Annales Geophysicae (2003) 21: 1405–1418 © European Geosciences Union 2003

A consistent thermodynamics of the MHD wave-heated two-fluid solar wind

I. V. Chashei[1], **H. J. Fahr**[2], **and G. Lay**[2]

[1]Lebedev Physical Institute, Leninskii pr. 53, 117924 Moscow, Russia
[2]Institut für Astrophysik und Extraterrestrische Forschung der Universität Bonn, Auf dem Hügel 71, D-53121 Bonn, Germany

Received: 23 August 2002 – Revised: 10 December 2002 – Accepted: 30 January 2003

Abstract. We start our considerations from two more recent findings in heliospheric physics: One is the fact that the primary solar wind protons do not cool off adiabatically with distance, but appear to be heated. The other one is that secondary protons, embedded in the solar wind as pick-up ions, behave quasi-isothermal at their motion to the outer heliosphere. These two phenomena must be physically closely connected with each other. To demonstrate this we solve a coupled set of enthalpy flow conservation equations for the two-fluid solar wind system consisting of primary and secondary protons. The coupling of these equations comes by the heat sources that are relevant, namely the dissipation of MHD turbulence power to the respective protons at the relevant dissipation scales. Hereby we consider both the dissipation of convected turbulences and the dissipation of turbulences locally driven by the injection of new pick-up ions into an unstable mode of the ion distribution function. Conversion of free kinetic energy of freshly injected secondary ions into turbulence power is finally followed by partial reabsorption of this energy both by primary and secondary ions. We show solutions of simultaneous integrations of the coupled set of differential thermodynamic two-fluid equations and can draw interesting conclusions from the solutions obtained. We can show that the secondary proton temperature with increasing radial distance asymptotically attains a constant value with a magnitude essentially determined by the actual solar wind velocity. Furthermore, we study the primary proton temperature within this two-fluid context and find a polytropic behaviour with radially and latitudinally variable polytropic indices determined by the local heat sources due to dissipated turbulent wave energy. Considering latitudinally variable solar wind conditions, as published by McComas et al. (2000), we also predict latitudinal variations of primary proton temperatures at large solar distances.

Key words. Interplanetary physics (interstellar gas, plasma waves and turbulence; solar wind plasma)

Correspondence to: H. J. Fahr (hfahr@astro.uni-bonn.de)

1 Introduction

For many years now it has been recognized that the solar wind dynamics and thermodynamics at larger distances are influenced by the creation and incorporation of pick-up ions, secondary ions which are produced from the ionization of neutral interstellar H-atoms and then move simultaneously with the solar wind bulk. On the one hand, it was already predicted very early that the solar wind decelerates due to both the pick-up ion loading and also due to the action of the pick-up ion pressure (Holzer, 1972; Fahr, 1973; Fahr, Ripken and Lay, 1981; Isenberg, 1986; Fahr and Fichtner, 1995; Lee, 1997; Whang, 1998; Whang, Lu and Burlaga, 1999; Fahr and Rucinski, 1999). While becoming decelerated the solar wind plasma taken as a mono-fluid is also heated, partly because of more adiabatic compression compared to a non-decelerated flow, and partly because of being loaded with suprathermal secondary ions (starting with early work by Holzer and Leer; 1973, Fahr, 1973). As an evident result the effective monofluidal solar wind sound velocity increases with distance in the outer heliosphere and the effective Mach numbers decrease (see Zank and Pauls, 1997, or Fahr and Rucinski, 1999, 2001).

Paying attention to the fact that primary and secondary solar wind ions do not quickly enough (i.e. within a convection time scale) assimilate their distribution functions, may influence one to consider the solar wind plasma as a two-fluid rather than a monofluid medium. In that case, however, primary and secondary ions, though in first order moving simultaneously with the solar wind bulk, behave differently concering their thermodynamics. From exact solutions of the transport equation of pick-up ions (see Isenberg, 1987; Chalov and Fahr, 1995, 1997; Fichtner et al., 1996), one can draw the theoretical conclusion (see Fahr, 2002b) that these ions behave isothermal at large solar distances, yielding temperatures of the order of $KT_2 \simeq 0.5\,\mathrm{keV}$. This simply expresses the fact that the distribution function of secondary ions as obtained from transport theories is found to be conformally invariant with respect to solar distance, expressing the fact that ratios of moments of this distribution function like $P_2/\rho_2 \sim T_2$ are constant.

Primary solar wind ions, on the other hand, at least at moderate distances r $\leq$ 20 AU, do not behave isothermal, but are nevertheless, non-adiabatic (see Gazis et al., 1994; Richardson et al., 1995; or Whang, 1998). As shown in VOYAGER-1/2 and PIONEER-11 data primary solar wind ion temperatures T_1 fall off with distance in a non-adiabatic manner inside of 20 AU and then stay rather constant at distances beyond, though their temperatures there definitively remain much below those of the secondary ions, i.e. $T_1 \leq T_2$. This clearly demonstrates that heat sources are operating at the expansion of both fluids, which, however, are different in magnitude for the two different ion species, with the energy injection rate to secondary ions obviously being greater.

In the following part of the paper we study these different heat sources and want to show that the different behaviours with solar distance r of the two temperatures $T_1(r)$ and $T_2(r)$ can thereby be explained.

2 Heating sources for the two-fluid solar wind

We consider heat sources acting upon primary and secondary species of solar wind ions at their propagation to large solar distances r. Direct heat conduction from secondary keV-energetic ions, or pick-up ions, to primary solar wind ions via Coulomb collisions can thereby safely be excluded at distances beyond 0.3 AU. The only immaginable and physically likely mechanism to heat solar wind protons of primary and secondary species is due to dissipation of MHD turbulent wave energy, as already anticipated by Parker (1964) and Coleman (1968), who expected that some extended heating due to dissipation of waves might cause a non-adiabatic expansion of the solar wind to regions beyond its critical point. The non-adiabatic temperature behaviour, in fact, is clearly manifest for the primary solar wind ions in data taken by the VOYAGER-1/2 spacecraft (see Richardson et al., 1995; Whang, 1998; Whang, Lu and Burlaga, 1999). Concering secondary ions, it has been seen in spectrometric measurements of plasma-analysers on board of ULYSSES and SOHO that high-energy shoulders are developping with increasing distance in their distribution functions which clearly show the action of a wave-driven energy diffusion process, also often denoted as Fermi-2 acceleration.

A quantitative study of the dissipation of turbulence energy to solar wind protons was carried out by Matthaeus et al. (1994), showing that wave energy is absorbed with a rate $q_{turb} \simeq \rho_s u^3/l$, where ρ_s, u, l are the solar wind mass density, the rms turbulent fluctuation speed, and the turbulent correlation scale. The dependence of these quantities u and l with distance r was investigated by Zank, Matthaeus and Smith (1996), describing the evolution of low-frequency turbulence power in the solar wind and of fluctuation amplitudes u and b about the mean solar wind bulk velocity V_w and the mean magnetic field B. The authors thereby took into account nonlinear dissipation terms and power sources, considering terms due to wave-driving by velocity shears associated with solar wind interaction regions and due to pick-up ions injected into unstable distribution functions. They demonstrated that the usual WKB approximations deviate far from what can be expected in the solar wind at large distances. Dislocated from solar wind interaction regions (or at higher heliographic latitudes) one does not expect shear-induced turbulent energy, but one should be aware, nevertheless, of pick-up ion induced turbulent energy. Matthaeus et al. (1994), Zank et al. (1998) and Smith et al. (2001) analysed the heating of the distant primary solar wind ions due to absorption of wave turbulent energy. They solve a system of coupled differential equations which describe the evolution with distance of the mean turbulent energy u^2, the correlation length l, and the proton temperature T_1. Comparison of results with VOYAGER data seem to show that the main features are explained by this approach, but the predicted values both for u^2 and the solar proton temperature T_1 at distances beyond 10 Au are too low with respect to VOYAGER-2 data, if heating by pick-up ions is not taken into account. Also due to mixing of high- and low-velocity solar wind structures, a physically straighforward data evaluation is complicated. Furthermore, the fact that some fraction of adiabatically cooled secondary ions copopulate the Maxwell tails of the primary solar wind ions may interfere the interpretation.

