal polarization of Alfvén waves and carry out a comparison with experimental data. The main results of this study are summarized in Sect. 7.

2 Basic equations

First, we introduce the following designations: the capital letters B, P and J stand for the equilibrium values of the magnetic field, pressure and current, the small letters b, p and j denote the wave-associated perturbations of these quantities, ξ is the displacement of plasma from the equilibrium position, ρ is equilibrium plasma density, E is the wave's electric field, and ω is the wave frequency. These quantities are related by the relation

$$\nabla P = (4\pi)^{-1} \boldsymbol{J} \times \boldsymbol{B} \tag{1}$$

(condition of hydromagnetic equilibrium),

$$\boldsymbol{J} = \nabla \times \boldsymbol{B}, \;\; \boldsymbol{j} = \nabla \times \boldsymbol{b} \tag{2}$$

(Ampere law),

$$i\omega \boldsymbol{b} = c\nabla \times \boldsymbol{E} \tag{3}$$

(Maxwell equation),

$$\boldsymbol{E} = -\frac{i\omega}{c} \boldsymbol{\xi} \times \boldsymbol{B} \tag{4}$$

(freezing-in condition). We consider the hydromagnetic waves in those magnetospheric regions where the plasma to magnetic pressure ratio $\beta \equiv 8\pi P/B^2$ is much less than unity. In these regions equilibrium plasma pressure across and along field lines differs no more than by 20% (Lui and Hamilton, 1992; Michelis et al., 1997); therefore, the anisotropy of the pressure tensor can be neglected. The pressure perturbation can then be found using the adiabaticity condition, the linearized form of which is written as

$$p = -\boldsymbol{\xi} \cdot \nabla P - \gamma P \, \nabla \cdot \boldsymbol{\xi} \tag{5}$$

(Kadomtsev, 1963). A linearized equation of small monochromatic oscillations in plasma has the form

$$-\rho\omega^2\boldsymbol{\xi} + \nabla p = \frac{1}{4\pi}\,\boldsymbol{J} \times \boldsymbol{b} + \frac{1}{4\pi}\,\boldsymbol{j} \times \boldsymbol{B}. \tag{6}$$

We now introduce a curvilinear coordinate system $\{x^1, x^2, x^3\}$, in which the field lines play the role of coordinate lines x^3, i.e. such lines, along which the other two coordinates are invariable (recall that the superscripts and subscripts denote counter-variant and covariant coordinates, respectively). In this coordinate system the stream lines are coordinate lines x^2, and surfaces of constant pressure (magnetic shells) are coordinate surfaces $x^1 = const$. This coordinate system is orthogonal if $\boldsymbol{J} \cdot \boldsymbol{B} = 0$ (Salat and Tataronis, 2000). The coordinates x^1 and x^2 have the role of the radial and azimuthal coordinates, and we shall use the McIlwain parameter L and the azimuthal angle φ, respectively, to represent them. The physical length along a field line is expressed in terms of an increase on the corresponding coordinate as $dl_3 = \sqrt{g_3}dx^3$, where g_3 is the component of the metric tensor, and $\sqrt{g_3}$ is the Lamé coefficient. Similarly, $dl_1 = \sqrt{g_1}dx^1$, and $dl_2 = \sqrt{g_2}dx^2$. The determinant of the metric tensor is $g = g_1 g_2 g_3$.

This paper considers the axisymmetric model of the magnetosphere. In this case all perturbed quantities can be specified in the form $\exp(-i\omega t + ik_2x^2)$, where k_2 is the azimuthal component of the wave vector. If the azimuthal angle φ is used as the coordinate x^2, then $k_2 = m$, where m is the azimuthal wave number. The "physical" value of the azimuthal component of the wave vector is $\hat{k}_2 = k_2/\sqrt{g_2}$. Unlike k_2, the value of $\hat{k}_2$ depends on the radial and longitudinal coordinates, because such a dependence is contained in the component of the metric tensor g_2; in the equatorial plane $\hat{k}_2(L, x^3_{eq}) = k_2/L = m/L$ in particular.

An important consequence of Eq. (6) is the smallness of the longitudinal component of the plasma displacement vector when compared with its transverse component when $\beta \ll 1$. Within the approximation of ideal plasma conductivity, the longitudinal component of the wave's electric field is zero, i.e. the electric field is a two-dimensional one; it lies on surfaces orthogonal to field lines. According to the Helmholz theorem (see, for example, Morse and Feshbach, 1953), an arbitrary vector field can split into the sum of the potential and vortical components. By applying this theorem to a two-dimensional field $\boldsymbol{E}$, we put

$$\boldsymbol{E} = -\nabla_\perp \Phi + \nabla_\perp \times \boldsymbol{e}_{||}\Psi, \tag{7}$$

where $\boldsymbol{e}_{||} = \boldsymbol{B}/B$. In a homogeneous plasma, the "potentials" Φ and Ψ describe the electric field of the Alfvén wave and fast magnetosound (FMS), respectively (Klimushkin, 1994; Glassmeier, 1995). Regarding the third MHD mode, slow magnetosound, it can be neglected for plasmas with $\beta \ll 1$. Let all perturbed quantities in Eq. (6) be expressed in terms of the wave's electric field written as Eq. (7). In obtaining the equations relating Φ and Ψ at finite but small pressure, we shall neglect the second and higher degrees of β. By letting the operator $\nabla_\perp$ act on the left-hand and right-hand sides of Eq. (6) (i.e. by taking its divergence from transverse coordinates), in view of Eqs. (1) – (5) we obtain the equation:

$$\hat{L}_A\Phi + \hat{L}_c\Psi = 0. \tag{8}$$

Here, $\hat{L}_A$ is the Alfvén operator defined as

$$\hat{L}_A \equiv -\partial_1\hat{L}_T(\omega)\partial_1 + m^2\hat{L}_P(\omega),$$

where $\hat{L}_T$ is the operator of the toroidal mode,

$$\hat{L}_T(\omega) = \partial_3\frac{g_2}{\sqrt{g}}\partial_3 + \frac{\sqrt{g}}{g_1}\frac{\omega^2}{A^2}$$

(here, $A = B/\sqrt{4\pi\rho}$ is the Alfvén velocity) and $\hat{L}_P$ is the operator of the poloidal mode,

$$\hat{L}_P(\omega) = \partial_3\frac{g_1}{\sqrt{g}}\partial_3 + \frac{\sqrt{g}}{g_2}\left(\frac{\omega^2}{A^2} + \eta\right),$$

where

$$\eta = -\frac{2}{R}\left(\frac{J}{B} + \frac{2}{R}\frac{s^2}{A^2}\right), \tag{9}$$

$1/R$ is the local curvature of a field line, and $s = \sqrt{\gamma P/\rho}$ is the sound velocity. $\hat{L}_c$ in Eq. (8) is the operator describing the FMS influence on the Alfvén mode,

$$\hat{L}_c = im\left(\partial_1\frac{\omega^2}{A^2}\right) + \left[\partial_1\partial_3\frac{g_2}{\sqrt{g}}\partial_3\frac{g_1}{\sqrt{g}}im - im\partial_3\frac{g_1}{\sqrt{g}}\partial_3\frac{g_2}{\sqrt{g}}\partial_1\right] - im\,\eta\,\partial_1 - im\frac{\sqrt{g_3}}{\sqrt{g_2}}\frac{R\eta}{2}\Delta_\perp$$

($\Delta_\perp \equiv \partial_1(g_2/\sqrt{g})\partial_1 - m^2(g_1/\sqrt{g})$ is the transverse Laplacian). This equation describes the Alfvén wave excited by FMS (a phenomenon that is often called the field-line resonance).

The second equation that relates the potentials Φ and Ψ, can be obtained by taking the longitudinal component of the curl of Eq. (6):

$$\hat{L}_F\Psi + \hat{L}_c^+\Phi = 0. \tag{10}$$

Here, $\hat{L}_F$ is the operator of the fast mode equal to

$$\hat{L}_F = -\Delta_\perp \frac{g_3}{\sqrt{g}} \frac{s^2 + A^2}{A^2} \Delta_\perp - \partial_1 \eta \frac{g_2}{\sqrt{g}} \partial_1 - \left[\partial_1 \frac{R\eta}{2} \frac{1}{\sqrt{g_1}} \nabla_\perp - \nabla_\perp \frac{R\eta}{2} \frac{1}{\sqrt{g_1}} \partial_1 \right] + \left[m^2 \frac{g_1}{\sqrt{g}} \hat{L}_T \frac{g_1}{\sqrt{g}} - \partial_1 \frac{g_2}{\sqrt{g}} \hat{L}_P \frac{g_2}{\sqrt{g}} \partial_1 \right].$$

The operator $\hat{L}_c^+$ (Hermitian conjugate to the operator $\hat{L}_c$) describes the back influence of the Alfvén mode on FMS.

In the limiting case of a homogeneous plasma, $\hat{L}_c, \hat{L}_c^+ = 0$, and Eq. (8) becomes

$$(\omega^2 - k_{||}^2 A^2)\Phi = 0.$$

This equation has an nontrivial solution when the dispersion relation of the Alfvén wave holds. It is for this reason that we refer to the potential Φ as a function describing the Alfvén wave field. Equation (10) for homogeneous plasma has the form

$$(\omega^2 - k_{||}^2 A^2 - k_\perp^2 A^2 - k_\perp^2 s^2)\Psi = 0,$$

i.e. the nontrivial solution exists provided that the dispersion relation for FMS in a plasma with $0 < \beta \ll 1$ is satisfied. Thus, the potential Ψ describes the field of the fast magnetosound.

For a further understanding of this system, we invoke the only conceivable method of analytical research, perturbation theory. To do this, assume that the operators $\hat{L}_c$ and $\hat{L}_c^+$ are small when compared with the operators $\hat{L}_A$ and $\hat{L}_F$ (see also Fedorov et al., 1998). This is true if the scale of variation of equilibrium magnetospheric parameters a, to which the operators $\hat{L}_c$ and $\hat{L}_c^+$ are inversely proportional, far exceeds the scales of variation of the functions Φ and Ψ. This assumption looks rather natural, since it is well known that near the Alfvén resonance surface there occurs a singularity of the wave field, and the function Φ changes quite drastically within a very short distance; moreover, the characteristic radial wavelength a/m when $m \gg 1$ is much smaller than the characteristic scale of space plasma inhomogeneity (or, roughly speaking, the size of the magnetosphere).

