

The spatio-temporal structure of impulse-generated azimuthal small-scale Alfvén waves interacting with high-energy charged particles in the magnetosphere

D. Yu. Klimushkin and P. N. Mager

Institute of Solar-Terrestrial Physics (ISTP), Russian Academy of Science, Siberian Branch, Irkutsk, P.O.Box 4026, 664033, Russia

Received: 23 June 2003 – Revised: 3 November 2003 – Accepted: 5 November 2003 – Published: 19 March 2004

Abstract. It is assumed to date that the energy source of azimuthal small-scale ULF waves in the magnetosphere (azimuthal wave numbers $m \gg 1$) is provided by the energetic particles interacting with the waves through the bounce-drift resonance. In this paper we have solved the problem of the bounce-drift instability influence on the spatio-temporal structure of Alfvén waves excited by a source of the type of sudden impulse in a dipole-like magnetosphere. It is shown that the impulse-generated Alfvén oscillation within a time $\tau \sim m/\Omega_{TN}$ (where Ω_{TN} is the toroidal eigenfrequency) is a poloidal one, and each field line oscillates with its own eigenfrequency that coincides with the poloidal frequency of a given L -shell. As time elapses, the wave becomes toroidally polarized because of the phase difference of the disturbance, and the oscillation frequency of field lines tends to the toroidal frequency. The drift-bounce instability growth rate becomes smaller during the wave temporal evolution, and the instability undergoes stabilization when the wave frequency coincides with the toroidal eigenfrequency. The total amplification of the wave can be estimated as $e^{\tilde{\gamma}\tau}$, where $\tilde{\gamma}$ is the wave growth rate at the beginning of the process, when it has its maximum value. The wave amplitude can increase only within a time $\sim \tau$, when it is poloidally polarized. After this time, when the wave becomes to be toroidally polarized, it goes damped because of the finite ionospheric conductivity. This is in qualitative agreement with the recent radar experimental data.

Key words. Magnetospheric physics (MHD waves and instabilities). Space plasma physics (kinetic and MHD theory; wave-particle interactions)

1 Introduction

This paper is devoted to the study of the generation of azimuthal small-scale (azimuthal wave number $m \gg 1$) Alfvén waves in the axisymmetric model of the magnetosphere, allowing for the field-line curvature. As is known, the main difference of high- m waves from small- m waves is that the former waves are assumed to be generated by intramagnetospheric sources, whereas the latter waves are produced by the resonance interaction with the magnetic field generated on the outer boundary of the magnetosphere or beyond it (Glassmeier, 1995). High- m waves are often observed in satellite and ground-based experiments (Fenrich and Samson, 1997; Glassmeier et al., 1999; Cramm et al., 2000; Baddaley et al., 2002).

Until the present time there have appeared a large number of theoretical publications devoted to the study of monochromatic Alfvén waves with $m \gg 1$ in realistic models of the magnetosphere (e.g. Leonovich and Mazur, 1993, 1997; Walker and Pekrides, 1996; Vetoulis and Chen, 1996; Klimushkin et al., 2004; Klimushkin, 1998a, b, 2000; Mager and Klimushkin, 2002). However, MHD waves in the magnetosphere are generated by some transient, nonstationary processes; therefore, the monochromatic approximation does not appear to be quite realistic (although it can serve as the basis for the theory of nonstationary (impulse-generated in particular) waves). Thus, Fenrich and Samson (1997), using the SuperDARN HF radar chain, found that the amplitude of oscillations with $m \gg 1$ increases with time, whereas the oscillations with $m \sim 1$ are damped ones. The authors explained the enhancement of high- m waves by the presence of a bounce-drift resonance with the energetic particle beam which is often assumed as the generation mechanism for these waves. For that reason, it is a currently central problem of constructing the theory of nonstationary Alfvén waves in realistic models of the magnetosphere with consideration for the various intramagnetospheric instabilities.

A considerable amount of research along this line has been done to date. Some publications (Tataronis and Grossman, 1973; Chen and Hasegawa, 1974; Radoski, 1974; Krylov et al., 1981; Mann and Wright, 1995; Mann et al., 1997) addressed the problem of the time evolution of the initial disturbance that is sufficiently widely distributed in radial coordinate and is poloidally polarized (where the azimuthal component of the electric field and the radial component of the magnetic field are dominant). It was shown that in the course of the evolution the structure becomes smaller scale in the radial direction, and the wave transforms to a toroidally polarized wave (by contrast, the radial component of the electric field and the azimuthal component of the magnetic field are dominant). This phenomenon is caused by the phase mixing of the wave field: at the initial instant of time all field lines oscillate with about the same phase; however, since each field line oscillates with its own eigenfrequency (Hasegawa et al., 1983), the oscillations on neighboring magnetic shells rapidly acquired a significant phase difference, the wave becomes strongly “indented” in radial coordinate and, hence, toroidally polarized. A similar problem was tackled by Heyvaerts and Priest (1983) and Dmitrienko and Mazur (1987), but, unlike previous work, the disturbance was specified on some surface normal to field lines, at all instances of time, and its spatial evolution was studied. All of the related above-mentioned studies used very simple one-dimensional inhomogeneous models of the medium; it was recently shown, however, that this phenomenon also occurs in a dipole-like, i.e. two-dimensional, inhomogeneous model of the magnetosphere (Antonova et al., 1999, 2000). Note also that a similar phenomenon is well known in the hydrodynamics of inhomogeneous fluid (Case, 1960). A considerable advance in the studying of impulse-generated high- m Alfvén waves has been made by Leonovich and Mazur (1998), who showed that these pulsations must represent field-line oscillations with a variable frequency changing from the local poloidal frequency to the local toroidal frequency; this phenomenon of frequency change occurs only when the field line curvature is taken into account. Subsequently, Leonovich (2000) studied the generation of Alfvén waves by the source localized not only in time but also in space. Leonovich and Mazur (1998), Leonovich (2000), and Antonova et al. (1999, 2000) regarded the source as a delta function of time, i.e. as having a zero duration. That led to the fact that the mode was widely distributed across magnetic shells, although with the passage of time, it was becoming increasingly smaller scale. Wright (1992) studied the oscillations generated by the source of a finite duration. He showed that the longer the source is at work, the narrower is the localization of the mode across magnetic shells. When the duration of the source operation tends to infinity, the mode is concentrated near the field-line resonance surface, as is the case in conventional theory of monochromatic oscillations.