Zank, Matthaeus and Smith (1996) and Smith et al. (2001) both assume as explicit distributed sources of turbulence pick-up ions, solar wind shear structures and shocks, and imposed at 1 AU a background fluctuation and correlation length scale which are based on observations. They describe MHD-waves by a common power spectrum characterized by two parameters, i.e. the turbulence level and the correlation length; in the approach we are presenting below we consider convected background MHD-waves and MHD waves locally generated by pick-up ions as two independent, superimposed contributions to the turbulence power spectrum, resulting in two additive heating sources. We use as a distance-dependent quantity the so-called outer scale $l_0 = 2\pi/k_0$, similar to the correlation length used by Smith et al. (2001), for the sake of characterizing the evolution of the background turbulence and of describing its contribution to the heating. In our approach here this outer scale increases monotonically with increasing solar distance. The main reason for treating the pick-up ion induced heating source separate from the one connected with the background turbulence is due to the fact that the wave number $k_i \simeq \Omega/V_w$ of maximum power generation by pick-up ions (see Huddleston and Johnstone, 1992) is only one order of magnitude smaller than the main dissipation wave number $k_{dis} = \Omega/v_A$, whereas the difference between k_i and k_0 amounts to more than three orders of magnitude.

2.1 A: Convected turbulences in a two-fluid solar wind

The solar wind transports turbulent energy distributed over a wide range of wave numbers k show a so-called “flicker” spectrum at the smallest wave numbers, up to a critical wave number k_0 (i.e. energy containing range), and a typical inertial spectrum from the critical wave number upwards to the solar proton dissipation wavenumber $k_{dis} \cong \Omega / v_A$. Here, Ω denotes the proton gyrofrequency, and v_A is the Alfvén velocity.

The spectrum of this convected MHD turbulence can be given by:
$W_k(k)=Dk^{-\eta}$ for wave numbers: $k =\leq k_0$ and
$W_k(k) = Dk_0^{\lambda-\eta}k^{-\eta}$ for wave numbers: $k =\geq k_0$,
where D is a structure constant. Here, $0 \leq \eta \leq 1$ is the spectral index of the so-called "flicker" spectrum, and λ is the spectral index of the inertial spectrum, i.e. $\lambda = 5/3$ for the Kolmogorov turbulence, or $\lambda = 3/2$ for the Kraichnan turbulence.

We consider now the locally constant wave-power flux in the inertial branch of the spectrum, as given by Zhou and Matthaeus (1990) or by Tu and Marsch (1995)

$$\Phi_k(r) = -D_{kk}\frac{\partial}{\partial k}W_k =$$

$$-\lambda C_{kk} k v_A (4\pi k W_k / B^2)^{\mu}(kW_k), \tag{1}$$

where D_{kk} is the nonlinear wave-wave diffusion coefficient for isotropic turbulence, $C_{kk} \approx 0.1 = const$, B is the static background magnetic field, and with $\mu = 1/2$ (or $\mu = 1$) for the Kolmogorov turbulence (or for the Kraichnan turbulence), respectively. This flux $\Phi_k(r)$, given by Eq. (1), represents a constant in the non-dissipative range, the magnitude of which can simply be found by evaluating the above equation at the critical wave number k_0, where the wave power is given by

$$W_k(k_0) = Dk_0^{-\eta}. \tag{2}$$

This then leads to the following value of the wave-power flux :

$$\Phi_k(k = k_0, r) = \lambda C_{kk} v_A k_0^{1+(\mu+1)(1-\eta)} D^{\mu+1}(4\pi/B^2)^{\mu}. \tag{3}$$

In order to further evaluate this expression we make use of the expected radial dependence of the critical wave number k_0 and the average field fluctuation energy. From Chashei and Shishov (1982) and Chashei (1986), requiring that at k_0 the rates of linear processes and nonlinear interactions should be equal, we then first obtain

$$V_w k_0 W_k(k_0) \approx r\Phi_k(k_0), \tag{4}$$

where V_w is the solar wind speed. Combining Eqs. (1) through (3), one can thus find the following relation for the turbulence critical scale k_0:

$$k_0^{1+\mu(1-\eta)} = (V_w/C_{kk} r v_A)(B^2/4\pi D)^{\mu}. \tag{5}$$

Taking into account Eq. (5) we can now evaluate $\Phi_k(k_0)$ given in Eq. (3):

$$\Phi_k(k_0) = (V_w/r)Dk_0^{1-\eta}. \tag{6}$$

Furthermore we consider the radial dependence of the critical wave number $k_0(r)$ and of the structure constant $D(r)$. For that purpose we assume, that based on Parker's field B and a spherically symmetric solar wind expansion geometry at distances $r \geq 1$ AU, the following relations are valid: $B^2 = B_0^2(r_0/r)^2$, $v_A = B/(4\pi\rho_w)^{1/2} = v_{A0} = const$.

Here, ρ_w denotes the total proton density given by $\rho_w = \rho_1 + \rho_2$. The structure constant D is proportional to the wave number average of the magnetic field fluctuation power, i.e. to $< \delta B^2 / 4\pi >$, and in the low frequency spectral range is dominated by linear processes, thus

$$D(r)/D(r_0) =< \delta b_{lin}^2(r) > / < \delta B^2(r_0) >, \tag{7}$$

where $< \delta B_{lin}^2(r) > = D(r)\ k_0^{1-\eta}(r_0)$. We assume below that

$$D(r) = D(r_0)(r_0/r)^2 \tag{8}$$

at distances $r \geq 1$ AU with the following arguments.

It is well known that the wave vectors of Alfvén waves have a tendency to approach the radial direction at increasing distances from the Sun (Völk and Alpers, 1973; Hollweg, 1975). However, the Alfvén waves propagating at large angles to the quasi-azimuthal ambient magnetic field become unstable with respect to an excitation of magnetosonic waves (Malara and Elaoufir, 1991; Grappin, Velli and Mangeney, 1993). If the magnetic field disturbances are mainly connected with Alfvén waves then their energy density has to fall off as $< \delta B^2(r)/4\pi >_A \sim r^{-3}$ in the WKB approach at distances $r \geq 1$ AU, while the radial dependence would be weaker for the fast magnetosonic waves, rather falling off as $< \delta B^2(r)/4\pi >_F \sim r^{-2}$. The radial dependence of the magnetic field fluctuations adopted in Eq. (8) for regions $r \geq 1AU$ was also supported by Jokipii and Kota (1989) and by Zank, Matthaeus and Smith (1996).

It is now thought that slab turbulence may perhaps only present a minor component (about 20%) of the solar wind fluctuations, whereas a major component (about 80%) of the fluctuation power may reside in 2-D turbulence. The basic theory for the propagation of this type of turbulence was developed by Zank and Matthaeus (1992), but it is too complicated to draw from it simple conclusions for our purposes here. For these reasons we may expect that the local MHD turbulence at $r \geq 1$ AU represents an undefined, but radially variable mixture of Alfvén and magnetosonic waves, with $< \delta B^2 >_A \approx < \delta B^2 >_F$ assumed as being valid at small distances, but the fast, magnetosonic waves finally being responsible for the successive radial transport of turbulent energy at larger distances. In this case the Eq. (7) is valid, and the radial dependence of $k_0(r)$ in Eq. (5) is defined only by the first factor, since the second factor turns out to be a constant, i.e. $B^2/\ D = const$. Assuming, futhermore, that $< \delta B_0^2/4\pi > \approx G_0\rho_0 v_{A0}^2$ with $G_0 = const \leq 1$ being the fractional turbulence level at $r = r_0 = 1$ AU, and, consequently, $k_0(r_0) \approx V_w\ /\ (v_{A0}\ r_0)$, we finally obtain:

$$\Phi_k(k = k_0, r) = \Phi_0(r/r_0)^{-s} \tag{9}$$

with:

$$\Phi_0 = \Phi_k(k = k_0, r_0) = G_0(V_w B_0^2)/(4\pi r_0) \tag{10}$$

and:

$$s = [3 + 3(\mu + 1)(1 - \eta)]/[1 + \mu(1 - \eta)]. \tag{11}$$

In the special cases of Kolmogorov ($\mu = 1/2$) turbulence or Kraichnan ($\mu = 1$) turbulence, we thus have from Eq. (11): $s = (11 - 5\eta)/(3 - \eta)$ at μ=1/2, or $s = (7 - 4\eta)\ /\ (2 - \eta)$ at μ=1. In the most interesting case of the low frequency "flicker" spectrum with $\eta = 1$ (Matthaeus and Goldstein, 1986 ; Horbury and Balogh, 2001) the radial profile of Eq. (9) does not depend on the turbulence model, and in both cases leads to $s = 3$. A decrease in η up to $\eta = 0$ in the case of a flat spectrum leads to an increase in s in Eq. (11), up to an extreme value of $s = 11/3$ in the case of the Kolmogorov turbulence and up to $s = 7/2$ in the case of the Kraichnan turbulence. In any case, the radial decrease of $\Phi_k(k = k_0, r)$ given in Eq. (9) is considerably slower than that for purely Alfvénic turbulences when $s > 4$.