First, we turn our attention to Eq. (10). Formally, it may be treated as an inhomogeneous differential equation, the general solution of which is the sum of the solution of the homogeneous equation and a partial solution of the inhomogeneous equation. The solution of the homogeneous equation describes the FMS structure without taking into account the interaction with the Alfvén mode. The solution of this equation in cold plasma was addressed in papers of Lee (1996), and Leonovich and Mazur (2000a, b; 2001), who established that at low frequencies the FMS transparent region lies at the edge of the magnetosphere, and as m increases, it is pressed even more strongly to the magnetopause. This solution does not contain any singularities. Obviously, the influence of small pressure implies merely a slight change in the shape of the FMS transparent region. In this paper, however, our concern is primarily with the Alfvén mode, whereas FMS is of our interest only because it is its source. Following perturbation theory, it is the solution of the homogeneous magnetosound equation which should be substituted, to represent Ψ, into the Alfvén Eq. (8). For a qualitative study of the partial solution of the inhomogeneous magnetosound equation, it is worthwhile to note that the region of localization of the Alfvén mode, at large m on the right-hand side of the equation, is dominated by the transverse Laplacian $\Delta_\perp$. It is this operator, however, which defines the longitudinal (compressional) component of the wave's magnetic field; as can be readily ascertained using formulas (3) and (7),

$$b_{||} = \frac{ic}{\omega} \frac{1}{\sqrt{g_1 g_2}} \Delta_\perp \Psi.$$

The potential of the Alfvén mode is not involved in the definition of the longitudinal magnetic field. However, the transverse Laplacian in the preceding formula is expressed in terms of Φ, i.e.

$$b_{||} = \frac{cm}{\omega} \frac{1}{\sqrt{g_1 g_2}} \frac{R\eta}{2} \Phi. \tag{11}$$

Thus, the coupling of FMS with the Alfvén mode in an inhomogeneous magnetic field gives rise to a marked longitudinal component of the magnetic field in the region of localization of the Alfvén wave; see also (Safargaleev and Maltsev, 1986). As far as the electric field and the transverse components of the FMS magnetic field are concerned, they are lost at the background of the corresponding components of the Alfvén wave because, as can be readily demonstrated,

$$\Psi \sim \beta m^{-1} \Phi \ll \Phi.$$

Note that magnetosound must not necessarily be the source of the Alfvén wave. In particular, when $m \gg 1$ this mode can be neglected altogether, i.e. its transparent region is very narrowly localized at the magnetopause (Leonovich and Mazur, 2001). That is why it is usually believed that high-m waves must be excited by a source inside the magnetosphere. Currents in the ionosphere (Leonovich and Mazur, 1993) and currents in the magnetosphere (Saka et al., 1992) can play the role of such a source. We now introduce the function q to describe all possible sources of the Alfvén mode. Equation (8) may then be written as

$$\partial_1 \hat{L}_T(\omega) \partial_1 \Phi - m^2 \hat{L}_P(\omega) \Phi = q. \tag{12}$$

3 Toroidal and poloidal modes

As is evident from the expression (7), when the condition $|\partial_1 \Phi / \sqrt{g_1}| \gg |m\Phi/\sqrt{g_2}|$ is satisfied, the electric field of the Alfvén wave is dominated by the radial component; otherwise the azimuthal component is dominant. On the contrary, in the former case the main contribution to the wave's magnetic field is made by the azimuthal component; in the latter case the radial component makes the main contribution. The wave structure in the former case is determined by the toroidal operator, and by the poloidal operator in the latter case. Let T_N and P_N denote the eigenfunctions of these operators. These functions alone do not describe the global structure of the mode because they are not the solution of Eq. (12), but they have the role of "scaffolding" for solving Eq. (12), as will be described in Sect. 4.

Let Ω_{TN} and Ω_{PN} denote the eigenfrequencies of the toroidal and poloidal operators. The difference between these eigenfrequencies is often referred to as the polarization splitting of the Alfvén oscillation spectrum. These quantities are functions of the radial coordinate. Plasma pressure influences the value of Ω_{TN}, as well as of Ω_{PN}, because the definition of both the toroidal and poloidal operators involves the coefficients of the metric tensor determined by the equilibrium magnetic field, which, in turn, depends on the current, i.e. on the derivative of pressure along the radial coordinate. However, pressure is involved explicitly only in the definition of the operator $\hat{L}_P$ in terms of the quantity η. For that reason, taking it into account has a greater influence on the value of Ω_{PN} compared with Ω_{TN}.

Further, we introduce the notion of the toroidal and poloidal surfaces defined by the equations

$$\Omega_{TN}(x^1) = \omega \tag{13}$$

and

$$\Omega_{PN}(x^1) = \omega. \tag{14}$$

The graphical solution of these equations is illustrated by Fig. 1.

Let us designate the distance between toroidal and poloidal surfaces in the equatorial plane as $\Delta_N(\omega) = x^1_{TN} - x^1_{PN}$. By the order of magnitude, this value is

$$\Delta_N \sim a\frac{\Omega^2_{TN} - \Omega^2_{PN}}{\Omega^2_{TN}} \tag{15}$$

(see Appendix A). The noncoincidence of the toroidal and poloidal surfaces is caused by the polarization splitting of the spectrum, i.e. ultimately, by the field line curvature. Thus the magnetospheric model under study now involves a parameter Δ_N which has no analog either in a homogeneous plasma or in the one-dimensionally inhomogeneous model with straight field lines.

To study the functions T_N and P_N and calculating the frequencies Ω_{TN} and Ω_{PN} and the coordinates of the toroidal and poloidal surfaces, we avail ourselves of the fact that when $\beta \ll 1$ the difference in the magnetosphere from a dipole one can be neglected. We considered three models of magnetospheric plasma: model I corresponds to a low level of magnetospheric disturbance when a significant time has elapsed after the storm; model II corresponds to a high level of disturbance, and model III which also corresponds to a low level of disturbance but when a short time has elapsed after the storm. We approximated the plasma pressure by the expression

$$P = P_0\left[1 - \tanh^2\left(\frac{L_0 - L}{D}\right)\right],$$

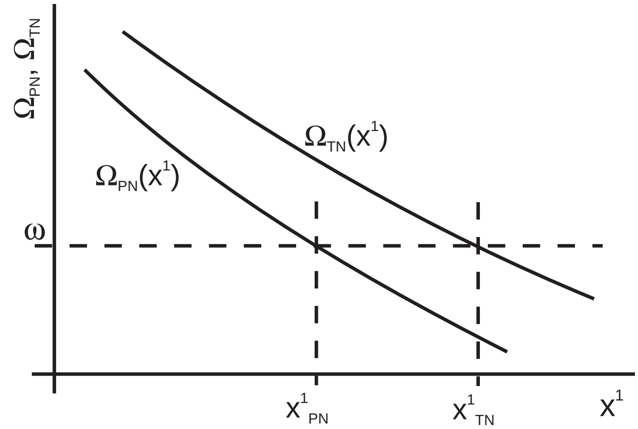

Fig. 1. The graphical solution of Eqs. (13 and 14): determination of the positions of the toroidal and poloidal surfaces in terms of the wave frequency ω and the functions $\Omega_{TN}(x^1)$ and $\Omega_{PN}(x^1)$.

where L_0 is the coordinate of the magnetic shell, on which pressure reaches its maximum value, and D is the parameter that determines the characteristic width of the pressure profile. The coordinate of maximum pressure is taken to be $L_0 = 3.5$ in the three models. We put $D = 2$ in model I and $D = 2$ in model II, in an attempt to reflect the fact that the higher the level of magnetic disturbance, the narrower the localization of the current across the magnetic shells (Sugiura, 1972; Lui et al., 1987; Lui and Hamilton, 1992; Michelis et al., 1997), which is related to pressure by the relation (1). For model III, we took $D = 0.7$, because in this case we take into account the strong current inside of the plasmapause (Williams and Lyons, 1974). It is worth noting that accorging to observational data (Sugiura, 1972) there is no jump of plasma pressure on the plasmapause. Figures 2a and b present the radial profiles of pressure P and the current J for models I, II, and III.

As far as the quantity P_0 is concerned, its specification is equivalent to specifying the parameter β on the shell with the coordinate L_0. In this case we started from the fact that the higher the level of magnetic disturbance, the higher the pressure, in general. Accordingly, in model II the maximum value of beta must be higher than that in model I and III. We chose the following numerical values. We put the value of the plasma parameter on the L-shell of maximum pressure $\beta(L_0) = 0.055$ for models II and III, and $\beta(L_0) = 0.4$ for model I. This parameter is plotted in Fig. 2c. Plots of the value of η, that determine the polarization splitting of the spectrum at finite pressure according to formula (A5) are presented in Fig. 3. Note that these figures plot the values of these quantities in the equatorial plane; β and η decrease rapidly with the distance from it, because, by virtue of the MHD equilibrium condition, plasma pressure is constant along a field line, whereas the magnetic field depends on the geomagnetic latitude of theta as

$$B(\theta) \propto f_B(\theta), \;\; f_B(\theta) \equiv (1 + 3\sin^2\theta)^{1/2}\cos^{-6}\theta.$$

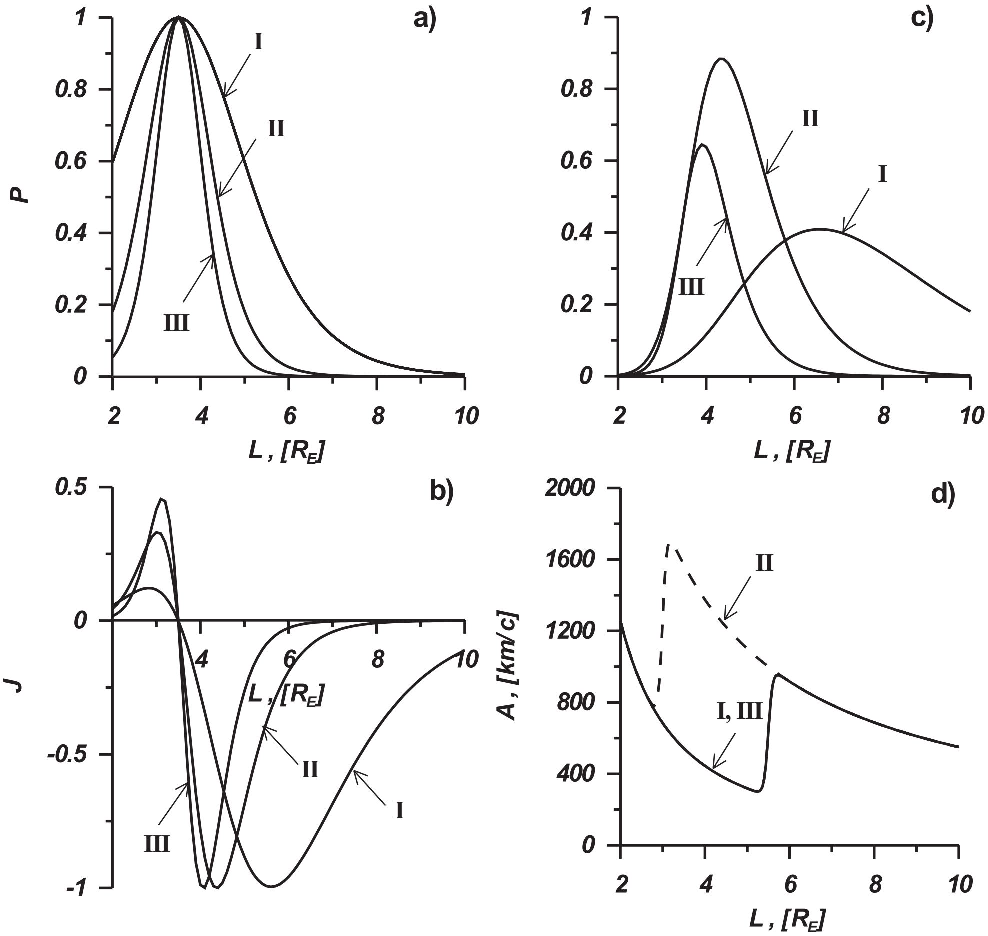

Fig. 2. Profiles of plasma pressure P **(a)**, the equilibrium current J **(b)** the equatorial value of β **(c)**, and the Alfvén velocity A in the equatorial plane **(d)** for models I, II, III. Pressure and current are shown in arbitrary units; they are normalized in terms of the value of β. When $J > 0$, the current flows in the eastward direction, when $J < 0$, the current flows in the westward direction.

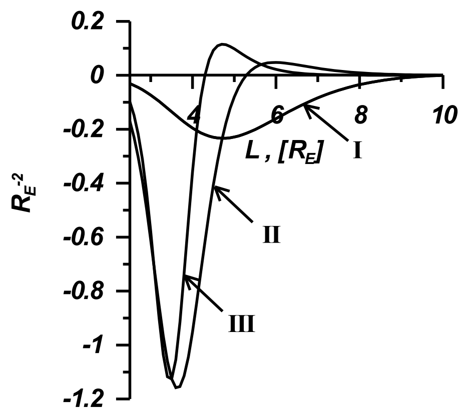

Fig. 3. Plot of the function $\eta(x^1)$ in the equatorial plane for models I, II, III.

Noteworthy is the fact that in some regions of the magnetosphere (especially in model II and III) the condition $\beta_{eq} \ll 1$ is violated. Even in such regions, however, the field line-averaged value of β is small compared to unity. Nevertheless, no calculations were performed for such regions. Note, by the way, that for the polarization splitting of the spectrum (Eq. A5) the value of η (which is proportional to the parameter β) is involved just in the form of an integral along a field line.