After all, the time is ripe to make sense of what we understand by the source of oscillations. It is customarily believed that the source of pulsations in the case of large m is provided by space plasma instabilities, such as the bounce-drift insta-

bility (Glassmeier, 1995), and this point of view has some experimental basis (e.g. Glassmeier et al., 1999; Baddaley et al., 2002). However, it would be more appropriate to regard the instability not as the source of the wave but as the mechanism of its enhancement. To set this mechanism to work now requires some triggering amplitude. Mathematically, this implies that the wave equations must have the right-hand side. It is this right-hand side which we understand by the source of the wave. In all references cited in the preceding paragraph, the source was also understood in this meaning of the word. This role cannot be played by high-energy particles, since the current produced by these particles is per se expressed in terms of the electric field of the wave (e.g. Karpman et al., 1977), i.e. it is not the right-hand side of the wave equation. Of course, this does not interfere with their serving as the wave energy sources: the wave that is excited by the external source propagates through the magnetosphere, loses its energy because of the finite ionospheric conductivity and acquires energy due to the bounce-drift resonance (Klimushkin, 2000). The role of the sources in our meaning of the word might be played by currents in the ionosphere (Leonovich and Mazur, 1993; Pokhotelov et al., 1999), currents in the magnetosphere (Pilipenko et al., 2001), etc. The physical origin of the sudden impulse source of high- m waves can involve the substorm injection of particles or an abrupt change on the magnetosphere-ionosphere current system.

This paper is concerned with the influence of the bounce-drift instability on nonstationary azimuthal small-scale Alfvén waves. We consider the high- m waves, where the coupling of the Alfvén wave to fast magnetosound is negligibly small. Within a low-pressure approximation, one can also neglect the coupling of the Alfvén mode to the drift-mirror wave and slow magnetosound. The formulation of the problem is as follows. At the time $t=0$, a source of the type of sudden impulse (delta function of time) is set to work in the magnetosphere, which excites the Alfvén wave with a given longitudinal wave number N and azimuthal wave number $m \gg 1$, a standing wave along a field line. The population of high-energy particles that resides in the magnetosphere takes up and enhances the arising oscillations. It is necessary to study the spatio-temporal structure of these oscillations. The source is assumed to be sufficiently widely distributed in space; for the sake of simplicity, the distribution function of particles is considered time-independent (these two assumptions do not limit very much the generality of our results, as pointed out in the Conclusion). Mathematically, our approach is based on using the ordinary differential equation describing the wave structure across magnetic shells that was derived in papers of Leonovich and Mazur (1997) and Klimushkin et al. (2004). This will enable us to reproduce, also relatively simply, the conclusions from an important paper of Leonovich and Mazur (1998), who used a rather heavy mathematical apparatus, which makes its comprehension substantially difficult.

In Sect. 2 we briefly describe the main conclusions of the theory of monochromatic waves which serves as the basis for the theory of nonstationary oscillations. In Sect. 3

we Fourier-transform over time the wave field of monochromatic oscillations and obtain the spatio-temporal structure of impulse-generated oscillations in the presence of energetic particle beams. In the Conclusion we formulate the main conclusions of our paper and consider their relation to the experimental data. In the Appendix we give some mathematical calculations omitted in the main text of the paper.

2 Input equations: excerpt from the theory of monochromatic waves

The theory of ULF waves in the axisymmetric model of the magnetosphere is rather complicated: as is known, when the field-line curvature and the 2-dimensional inhomogeneity of plasma are taken into account, the MHD waves are described by a system of coupled equations. The problem is somewhat alleviated in the case of $m \gg 1$, $\beta \ll 1$ when this system can be brought to a single differential equation describing the Alfvén mode only (Leonovich and Mazur, 1993; Mager and Klimushkin, 2002). But this equation is still relatively unwieldy, as it contains interrelated problems of determining the longitudinal and transverse structures of the wave. Nonetheless, Vetoulis and Chen (1996), Leonovich and Mazur (1997), and Klimushkin et al. (2004) found that the radial structure of the wave can be qualitatively described by an ordinary differential equation containing derivatives with respect to the coordinate x only, the unit vector of which is directed across magnetic shells. Before putting down this equation, we give some definitions. Let $\Omega_{TN}(x)$ and $\Omega_{PN}(x)$ be the eigenfrequencies of the N -th harmonic of toroidal and poloidal oscillations which are functions of radial coordinate x only. Let x_{TN} and x_{PN} designate the coordinates of the magnetic surfaces on which the wave frequency ω equals the toroidal or poloidal frequency, respectively, i.e. x_{TN} and x_{PN} are the solutions to the equations $\omega = \Omega_{TN}(x)$ and $p\omega = \Omega_{PN}(x)$. As is apparent from this definition, x_{TN} and x_{PN} are functions of the wave frequency. The inequalities $x_{TN} \neq x_{PN}$ and $\Omega_{TN}(x) \neq \Omega_{PN}(x)$ are caused by the curvature of field lines (Krylov et al., 1981; Leonovich and Mazur, 1993); they are strongly influenced by plasma pressure finiteness as well (Mager and Klimushkin, 2002; Klimushkin et al., 2004). The electric field of the wave can be expressed in terms of a scalar function Φ as