The turbulent energy $\Phi_k(k = k_0, r)$ is cascading down to the proton dissipation scale, i.e. to the wave number $k_{dis} = \Omega/v_A$ (Smith et al., 2001), and thereby is absorbed in parts both by primary and secondary solar wind protons. It thus serves as one of two relevant heat sources for these two ion species. It should also be mentioned here that Leamon et al. (1998), using WIND magnetic data, argued that the dissipation scale at heliocentric distances of about 1 AU is possibly connected with the value $k_{dis} \simeq \Omega/v_{th}$, where v_{th} is the ion thermal speed. In the outer heliosphere at $r \gg 1\ AU$ this scale is of the same order as the inverse inertial scale $k_{dis} = \Omega/v_A$, which leaves the possibility of using one of these scales without detailed consideration of the exact dissipation mechanism.

2.2 B: Pick-up ion generated turbulences in the two-fluid solar wind

Secondary ions are produced from ionizations of interstellar neutral atoms in the heliosphere. After creation and pitch-angle isotropization these ions are convected outwards mainly with the solar wind flow constituting a separate suprathermal ion fluid. The thermodynamic action of this energetic ion fluid at its motion towards the outer heliosphere was discussed in more recent papers by Williams, Zank and Matthaeus (1995), Fahr and Rucinski (1999, 2001, 2002) and Smith et al. (2001). As explained in more detail by Fahr and Chashei (2002), it is expected that freshly injected secondary ions excite waves by virtue of their initial distribution function, which is unstable with respect to the excitation of wave power. While immediately after their injection these ions start generating wave power via kinetic instabilities, they,after some first-order pitch-angle isotropization has taken place, then rearrange on longer time scales in velocity space due to Fermi-2 energizations (energy diffusion) by nonlinear wave-particle interaction with wind-entrained wave turbulences (Chalov, Fahr and Izmodenov, 1995, 1997; Fichtner et al., 1996; Le Roux and Fichtner, 1997).

In the following we consider energy gain and loss mechanisms connected with the above-mentioned wave-particle interaction processes and then study the effect of ion-generated turbulent energy on primary protons due to a consecutive absorption of such energy. A purely kinetic description of this situation is not aimed at here, but we simply want to describe the coupled thermodynamics of primary and secondary solar wind protons on the basis of a two-fluid approximation characterized by pressures P_1 and P_2. Hereby it suffices to take into account the total dissipation of turbulent energy to primary and secondary ions at whatever wave number connected with ion-induced wave power. For the highly subsonic primary ions one can assume that turbulent energy cascades down to the main dissipation scale, i.e. up to $k_{dis} \simeq \Omega_p/v_A$. For the marginally sonic secondary ion population dissipation takes place between wave numbers $\Omega_p/V_w \le k_{dis} \le \Omega_p/v_A$.

We assume that secondary ions just after their injection undergo fast pitch-angle scattering from an initial torus configuration in velocity space onto a bi-spherical hemispheric shell configuration (see Huddleston and Johnstone, 1992; Williams and Zank, 1994). Rigorously taken, even in this bi-spherical, but infinitely thin shell mode these ions have not yet attained a strictly stable distribution function, but still may continue on longer time scales to generate some more turbulent energy (see, e.g. Lee and Ip, 1987; Freund and Wu, 1988), a process which, however, is hardly quantifyable. Thus, it is not taken into account in our following calculations. Instead, we only take into account the free energy of the freshly injected secondary ions pumped into the turbulent wave field until they arrive at a quasi-static, bi-spherical distribution.

At heliospheric regions beyond 5 AU the tilt of the magnetic field $\boldsymbol{B}$ with respect to the radial solar wind flow direction (especially in the ecliptic) becomes nearly orthogonal. Thus, the associated bi-spheres centered around the reference systems of upgoing and downgoing (i.e. with respect to $\boldsymbol{B}$) Alfvén waves become symmetrically populated. Under these conditions at the event of injection the new secondaries have an ion velocity v, which in the Solar Wind (SW) frame is equal to the solar wind velocity V_w. Consequently, after pitch-angle scattering to the accessible bi-spheres (only a loss of energy is permitted!) within each of these fractional shells for upstream and downstream waves, one expects the following pick-up ion velocity v as judged in the SW frame:

$$v^2 = v_A^2 + (V_w^2 + v_A^2) - 2v_A\sqrt{V_w^2 + v_A^2}\cos\vartheta, \tag{12}$$

where ϑ is the pitch angle of the resulting velocity v in the upstream wave frame. Since the newly generated secondary ions quickly become randomly distributed on the accessible spherical shells, the distribution function for the populated velocities v is then simply given by the associated velocity space volume $\Delta v = vd\cos\vartheta(v)$, i.e. is given by the expression:

$$f(v) = \frac{-d\cos\vartheta(v)}{1-\cos\vartheta_{\max}}, \tag{13}$$

where the maximum possible pitch angle $\vartheta_{\max}$ is simply given by:

$$\cos\vartheta_{\max} = \frac{v_A}{\sqrt{V_w^2+v_A^2}}. \tag{14}$$

With expression (13) one obtains the mean-squared velocity $\left\langle v^2(\vartheta)\right\rangle$ of the bi-spherically distributed pick-up ions by the following expression:

$$\left\langle v^2(\vartheta)\right\rangle = \frac{1}{1-\frac{v_A}{\sqrt{V_w^2+v_A^2}}}\times$$

$$\int_{\cos\vartheta_{\max}}^{1}\left[v_A^2+(V_w^2+v_A^2)-2v_A\sqrt{V_w^2+v_A^2}X\right]dX, \tag{15}$$

which easily evaluates to the following expression:

$$\left\langle v^2(\vartheta)\right\rangle = V_w^2\left[1-\frac{v_A}{\sqrt{V_w^2+v_A^2}}\right]+2v_A^2 \le V_w^2. \tag{16}$$

This now permits to account for the fact that the initial energy $\Xi_i = \frac{1}{2}m_pV_w^2$ of freshly injected secondary ions, after a first violent period of wave-driving, is then reduced to an average energy $\epsilon_i = \frac{1}{2}m_p\left\langle v^2(\vartheta)\right\rangle$. This also means, on the other hand, that the energy $\Delta\epsilon_i = \Xi_i - \epsilon_i$ is pumped into the ambient turbulent wave field, mainly at or around the injection wave number $|k_i| \simeq \Omega_p/V_w$ (see the analysis carried out by Huddleston and Johnstone, 1992; and Williams and Zank, 1994). The loss of free energy to the wave fields at the occasion of the redistribution from the initial torus configuration onto the bi-spherical configuration is thereby properly taken into account.

Now we may assume that this energy input $\Delta\epsilon_i$ into the turbulent wave power produces a local power peak at k_i with decreasing spectral powers on the left and the right spectral side of this injection wave number. In our view it can be expected that due to nonlinear wave-wave interactions, i.e. diffusion in k-space, this energy input cascades both up and down from k_i roughly at equal parts, like in the case of an oscillator that couples its enforced oscillations to nearby oscillator modes by coupling strengths only dependent on $[k_i^2-k^2]^{-2}$. This means that without further dissipation processes about one half would cascade to smaller wave numbers, where strongly Doppler-shifted secondary ions still can resonantly interact and here may contribute to ongoing Fermi-2 accelerations of energetic secondary ions to even higher energies (see Chalov, Fahr and Izmodenov, 1995), and the other half, to the opposite, would cascade up to larger wave numbers until it finally is absorbed by primary solar wind protons at the dissipation wave number $k_{dis} = \Omega_p/v_A$. One could also take into account the process of perpendicular cascading which has been seen in many simulations and also lies at the core of works published by Zank, Matthaeus and Smith (1996), Matthaeus et al. (1994) or Smith et al. (2001).