The function that approximates the Alfvén velocity profile, with the plasmapause taken into account, was taken from a paper of Leonovich and Mazur (2000b), with minor modifications:

$$A = \left\{ \frac{1}{2} \left[A_1 \left(\frac{L_1}{L} \right)^{c_1} + A_2 \left(\frac{L_2}{L} \right)^{c_2} \right] - \frac{1}{2} A_1 \right.$$
$$\left. \left[\left(\frac{L_1}{L} \right)^{c_1} - A_2 \left(\frac{L_2}{L} \right)^{c_2} \right] \tanh \left(\frac{L - L_{pp}}{D_{pp}} \right) \right\} f_B(\theta),$$

where $A_1 = 250\ km/s$, $A_2 = 500\ km/s$, $L_1 = 2.5$, $L_2 = 5$, $c_1 = 1.5$, $c_2 = 1$. The parameter $D_{pp} = 0.1$ determines the width of the plasmapause, and the quantity L_{pp} determines its coordinate. Since, with increasing magnetospheric disturbance, the magnetopause is displaced toward the Earth (see, for example, Chappell et al., 1970), we put $L_{pp} = 5.5$ for models I and III, and $L_{pp} = 3.0$ for model II. Figure 2d

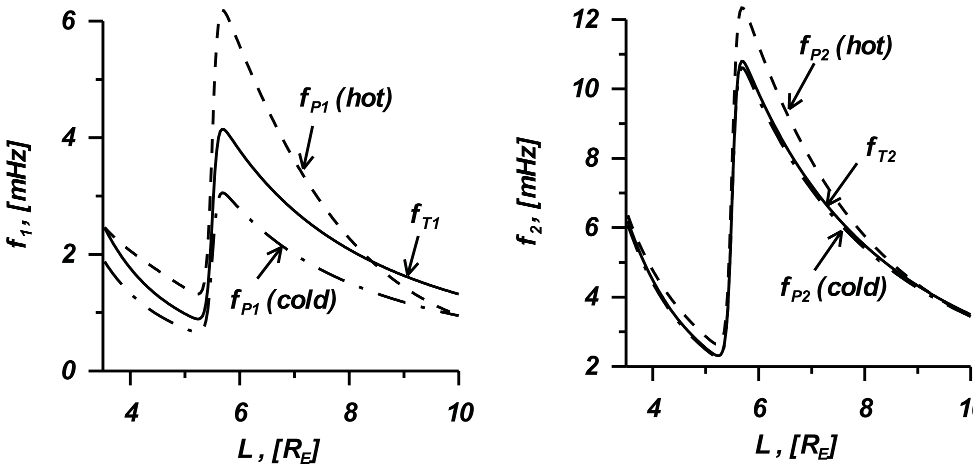

Fig. 4. Toroidal $f_{TN} = \Omega_{TN}(x^1)/2\pi$ and poloidal $f_{PN} = \Omega_{PN}(x^1)/2\pi$ frequencies when $N = 1, 2$ for model I.

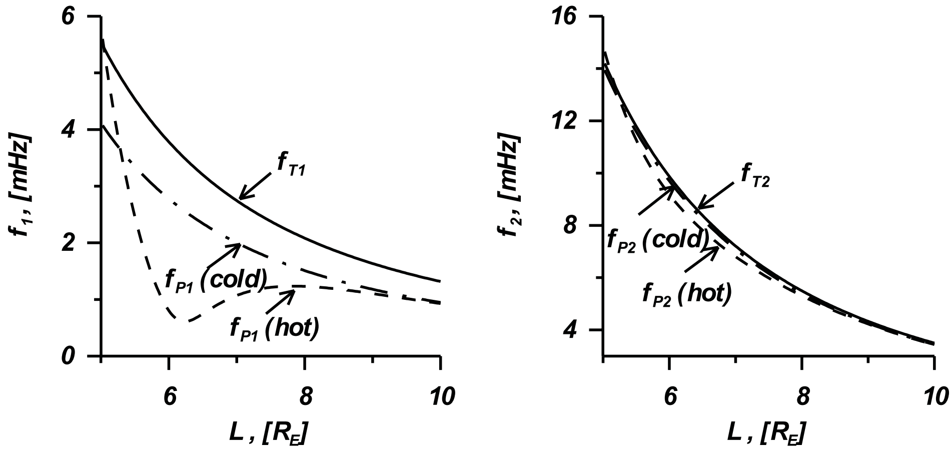

Fig. 5. Toroidal $f_{TN} = \Omega_{TN}(x^1)/2\pi$ and poloidal $f_{PN} = \Omega_{PN}(x^1)/2\pi$ frequencies when $N = 1, 2$ for model II.

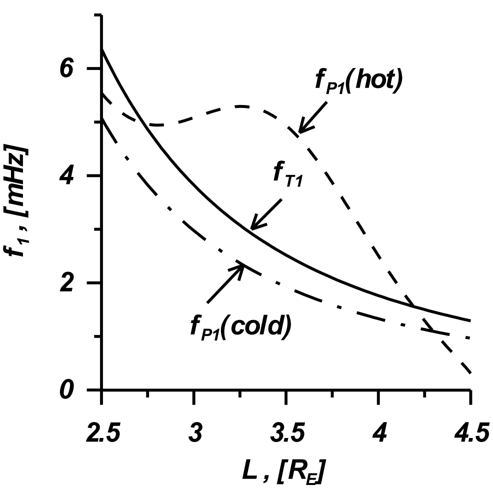

Fig. 6. Toroidal $f_{TN} = \Omega_{TN}(x^1)/2\pi$ and poloidal $f_{PN} = \Omega_{PN}(x^1)/2\pi$ frequencies when $N = 1$ for model III.

presents the radial profile of the Alfvén velocity A for models I, II, III.

It must be added that the models which we have used do not even reach the limit of the whole variety of conditions in the magnetosphere. Even at the same value of the K_p-index, the profiles of equilibrium quantities can be quite different; situations are possible with several plasmapauses, with the maximum of pressure shifted onto the outer L-shells, etc. Numerical values in these formulas can also be open to argument. Nevertheless, these models are, in a sense, extreme ones and permit the value of the poloidal and toroidal eigenfrequencies to be judged qualitatively in some limiting and most interesting cases.

Results of our calculations of the frequencies are shown in Figs. 4–6 (to ease the comparison with observational data, the frequencies $f_{TN,PN} = \Omega_{TN,PN}/2\pi$ are presented); for the purposes of illustration we also give the values of these quantities in a cold plasma for $\beta = 0$. We note once more that for models II and III, we studied only those regions where $\beta \ll 1$, otherwise we violate the limits of applicability of our theory. As is evident, pressure has quite a substantial influence on the value of the poloidal frequency. For the fundamental harmonic ($N = 1$), at positive values of η, the difference of the poloidal and toroidal frequencies is considerably higher than that in a cold plasma, and when $\eta < 0$, pressure leads to a change in the sign of the polarization splitting of the spectrum in some regions of the magnetosphere (as is evident from Figs. 4 and 5; in a cold plasma the poloidal frequency is always less than the toroidal frequency). With an increase in the harmonic number, the polarization splitting of the spectrum decreases, but much more slowly than in the case of zero pressure. For the second and higher harmonics, the curves $\Omega_{TN}(x^1)$ and $\Omega_{PN}(x^1)$ in a cold plasma practically coinside, whereas in the presence of pressure the difference $\Omega_{TN} - \Omega_{PN}$ is quite pronounced, even for $N \geq 2$ (the plots for $N > 2$ for models I and II and for $N > 1$ for model III are not given here, to save room). Notice also the appearance of additional extrema of the function $\Omega_{PN}(x^1)$ in models II and III in regions of strong current which are not accompanied by extrema of the function $\Omega_{TN}(x^1)$; in a cold plasma, the extrema of these two functions occurred at the plasmapause only.

It is also of interest to investigate the functions $\Delta_N(\omega)$ for different N, for different models. Here we confined ourselves only to model I and II (Figs. 7 and 8). Figure 7 shows that in model I, plasma pressure makes the poloidal surface shift to more distance magnetic shells compared to the toroidal surface, whereas in the cold plasma case the poloidal surface is always closer to the Earth than the toroidal surface. Furthermore, when $N = 1$, even in cold plasma, the value of Δ_N is relatively large, so that plasma pressure can contribute to a decrease in the distance between the toroidal and poloidal surfaces (see Fig. 7a, b). But, on the other hand, when $N = 2$ in cold plasma this distance is very small, and pressure contributes greatly to its increase. In model II (see Fig. 8) pressure generally increases the width of the interface between the toroidal and poloidal surfaces, and it shifts the poloidal surface even closer to the Earth than it does in cold plasma (although its behavior may be the opposite for higher frequencies).

Thus, we can draw the following general conclusion: usually, pressure contributes to an increase in the polarization splitting of the spectrum and, hence, to an increase in the distance between the toroidal and poloidal surfaces.

We now turn our attention to the question of the toroidal and poloidal eigenfunctions. Using the WKB approximation in longitudinal coordinate it is possible to find that when $N \gg 1$, even in a cold plasma, these functions differ rather strongly from one another (Leonovich and Mazur, 1993). At small N, the form of the functions $T_N(x^3)$ and $P_N(x^3)$ can only be determined numerically, but it is the waves with small N which manifest themselves in the form of geomagnetic Pc 3-5 pulsations addressed in this paper. Results of our calculations of these functions for $N = 1, 2$ for model I are presented in Fig. 9. It is evident from the plots that for the first two harmonics, the differences between the poloidal and toroidal eigenfunctions are reasonably small. In models II and III, this conclusion remains valid. Based on the fact of the small difference in the functions $T_N(x^3)$ and $P_N(x^3)$ that determine the longitudinal structure of long-period Alfvén waves, in the next section we shall bring the partial differential Eq. (12) to an ordinary differential equation, describing the structure of the wave across magnetic shells.

The question arises as to whether it is possible to extend our results to a more general case where the inequality $\beta \ll 1$ does not hold. Klimushkin (1998a) studied the structure of MHD waves for arbitrary β, but with $m \gg 1$. There exist two modes of MHD oscillations in that limit: the Alfvén mode and the slow magnetosound mode (SMS); in the $m \gg 1$ case fast magnetosound (FMS) can be neglected (whereas at this point we consider arbitrary m, but $\beta \ll 1$, so Alfvén mode and FMS exist, but SMS is unimportant). The coupled Alfvén and SMS modes are described by the system of Eqs. (36) and (37) of the cited reference. A study of this system showed that when $\beta \sim 1$, in addition to the Alfvén resonance surface (toroidal surface), there arises the SMS resonance surface, with which one more poloidal surface is associated. It is easy to show, however, than when the toroidal frequency far exceeds the SMS resonance frequency, the equation, corresponding to the Alfvén, reduces approximately to Eq. (12) of this paper, but with no terms containing Ψ (the absence of these terms is, of course, accounted for by the fact that they are responsible for the FMS which is absent in the limit $m \gg 1$). Further, numerical calculations performed by Cheng et al. (1993) and Lui and Cheng (2001) showed that the SMS resonance frequency is indeed much lower than Ω_{TN} (and the cited authors did not introduce the limitation $\beta \ll 1$). The reason seems to lie in the above-mentioned fact that even if at the equator $\beta_{eq} \sim 1$, at high latitudes beta decreases rapidly, due to the crowding of field lines. Hence, we can conclude that the Eq. (12) describes qualitatively the Alfvén waves, even if $\beta_{eq} \leq 1$.