$$\mathbf{E} = -\nabla_{\perp} \Phi, \quad (1)$$

where ∇_{\perp} is the transverse nabla operator. The azimuthal component of the wave vector is defined in terms of the wave number m and the normalized distance to the top of the field line L as $K = m/L$. The damping decrement of the mode due to finite ionospheric resistivity is described by the dimensionless quantity δ . The quantity $a\delta \equiv |d\Omega_{TN}/dx|^{-1}\delta$ defines the characteristic radial scale produced by the ionospheric damping (Southwood and Hughes, 1983). If the source of the wave q is also introduced, then the equation of the radial structure becomes

$$\frac{\partial}{\partial x} (x - x_{TN} + ia\delta) \frac{\partial \Phi(x, \omega)}{\partial x} -$$

$$K^2 (x - x_{PN} + ia\delta) \Phi(x, \omega) = q. \quad (2)$$

This equation describes all the main characteristics of the structure of Alfvén waves in a dipole-like magnetosphere: the logarithmic behavior near the field-line resonance surface (toroidal surface), the Alfvén wave structure in the form of the Airy function in the region of poloidality of the mode (where $x \simeq x_{PN}$) (Leonovich and Mazur, 1993; Vetoulis and Chen, 1996), and the propagation of the wave across magnetic shells caused by the curvature of field lines (Leonovich and Mazur, 1993). If we apply the WKB approximation to Eq. (2), we obtain a wave frequency-dependent radial component of the wave vector,

$$k_x^2 = K^2 \frac{x - x_{PN}(\omega)}{x_{TN}(\omega) - x},$$

i.e. the appearance of the transverse dispersion. When the distance between toroidal and poloidal surfaces $\Delta_N = x_{TN} - x_{PN}$ tends to zero, this approximation is no longer valid, and it would not make any sense to speak of the transverse dispersion. So, parameter Δ_N plays the role of the dispersion parameter. To avoid a misunderstanding, we notice that Eq. (2) is applicable only in those regions of the magnetosphere where the functions $\Omega_{TN}(x)$, $\Omega_{PN}(x)$ are monotonic ones.

In the presence of the bounce-drift resonance with the beam of energetic particles, Eq. (2) must be modified, since in addition to the damping decrement of the mode at the ionosphere δ , there arises an instability growth rate γ . As is known, if the mode is toroidally polarized, i.e. $\omega = \Omega_{TN}$, this growth rate is zero, and it is maximal for the poloidally polarized mode when $\omega = \Omega_{PN}$ (see, e.g. Klimushkin, 2000). Since Ω_{TN} and Ω_{PN} are both dependent on the radial coordinate, from this it follows that the growth rate is also a function of x , and it can be approximated by the expression

$$\gamma = \tilde{\gamma} \frac{x_{TN} - x}{\Delta_N} \quad (3)$$

(Klimushkin, 2000). Upon substituting in Eq. (2) δ for $\delta - \gamma$, we obtain the equation for the radial structure of the wave in the presence of a bounce-drift instability:

$$\frac{\partial}{\partial x} \left[x - x_{TN} + ia \left(\delta - \tilde{\gamma} \frac{x_{TN} - x}{\Delta_N} \right) \right] \frac{\partial \Phi(x, \omega)}{\partial x} - K^2 \left[x - x_{PN} + ia \left(\delta - \tilde{\gamma} \frac{x_{TN} - x}{\Delta_N} \right) \right] \Phi(x, \omega) = q. \quad (4)$$

Note that the curvature of field lines is taken into account here by the fact of the inequality $x_{TN} \neq x_{PN}$. Natural boundary conditions for this equation are provided by the boundedness of the mode in radial coordinate, i.e.

$$|\Phi(x \rightarrow \pm\infty)| < \infty. \quad (5)$$

The quantity K in Eq. (4) can be considered constant, since the distance between the toroidal and poloidal surfaces

is much smaller than the characteristic size of the magnetosphere: $\Delta_N \ll L$. Further, the right-hand side of the equation q is taken to be independent of the radial coordinate. The decrement and growth rates are considered small compared with the wave frequency, in which case the conditions $a\delta/\Delta_N \ll 1$ and $a\tilde{\gamma}/\Delta_N \ll 1$ are satisfied. Then, by applying Fourier-transform in radial coordinate, the solution of Eq. (4) with the boundary conditions of Eq. (5) may be represented as (see also Leonovich and Mazur, 1999)

$$\Phi(x, \omega) = \frac{iq}{K} \int_0^\infty \frac{dk}{\sqrt{k^2 + K^2}} \exp \left[ik(x - x_{TN}) + iK\Delta_N \arctan \frac{k}{K} - ak\delta + Ka\tilde{\gamma} \arctan \frac{k}{K} \right]. \quad (6)$$

Furthermore, when $K\Delta_N \gg 1$ (which corresponds to large values of the azimuthal wave number m), Eq. (4) can also be solved within the WKB approximation. From this solution it follows that the monochromatic wave with $m \gg 1$, constituting a standing wave along field lines, travels across magnetic shells from the toroidal to poloidal surface, loses its energy because of the interaction with the ionosphere and acquires energy due to the interaction with the beam of energetic particles; in the course of this process the polarization changes from poloidal to toroidal. The mode is localized between the surfaces x_{TN} and x_{PN} ; we shall also refer to this interval as the transparent region of the wave, since the square of the radial component of the wave vector is positive therein. All this is consistent with the conclusions from a more rigorous investigation (Klimushkin, 2000). Note that the aforesaid transformation has nothing to do with the transformation of nonstationary poloidal waves to toroidal waves caused by the phase mixing discussed in the Introduction: transformation in monochromatic high- m waves is possible only in the presence of a curvature of field lines, whereas nonstationary Alfvén waves transform from poloidal to toroidal waves even in the plane model of the magnetosphere.