The question is really how the typical time periods for diffusion in k-space and for power dissipation to protons compare with each other. For the typical diffusion period τ_{kk}, one can derive from the one-dimensional k-space diffusion equation the following value:

$$\tau_{kk} \simeq \frac{(k_{dis}-k_i)^2}{4D_{kk}} = (1-\frac{v_A}{V_w})^2\frac{\Omega^2}{4D_{kk}v_A^2}, \tag{17}$$

where the diffusion coefficient D_{kk}, according to Zhou and Matthaeus (1990), can be given in the following form:

$$D_{kk} = Cv_Ak^{7/2}\sqrt{\frac{W_k(k)}{\left(B_0^2/4\pi\right)}} \tag{18}$$

and where the spectral power at k_i for stationary conditions can be estimated to be given by:

$$W_k(k_i) \simeq \beta_{ex}\Delta\epsilon_i\tau_{kk}/(k_{dis}-k_i) \tag{19}$$

and thus leads to:

$$\tau_{kk}^{3/2} \simeq \frac{(1-\frac{v_A}{V_w})^2}{4Cv_Ak^{3/2}\sqrt{\frac{\beta_{ex}\Delta\epsilon_i}{(B_0^2/4\pi)}}}. \tag{20}$$

Now we study the process competing with k-number diffusion which is the dissipation of wave power to ambient protons. For this purpose we follow the analysis carried out by Gray et al. (1996) which is fully applicable here and shows that on the basis of the Alfvén ion cyclotron instability, the absorption time period τ_{dis} can be estimated with:

$$\tau_{dis} \simeq 40\cdot\Omega^{-1}. \tag{21}$$

To decide which of the above periods is larger requires prior knowledge of the injection rate β_{ex}, which we, however, at this part of the paper want to keep as an unfixed quantity and therefore we postpone the decision as to which of these time periods is the larger one.

We consequently take into account two possible versions of a distribution of free energy between the two ion populations. We assume that the energy input to primary and secondary ions due to absorption of ion-induced wave energy is equally split into two halves and hence, is given by:

$$Q_{1,2}(r) = \frac{1}{2}\beta_{ex}(r)\Delta\epsilon_i =$$

$$\frac{1}{4}\beta_{ex}(r)m_p(V_w^2\frac{v_A}{\sqrt{V_w^2+v_A^2}}-2v_A^2), \tag{22}$$

if the case $\tau_{kk} \le \tau_{dis}$ can be faced. On the other hand, if the case $\tau_{kk} \ge \tau_{dis}$ has to be faced, then the turbulent energy input to primary and secondary protons, respectively, is split according to the relative abundances $\xi_{1,2}$ of these proton species and hence, one would obtain:

$$Q_{1,2}(r) = \frac{1}{2}\beta_{ex}(r)\Delta\epsilon_i \xi_{1,2}. \tag{23}$$

In the two above equations $\beta_{ex}(r)$ denotes the local charge exchange rate of solar wind protons and interstellar H-atoms also describing the local rate of pick-up proton injections. This rate is given by:

$$\beta_{ex}(r) = n_w(r) n_H(r) \sigma_{ex} V_w, \tag{24}$$

where n_w and n_H are the local proton and H-atom densities, respectively, σ_{ex} is the charge exchange cross section and V_w is the solar wind velocity.

For the upwind hemisphere the following approximative, so-called "cold" representation of the H-atom density can be used here (see Fahr, 1971, Axford, 1972):

$$n_H(r,\Theta) = n_{H\infty} \exp\left(-\frac{\beta_0 r_0^2 \Theta}{V_{H\infty} r \sin\Theta}\right), \tag{25}$$

where Θ denotes the off-wind angle, $\beta_0 \simeq n_{w0}\sigma_{ex} V_w$ is the ionization frequency of H-atoms at the reference distance r_0, and where $V_{H\infty}$ is the bulk velocity of the inflowing interstellar H-atoms at large distances.

For applying formula (25) to the upwind axis with an off-axis inclination angle $\Theta = 0$, one simply considers this formula for the limit $\Theta \to 0$ and then obtains:

$$n_H(r, \Theta = 0°) = n_{H\infty} \exp\left(-\frac{n_{w0}\sigma_{ex} V_w r_0^2}{V_{H\infty} r}\right). \tag{26}$$

For the downwind axis, i.e. for $\Theta = \pi$, one has, however, to take into account the interstellar H-atom temperature $T_{H,\infty}$ and here, when solar gravity is compensated by solar H-Lyman-Alpha radiation pressure, one can use the following approximative representation:

$$n_H(r, \Theta = \pi) = n_{H\infty} \exp\left(-\frac{\beta_0 r_0^2 \Theta^t}{V_{H\infty} r \sin\Theta^t}\right) \tag{27}$$

with the angle Θ^t given by:

$$\Theta^t = \pi - \mathrm{arctg}\left(\sqrt{\frac{2KT_{H\infty}}{m_p V_{H\infty}^2}}.\right) \tag{28}$$

Coming back to the Eq. (22) one can simplify the expression for Q_i by setting:

$$Q_{1,2}(r) = \frac{1}{2}\beta_{ex}(r)\Delta\epsilon_i = \frac{1}{4} m_p . V_w^2 \epsilon_f. \tag{29}$$

Here, the following shorthand notation for the factor ϵ_f has been introduced:

$$\epsilon_f = \frac{\mu}{\sqrt{1+\mu^2}} - 2\mu^2 \simeq \mu - 2\mu^2 - \frac{1}{2}\mu^3 \simeq \mu_0 (1 - 2\mu_0), \tag{30}$$

where $\mu = v_A/V_w \simeq \mu_0 = v_{A0}/V_{w0} \ll 1$ was used.

3 Thermodynamics of the wave-heated two-fluid solar wind

We first formulate the coupled system of enthalpy flow conservation equations to describe the thermodynamics of the two-fluid solar wind plasma, consisting of primary and secondary protons in terms of pressures P_1 and P_2, taking into account the effects of adiabatic cooling and of heating both by the diffusive energy flux $\Phi_k(k = k_0, r)$ in the convected turbulence spectral power and by locally generated turbulent energy $Q_{1,2}$ locally pumped into the wave field by freshly injected secondary ions. For these pressures we then obtain the following two coupled equations (see also Fahr and Chashei, 2002):

$$div\left(\frac{\gamma}{\gamma-1} P_1 \boldsymbol{V_w}\right) - (\boldsymbol{V_w} \cdot grad) P_1 =$$

$$-\frac{3}{2}\xi_1 \beta_{ex}(KT_1) + Q_{b1}(r) + Q_{i1}(r), \tag{31}$$

$$div\left(\frac{\gamma}{\gamma-1} P_2 \boldsymbol{V_w}\right) - (\boldsymbol{V_w} \cdot grad) P_2 =$$

$$\xi_1 \beta_{ex}(1-\epsilon_f)\frac{1}{2} m_p V_w^2 + Q_{b2}(r) + Q_{i2}(r), \tag{32}$$

where, at the application to the case $\tau_{kk} \geq \tau_{dis}$ which first is adopted here, according to Eqs. (20) and (21), the following definitions were used:

$$Q_{b1,2}(r) = \xi_{1,2}\Phi_k(k = k_0, r), \tag{33}$$

and:

$$Q_{i,1,2}(r) = \frac{1}{2}\xi_{1,2}\beta_{ex}(r) m_p V_w^2 \epsilon_f. \tag{34}$$

While the first term on the right-hand side of Eq. (31) describes the removal of thermal energy from the primary ion fluid due to charge exchange reactions with H-atoms, the first term on the right-hand side of Eq. (32) describes the simultaneous input of kinetic energy to the secondary ion fluid due to freshly injected ions. Furthermore, K is the Boltzmann constant, and m_p is the proton mass. The quantities $\xi_{1,2}$ are the relative abundances of primary or secondary ions in the two-fluid solar wind, respectively, defined by:

$$\xi_{1,2} = \frac{n_{1,2}}{n_1 + n_2}. \tag{35}$$

These abundances can be found with the help of the following continuity equations:

$$div(n_2 \boldsymbol{V_w}) = -div(n_1 \boldsymbol{V_w}) = \xi_1 \beta_{ex} \tag{36}$$

and are given by (see Fahr and Rucinski, 1999):

$$\xi_1 = \frac{n_1}{n_1 + n_2} = \exp\left[-\int_{r_0}^{r} \sigma_{ex} n_H' dr'\right] \tag{37}$$

and

$$\xi_2 = \frac{n_2}{n_1 + n_2} = 1 - \exp\left[-\int_{r_0}^{r} \sigma_{ex} n_H' dr'\right]. \quad (38)$$

The two above expressions (37) and (38) can be simplified for regions $r \geq r_0 \geq 5AU$ with the assumption $n_H \simeq const \simeq n_{H\infty}$ and then yield:

$$\xi_1 = \exp\left[-\Lambda\left(\frac{r}{r_0} - 1\right)\right] \quad (39)$$

and:

$$\xi_2 = 1 - \exp\left[-\Lambda\left(\frac{r}{r_0} - 1\right)\right] \quad (40)$$

with the denotation $\Lambda = n_{H\infty}\sigma_{ex} r_0$.

The left-hand sides of the above Eqs. (31) and (32) describe the divergences of the primary and secondary proton enthalpy flows and the work done by the pressures P_1 and P_2 at the expansion of the two plasmas.