4 The equation for the Alfvén wave structure across magnetic shells

The toroidal and poloidal modes are two limiting cases of Alfvén waves in the magnetosphere. If their longitudinal structure differs little from one another, then it can be suggested that the longitudinal structure of field line oscillations differs little from the toroidal function in the general case as well. Then Φ may be represented as

$$\Phi = R_N(x^1)T_N(x^1, x^3) + \delta\Phi_N, \tag{16}$$

where $\delta\Phi_N$ is a small correction. Let us assume that the Alfvén wave is sufficiently narrowly localized across magnetic shells, and the regions of localization of different N-harmonics do not cross each other. Since the characteristic scale of variation of the function T_N across magnetic shells coincides by the order of magnitude with the scale of variation of the equilibrium parameters a (roughly speaking, with

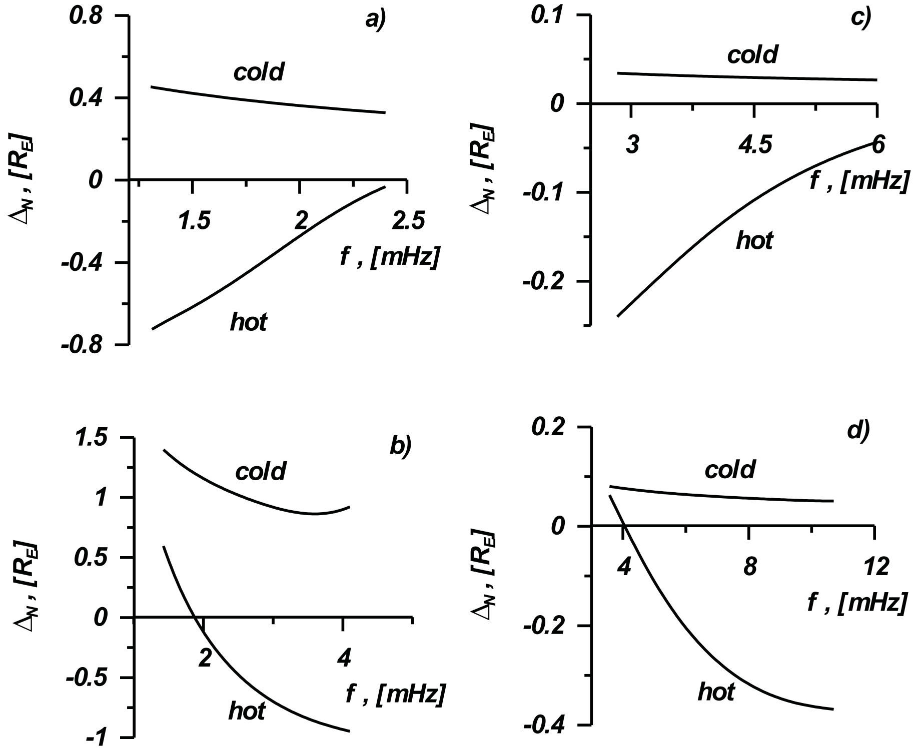

Fig. 7. Distance between the toroidal and poloidal surfaces $\Delta_N(\omega)$ for model I: **(a)** inside the plasmasphere when $N = 1$, **(b)** outside the plasmasphere when $N = 1$, **(c)** inside the plasmasphere when $N = 1$, and **(d)** outside the plasmasphere when $N = 2$.

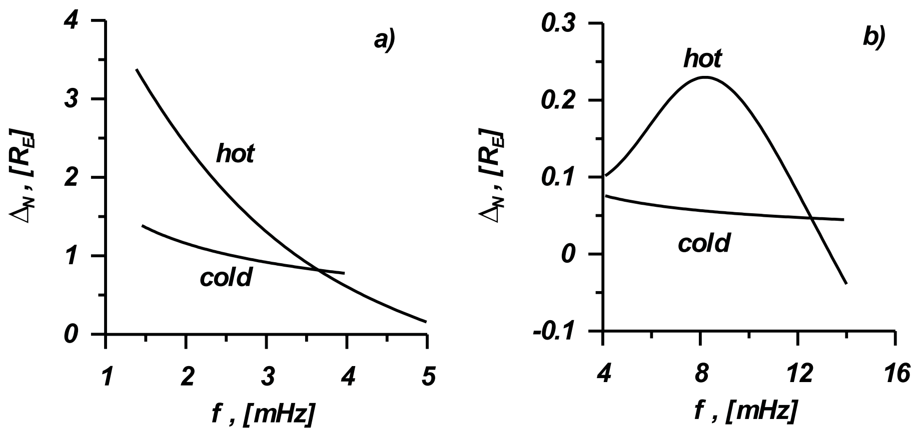

Fig. 8. Distance between the toroidal and poloidal surfaces $\Delta_N(\omega)$ for model II outside the plasmasphere: **(a)** when $N = 1$, and **(b)** when $N = 2$.

the size of the magnetosphere), we can formulate a limitation on the function R_N:

$$\left|\frac{1}{R_N}\partial_1 R_N(x^1)\right| \gg \left|\frac{1}{T_N}\partial_1 T_N(x^1, x^3)\right|. \tag{17}$$

To determine the radial structure of the wave specified by the function R_N, we use the method of successive approximations by treating the deviation of the function Φ from the toroidal function as a small perturbation. We substitute Eq. (16) into Eq. (12), multiply the resulting expression by T_N and integrate along the field line from the point x^3_- to x^3_+ of the intersection of a field line with the ionosphere; in doing this, we neglect small terms:

$$\partial_1(\omega^2 - \Omega^2_{TN})\partial_1 R_N - m^2 R_N \left\langle T_N \hat{L}_P(\omega) T_N \right\rangle = \langle T_N q \rangle. \tag{18}$$

The derivation of this equation was based on using the normalization conditions for the function T_N (Eq. A2) and the Hermitian character of the operator $\hat{L}_T$.

We transform the second term on the left-hand side of Eq. (18) by making use of the Hermitian nature of the operator $\hat{L}_P$ and introducing a difference between the

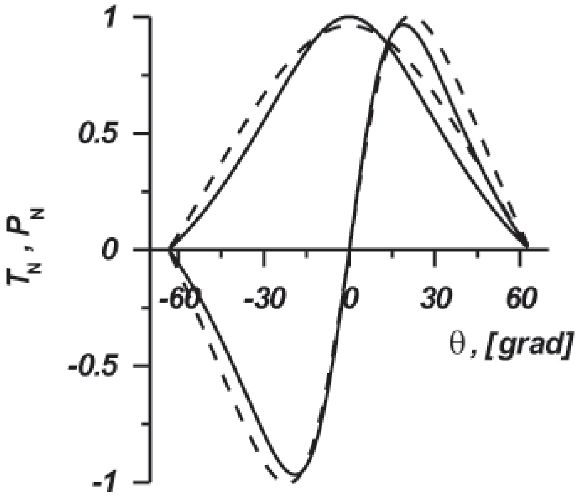

Fig. 9. Toroidal T_N and poloidal P_N eigenfunctions when $N=1, 2$ (model I, $L=6$).

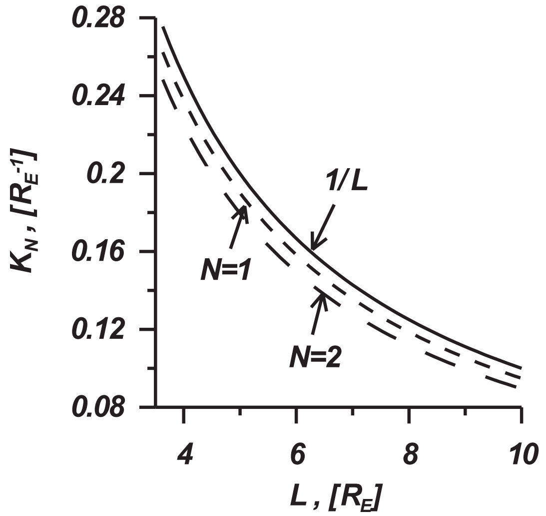

Fig. 10. Plot of the function K_N, an analogy of the physical value of the wave vector azimuthal component, when $m=1$, and $N=1, 2$. For comparison, the plot of the function $1/L$ is shown.

toroidal and poloidal eigenfunctions $\phi_N = P_N - T_N$:

$$\left\langle T_N \hat{L}_P(\omega) T_N \right\rangle = \left\langle \frac{\sqrt{g}}{g_2} \frac{T_N^2}{A^2} \right\rangle (\omega^2 - \Omega_{PN}^2) + \left\langle \phi_N \hat{L}_P(\Omega_{PN}) \phi_N \right\rangle .$$

Since the value of ϕ_N is assumed small, the second term on the right-hand side can be neglected. As a result, we obtain an ordinary differential equation describing the radial structure of the wave:

$$\partial_1(\omega^2 - \Omega_{TN}^2)\partial_1 R_N - K_N^2(\omega^2 - \Omega_{PN}^2) R_N = q_N . \qquad (19)$$

Here the following abbreviations are used:

$$K_N^2 = m^2 \left\langle \frac{\sqrt{g}}{g_2} \frac{T_N^2}{A^2} \right\rangle ,$$

$$q_N = \langle T_N q \rangle .$$

Using numerical calculations it was established that the value of K_N coincides by the order of magnitude with the azimuthal component of the wave vector in the equatorial plane m/L (Fig. 10).

In publications on MHD waves in a two-dimensionally inhomogeneous magnetosphere, Eq. (19) was, for the first time, reported by Leonovich and Mazur (1997), who also solved it numerically. An important difference in our article from that paper is the fact that we obtained this equation for plasma with finite pressure.

It remains to add a boundary condition for this equation. A natural boundary condition with respect to the radial coordinate is the absence of any increase in the potential when $x^1 \to \infty$:

$$|R_N\,(x^1 \to \pm\infty)| < \infty . \qquad (20)$$

5 Alfvén waves with the toroidal and poloidal polarization in different regions of the magnetosphere

At this point we introduce the quantity $\nu_N \equiv K_N \Delta_N$, the number of azimuthal wavelengths fitting into the transparent regions. Since the estimation (Eq. 15) holds and, by the order of magnitude, $K_N \sim m/a$, one has

$$\nu_N \sim m \, \frac{\Omega_{TN}^2 - \Omega_{PN}^2}{\Omega_{TN}^2} .$$

There are two possible limiting cases: $\nu_N \ll 1$, and $\nu_N \gg 1$, which will be considered in Sects. 5.1 and 5.2. Section 5.3 addresses the waves in those regions where the function $\Omega_{PN}(x^1)$ reaches its extreme values.

5.1 Case $\nu_N \ll 1$: localized toroidal modes

Within the $\nu_N \ll 1$ approximation, the differences between the toroidal and poloidal surfaces can be neglected. This means that within this approximation the field line curvature is unimportant, and the wave structure qualitatively coincides with the wave field described in earlier publications on field-line resonance (Tamao, 1965; Southwood, 1974; Chen and Hasegawa, 1974). Since in most of the magnetosphere the functions $\Omega_{TN}^2(x^1)$ and $\Omega_{PN}^2(x^1)$ are monotonically decreasing ones, we can avail ourselves of the linear expansion (Eq. A6). Equation (19) then becomes

$$\partial_1(x^1 - x_{TN}^1)\partial_1 R_N - K_N^2(x^1 - x_{TN}^1) R_N = q_N \omega^{-2} a \qquad (21)$$

(cf. Tataronis and Grossman, 1973). Note that x_{TN}^1 is a function of the wave frequency ω. Next, we introduce a new variable $\zeta = K_N(x^1 - x_{TN}^1)$. The solution to Eq. (21) bounded in the radial coordinate, according to Eq. (20), is then written in terms of the modified Bessel functions $I_0(\zeta)$ and $K_0(\zeta)$:

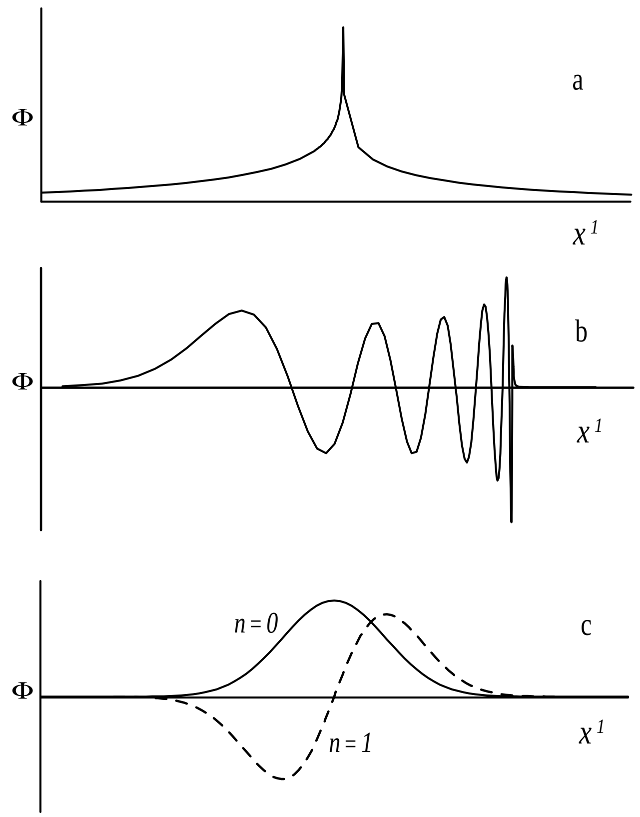

Fig. 11. Three kinds of wave structure across magnetic shells: localized resonance **(a)**, traveling wave **(b)**, and standing wave in the resonator **(c)**. The relation between the Alfvén wave "potential" Φ, shown in the figure, and the function R_N used in the text is given by formula (16).

$$R_N = q_N K_N^{-1} \omega^{-2} a \cdot \begin{cases} \left[K_0(\zeta)\left(i - \int\limits_0^{\zeta} I_0(\zeta')d\zeta' \right) - I_0(\zeta) \int\limits_{\zeta}^{\infty} K_0(\zeta')d\zeta' \right] & \text{when } x^1 > x^1_{TN}, \\ \left[K_0(-\zeta)\left(i - \int\limits_0^{\zeta} I_0(-\zeta')d\zeta' \right) + I_0(-\zeta) \int\limits_{-\infty}^{\zeta} K_0(-\zeta')d\zeta' \right] & \text{when } x^1 < x^1_{TN}. \end{cases} \tag{22}$$

The relation between R_N and the "potential" of the Alfvén wave is given by the formula (16), where the function T_N (it will be recalled) depends relatively slowly on the radial coordinate. In Fig. 11a, we show the transverse structure of the wave field described by the solution (22).

On the toroidal surface this solution has a logarithmic singularity,

$$R_N \sim \frac{q_N\, a}{K_N \omega} \ln\left[\omega^2 - \Omega^2_{TN}(x^1)\right], \tag{23}$$

which, since classical publications of Chen and Hasegawa (1974) and Southwood (1974), has been regarded as the distinctive property of Alfvén resonance. The singularity can be regularized by taking into account the presence of finite conductivity of the ionosphere, in view of which the boundary condition on the function $\delta\Phi_N$ is formulated thus:

$$\delta\Phi_N|_{x^3_\pm} = \mp \left(i \frac{c^2 \cos\chi}{4\pi\, \Sigma_p}\, R_N\, \mathbf{e}_{||} \cdot \nabla T_N \right)_{x^3_\pm}, \tag{24}$$

where χ is the angle between the field line and a normal to the ionosphere, and Σ_p is global Pedersen ionospheric conductivity (Leonovich and Mazur, 1993). Then in Eq. (19) there appears an additional term

$$\left\langle T_N \hat{L}_T(\Omega_{TN}) \partial_1^2 \delta\Phi_N \right\rangle = 2i\gamma_N \omega \equiv i \sum_{j=x^3_\pm} \left[\frac{g_2}{\sqrt{g}} \frac{c^2 \cos\chi}{4\pi\, \Sigma_p} (\partial_3 T_N)^2 \right]_j,$$

where γ_N is the mode damping decrement at the ionosphere (its value is assumed small compared to the wave frequency, which reflects high ionospheric conductivity). This term vanishes in the case of infinite ionospheric conductivity. This gives rise to a small imaginary addition to x^1 in formula (21), $\mathrm{Im}\, x^1 = \epsilon_N \equiv 2\gamma_N a/\omega$ (since $\gamma_N/\omega \ll 1$, then $\epsilon_N/a \ll 1$). In view of this correction when $x^1 \simeq x^1_{TN}(\omega)$, the solution behaves as $\Phi_N \propto \ln[x^1 + i\epsilon_N - x^1_{TN}(\omega)]$. Hence, it follows that on the toroidal surface (that is, on a magnetic surface, where toroidal eigenfrequency is equal to the wave frequency) there occurs a sharp wave amplitude peak, the characteristic scale of localization of which $\epsilon_N/a \ll 1$. And, on the contrary, at a given magnetic shell the wave has a maximum amplitude in the case where the toroidal eigenfrequency at it coincides with the wave's frequency. As is apparent from Fig. 11a, the wave, when $\nu \ll 1$, may be described as a localized resonance, having a toroidal polarization throughout the region of its existence.

Notice that the mode can be toroidal even when $m \gg 1$, provided only that the inequality $\nu_N \ll 1$ holds. An example of this is just the magnetospheric model with straight parallel field lines where the polarization splitting of the spectrum is absent altogether, i.e. $\nu_N = 0$ for any azimuthal wavelengths. Thus, a large value of the azimuthal wave number is not a sufficient condition of the poloidal polarization of the Alfvén wave.

Another feature of this solution is the change in the wave phase by 180°, i.e. the change in sign of the ratio E_2/E_1 at the crossing of the toroidal surface. This is obvious from the fact that for the Alfvén wave we have $\partial_2 E_1 - \partial_1 E_2 = 0$, whence it follows that (Southwood, 1974)

$$\frac{E_1}{E_2} = \frac{1}{im} \frac{\partial_1 E_2}{E_2}. \tag{25}$$

In this case, $E_1 \propto (x^1 - x^1_{TN})^{-1}$, i.e. when $x^1 > x^1_{TN}(\omega)$ and $x^1 < x^1_{TN}(\omega)$ the logarithmic derivative of the function $E_1(x^1)$ has a different sign. We shall return to the relation (25) in Sect. 3.

5.2 Case $\nu_N \gg 1$: poloidal modes that transform into toroidal ones

To solve Eq. (21) for $\nu_N \gg 1$, we can avail ourselves of the method of matching asymptotic expansions. For the time being, we consider the situation where the toroidal frequency is larger than the poloidal frequency. In this case $x^1_{TN} > x^1_{PN}$. The magnetospheric regions in which this inequality is realized in models I–III is evident from Figs. 4–8. The details of the calculations are given in Appendix B, and here we restrict ourselves to the final answer only.

In the region $|x^1 - x^1_{TN}| \ll \Delta_N$ the solution is

$$R_N = C_T \cdot K_0\left(2\sqrt{\frac{x^1 - x^1_{TN}}{\lambda_{TN}}}\right), \tag{26}$$

where

$$\lambda_{TN} = \Delta_N \nu_N^{-2}$$

is the characteristic wavelength near the toroidal surface and C_T is a constant defined by Eq. (B3). In the region $|x^1 - x^1_{PN}| \ll \Delta_N$ the solution can be written in the integral form (Leonovich and Mazur, 1993):

$$R_N(z_P) = iq_N K_N^2 \Delta_N \int_0^\infty dt \exp\left(it\,\frac{x^1 - x^1_{TN}}{\lambda_{PN}} - \frac{it^3}{3}\right), \tag{27}$$

where

$$\lambda_{PN} = \Delta_N \nu_N^{-2/3} \tag{28}$$

is the characteristic wavelength near the poloidal surface. In the region $x^1_{PN} < x^1 < x^1_{TN}$, where the WKB approximation is applicable, the solution is

$$R_N = C_W\left[K_N^2(x^1 - x^1_{PN})(x^1_{TN} - x^1)\right]^{-1/4} \exp i \int_{x^1_{PN}}^{x^1} k_1(x^{1\prime})dx^{1\prime}, \tag{29}$$

where C_W is a constant defined by Eq. (B4) and

$$k_1^2 = K_N^2\,\frac{\omega^2 - \Omega^2_{PN}(x^1)}{\Omega^2_{TN}(x^1) - \omega^2} \tag{30}$$

is a radial component of the wave vector squared. The function $\Phi = T_N R_N$ determined by Eqs. (26)–(29) is plotted in Fig. 11b. We emphasize once again that the functions T_N and P_N introduced in Sect. 3 and used in many other publications do not describe on their own accord the wave structure in the magnetosphere, as they are not the solutions of the wave Eq. (12).

Let us discuss the main features of this solution. As in the case $\nu_N \ll 1$, the wave field in the case $\nu_N \gg 1$ has a logarithmic singularity on the surface $x^1 = x^1_{TN}$

$$R_N \sim \frac{2q_N\Delta_N a}{\nu_N^2\omega^2}\left(\frac{\lambda_{PN}}{\lambda_{TN}}\right)^{1/4} \ln\left[\omega^2 - \Omega^2_{TN}(x^1)\right], \tag{31}$$

which is also regularized by taking into account the finite ionospheric conductivity. However, the term before the logarithm differs from the one in the case $\nu_N \ll 1$ (cf. Eq. (23)). Besides, in that case the function $\Phi(x^1)$ was a monotonic one on both sides of the resonance surface (see Fig. 11a), whereas in the case $\nu_N \gg 1$ this function is an oscillating one in the interface between the surfaces x^1_{TN} and x^1_{PN}, including in the region of toroidal polarization of the mode, as is clearly seen from the asymptotic representation (Eq. (B1)) given in Appendix B, as well as from Eq. (30).

As is evident, k_1 is a function of the wave frequency ω, i.e. the field line curvature also leads, along with the polarization splitting of the spectrum, to the appearance of the Alfvén wave dispersion across magnetic shells. The wave's transparent region (i.e. the region where $k_1^2 > 0$) lies between the toroidal and poloidal points. This solution describes the wave, the phase velocity of which is directed from the poloidal to the toroidal surface. The wave's group velocity is determined from the relation

$$v^1_{gN} = \left(\frac{\partial k_1}{\partial \omega}\right)^{-1} = \frac{(\omega^2 - \Omega^2_{PN})^{1/2}(\Omega^2_{TN} - \omega^2)^{3/2}}{2\omega K_N(\Omega^2_{TN} - \Omega^2_{PN})}. \tag{32}$$

As is apparent, $v^1_{gN} > 0$, i.e. the wave energy is also transported from the poloidal to the toroidal surface. By the order of magnitude, the group velocity

$$v^1_{gN} \sim A\left(\frac{\Delta_N}{a}\right)^2 \nu_N^{-2},$$

i.e. it is much less than the Alfvén velocity. On the poloidal and toroidal surfaces the group velocity becomes zero.

If the poloidal surface is farther away from the Earth than the toroidal surface, then the solution coincides qualitatively with the solution for $x^1_{TN} > x^1_{PN}$. But there is one difference: the phase velocity of the wave is directed from the toroidal to poloidal surface. Nevertheless, energy is transferred, as before, from the poloidal to the toroidal surface. This is evident from the fact that when $\Omega_{TN} < \Omega_{PN}$, the group velocity is negative.

Thus, we arrive at the following picture. The wave is generated near the poloidal surface and propagates toward the toroidal surface where it is totally attenuated, transferring its energy to the ionosphere due to its finite conductivity. Furthermore, the wave is a standing wave along field lines. As the wave is propagating, the radial wavelength decreases and its polarization changes from poloidal to toroidal. We can call this phenomenon the transformation of the poloidal mode to a toroidal mode. Leonovich and Mazur (1993) were the first to establish this picture for the case of a cold plasma ($\beta = 0$). The propagation of Alfvén waves across the L-shells in a finite-β plasma was studied by Safargaleev and Maltsev (1986), Kouznetsov and Lotko (1995), and Klimushkin (1997, 1998a). Besides, Klimushkin et al. (1995) explored the transverse propagation within the approximation $\beta = 0$, but with the three-dimensional inhomogeneity of the magnetosphere taken into account.