At this point we can finish our excursus into the theory of monochromatic waves and proceed to considering the impulse-generated waves.

3 Spatio-temporal structure of impulse-generated oscillations

For studying the nonstationary oscillations, we apply a Fourier-transform over time:

$$\Phi(x, t) = \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \Phi(x, \omega). \quad (7)$$

The source of the wave is assumed to be of the type of sudden impulse that turns on at the time $t=0$ and immediately turns off, i.e. $q(t)=\delta(t)$, and

$$q(\omega) = -i. \quad (8)$$

Let us now adopt some simple models for the frequency dependencies of all quantities involved in Eq. (6). First of

all, the width of the transparent region Δ_N and the characteristic scale determined by the ionospheric damping $a\delta$ will be considered frequency-independent. Further, we approximate the toroidal and poloidal frequencies by the functions

$$\Omega_{TN}(x) = \Omega_0 \left(1 - \frac{x}{l} \right) \quad (9)$$

and

$$\Omega_{PN}(x) = \Omega_0 \left(1 - \frac{x + \Delta_N}{l} \right), \quad (10)$$

implying that in most of the magnetosphere these frequencies are decreasing functions of radial coordinate; linear expansions of these functions are obviously applicable in the region of localization of the monochromatic wave, as its width is small compared with the characteristic scale of the magnetosphere. From this we obtain the solution for the equation $\omega = \Omega_{PN}(x)$ that defines the position of the toroidal surface:

$$x_{TN}(\omega) = l \left(1 - \frac{\omega}{\Omega_0} \right). \quad (11)$$

The situation with the instability growth rate γ is somewhat more complicated. On a given magnetic shell the growth rate depends on frequency, at least in view of the fact, that if the frequency coincides with the toroidal eigenfrequency of this L -shell, then $\gamma(x_{TN}(\omega))=0$, which is reflected in Eq. (3). The quantity $\tilde{\gamma}$ that defines the maximum value of the growth rate in the transparent region (i.e. on the poloidal surface) can also be frequency dependent. Indeed, the bounce-drift instability growth rate is nonzero only when the following condition is satisfied:

$$\omega - n\omega_b - m\omega_d = 0. \quad (12)$$

Here, ω_b is the bounce-frequency, and ω_d is the bounce-averaged drift frequency; it is not too important for us whether the number n is an integer, as is usually considered, or whether it can have a non-integer value (Glassmeier et al., 1999). In any case, some dependence $\tilde{\gamma}(\omega)$ must exist. Its character and the parameters of its maximum, in particular (height, characteristic width), are determined by the properties of the distribution function of energetic particles; unfortunately, such an investigation has not been carried out yet. Therefore, we have to confine ourselves to qualitative reasoning concerning $\Delta\omega$, the characteristic width of the peak of the function $\tilde{\gamma}(\omega)$. The dependence $\tilde{\gamma}(\omega)$ has a relatively weak influence on the growth rate as a function of frequency, if the condition $\Delta\omega \gg \Delta\Omega_N$ is satisfied, where $\Delta\Omega_N$ is the difference between the toroidal and poloidal frequencies; when this condition is satisfied, the first term ($\tilde{\gamma}$) in Eq. (3) depends much less on frequency than does the second term ($x_{TN}-x/\Delta_N$). This condition appears to be quite natural, as the difference between the toroidal and poloidal frequencies is known to be much smaller than each of them separately, $\Delta\Omega_N \ll \Omega_{TN}, \Omega_{PN}$. Of course, an alternative situation where $\Delta\omega \ll \Delta\Omega_N$ is also quite conceivable, although we believe it is much less realistic and less interesting. It will be briefly considered in the Conclusion.

Another possible cause for the frequency dependence of the growth rate is associated with the time evolution of the distribution function of beam particles which can be brought about by two factors. First, populations of high-energy particles are present in the magnetosphere but not permanently – they are generated by some nonstationary magnetospheric processes, perhaps by those which are the sources of the pulsations themselves, and disappear due to precipitations into the ionosphere. Second, the distribution function is gradually evolving due to the interaction with the wave which it enhances. Besides, the distribution function of charged particles can also vary due to the fact that a global convection electric field can eject particles from the region of localization of the wave (Ozeke and Mann, 2001). These factors lead to instability stabilization, which must also influence the dependence $\tilde{\gamma}(\omega)$. In this paper we neglect these factors in view of the fact that there is a much simpler cause for instability stabilization discussed in this section. In the Conclusion we consider qualitatively the changes in the picture of the spatio-temporal structure of impulse-generated oscillations, which can be caused by the time evolution of the energetic particles' distribution function.

Thus, it is assumed that the frequency dependence of the growth rate is defined by Eq. (3), where $\tilde{\gamma}(\omega)=const$, and the function $x_{TN}(\omega)$ is defined by Eq. (11).