Below we will now make use of the assumption $V_w \simeq V_{w0} \gg v_A \simeq v_{A0}$, valid at distances $r \geq 1$ AU. We further assume that the two components of the solar wind plasma can be considered as one-atomic ion gases, suggesting $\gamma_1 = \gamma_2 = 5/3$. Then, Eqs. (31) and (32), when reduced to equations for the temperatures $T_{1,2}$ by setting $P_{1,2} = \xi_{1,2} n_w K T_{1,2}$ with $n_w = n_{w0}(r_0/r)^2$, allow one to find the following equations for T_1 and T_2, respectively:

$$\frac{d}{dx}T_1 + \frac{4}{3}\frac{T_1}{x} = T_s\left[G_0 x^{2-s} + g_0 \exp\left(-\frac{\Lambda_1}{x}\right)\right] \quad (41)$$

and:

$$\frac{d}{dx}T_2 + \frac{4}{3}\frac{T_2}{x} + \frac{\beta_{ex} T_2 r_0}{n_2 V_w} =$$
$$\frac{\beta_{ex} m_p V_w r_0 (1-\epsilon_f)}{3 n_2 K} + T_s\left[G_0 x^{2-s} + g_0 \exp\left(-\frac{\Lambda_1}{x}\right)\right], \quad (42)$$

where we have introduced the following new denotations: $x = r/r_0$; $T_s = 2m_p v_A^2/(3K)$; $\Lambda_1 = \Lambda(V_w/V_{H\infty})(n_0/n_{H\infty})$ and: $g_0 = \Lambda(V_w/2v_A)(1 - 2v_A/V_w)$.

It should be noted that the assumptions made to obtain Eqs. (33) and (34) describe the energy equipartition per ion of the available energy sources to primary and secondary ions. This consequently then requires as evident that the right-hand side of Eq. (41) is equal to the second term on the right-hand side of Eq. (42).

3.1 The primary solar wind ion fluid

Equation (41) which describes the distance-dependence of the temperature T_1 of primary solar wind ions unsurprisingly turns out to be identical with the equation already found and analysed by Fahr and Chashei (2002) and thus, its solutions in some respects can already be found by the reader there. Here, we only want to investigate solutions for the temperatures $T_1(x)$ which one should expect to find under different solar wind conditions, specified by associated different solar wind bulk velocities V_w and Alfvén velocities v_A.

First, we intend to calculate upwind temperature profiles $T_1(x)$, however, for different solar wind velocities V_w for the purpose of comparing them with data shown by Gazis et al. (1994) and Richardson et al. (1995). These data show monthly averages of T_1- and V_w, observations obtained with PIONEER 11 and VOYAGER 1/2 in the upwind heliosphere and clearly reveal a strong, positive correlation of temperature and velocity measurements, with higher temperatures occuring at higher bulk velocities. Since temperature fluctuations occuring at some distance x, as those shown in many figures of papers by Williams, Zank and Matthaeus (1995) or by Smith et al. (2001), are mainly due to associated fluctuations in solar wind bulk velocities V_w, we therefore want to show in Fig. 1 temperature profiles $T_1(x)$ calculated on the basis of Eq. (41), however, for various solar wind velocities V_w.

For that purpose we take into account the direct dependence on V_w of the quantities Λ_1 and g_0, as well as the indirect dependence on V_w of the quantities T_s, g_0 and $G_0 = G_{00} n_{00} v_{A00}^2/(n_0 v_{A0}^2)$ via dedendence on v_A. As a consequence of Parker's spiral field producing an increasing azimuthal field component B_ϕ, dependent on the solar wind velocity, with increasing distance, one therefore first obtains as a parametrization with respect to V_w the following approximate relation $B \simeq B_\phi(r, V_{w0})(V_{w0}/V_w)$. For the Alfvén velocity one consequently obtains:

$$v_A = v_{A00}\frac{V_{w0}}{V_w}\sqrt{\frac{n_{00}}{n_0}}, \quad (43)$$

where v_{A00} denotes the Alfvén reference velocity at 1 AU for the solar wind reference conditions: i.e. bulk velocity V_{w0} = 400 km/s ; proton density n_{00} = 10 cm^{-3}, proton temperature $T_{w00} = 4 \cdot 10^4$ K.

For the relation between solar wind bulk velocity and density we base our calculations on the measurements published by McComas et al. (2000) which show both a slight decrease in the solar wind mass flow and a pronounced increase in the solar wind temperature with increasing values of V_w. According to these results in the range 400$\cdot$ km/s$\leq V_w \leq$ 800$\cdot$ km/s the following relations appear to be fulfilled for the solar wind density n_0 and temperature T_{w0}, respectively, at 1AU (see Richardson et al., 1995, McComas et al., 2000):

$$n_0 = \left[n_{00}\frac{V_{w0}}{V_w} - 2.5\left(1 - \frac{V_{w0}}{V_w}\right)\right] cm^{-3} \quad (44)$$

and:

$$T_{w0} = T_{w00}\left(\frac{V_w}{V_{w0}}\right)^2. \quad (45)$$

In the following we shall discuss the most important results of the above theoretical derivations. For that purpose we first select appropriate values for the relevant

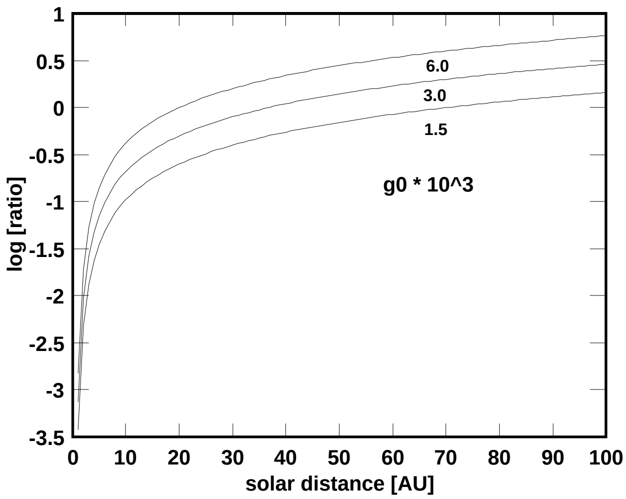

Fig. 1. Shown is the ratio $R_{i/b} = Q_{i1}/Q_{b1}$ as a function of the solar distance in units of AU for various values of $g_0 = 1.5$; 3.0; 6.0 $\cdot 10^{-3}$. The other relevant parameters in the calculations hereby are taken to be equal to $G_0 = 0.1$ and $s = 9/3$.

parameters occuring in the calculations. With $n_{H\infty} = 0.05\,\mathrm{cm}^{-3}$ (see, e.g. Izmodenov, 2000, Fahr, 2000) and σ_{ex}(440km/s)=$2{\cdot}10^{-15}\mathrm{cm}^2$ one obtains for the standard value of $\Lambda = n_{H\infty}\sigma_{ex} r_0$=$1.5{\cdot}10^{-3}$. With n_{w0}=7 cm^{-3} and V_w=440 km/s (see Whang, 1998) one obtains the critical ionization distance by $\Lambda_1 \cdot r_0 = \Lambda \cdot (n_{w0} V_w / n_{H\infty} V_{H\infty}) \cdot r_0 =$ 3.7 AU. The parameter $g_0 = \Lambda \cdot (V_w / 4 v_A)$ for these standard values thus amounts to $g_0 = 3.3 \cdot 10^{-3}$, adopting the Alfvén speed with v_A=50 km/s. As the standard value for $G_{00} = \langle \delta B^2 / 4\pi \rangle / (\rho_0 v_A^2)$ we may take here $G_{00} = 0.1$. Using these above relations and adopting the values $V_{H\infty}$=25km/s; $s = 9/3$; and $G_{00} = 0.1$, we find the ratio of primary ion heating by pick-up ion induced - over convected - turbulence power, i.e. $R_{i/b} = Q_{i1}/Q_{b1}$, as shown in Fig 1. and the temperature profiles $T_1(x, V_w)$ as shown by Fig 2.

As one can see in Fig. 1 heating by pick-up ion induced turbulence, depending on the value for the quantity g_0, i.e. for Λ, starts to dominate over heating by convected turbulences from distances of 15 AU to 50 AU outwards.