When $\nu_N \gg 1$ the mode is confined between the poloidal and toroidal surfaces, i.e. its scale of localization is determined by the field line curvature. This contrasts with the case $\nu_N \ll 1$ when the scale of localization of the wave is determined by the mode dissipation from the ionosphere. It is of interest to consider the situation where $\nu_N \gg 1$ but $\Delta_N \ll \epsilon_N$, i.e. the scales of localization that are determined by the curvature and attenuation, compete with each other. It is easy to see that in this case the attenuation at the ionosphere is so strong that while propagating across field lines, the mode is now dissipated within a small distance from the poloidal surface without reaching the toroidal surface (Klimushkin, 2000).

5.3 Waves with $m \gg 1$ in the range of extreme values of the function $\Omega_{PN}(x^1)$: localized poloidal modes

In some magnetospheric regions the mode is bounded on either side by poloidal surfaces. They are magnetic shells near minima of the function $\Omega_{PN}(x^1)$, if $\Omega_{PN} < \Omega_{TN}$ holds there, and regions near maxima of this function, if an inverse inequality holds there (see Figs. 4–6). The cavity between two poloidal surfaces will be henceforth referred to as the Alfvén resonator. At zero pressure the resonator can lie on the inner plasmapause edge only. Finite pressure in models I and II leads to the elimination of the resonator, because the poloidal frequency becomes larger than the toroidal frequency; instead, there arises a resonator on the outer edge of the plasmapause. In model II, the resonator is produced in the westward current region. In model III, the resonator arises inside of the plasmasphere in the eastward current region; the westward current in model III that was accidentally coincident with the plasmapause led to a deepening of the resonator on the inner edge of the plasmapause (the situation is even possible where $\Omega_{PN}^2 < 0$ in this model, and this will be discussed below). Note that the appearance of cavities in the region of currents requires a rather rigorous selection of equilibrium conditions, unlike the cavities in the plasmapause region.

We now derive the equation describing the radial structure of the mode within the resonator near the extremum of the function $\Omega_{PN}(x^1)$; we designate this value by Ω_0. For definiteness, we consider the resonator on the outer edge of the plasmapause where the following representation can be used:

$$\Omega_{PN}^2(x^1) = \Omega_0^2 \left[1 - \left(\frac{x^1}{l} \right)^2 \right], \tag{33}$$

where the quantity l defines the characteristic width of the resonator, and the coordinate x^1 is measured from the point of extremum. The coordinates of the poloidal surfaces that bound the mode within the resonator are

$$b = \pm l \left(\frac{\omega^2 - \Omega_0^2}{\Omega_0^2} \right)^{1/2}.$$

When $x^1 \simeq 0$ the toroidal frequency can be considered approximately constant if $\Omega_0^2 - \omega^2 \ll \Omega_0^2 - \Omega_{TN}^2$. We introduce a new variable $\xi = x^1/\lambda_{RN}$, where

$$\lambda_{RN} = \left(\frac{l}{K_N} \right)^{1/2} \left(\frac{\Omega_0^2 - \Omega_{TN}^2}{\Omega_0^2} \right)^{1/4}. \tag{34}$$

Equation (19) then becomes

$$\frac{d^2 R_N}{d\xi^2} + (\sigma - \xi^2) R_N = \frac{q_N \lambda_{RN}}{\Omega_0^2 - \Omega_{TN}^2}, \tag{35}$$

where the designation $\sigma = b^2/\lambda_{RN}^2$ is introduced. It is an easy matter to show that this equation defines the structure of the mode within the resonator in the general case, and not only on the outer edge of the plasmapause.

In contrast to the situations considered in two previous subsections, this equation has the solution that satisfies the boundary condition – Eq. (20) – even without a source, $q_N = 0$. In this case Eq. (35) has the same form as one of the best known equations of physics, the Schrödinger equation for the harmonic oscillator. As is known, the existance of the solution requires that the parameter σ be quantized, $\sigma = 2n + 1$, where $n = 0, 1, 2, \ldots$ is an integer number. From this follows the quantization condition for the wave frequency:

$$\omega^2 = \omega_n^2 \equiv \Omega_0^2 \mp \Omega_0^2 \frac{\lambda_{RN}^2}{l^2}(2n+1). \tag{36}$$

Here the "−" sign refers to the case where the resonator is localized near a maximum of the function $\Omega_{PN}(x^1)$, and the "+" sign corresponds to the opposite case. The solution of Eq. (35) is expressed in terms of Hermitian polynomials H_n:

$$R_N = const \cdot H_n(\xi)\, e^{-\xi^2/2}. \tag{37}$$

This solution describes the standing wave confined within the transparent region between poloidal surfaces (Fig. 11c).

When the right-hand side is nonzero, $q_N \neq 0$, Eq. (35) has the solution bounded when $x^1 \to \pm\infty$, at any frequency. However, the amplitude of the solution of the inhomogeneous equation is still maximal when $\omega \simeq \omega_n$, and under this condition function Eq. (37) is an approximate solution of Eq. (35). We do not give here any mathematical details, as they may be found in, for example, a paper of Leonovich and Mazur (1995).

Since H_0, when $n = 0$ the wave equation is described by the Gaussian function with the half-width b. It is a very important result, because in many observed cases of poloidal pulsations the amplitude is indeed close to a Gaussian (e.g. Chisham et al., 1997; Cramm et al., 2000). Note that this result is the solution if the wave equation and is not a consequence of any assumptions of the initial conditions. The derivative of the function $E_2(x^1) = -im\Phi$ on different slopes of the Gaussian has a different sign; therefore, in accordance with formula (25), the transition through the region of localization of the mode must be accompanied by a change in the wave phase by 180°. This phenomenon has already been pointed out by considering an example of the localized resonance (Sect. 5.1). In this case, however, the mode can not be toroidally polarized. Indeed, it is an easy matter to check that the inequality $|E_1/\sqrt{g_1}| \gg |E_2/\sqrt{g_2}|$ has as a consequence the inequality $\omega_0^2 - \omega_1^2 \gg \Omega_0^2 - \Omega_{TN}^2$, indicating that the resonator is so shallow that no harmonic is accommodated in it. On the contrary, the mode in the resonator is poloidal if the resonator is deep enough. The question of the conditions of the poloidal and toroidal polarization of the wave will be discussed in greater detail in the next section.

In the plasmapause region the situation is also possible where the transparent region is bounded on either side by two toroidal surfaces (see, for example, Fig. 4b). This may produce the impression that the solution of the wave equations in this case describes a double resonance when two maxima of the amplitude lying at $x^1 = x^1_{TN}$ are interconnected by a continuous transparent region. However, such a solution does not satisfy the natural boundary conditions of the decrease in the opaque region. Indeed, as has been pointed out in the preceding subsection, the solution bounded by the opaque region which describes a resonance singularity, has the form of a wave arriving at the singular turning point. But there cannot be a wave arriving at two turning points simultaneously. In fact, the solution in this region does not contain any resonance singularities and, in essence, describes the noise background of hydromagnetic oscillations of the magnetosphere (Klimushkin, 1998b).

Previous studies of the resonator in the plasmapause region were carried out by (Leonovich and Mazur, 1990, 1995; Vetoulis and Chen, 1996; Klimushkin, 1998b; Denton and Vetoulis, 1998). The possibility of existence of the resonator on the current inside the plasmasphere was showed by Klimushkin (1998b).

6 Discussion

6.1 The conditions of the poloidal and toroidal polarization of Alfvén waves

The poloidality condition of the Alfvén mode in a general form implies that the radial wavelength λ_r far exceeds the azimuthal wavelength λ_a. For the toroidal polarization, an inverse inequality, $\lambda_r \ll \lambda_a$, must hold. This may produce the impression that the toroidal and poloidal polarizations are equivalent. This is in fact not the case.

If somewhere in the magnetosphere the equality (Eq. 13) holds, i.e. at some wave frequency there is a toroidal surface, then on this magnetic shell there is a wave field singularity, in the area of which the mode has a toroidal polarization. But the existence of Eq. (14) is only necessary but not sufficient for the poloidal polarization of the mode. As an example, we consider the case of the wave traveling from the poloidal the to toroidal surface. In this case the radial wavelength near the poloidal surface is given by formula (28). By the order of magnitude, $\lambda_a \sim a/m$, we avail ourselves of the estimation (Eq. 15) to obtain the poloidality condition of the mode $\lambda_r \gg \lambda_a$ in the form

$$\frac{|\Omega_{TN}^2 - \Omega_{PN}^2|}{\Omega_{TN}^2} \gg \frac{1}{m}, \tag{38}$$

which coincides with the condition of applicability of the WKB approximation in the radial coordinate ($\nu_N \gg 1$). If this approximation is applicable and if there exists the solution of Eq. (14), the Alfvén wave must have a poloidal polarization in a part of its transparent region, near the poloidal surface. If, however, the inequality does not hold, then even near the surface x^1_{PN} the mode is not poloidal. Hence, more stringent conditions are required for the poloidal polarization of the wave than for the toroidal polarization.

This gives us a clue to an understanding of the situation when $\nu_N \sim 1$, where it is impossible to develop approximate methods for solving Eq. (19). In this case there also occurs an Alfvén resonance accompanied by the toroidal polarization of the mode, and since the poloidality condition does not hold anywhere, the mode in the region of its existence has predominantly a toroidal polarization with some addition of the poloidal component in some places where the wave amplitude is substantially smaller. This is also confirmed by numerical calculations performed by Leonovich and Mazur (1997). Thus, we can conclude that when $\nu_N \leq 1$ the mode has predominantly a toroidal polarization throughout the region of its existence.

Generally, plasma pressure contributed to the poloidal polarization of the mode, as it leads to an increase in the polarization splitting of the spectrum and to an increase in the width of the transparent region. Moreover, in the case of finite pressure in some regions of the magnetosphere the poloidality condition can be satisfied, even for m values ($m \simeq 10$, say), that are not very large. Alfvén waves with such m values still can be generated through the interaction with FMS, i.e. the resonance excitation of Alfvén oscillations by magnetosound can also give rise to poloidally polarized waves. Such a possibility was, for the first time, pointed out by Kouznetsov and Lotko (1995), who considered the possibility that the poloidal surface can lie between the toroidal surface and the transparent region of FMS (they called the wave propagating across magnetic shells as the "Alfvén buoyancy wave"). But the widest transparent region accommodating even low-m waves is produced in the case where plasma pressure causes the poloidal surface to be displaced significantly toward the Earth. Just such a situation arises in model III when $N = 1$ (see Fig. 5). The width of the transparent region in this case can reach several terrestrial radii (see Fig. 8). Note, by the way, that in the case of very wide transparent regions, our results should be regarded with caution, because when deriving Eq. (19), it was assumed that the transparent region was significantly narrower than the magnetosphere. Leonovich and Mazur's (1993) two-dimensional WKB approximation is more suited for investigating wide transparent regions.

In the case where the mode is confined within the resonator, the general poloidality condition $\lambda_r \gg \lambda_a$ is transformed in a somewhat different manner: $\lambda_{RN} \gg m/L$, where the characteristic wavelength in the resonator, λ_{RN}, is defined by the equality (34). After some arithmetic, from this we obtain

$$\frac{l}{L}\left|\frac{\Omega_0^2}{\Omega_0^2-\Omega_{TN}^2}\right|^{1/2} \gg \frac{1}{m}. \quad (39)$$

A maximum width of the resonator is

$$b_{\max} = l\left|\frac{\Omega_0^2-\Omega_{TN}^2}{\Omega_0^2}\right|^{1/2}.$$

By combining the two last formulas, we obtain the poloidality condition in the resonator at the plasmapause in the form

$$\frac{mb_{\max}}{L} \gg 1, \quad (40)$$

i.e. the resonator must accommodate many azimuthal wavelengths. The same poloidality condition can also be obtained for the resonator in the ring current region. As is evident, the condition $\nu_N \gg 1$ is also satisfied for the poloidal mode in the resonator, if the width of the transparent region is meant to be a maximum width of the resonator. Again, finite pressure favors the fulfilment of the poloidality condition: rather wide cavities appear on the outer edge of the plasmapause in models I and III ($b_{\max} \sim 1\,R_E$) for all N that have been studied, and in the wing current region in models II and III ($b_{\max} \sim 2\,R_E$, and $\sim 0.5\,R_E$, respectively) when $N = 1$.