Upon substituting the solution of Eq. (6) into Eq. (7) and using Eqs. (8) and Eq. (11), we obtain the following dependence of the wave field on the time and radial coordinate:

$$\Phi(x, t) = -\frac{2\pi}{K^2} \frac{\theta(t)}{\sqrt{1+(t/\tau)^2}} \exp \left[-i\Omega_{TN}(x)t + iK \arctan \frac{t}{\tau} - aK\delta \frac{t}{\tau} + a\tilde{\gamma}K \arctan \frac{t}{\tau} \right]. \quad (13)$$

Here we have designated

$$\tau = \frac{lK}{\Omega_0} \quad (14)$$

(on the order of magnitude, $\tau \sim m/\Omega_{TN}$); $\theta(t)$ is a Heaviside step function, implying the absence of oscillations before the source is turned on (when $t < 0$). Some technical details of the derivation of (13) are given in the Appendix.

Let us consider two limiting cases of the expression (13). When $0 < t \ll \tau$ it can be represented as

$$\Phi(x, t) = -\frac{2\pi}{K^2} \exp \left[-i\Omega_{PN}(x)t - \frac{a\Omega_0}{l}(\delta - \tilde{\gamma})t \right], \quad (15)$$

where the poloidal frequency is represented by Eq. (10). In the opposite limiting case, $t \gg \tau$, the solution of Eq. (13) reduces to the expression

$$\Phi(x, t) = -\frac{2\pi}{K^2} \frac{\tau}{t} \exp \left[-i\Omega_{TN}(x)t - a\delta \frac{\Omega_0 t}{l} + iK \Delta_N \frac{\pi}{2} + a\tilde{\gamma}K \frac{\pi}{2} \right]. \quad (16)$$

The electric field components of the wave are defined in terms of the potential Φ , in accordance with Eq. (1).

Obviously, the wave field may be imagined as independent oscillations of each field line with the x -dependent instantaneous frequency varying from Ω_{PN} to Ω_{TN} . The field-line oscillations can be considered quasi-monochromatic if the characteristic time of change in the instantaneous frequency τ far exceeds the oscillation period (which can be taken to be approximately equal to both to $2\pi/\Omega_{PN}$ and to $2\pi/\Omega_{TN}$, since the toroidal and poloidal frequencies differ little from each other), i.e. when the following condition is satisfied

$$|K| \gg \frac{2\pi}{\Omega_{TN}} \left| \frac{\partial \Omega_{TN}}{\partial x} \right|.$$

This inequality is satisfied for high- m oscillations, i.e. only for those which are assumed to be generated by intramagnetospheric sources. Note in passing that in the model with straight field lines we have $\Omega_{PN}=\Omega_{TN}$, and the oscillations of field lines are quasi-monochromatic for any azimuthal wavelengths. The phenomenon of the instantaneous frequency change was found for the first time by Leonovich and Mazur (1998).

The role of the radial component of the wave vector of impulse-generated oscillations is played by the derivative (with respect to x) of the imaginary part of the exponent (13,15,16), i.e.

$$k_x = \left| \frac{\partial \Omega_{TN}}{\partial x} \right| t.$$

Alfvén waves are poloidally polarized when $k_x \ll K$ and toroidally polarized when the inverse inequality holds. It is easy to see that the characteristic time of polarization reversal also coincides with τ . Thus, the wave is poloidally polarized at the time of its generation and becomes toroidally polarized after a lapse of the characteristic time τ . If there is neither dissipation nor instability ($\delta=\tilde{\gamma}=0$), then the azimuthal component of the electric field of the wave $E_a=-iK\Phi$ decreases as $E_a \propto t^{-1}$, whereas the radial component $E_r=-\partial\Phi/\partial x$ tends to a constant value. This phenomenon is caused by the phase mixing of the wave field, as was stated in the Introduction. We notice once again that transformation of impulse-generated poloidal Alfvén waves to toroidal waves occurs even with the zero curvature of field lines, unlike the phenomenon of the frequency change from poloidal to toroidal, which is possible only when the curvature is taken into account.

Next, we consider the contribution from the instability to this picture. As is evident from Eq. (13), the instability-induced enhancement of the wave is described by the term $\tilde{\gamma}Ka \arctan(t/\tau)$. It is easy to see that when $t \gg \tau$ there is instability stabilization. In the course of evolution of a impulse-generated disturbance, its instantaneous frequency tends to the toroidal frequency, but the instability growth rate is zero when the wave frequency coincides with the toroidal eigenfrequency. In other words, there exists a mechanism of instability stabilization associated not with the evolution of the distribution function of particles but with the properties of the

wave itself. The total enhancement of the wave amplitude is determined by the factor

$$\Gamma = \exp(a\tilde{\gamma}K\pi/2),$$

that is after the instability is stabilized, the wave has Γ times larger amplitude than without the instability. The order of magnitude estimation of this factor is

$$\Gamma \sim \exp(\tilde{\gamma}m/\Omega_{TN}). \quad (17)$$

It can be obtained from the following simple considerations. By the definition, $\Gamma \simeq \exp(\tilde{\gamma}t_0)$, where t_0 is the temporal scale, on which a wave can gain the energy from the particles. In our case it is $\tau \sim m/\Omega_{TN}$. After substitution of this ordering into the former expression, we obtain Eq. (17). We can evaluate the total enhancement for such reasonable values as $\tilde{\gamma}/\Omega_{TN}=0.1$, and $m=100$. Then $\Gamma \sim 2 \cdot 10^4$.