Inspection of Fig. 2 shows that the temperatures T_1 first fall off rapidly with increasing solar distances r. Then in the region between 20 through 30 AU they reach a minimum and beyond they start even to slightly increase with increasing distance. The sensitivity of the temperatures T_1 to the actual solar wind bulk velocities V_w thereby is fairly mild, however, clearly recognizable. As we can show by comparing the results of Fig. 1 with 50-day averages of VOYAGER-2 temperature data (see Gazis et al., 1994) our theoretical curves nicely guide the observed fluctuating temperature values in the ranges 5 AU through 15 AU and 25 AU through 40 AU whereas in the region in between the measured temperature values for some reason seem to be fairly on the low side of the theoretical values. The T_1-fluctuations, nevertheless, are influenced by associated fluctuations of the velocities V_w. For the latter to be in fact nearly quantitatively true there, however, is a clear need that only a minor fraction ϵ_f as derived in Eq. (30) of the initial pick-up ion energy originally incorporated into the two-fluid solar wind flow finally and exclusively be transfered to the primary solar wind ions. Factors $\epsilon_f \geq 0.1$ appear to be clearly ruled out from the data, as was already recognized earlier by Williams, Zank and Matthaeus (1995).

The velocity-dependence of $T_1(x)$ emphasized in the results of Fig. 2 also becomes especially interesting in view of the fact that, at least under solar minimum conditions, strong systematic variations of the solar wind velocity with latitude have been recognized (see McComas et al., 2000, 2002). This invites us here to study in a little more detail the latitudinal temperature $T_1(\vartheta)$-variation to be expected from this fact. In order to also be thereby based as much as possible on experimental data we start again from ULYSSES plasma measurements published by McComas et al. (2000), representing typical solar minimum conditions of the heliosphere. As obtained from the first full polar orbit passage of ULYSSES the essential solar wind parameters scaled to a reference distance of r_0 =1 AU, like bulk velocity V_{w0}, density n_{w0}, proton temperature $T_{1,0}$, have been obtained as functions of solar latitude angle ϑ.

Concerning the bulk velocity $V_w(\vartheta)$ one can derive the following best-fitting analytic representation of these data:

$$V_w(\vartheta) = V_{ws} + \frac{V_{wf} - V_{ws}}{2}$$

$$[1 - \tanh(\alpha_c(\cos\vartheta - \cos\vartheta_c)] \,, \tag{46}$$

where V_{ws}=380 km/s and V_{wf}=760 km/s are the observed wind speeds for slow and fast solar wind, respectively, and where $\vartheta_c = 40°$ and $\alpha_c = 45$ characterize the critical latitude and the rapidity of the change from slow to fast solar wind.

The solar wind density n_{w0} can be represented by the following analytic function:

$$n_{w0}(\vartheta) = n_{ws} + \frac{n_{wf} - n_{ws}}{2}.$$

$$[1 - \tanh(\alpha_c(\cos\vartheta - \cos\vartheta_c)] \,, \tag{47}$$

where the values n_{ws}=7 cm^{-3} and n_{wf}=3 cm^{-3} have to be used as typical densities for the slow and the fast solar wind, respectively. For the temperature $T_{1,0}$ of the primary solar wind protons one analogously obtains from the ULYSSES data when scaled to r_0=1 AU by a factor $(1/r)$ (for justification see McComas et al., 2000), the following representation:

$$T_{1,0}(\vartheta) = T_{1s} + \frac{T_{1f} - T_{1s}}{2}.$$

$$[1 - \tanh(\alpha_c(\cos\vartheta - \cos\vartheta_c)] \,, \tag{48}$$

with the typical temperature values $T_{1s} = 7{\cdot}10^4$ K and $T_{1f} = 3 \cdot 10^5$ K for the slow and the fast solar wind, respectively.

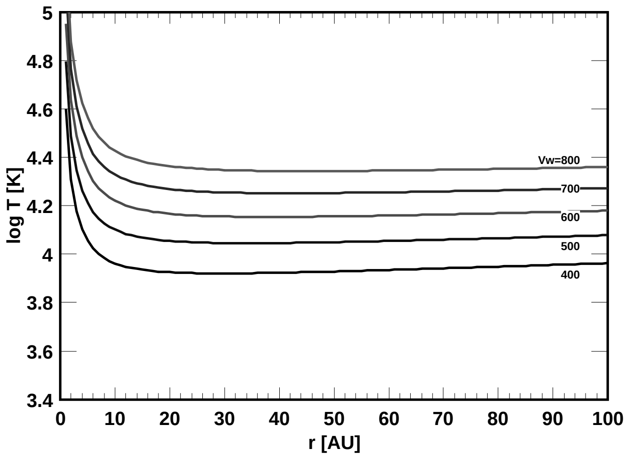

Fig. 2. Shown versus solar distance r along the upwind axis is the temperature T_1 of primary solar wind ions in units of [Kelvin] for various solar wind bulk speeds V_w. Consistent with the variation of V_w, solar wind density, the Alfvén speed and the energy input parameters g_0 and G_0 have also been changed according to prescriptions derived in the text.

From Eq. (41) one now derives the following solution:

$$T_1(x,\vartheta) = x^{-4/3}$$

$$[T_{1,0}(\vartheta) + T_S(\vartheta)(I_b(x,\vartheta) + I_i(x,\vartheta))], \tag{49}$$

where $T_s(\vartheta)$ is given by:

$$T_s(\vartheta) = 2m_p v_A^2(\vartheta)/(3K)$$

with:

$$v_A(\vartheta) = v_{A0}\frac{B_0(\vartheta)}{B_0(\vartheta=0)}\sqrt{n_{w0}(\vartheta=0)/n_{w0}(\vartheta)}.$$

Adopting Parker's spiral magnetic field solution one obtains for the ratio of the magnetic fields:

$$\frac{B_0(\vartheta)}{B_0(\vartheta=0)} = \frac{V_w(\vartheta=0)\cos\vartheta}{V_w(\vartheta)}.$$

Furthermore, the expressions $I_b(x,\vartheta)$ and $I_i(x,\vartheta)$ in Eq. (49) are defined by:

$$I_b(x,\vartheta) = \frac{3G_0(\vartheta)}{13-3s}\,(x^{(13-3s)/3}-1) \tag{50}$$

and:

$$I_i(x,\vartheta) = g_0(\vartheta)\int_1^x \varsigma^{4/3}\exp(-\Lambda_1(\vartheta)/\varsigma)d\varsigma \tag{51}$$

with the following definitions:

$$\Lambda_1(\vartheta) = \Lambda\frac{\vartheta}{\sin\vartheta}\frac{V_w(\vartheta)}{V_{H\infty}}\frac{n_{w0}(\vartheta)}{n_{H\infty}} \tag{52}$$

and

$$g_0 = \Lambda\frac{V_w(\vartheta)}{2v_A(\vartheta)}(1-2\frac{v_A(\vartheta)}{V_w(\vartheta)}) \tag{53}$$

and

$$G_0(\vartheta) = \left|\frac{\langle\delta B_0^2/4\pi\rangle}{\rho_{w0}v_{A0}^2}\right|_\vartheta = \left\langle\frac{\delta B_0^2}{B_0^2}\right\rangle_\vartheta . \tag{54}$$

The latter assumption is similar to the argumentation given by Zank, Matthaeus and Smith (1996). Here it has to be mentioned that Eq. (52) is only correct in that plane containing both the upwind axis and the solar polar axis, since only then the latitude angle ϑ and the off-axis angle Θ are identical and with Eq. (25) lead to the given expression for $\Lambda_1(\vartheta)$. For regions outside of this plane the angles ϑ and Θ are different from each other, which then would slightly complicate the expression for $\Lambda_1(\vartheta)$ given by Eq. (52).

To evaluate the expression for $G_0(\vartheta)$ given in Eq. (54), one may study observational results taken by ULYSSES on magnetic field fluctuations and published by Horbury and Balogh (2001). From these results one can draw the conclusion that the dependence on the latitude angle ϑ in the relevant range of turbulence scales is sufficiently weak. For this reason we adopt here in this study $G_0(\vartheta) = G_{00} = const.$

Now we want to use Eq. (49) to calculate the primary proton temperature $T_1(r,\vartheta)$, but prior to doing so one should keep in mind the fact that this equation concerning the effect of a heating due to dissipated convected turbulences (see Sect. 4.1) was derived for heliospheric conditions only valid beyond some critical distance r_c at which the tilt Ψ of the Archimedian magnetic field with respect to the radial direction becomes larger than $tg\Psi = B_a/B_r \geq 1$. This critical distance on the basis of the Parker spiral field solution evaluates to an expression given by:

$$r_c(\vartheta) = \frac{V_w(\vartheta)}{\Omega_s\cos\vartheta}$$

where Ω_s is the angular speed of the solar rotation. For latitudes $\vartheta \geq \vartheta_c = 20°$ the above relation roughly yields the following result:

$$r_c(\vartheta) \simeq \frac{V_w(\vartheta=0)}{\Omega_s}\frac{V_w(\vartheta_c)}{V_w(\vartheta=0)\cos\vartheta} \simeq$$

$$\frac{760}{380}\frac{r_0}{\cos\vartheta} = \frac{2AU}{\cos\vartheta}.$$

In order to be able to rely on data which were taken by ULYSSES at $r \leq 5AU$ one should thus make sure that only latitudes are considered at which $r_c(\vartheta) \leq 5$ AU, meaning that with the above derivations, in principle, we are restricted to latitudes $\vartheta \leq 65°$. In the following we thus make use of Eq. (41), however, starting from an inner boundary with $r_5(\vartheta)$=5 AU. At this boundary we obtain the observationally supported temperature $T_{1,5}(\vartheta)$ using relation (48) but rescaling temperatures to 5 AU with the scaling law $T_{1,5}(\vartheta) = (1/5)T_{10}(\vartheta)$ applied by the authors of the paper by McComas et al. (2000).