It is also important to remark that the higher the harmonic number N, the smaller the relative polarization splitting of the spectrum $|\Omega_{TN}^2 - \Omega_{PN}^2|/\Omega_{TN}^2$, and the more difficult it is to satisfy the poloidality condition. This is obvious from our Figs. 4 and 5, showing how much the difference between the poloidal and toroidal frequencies decreases when passing from $N = 1$ to $N = 2$; at even higher N, this difference is still smaller. Hence, the lower the wave frequency, the smaller the values of the azimuthal wave number, at which it can be poloidal.

Noteworthy is also the fact that in the case of large and small ν_N at a given value of the source q, the wave amplitude near the resonance (toroidal) surface is different, because the terms of the resonance logarithm are different in these two cases. As is seen from Eq. (23), the wave amplitude is independent of ν_N, whereas when $\nu_N \gg 1$ it decreases with increasing ν_N (Eq. 31). From this it is easy to find that in the case of large ν_N the wave amplitude near the resonance surface is by a factor of $\nu_N^{2/3} \gg 1$ smaller compared to small ν_N. Thus, if magnetospheric conditions are conducive to the existence of poloidally polarized waves, at the same time they make the toroidally polarized waves less clearly pronounced.

6.2 On the observation of toroidal and poloidal Alfvén waves in the magnetosphere

We now compare the picture outlined above with the experiment. However, we are not yet fully prepared for this endeavor, because currently, available theories are still too concerned with simplified models. We still do not fully know what changes can be introduced into this picture by the azimuthal inhomogeneity of the magnetosphere and the associated field-aligned currents, the wide-band character and the possible narrow localization of oscillation sources, the interaction of waves with particles drifting in the magnetosphere, and the active role of the ionosphere. Work in this direction is underway, and this is testified by recently published papers addressing these issues (Salat and Tataronis, 1999; Klimushkin et al., 1995; Mann et al., 1997; Leonovich, 2000; Antonova et al., 2000; Vetoulis and Chen, 1996; Klimushkin, 2000; Glassmeier et al., 1999a; Leonovich and Mazur, 1996). However, the creation of a unified realistic model of MHD waves in the magnetosphere is a long way from now. Observations and experiments can have a leading role in such efforts, and at the present stage we need at least to understand whether the picture available to us has anything to do with the information provided by experiments.

Observations from the ground recorded repeatedly nearly monochromatic toroidal Alfvén waves in the Pc 4-5 range, which showed characteristic properties of a localized resonance described in Sect. 5.1: a strong localization of the wave across L-shells, toroidal polarization, and a phase change by 180° at the passage across the resonance peak (Samson et al., 1971; Walker et al., 1979; see also Fenrich and Samson, 1997; references therein). On the other hand, it was pointed out earlier (Glassmeier et al., 1999b) that when observed from satellites, these features of the localized resonances were never identified, in spite of the vast occurrence of toroidal pulsations. A likely explanation for this paradox would be to assume that most of the monochromatic ULF waves in the magnetosphere in the Pc 5 range have large azimuthal wave numbers and $\nu_N \gg 1$. In this case the oscillations are no longer a localized resonance, and the behavior of their phase is much more complicated than in the case $\nu_N \le 1$, which corresponds to a localized resonance: when $\nu_N \gg 1$ the Alfvén wave travels across magnetic shells, having close to the toroidal surface a very small radial component of the wave vector. If this is indeed the case, then the chance to capture in the magnetosphere a localized resonance is relatively poor. Further, the atmosphere comes into play which has the role of a filter transmitting to the ground only waves with a sufficiently smooth dependence of the field on transverse coordinates; thus, the waves with $m \gg 1$ almost do not penetrate through the atmosphere (e.g. Hughes, 1974; Glassmeier and Stellmacher, 2000). For that reason, observations from the ground provide a distorted picture of the wave processes in the magnetosphere. Because of the influence of the ionosphere, only localized resonances are able to penetrate to the ground, and they are the ones that are observed from the radars and magnetometers. In the absence of a detailed theory that would take into account the factors mentioned in the preceding paragraph, this hypothesis must be regarded only as a preliminary explanation for this paradox. But it clearly demonstrates how important it is to take into account the entire body of theoretical knowledge when interpreting experimental data.

We now turn our attention to poloidal pulsations. Space experiments show that they occur much more rarely than toroidal pulsations. For instance, according to the AMPTE/CCE data (Anderson et al., 1990), about five toroidal pulsations correspond to one poloidal pulsation. This is consistent with our conclusion that for the poloidal polarization of the wave, more stringent conditions are required than those for the toroidal wave. Furthermore, the occurrence rate of radially-polarized pulsations decreases with the increasing harmonic number N. This is readily illustrated by dynamic spectrograms obtained by Takahashi et al. (1984) from the ATS 6 and SMS 1 and 2 satellite data: when $N \geq 3$ the azimuthal component of the magnetic field of the pulsations is distinguished much more clearly than the radial component. Within the framework of our theory, this fact is readily explained, because with the increasing harmonic number, the poloidality conditions are satisfied even less. On the other hand, the second harmonic of radially polarized waves with azimuthal wave numbers from 20 to 150 is very often recorded in the magnetosphere. In this case, in a cold plasma the left-hand side of the inequality involved in the poloidality condition (38) for $N = 2$ makes up no more than 1%; therefore, this condition can only be satisfied for pulsations with unrealistically large azimuthal wave numbers, $m \gg 100$. Taking finite pressure into account saves the situation, since in this case the left-hand side readily reaches the values 10–20%, and the waves with $N = 2$ and $m \sim 20 - 150$ may well have a poloidal polarization. An indication of the important role of finite pressure in the formation of these waves is also provided by the existence of a substantial longitudinal component of the magnetic field observed in a number of poloidal pulsations (e.g. Hughes et al., 1979), as it can reach marked values for Alfvén waves only in the case of finite β (see Eq. 11).

Singer et al. (1982) and Engebretson et al. (1992) considered the radially polarized Pc 4 pulsations events which are strongly localized across magnetic shells. One would expect that these pulsations were the excitations of the Alfvén resonator described in Sect. 5.3. Cramm et al. (2000) explored a poloidal Pc 4 pulsation observed by the Equator-S satellite. An analysis showed that this pulsation was nearly monochromatic and very narrowly localized across magnetic shells (a Gaussian with the half-width of about 0.1 R_E), and there was a phase change by 180° at the transition through the region of localization. Such a behavior is characteristic for poloidal waves confined within the resonator. The authors made an estimate of the azimuthal wave number using reasoning similar to ours in Sect. 6.1, and obtained the value of $m \simeq 150$. Of course, it is necessary to understand where this resonator was localized in any particular case, but the relevant information is not always available.

One of the resonators must be localized on the outer boundary of the plasmapause. In all likelihood, radially polarized waves that are confined within this resonator represent a reasonably widespread phenomenon. Singer et al. (1982) reported ISEE-1, 2 satellite observations of poloidal waves which were strongly localized across the L-shells in this region. Takahashi and Anderson (1992) showed the presence of a marked increase in intensity of poloidal waves near the plasmapause using AMPTE/CCE data. In all of these cases the scale of localization across the magnetic shells was $\leq 1\,R_E$. This is in good agreement with the assumption that in these cases, the poloidal waves were eigenmodes of the resonator in the plasmapause region.

It seems likely that the same can also be said of one of the most interesting varieties of poloidally polarized waves, giant pulsations (Pg). These nearly monochromatic waves are usually observed during quite geomagnetic conditions, when the plasmapause lies somewhere at $L \sim 5.5 - 6$, and giant pulsations (Pg) are recorded just there. Rostoker et al. (1979) were the first to notice this. The assumption that Pg are resonator modes on the outer edge of the plasmapause is consistent with the strong localization of Pg across magnetic shells accompanied by a phase change by 180 degrees (Green, 1979; Rostoker et al., 1979; Glassmeier, 1980), and the amplitude distribution in L is described by the Gaussian function with the halfwidth of about 1 R_E (Chisham et al., 1997), and this is indeed expected for the fundamental radial harmonic within the resonator on the outer edge of the plasmapause. The poloidality condition (40) for the values of $m \sim 20$ observed in Pg in the models which we have studied, is satisfied, though without a very large reserve. On the other hand, it seems feasible to rule out the possibility that Pg are Alfvén waves traveling across magnetic shells. Satellite observations do not show any indications of the transformation of poloidal Pg-wave to toroidal waves (Takahashi et al., 1992; Glassmeier et al., 1999a). Here it is very important to make reference to satellite experiments, because such a transformation is also impossible to notice from the ground: at the same m the transverse component of the wave vector $k_\perp = \sqrt{(k_1^2/g_1) + (m^2/g_2)}$ in poloidal waves ($k_1 = 0$) is much smaller than that in toroidal waves ($k_1 \to \infty$); therefore, when $\nu_N \gg 1$, only the oscillations near the poloidal surface have a chance to be transmitted through the ionosphere (Leonovich and Mazur, 1996).

On the other hand, Green (1985) detected several Pg events deep inside the plasmasphere. The geomagnetic conditions where these pulsations were observed, were characterized by the presence of a significant ring current inside the plasmasphere, exactly as in the case of our model III. But this model assumes a resonator inside the plasmasphere. Thus, we can conclude that the events observed by Green (1985) were the eigenmodes of this resonator.

At the same time, the L-dependence of averaged spectra of poloidal oscillations observed by AMPTE/CCE (Takahashi and Anderson, 1992) shows that there exists a rather clearly pronounced population of radially polarized waves, not associated with regions of poloidal frequency extrema. These waves ought to be traveling across magnetic shells. It seems likely that the radially polarized waves that are standing waves across magnetic shells are generally more accessible to observations when compared with poloidal waves. At present, the concept is widely held that high-energy particles drifting in the magnetosphere supply energy to observed poloidal pulsations via bounce-drift resonance. Indeed, in some cases unstable distribution functions of particles associated with poloidal waves were observed (Hughes et al., 1978, 1979; Glassmeier et al., 1999a; Wright et al., 2001); there are also a number of indirect arguments in favor of this concept (Takahashi et al., 1990; Fenrich and Samson, 1997; Ozeke and Mann, 2001). It can be suggested that, were it not for the high-energy particles, the waves with large m would simply not have had a sufficiently large amplitude in order to be observed. But in the case of the displacement of the azimuthally small-scale waves across magnetic shells, the most enhanced waves would be the ones near the toroidal surface, because the waves were to accumulate the particle energy in the process of their propagation from the poloidal to toroidal surface (Klimushkin, 2000), although the build-up rate of the wave energy decreases as the wave detaches itself from the poloidal surface. And only when high-m waves are confined within the resonator, is the transfer of energy from particles able to enhance the poloidal pulsations.

7 Conclusion

In conclusion, we briefly restate the logic of our paper and describe the main results. Our principal intent was to study the conditions where the Alfvén waves in the magnetosphere can be toroidally or radially polarized. Since the toroidal (poloidal) polarization of Alfvén waves implies that the radial wavelength of the wave λ_r is significantly smaller (larger) than the azimuthal wavelength λ_a, it is impossible to study the polarization without studying the structure of the wave field across magnetic shells. To do this, we made use of the system of MHD equations by writing them for plasma of finite but small pressure residing in a curved magnetic field.

As a consequence of this system, we obtained Eq. (12), the basic equation of our paper. It describes the Alfvén wave excited by the magnetosound and, perhaps, by some other sources. This equation defines both the transverse and longitudinal structure of the wave. This is described in the limit $\lambda_r \ll \lambda_a$ by a toroidal longitudinal function, otherwise it is described by a poloidal function. Using numerical calculations we found that when $N = 1-3$ (with these longitudinal harmonic numbers were our prime interest), these functions differ relatively slightly from one another. That permitted us to separate the longitudinal and transverse structures by the method of successive approximations. Thus, we obtained Eq. (19), describing the structure of the wave across magnetic shells. The solution of this equation allowed us to determine both the spatial structure of the wave and the conditions of toroidal and poloidal polarization.