Obviously, if $\delta > \tilde{\gamma}$, then the dissipation will always predominate over the instability, and this can be neglected altogether. Of the greatest interest is the case $\delta < \tilde{\gamma}$. In this case the wave will be enhanced within a time on the order of τ , i.e. when it is poloidally polarized. After a lapse of this time the decrement becomes predominant over the growth rate, and the wave starts to attenuate, and its polarization will tend to the toroidal polarization. This is well seen from Eqs. (15) and (16). Thus, the impulse-generated poloidal waves will be enhanced by the instability, and the toroidal waves, on the contrary, will be attenuated.

4 Conclusion

Thus, the following picture is beginning to emerge. An instantaneous impulse generates an Alfvén oscillation which is poloidal within a time $t \lesssim \tau$ and increasing with time, if there are appropriate bounce-drift instability conditions. Furthermore, each field line oscillates with its own eigenfrequency coincident with the poloidal frequency of a given L -shell. If $m \gg 1$, then the oscillations of each field line are quasi-monochromatic ones. With the passage of time, because of the phase difference of the emerging poloidal disturbance, the wave acquires a toroidal polarization, and the oscillation frequency of field lines tends to the toroidal frequency. And there occurs instability stabilization, as the instantaneous frequency tends to the toroidal eigenfrequency, and the wave becomes damped due to finite ionospheric conductivity. The total amplification of the wave is estimated as $\Gamma \sim \exp(\tilde{\gamma}m/\Omega_{TN})$.

We can give a simple physical treatment of this picture, following the papers by Leonovich and Mazur (1998) and Klimushkin (2000). From a formal point of view, a sudden impulse type source formally excites a continuous set of monochromatic waves instantaneously on all magnetic shells. On a given magnetic shell x a monochromatic wave is excited, for which this shell is a poloidal one. Consequently, an oscillation of the poloidal type with the frequency equal to the poloidal frequency of this magnetic shell x will be excited. Correspondingly, the wave's frequency ω equals the

poloidal frequency of this shell Ω_{PN} in the beginning, and the wave is poloidally polarized. After that, the magnetic surface x is occupied by oscillations arriving from increasingly distant magnetic shells that originated as poloidal ones there; but as they travel towards this given magnetic surface, they transform progressively to toroidal ones. In the course of this propagation, the wave loses its energy because of the finite ionospheric conductivity and acquires energy due to the bounce-drift resonance. If the instability is stronger than the attenuation ($\tilde{\gamma} > \delta$), then the wave is amplified, but only in the beginning of the process, because the decrement δ is more or less constant, but the local wave growth rate γ decreases during the propagation. Finally, an oscillation reaches this magnetic surface x , for which this surface is toroidal, and hence, the frequency of this oscillation is Ω_{TN} , and the polarization is toroidal. When this wave has reached the surface x , it is no more amplified, because $\gamma=0$ on the toroidal surface; hence, this wave must be attenuated.

Thus, a physical reason for the stabilization of the instability is a difference between toroidal and poloidal frequencies. The reason for it is a plasma and magnetic field inhomogeneity (Krylov et al., 1981; Leonovich and Mazur, 1993; Klimushkin et al., 2004). Hence, we can conclude that it is the inhomogeneity that facilitates the wave growth ceasing.

We now compare these conclusions with experimental data reported by Fenrich and Samson (1997). Based on investigations using the SuperDARN HF radar chain, they showed that poloidal waves are always enhanced with time, whereas toroidal waves are damped, in full agreement with our theoretical picture. It is interesting that the observed poloidal waves had large azimuthal wave numbers, and the toroidal waves had small m . In our theory impulse-generated waves must be poloidally polarized at the time of their generation, regardless of the value of m , and only when $m \gg 1$ can they retain this polarization within a relatively long time, unlike the waves with $m \sim 1$ which must very rapidly acquire the toroidal polarization. Thus, one can speak of an agreement between theory and experiment.

Incidentally, not all properties of the experiment reported by Fenrich and Samson (1997) match our theoretical picture. Their data include a rather marked latitudinal localization of the disturbance, and the low- m and high- m waves showed a different sign of phase change across the region of localization. In our picture, however, the wave is very broadly localized across magnetic shells. This is because the source was considered to be a delta-function of time, i.e. had a zero duration. Obviously, in a more realistic case of the source of a finite duration, the disturbance must acquire a wider localization across the L -shells, which was pointed out by Wright (1992).

Some other limitations of our model also deserve mention. It was assumed in this study that the oscillation source is sufficiently widely distributed in space. Physically, this means that the region of localization of the source far exceeds the distance between the toroidal and poloidal surfaces. This is not such a strong limitation, as this distance is rather small –

it does not exceed a few tenths of the terrestrial radius when projected onto the equatorial plane (Klimushkin et al., 2004).

Furthermore, it was assumed that the inequality $\Delta\omega \gg \Delta\Omega_N$ holds, where $\Delta\omega$ is the characteristic value of frequency mismatch, at which the resonance condition (12) still holds, and the growth rate is not small. This condition appears rather natural, as the difference between the toroidal and poloidal frequencies is fairly small. Nevertheless, it is also easy to consider the opposite variant where $\Delta\omega \ll \Delta\Omega_N$. In this case instability turns on not immediately after the start of the oscillation but somewhat later, and stabilization sets in not after a lapse of a time τ , as is the case in the theory considered here, but earlier. It is unlikely that this would be an exceedingly interesting modernization of our picture.