In Fig. 3 we have shown the temperatures $T_1(r,\vartheta)$ obtained with Eq. (49) as a function of solar distance for different solar latitudes ϑ. As can be seen there, within latitudes

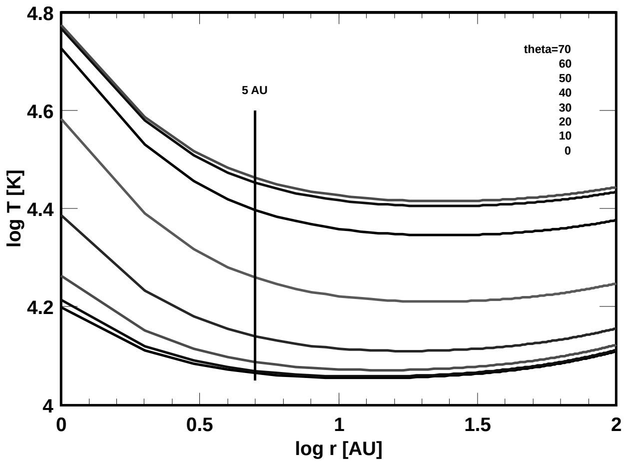

Fig. 3. Shown versus solar distance r is the temperature T_1 of primary solar wind ions in units of [Kelvin] for various heliographic latitudes, i.e. for $\vartheta = 0, 10, 20, 30, 50, 60, 70$ degrees. Consistent with the variation of ϑ, solar wind velocity, density, the Alfvén speed and the energy input parameters g_0 and G_0 have been changed according to prescriptions derived in the text. The initial boundary values for T_1 at 5 AU have been taken from McComas et al. (2002).

$0° \leq \vartheta \leq 20°$ temperatures are nearly insensitive to latitude and distance. However, then with increasing latitudes $30° \leq \vartheta \leq 60°$ there appears a fairly strong latitude dependence. As a hint for a better understanding of the data it is perhaps important to recognize that the latitudinal gradient in T_1 is much more pronounced than the radial gradient, at least during solar minimum conditions.

3.2 The secondary solar wind ion fluid

Now we want to investigate the temperature T_2 of secondary protons, but for that purpose werestrict again our considerations to the ecliptic, i.e. keeping $\vartheta = \Theta = 0$. Using Eqs. (36), (49) and (50) the Eq. (48) is then represented in the form:

$$\frac{dT_2}{dx} + \frac{4}{3}\frac{T_2}{x} + \Lambda\frac{\xi_1}{\xi_2}\exp\left(-\Lambda_1/x\right) T_2 =$$

$$\Lambda\frac{\xi_1}{\xi_2}\exp\left(-\Lambda_1/x\right)\left(\frac{m_p V_w^2(1-\epsilon_f)}{3K}\right) +$$

$$+T_S\left[G_0 x^{2-s} + g_0\exp\left(-\Lambda_1/x\right)\right], \tag{55}$$

where the relative abundances ξ_1 and ξ_2 of primary protons and secondary protons are given by Eqs. (39) and (40).

The ratio R_h^{ex} of the first and the third term on the right-hand side of Eq. (55) evaluates to:

$$R_h^{ex} = \frac{\left[\Lambda\frac{\xi_1}{\xi_2}\exp\left(-\Lambda_1/x\right)\left(\frac{m_p V_w^2}{3K}\right)(1-\epsilon_f)\right]}{\left[g_0 T_S\exp\left(-\Lambda_1/x\right)\right]} =$$

$$(2\xi_1 V_w)\,/\,(\xi_2 v_A) \gg 1, \tag{56}$$

and thus expresses the fact that heating of secondary ions by damping of self-induced waves can tacticly be neglected in comparison with the direct energy injection connected with charge exchange induced injection of new secondary ions. This does not imply that Fermi-2 acceleration of secondary ions, as discussed in Chalov, Fahr and Izmodenov (1995, 1997) or Fichtner et al. (1996), is unimportant, but it has to be seen as connected with energy absorption from convected turbulences (see the term on the right-hand side of Eq. (55) affiliated with the factor G_0).

In the following we first investigate the asymptotic behaviour of the secondary ion temperature T_2 with solar distance at heliocentric distances, where $100 \leq x > \Lambda_1 \gg \Lambda$ is valid. In this range we can, on the one hand, neglect both self-induced and background heating sources on the right-hand side of Eq. (55) (see also Fahr and Chashei, 2002), but, on the other hand ,one can expand Eqs. (39) and (40) for small values $\Lambda x \ll 1$ and can easily find that $\xi_1/\xi_2 \approx 1/(\Lambda x)$. Thus the asymptotic form of Eq. (55) is obtained by:

$$\frac{dT_2}{dx} + \frac{7}{3}\frac{T_2}{x} = \frac{T_{ex}}{x}, \tag{57}$$

where $T_{ex} = m_p V_w^2(1-\epsilon_f)\,/\,3K$ is the injection temperature corresponding to the energy imported into the secondary ion fluid by freshly injected secondary ions immediately after the charge exchange process and the incorporation into the solar wind bulk has occured. Equation (57) has the following solution

$$T_2^{as} = x^{-7/3}\left(T_{20} + \frac{3}{7}x^{7/3}T_{ex}\right). \tag{58}$$

Here T_{20} is the secondary ion temperature at some inner heliocentric reference distance $r_{20} \geq 5AU$ beyond which Eq.(57) is valid. This boundary temperature most probably is determined by turbulent heat sources as those discussed above.

One can see from the solution 58 that, if these turbulence-induced heating sources are neglected completely, then the secondary ion temperature is approaching from above the constant value $T_{2\infty} = m_p V_w^2(1-\epsilon_f)/7\mathrm{K}$ which is determined by a balance between the energy input due to locally generated secondary ions and the quasi-adiabatic cooling at the motion to larger distances. This asymptotically isothermal behaviour of the secondary ions, interestingly enough, was already anticipated as a well suggested fact from kinetic studies and even was used as a suggested assumption in the paper by Fahr (2002b). In view of Eq. (58) the asymptotic value $T_{2\infty}$ can be refered to as the minimal possible temperature of secondary or pick-up ions which can result under specific solar wind conditions, i.e. given by corresponding values of V_w and v_A.

A value for the secondary proton temperature at small solar distances of $r_0 \simeq 1\,\mathrm{AU}$ can also be easily found from

Eq. (48) by setting, in view of the vanishingly small abundances of secondary protons there, $\xi_1/\xi_2 \gg 1$. Using this result one simply obtains:

$$T_{20} = \frac{m_p V_w^2 (1-\epsilon_f)}{3K} \simeq \frac{7}{3} T_{2\infty}. \tag{59}$$

The general profile of the secondary ion temperature $T_2(x)$ now has to be found from the general solution of Eq. (55) given by:

$$T_2(x) = \exp[-\int_5^x A_2(\acute{x}) d\acute{x}] \times$$

$$\left\{ T_{20} + \int_5^x B_2(x') \exp[+\int_5^{x'} A_2(x'')dx'']dx' \right\}. \tag{60}$$

The lower border x_{20}=5 AU has been selected because here the boundary value for T_{20} can still well be taken from relation (59) and from here outwards the approximations (39) and (40) are valid. In the above expression the following functions $A_2(x)$ and $B_2(x)$ were introduced:

$$A_2(x) = \frac{4}{3x} + \frac{\Lambda \exp(-\Lambda(x-1))}{1-\exp(-\Lambda(x-1))} \exp(-\frac{\Lambda_1}{x}) \tag{61}$$

and:

$$B_2(x) = T_{20} \frac{\Lambda \exp(-\Lambda(x-1))}{1-\exp(-\Lambda(x-1))}$$

$$\exp(-\frac{\Lambda_1}{x}) + T_s G_0 x^{2-s}, \tag{62}$$

where the expression $T_{20} = m_p V_w^2(1-\epsilon_f)/3K$ developed in Eq. (59) has been used again here.