In order for the wave to be toroidally polarized on the magnetic shell with the radial coordinate x^1, it is necessary and sufficient that the condition $\omega = \Omega_{TN}$ is satisfied, where Ω_{TN} is the toroidal eigenfrequency on a given shell. A similar condition $\omega = \Omega_{PN}$, developed for the poloidal frequency, is not a sufficient condition of poloidal polarization – it is also necessary that condition (38) is satisfied, which implies that many azimuthal wavelengths are accommodated between the toroidal and poloidal surfaces. If this condition is not satisfied, then the mode is toroidally polarized throughout the region of its existence. Furthermore, it is sharply localized across magnetic shells, having a singularity on the toroidal surface (regularized by taking into account the ionospheric dissipation). If the poloidality condition is satisfied, then the wave is poloidally polarized in the part of its transparent region. It propagates slowly across the magnetic shells and changes its polarization from poloidal to toroidal. Finally, there exist regions in which the poloidal frequency Ω_{PN} reaches its extreme values. The poloidality condition for these regions is written as Eq. (39). In this case the wave is a standing wave across the magnetic shells, having a poloidal polarization throughout the region of its existence. The fundamental (most easily excited) harmonic of this resonator is described by a Gaussian function.

It is progressively easier to satisfy the poloidality condition with the increasing difference between the toroidal and poloidal frequencies (polarization splitting of the spectrum) and with the increasing azimuthal wave number m. The former of these quantities is determined by geospace plasma and magnetic field parameters, and by the longitudinal harmonic number N. We studied three models of the magnetosphere:

(I) low level of disturbance when a significant time has elapsed after the storm;

(II) high level of disturbance; here is a well-developed ring current; and

(III) low level of disturbance, but when a short time has elapsed after the storm (significant ring current inside of the plasmasphere).

The main conclusion drawn by considering these models implies that an increasing plasma pressure contributes to satisfying the poloidality condition at fixed m. It was ascertained that with β actually observed in the magnetosphere, this condition is satisfied for poloidal Alfvén waves with $N = 2$ and $m \sim 50 - 100$ that are routinely observed in the magnetosphere. The presence of a special criterion of poloidality explains the scarcity of poloidal pulsations compared to toroidal pulsations, especially when $N > 2$.

A further important result is the inferred possible existence of the resonator for poloidal waves in the plasmapause region. We adduced arguments in support of the fact that oscillations that are modes of this resonator are indeed observed. Possibly, they include, among others, giant pulsations (Pg).

At the same time our conclusion about the agreement of theory and observations is a preliminary one, because there are a large number of factors which are neglected by our theory and which can have a substantial influence on the behavior of MHD waves in the magnetosphere. Specifically, they include the azimuthal inhomogeneity of the magnetosphere, field-aligned currents, the non-stationarity of the oscillations, the narrow localization of their sources, the interaction of waves with particles drifting in the magnetosphere, and the active role of the ionosphere. Hence, further efforts are needed, in order to create the more realistic models of ULF waves in the magnetosphere.

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Appendix A Definitions and basic properties of toroidal and poloidal modes

Let T_N and P_N denote the eigen-functions of toroidal and poloidal operators satisfying the boundary conditions

$$T_N,\ P_N|_{x^3=x^3_\pm}=0, \tag{A1}$$

where $x^3_\pm$ stands for the intersection points of a field line with the upper ionospheric boundary. Toroidal and poloidal functions are conveniently normalized in the following manner:

$$\left\langle \frac{\sqrt{g}}{g_1}\frac{T_N^2}{A^2}\right\rangle=1,\ \left\langle \frac{\sqrt{g}}{g_2}\frac{P_N^2}{A^2}\right\rangle=\left\langle \frac{\sqrt{g}}{g_2}\frac{T_N^2}{A^2}\right\rangle \tag{A2}$$

(here the angle brackets designate integration along the field line between the ionsospheres, $\langle ...\rangle=\int_{x^3_-}^{x^3_+}(...)dx^3$). With such a normalization of these functions, they have identical dimensions and can be compared with one another.

Let Ω_{TN} and Ω_{PN} denote the eigenfrequencies of the toroidal and poloidal operators, where

$$\hat{L}_T(\Omega_{TN})T_N=0 \tag{A3}$$

and

$$\hat{L}_P(\Omega_{PN})P_N=0 \tag{A4}$$

hold. The difference between the toroidal and poloidal eigenfrequencies is often referred to as the polarization splitting of the Alfvén oscillation spectrum. To find an analitical expression for it we multiply Eq. (A3) by P_N and Eq. (A4) by T_N, extract one from the other, and integrate along the field line. After the integration by parts, we obtain the difference between the squares of these eigenfrequencies:

$$\Omega_{TN}^2-\Omega_{PN}^2=\left[\left\langle\sqrt{g_3}\,\eta\,P_NT_N\right\rangle\right.$$
$$\left.+\left\langle P_NT_N(\boldsymbol{e}_{||}\cdot\nabla)\left(\ln\sqrt{\frac{g_2}{g_1}}\right)\right\rangle\right]\cdot\left\langle\frac{\sqrt{g_3}}{A^2}P_NT_N\right\rangle^{-1}. \tag{A5}$$

The polarization splitting of the spectrum is caused by the presence of the field line curvature. This is obvious if finite plasma pressure is taken into account, because the first term of the expression (A5) that takes this factor into account, contains explicitly the field line curvature R^{-1} according to formula (9). The situation is somewhat more complicated in cold plasma, where the second term of this formula is responsible for the splitting of the spectrum. The quantity $\sqrt{g_2/g_1}$ involved in the formula has a simple geometrical meaning. If we take a flux tube with the cross section $dx^1=1$, $dx^2=1$, then the physical dimensions in these directions will be, respectively, $d\hat{x}^1=1\cdot\sqrt{g_2}$, $d\hat{x}^2=1\cdot\sqrt{g_2}$. Thus the quantity $\sqrt{g_2/g_1}$ describes the variation of the ratio of these physical dimensions along the tube, i.e. the change of the form of this cross-section (Leonovich and Mazur, 1990). One may well imagine the magnetic field configurations in which this quantity varies even along straight field lines. In these configurations field lines must become increasingly sparser with the advance along them. Such configurations, however, are unlikely to be relevant to magnetosphere physics, where it is assumed that field lines become sparser when leaving one magnetic flux and become denser when entering another flux. Obviously, in this case the derivative $(\boldsymbol{e}_{||}\cdot\nabla)\sqrt{g_2/g_1}$ can be nonzero only when field lines are curved. Moreover, in this case the curvature is only a necessary rather than sufficient condition of the polarization splitting of the spectrum. Indeed, it can be shown (Krylov et al., 1981; Krylov and Lifshitz, 1984) that the following relation holds:

$$(\boldsymbol{e}_{||}\cdot\nabla)\ln\sqrt{g_2/g_1}=K_+-K_-,$$

where K_+ and K_- are a maximum and minimum curvature of the surfaces that are orthogonal to field lines (i.e. of the $x^3=const$ surfaces). As an example of the model in which there is a curvature but no polarization splitting, we consider the situation where the magnetic shells are semicylinders and the field lines are circles. The surfaces $x^3=const$ are plane in this model, $K_+-K_-=0$, and, hence, the toroidal and poloidal eigenfrequencies coincide in this model. For further discussion of this issue see paper of Leonovich and Mazur (1990). The final conclusion from this discussion is thus: in geomagnetic field models the polarization splitting of the spectrum is possible only in the case of curved field lines.

To make a rough estimate of the distance between the toroidal and poloidal surfaces Δ_N, we assume that it is small compared to the typical size of the magnetosphere. We can then avail ourselves of the expansions

$$\omega^2-\Omega_{TN}^2=\omega^2\,\frac{x^1-x^1_{TN}}{a} \tag{A6}$$

and

$$\omega^2-\Omega_{PN}^2=\omega^2\,\frac{x^1-x^1_{PN}}{a}. \tag{A7}$$

Because the difference between the toroidal and poloidal eigenfrequencies is rather small, $\Omega_{TN}-\Omega_{PN}\ll\Omega_{TN},\Omega_{PN}$, and $\Omega_{TN}\sim\omega$ in the mode localization region, we then obtain from Eqs. (A6, A7) the ordering (15).

Appendix B The asymptotic solution of the radial structure equation when $\nu_N \gg 1$

The interval between the toroidal and poloidal surfaces can be broken up into three regions: near the toroidal surface ($|x^1-x^1_{TN}| \ll \Delta_N$), near the poloidal surface ($|x^1-x^1_{PN}| \ll \Delta_N$), and sufficiently far away from these surfaces where the WKB approximation is applicable. Here we consider only the situation where the toroidal frequency is larger than the poloidal frequency. In this case $x^1_{TN} > x^1_{PN}$.

In the region $|x^1 - x^1_{TN}| \ll \Delta_N$, the expansion (A6) can be used. Then Eq. (19), through the substitution

$$z_T = \sqrt{\frac{x^1 - x^1_{TN}}{\lambda_{TN}}},\ \lambda_{TN} = \Delta_N \nu_N^{-2},$$

is transformed to the zero-order Bessel equation. The solution of this equation, bounded when $x^1 > x^1_{TN}$, is

$$R_N = C_T \cdot K_0(2z_T),$$

where C_T is an arbitrary constant yet to be determined. We now put the asymptotic representation of the solution (26) for $x^1_{TN} > x^1$, $(x^1_{TN} - x^1)/\lambda_{TN} \gg 1$:

$$R_N = C_T \sqrt{\frac{\pi}{4}} \left(\frac{x^1_{TN} - x^1}{\lambda_{TN}}\right)^{-1/4} \exp\left(-2i\sqrt{\frac{x^1_{TN} - x^1}{\lambda_{TN}}} - \frac{i\pi}{4}\right). \tag{B1}$$

This expression describes the wave propagating toward the increase of the coordinate x^1.

Near the poloidal surface, when $|x^1-x^1_{PN}| \ll \Delta_N$, we can make use of the linear expansion (A7). Then, we introduce a new variable

$$z_P = \frac{x^1 - x^1_{TN}}{\lambda_{PN}},\ \lambda_{PN} = \Delta_N \nu_N^{-2/3}.$$

Then Eq. (19) is brought to the inhomogeneous Airy equation. We need to find such a solution to this equation that is bounded when $x^1 < x^1_{PN}$ and represents a wave propagating toward the increase in the coordinate x^1 (in order that it can be matched with the solution when $x^1 \simeq x^1_{TN}$). We give this solution in the integral form (Eq. 27) (see Leonovich and Mazur, 1993). The asymptotic representation of this solution when $z_P > 0$, $z_P \gg 1$ is

$$R_N = q_N K_N^2 \Delta_N \frac{\pi^{1/2}}{z_P^{1/4}} \exp\left(\frac{2i}{3} z_P^{3/2} + \frac{i\pi}{4}\right). \tag{B2}$$

When $x^1_{PN} < x^1 < x^1_{TN}$ in the region where the WKB approximation is applicable, the solution is given by Eq. (29). The asymptotic representations (Eqs. (B1), (B2), (29)) are matched in regions of their common applicability, defining the constants C_T and C_W:

$$C_T = \frac{2q_N \Delta_N a}{\nu_N^2 \omega^2} \left(\frac{\lambda_{PN}}{\lambda_{TN}}\right)^{1/4} \exp\left(\frac{i\pi}{2} + i\int_{x^1_{PN}}^{x^1_{TN}} k_1 dx^1\right), \tag{B3}$$

$$C_W = \sqrt{\pi} q_N a \Delta_N /, \nu_N^{-5/3} \omega^{-2} e^{i\pi/4}. \tag{B4}$$

Thus, the asymptotic solution of Eq. (19) is given by the expressions (26, 27, 29) with constants defined by formulas (B3, B4).

Noteworthy is the importance of taking into account the right-hand side of Eq. (19), the source of oscillations q_N. Without the source, this equation would not have any solutions at all, which are bounded in the opaque region, according to Eq. (20), because it would be impossible to match the solutions near the poloidal and toroidal surfaces.

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