Another factor that can be important for the high- m waves is finite Larmor radius ρ_L . It can lead to a nonzero $k_{\perp}\rho_L$ and modify the wave growth rate (e.g. Dettrick et al., 2003). But finite Larmor radius effects can be sufficient only where the radial component of the wave vector tends to infinity, i.e. is in the region of the toroidal surface. It is unlikely that this would drastically change our conclusions, since the fact that the growth rate of the wave tends to zero with an increase in the transverse wave vector is a very general property of the Alfvén mode that is eventually related to the huge energy concentration near the resonance surface.

It would be more interesting to consider the changes introduced by the evolution of the distribution function of particles. As pointed out above, such an evolution must occur because of the back influence of the Alfvén wave on particles and due to the fact that the population of high-energy particles is per se generated by some transient magnetospheric process. These factors must also lead to instability stabilization. In our paper, however, stabilization is taken for granted, so that taking into account the evolution of the distribution function cannot have any essential influence on the picture of disturbance development. Conceivably this would lead to some change in the characteristic time of stabilization. In our paper this time, τ , coincides with the time of polarization reversal of the wave. If the evolution of the distribution function led to a substantial decrease in the stabilization time, the sum of the poloidal pulsations would be damped with time. However, the poloidal waves observed by Fenrich and Samson (1997) were found to be enhanced all the time. Hence it follows that the evolution of the distribution function is unlikely to have time to drastically change the spatio-temporal picture of the disturbance.

There is also yet another very important factor which we neglected here. We operated with the equation of the radial structure (4), which is applicable only in those magnetospheric regions where the functions Ω_{TN} , Ω_{PN} are monotonic. This is indeed valid in almost the entire magnetosphere, except for the plasmopause and ring current regions. However, it is these regions that are very interesting in terms of the theory of MHD waves, because the existence of modes is possible in these regions, which are bounded on both sides by poloidal surfaces (Vetoulis and Chen, 1996; Klimushkin,

1998b). It seems likely that some of the observed poloidal pulsations are, in fact, modes enclosed within this resonance (Klimushkin et al., 2004). Impulse-generated oscillations in this region are in need of their own theory which may turn out to be significantly more complicated than the one presented in this paper.

Appendix A On the derivation of Eq. (13)

Equation (7) contains two integrals: one is the time integral, and the other is the integral over the radial coordinate, since the function $\Phi(x, \omega)$ is defined by Eq. (6). However, the Eq. (13) does not involve any integration at all. We now explain how this came about. When calculating $\Phi(x, \omega)$, first we changed the order of integration:

$$\Phi(x, \omega) = \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \frac{iq}{K} \int_0^{\infty} dk F(k) e^{-ikx_{TN}(\omega)} =$$

$$\frac{1}{K} \int_0^{\infty} dk F(k) e^{-ikl} \int_{-\infty}^{\infty} d\omega e^{-i\omega t + i\omega kl/\Omega_0}$$

and the internal integral turned out to be an integral representation of the delta function, which, in turn, removed the integration over the variable k . This gives directly Eq. (13).

Acknowledgements. We are grateful to I. S. Dmitrienko, A. S. Leonovich, V. A. Mazur, and K.-H. Glassmeier for fruitful discussions, and to V. G. Mikhalkovsky for his assistance in preparing the English version of the manuscript. D. K.'s work was supported by INTAS grant YSF 01/018. P. M.'s work was supported by RFBR grant 03-05-64545.

Topical Editor T. Pulkkinen thanks a referee for his help in evaluating this paper.

References

- Antonova, A. E., Gubar', Yu. I., and A. P. Kropotkin: Towards the model of relativistic electron fluxes: acceleration in the field of strong Alfvénic disturbances, *Radiation measurements*, 30, 515–521, 1999.
- Antonova, A. E., Gubar', Yu. I., and Kropotkin, A.P. A model of spatio-temporal structure of the substorm electromagnetic disturbance and its consequences, *Phys. Chem. Earth (C)*, 25, 43–46, 2000.
- Baddeley, L. J., Yeoman, T. K., Wright, D. M., Davies, J. A., Trattner, K. J., and Roeder, J. L.: Morning sector drift-bounce resonance driven ULF waves observed in artificially-induced HF radar backscatter, *Ann. Geophys.*, 20 1487–1498, 2002.
- Case, K. M.: Stability of an idealized atmosphere. I. Discussion of results, *Phys. Fluids*, 3, 149–154, 1960.
- Chen L. and A. Hasegawa: A theory of long period magnetic pulsations. 2. Impulse excitation of surface eigenmode, *J. Geophys. Res.*, 79, 1033–1037, 1974.
- Cramm, R., Glassmeier, K. H., Othmer, C., Fornaçon, K. H., Auster, H. U., Baumjohan, W., and Georgescu, E.: A case study of a radially polarized Pc 4 event observed by the Equator-S satellite, *Ann. Geophys.*, 18, 411–415, 2000.

- Dettrick, S., Zheng, L., and Chen, L.: Kinetic theory of geomagnetic pulsations: 4. Hybrid gyrokinetic simulation of drift-bounce resonant excitation of shear Alfvén waves, *J. Geophys. Res.*, 108(A4), 1150, doi:10.1029/2002JA009650, 2003.
- Dmitrienko, I. S. and V. A. Mazur: Spatial evolution of the MHD disturbance in cold inhomogeneous plasma, *Issledovania po geomagnetizmu, aeronomii i fizike Solntsa* (in Russian), 78, 3–18, 1987.
- Fenrich, F. R. and Samson, J. C.: Growth and decay of field line resonances, *J. Geophys. Res.*, 102, 20 031–20 039, 1997.
- Glassmeier, K. H.: ULF pulsations, *Handbook of Atmospheric Electrodynamics*, Vol. II, 463–502, 1995.
- Glassmeier, K. H., Buchert, S., Motschmann, U., Korth, A., and Pedersen, A.: Concerning the generation of geomagnetic giant pulsations by drift-bounce resonance ring current instabilities, *Ann. Geophys.*, 17, 338–350, 1999.
- Hasegawa, A., Tsui, K. N., and Assis, A. S.: A theory of long period magnetic pulsations. 3. Local field line oscillations, *Geophys. Res. Lett.*, 10, 765–767, 1983.
- Heyvaerts, J. and Priest, E. R.: Coronal heating by phase-mixed shear Alfvén waves, *Astron. Astrophys.*, 117, 220–234, 1983.
- Karpman, V. I., Meerson, B. I., Mikhailovsky, A. B. and Pokhotelov, O. A.: The effects of bounce resonances on wave growth rates in the magnetosphere, *Planet. Space Sci.*, 25, 573–585, 1977.
- Klimushkin, D. Yu.: Theory of azimuthally small-scale hydromagnetic waves in the axisymmetric magnetosphere with finite plasma pressure, *Ann. Geophys.*, 16, 303–321, 1998a.
- Klimushkin, D. Yu.: Resonators for hydromagnetic waves in the magnetosphere, *J. Geophys. Res.*, 103, 2369–2378, 1998b.
- Klimushkin, D. Yu.: The propagation of high- m Alfvén waves in the Earth's magnetosphere and their interaction with high-energy particles, *J. Geophys. Res.*, 105, 23 303–23 310, 2000.
- Klimushkin, D. Yu., Mager, P. N., and Glassmeier, K.-H.: Toroidal and poloidal Alfvén waves with arbitrary azimuthal wave numbers in a finite pressure plasma in the Earth's magnetosphere, *Ann. Geophys.*, 22, 267–288, 2004.
- Krylov A. L., Lifshitz, A. F., and Fedorov, E. N.: On resonance properties of magnetosphere, *Izv. Akad. Nauk SSSR. Fizika Zemli*, 8, 49–58, 1981.
- Leonovich, A. S.: Magnetospheric MHD response to a localized disturbance in the magnetosphere, *J. Geophys. Res.*, 105, 2507–2520, 2000.
- Leonovich, A. S. and Mazur, V. A.: A theory of transverse small-scale standing Alfvén waves in an axially symmetric magnetosphere, *Planet. Space Sci.*, 41, 697–717, 1993.
- Leonovich, A. S. and Mazur, V. A.: A model equation for monochromatic standing Alfvén waves in the axially-symmetric magnetosphere, *J. Geophys. Res.*, 102, 11 443–11,456, 1997.
- Leonovich A. S. and Mazur, V. A.: Standing Alfvén waves in an axisymmetric magnetosphere excited by a non-stationary source, *Ann. Geophys.*, 16, 914–920, 1998.
- Leonovich A. S. and V. A. Mazur: Standing Alfvén waves in the magnetosphere from a localized monochromatic source, *J. Geophys. Res.*, 104, 2411–2420, 1999.
- Mager, P. N. and D. Yu. Klimushkin: Theory of azimuthally small-scale Alfvén waves in an axisymmetric magnetosphere with small but finite plasma pressure, *J. Geophys. Res.*, 107, 1356, doi:1029/2001JA009137, 2002.
- Mann, I. R. and Wright, A. N.: Finite lifetimes of ideal poloidal Alfvén waves, *J. Geophys. Res.*, 100, 23 677–23 686, 1995.
- Mann I. R., Wright, A. W., and Hood, A. W.: Multiple-timescales analysis of ideal poloidal Alfvén waves, *J. Geophys. Res.*, 102, 2381–2390, 1997.
- Ozeke, L. G. and Mann, I. R.: Modeling of the properties of high- m Alfvén waves driven by the drift-bounce resonance mechanism, *J. Geophys. Res.*, 106, 15 583–15 597, 2001.
- Pilipenko, V., Kliemenova, N., Kozyreva, O., Engebretson, M., and Rasmussen, O.: Long-period magnetic activity during the May 15, 1997 storm, *J. Atmos. Terr. Phys.*, 63, 489–501, 2001.
- Pokhotelov, O. A., Pilipenko, V. A., and Parrot, M.: Strong ionospheric disturbance as a possible origin of inner zone particle diffusion, *Ann. Geophys.*, 17, 526–532, 1999.
- Radoski, H. R.: A theory of latitude dependent geomagnetic micropulsations: the asymptotic fields, *J. Geophys. Res.*, 79, 595–613, 1974.
- Southwood, D. J. and Hughes, W. J.: Theory of hydromagnetic waves in the magnetosphere, *Space Sci. Rev.*, 35, 301–366, 1983.
- Tataronis, J. A. and Grossman, W.: Decay of MHD waves by phase mixing, 1, The sheet-pinch in a plane geometry, *Z. Phys.*, 261, 203–216, 1973.
- Vetoulis, G. and Chen, L.: Kinetic theory of geomagnetic pulsations, 3, Global analysis of drift Alfvén-ballooning modes, *J. Geophys. Res.*, 101, 15 441–15 456, 1996.
- Walker, A. D. M. and Pekrides, H.: Theory of magnetospheric standing hydromagnetic waves with large azimuthal wave number, 4, Standing waves in the ring current region, *J. Geophys. Res.*, 101, 27 133–27 148, 1996.
- Wright, A. N.: Asymptotic and time-dependent solutions of magnetic pulsations in realistic magnetic field geometries, *J. Geophys. Res.*, 97, 6429–6450, 1992.