In Fig. 4 we have shown the temperature $T_2 = T_2(x)$ calculated as a function of the solar distance $x \geq x_{20} = 5AU$ along the upwind axis for different values of V_w, taking into account the associated functions $n_w(V_w)$, $v_A(V_w)$, $\Lambda_1(V_w)$ as given by Eqs. (44), (43) and (45). As it can be seen in Fig. 3 the temperatures T_2 at all distances are clearly related to the solar wind bulk velocity and at larger solar distances behave with distance fairly isothermal, as was already pointed out earlier in papers by Fahr (2002a, b). In view of the latitudinal solar wind velocity gradient, which is evident at solar activity minimum conditions, analogous to the results shown in Figure 3, we can thus in view of Fig. 4 also expect that strong latitudinal gradients in the temperatures T_2 should be present. The secondary ion fluid definitely will show higher temperatures T_2 at higher heliographic latitudes.

As one can clearly see in Fig. 4 the secondary ion temperatures $T_2(x)$ show a pronounced reaction to the solar wind bulk speeds V_w with initial and asymptotic values being pure functions of V_w and with the change-over profile being determined by other competing parameters like the values for g_0, G_0, v_A and s.

Finally, we only want to mention briefly here that at fairly large ion production rates β_{ex}, the comparison of diffusion time periods given by Eq. (30) and dissipation time periods given by Eq. (20) may tell that diffusion becomes the more effective process, meaning that one can expect the free energy of freshly injected secondary ions to be distributed by equal amounts to primary and secondary protons, and not according to the relative abundances. For this alternative version of the heating model presented by us here, one can thus assume that the wave energy generated by freshly injected secondary ions passes by equal amounts to primary and secondary protons. In this case the heating source in Eqs. (48) and (49) should be modified accordingly, i.e. the factor g_0 should be changed to $(1/2)g_0\,(\xi_1/\xi_2)$. However, this modification does not change the asymptotic solution (59) because similar to Eq. (50) the ratio $R_{ex/h}$ now remains quite large, which again yields: $R_{ex/h} = 2V_w/v_A \gg 1$.

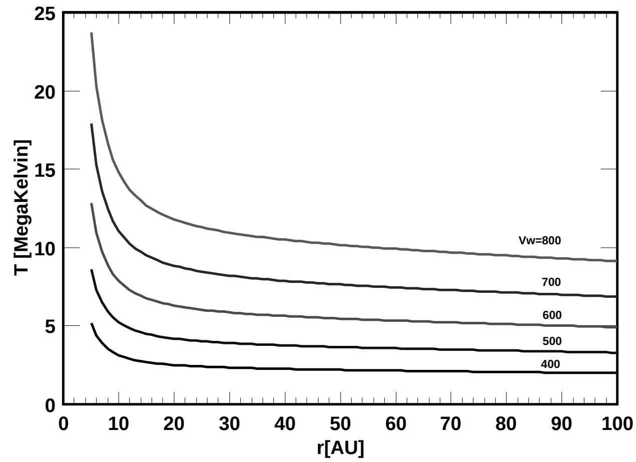

Fig. 4. Shown versus solar distance r along the upwind axis is the temperature T_2 of secondary solar wind ions in units of [Mega Kelvin] for various solar wind bulk speeds V_w. Consistent with the variation of V_w, solar wind density, the Alfvén speed and the energy input parameters g_0 and G_0 have also been changed according to prescriptions derived in the text.

4 Concluding remarks

In the following we shall discuss the most important results of the theoretical derivations presented in the aforegoing sections. We have described a two-fluid solar wind plasma consisting of primary and secondary (i.e. pick-up) solar wind ions. The two fluids are thermodynamically coupled by their enthalpy source terms, i.e. dissipation of convective turbulent energy and pick-up ion induced turbulent energy. As a result one can see in Fig. 2 that the primary solar wind ion temperatures T_1, even at small distances, do not fall off adiabatically, but polytropically, with distance-variable polytropic indices. They are also noticeably dependent on the solar wind speeds V_w, and thus also a clear latitude dependence of T_1 as shown in Fig. 3, has to be expected at least for solar minimum conditions. The secondary ion fluid temperature T_2, as shown in Fig. 4, reflects a clear dependence on the solar wind speed

V_w, demonstrating a quasi-isothermal behaviour at distances beyond 30 AU. Though not very many observations of pick-up ions in the outer heliosphere are at the present time available, which could enlighten their thermal behaviour, it nevertheless turns out that at large distances these ions definitely appear to be energized and show suprathermal tails (Krimigis and Decker, 2001, Decker et al., 2002, Gloeckler, 2003),thus supporting our theoretical conclusions.

As shown in Figs. 2 and 3 the effect of heating due to absorption of convected and self-generated turbulences is such that at larger distances in all cases shown the polytropic index falls below 1, reflecting a slight temperature increase with the expansion of the solar wind to larger distances. This latter result differs from a similar study carried out by Fahr (2002a/b) where only a drop-off of the effective polytropic index $\gamma_{eff}\ (x)$ with distance x to asymptotic values of $\gamma_{eff}(x \to \infty) \simeq 1.1 \geq 1$ was obtained. This study by Fahr (2002a, b) was, however, based on two assumptions: first, it was asssumed that pick-up ions when being convected outwards with the solar wind strictly behave isothermal, and second, that the two-fluid solar wind plasma, consisting of original solar wind protons and pick-up ions, only have available the kinetic energy of freshly injected pick-ups as their joint energy source. Here we instead obtain the quasi-isothermal behaviour of pick-up ions as a result of the consistent two-fluid thermodynamics and we have taken into account that solar wind protons not only profit from the pick-up ion induced heat source, but also from convected turbulence energy absorbed by them.

From VOYAGER-1/2 and PIONEER-11 data maybe no clear indication of a temperature increase with distance in the upwind hemisphere can be gained, but it nevertheless seems to be a well established fact that proton temperatures do hardly fall off, if at all, in the region between 20 to 40 AU, but rather stay constant there (see Gazis et al., 1994). This seems to be nicely represented by the curves shown in Figs. 2 and 3, where it is only evident that the level of the constant temperature is variable, perhaps depending on the level of turbulence parametrized by G_0 or the solar wind bulk speed V_w. The theoretical values of T_1 for heliocentric distances larger than 10 AU given in Fig. 4 are also in reasonably good agreement with the corresponding values found by Gazis et al. (1994) with VOYAGER-1/2 and PIONEER-11.

Finally, it should be mentioned here that our analysis presented in this paper, as those carried out earlier by authors like Zank, Matthaeus and Smith (1996) or Smith et al. (2001), is based on the assumption that the solar wind bulk velocity V_w is constant with solar distance in the outer regions of the heliosphere. Though this is not too far from being the truth (see, e.g. observations by Richardson et al., 1995, or predictions by Fahr and Rucinski, 1999, 2001), one nevertheless should bear in mind that this assumption in order to be fulfilled requires a specific relation of the secondary ion pressure gradient ∇P_2 and the momentum loading of the solar wind pick-up ions. Inspection of the equation of motion for the two-fluid solar wind tells one that a vanishing of the bulk velocity gradient $d/dr(V_w)$ requires that forces due to the action of the secondary ion pressure gradient and due to pick-up ion momentum loading just have to cancel each other (see Fahr, 2002b), meaning that:

$$\nabla P_2 = \rho_w V_w^2 \sigma_{ex} n_H \qquad (63)$$

should be fulfilled. Otherwise, if Eq. (63) is not fulfilled, for a decelerating solar wind, some additional terms Γ_1 and Γ_2 should be included in Eqs. (31) and (32) given by:

$$\Gamma_{1,2} = \frac{\gamma}{\gamma - 1} P_{1,2} \frac{d}{dr} V_w. \qquad (64)$$

In this respect the analysis of this paper is still not fully consistent, since in the two-fluid thermodynamics presented here the solar wind dynamics was inconsistently prescribed by the requirement that $V_w = const$.

Acknowledgements. This work was partially funded by a NATO Scientific Research Grant (CLG number: 69-5109) and by a binational cooperation project numbered 00-02-04022 jointly sponsored by the Russian Foundation of Basic Research (RFBR) and the Deutsche Forschungsgemeinschaft (DFG). The authors are also grateful to financial support granted in the frame of the project Fa 97/26-1 by the Deutsche Forschungsgemeinschaft. The authors greatly appreciate the useful comments made by C. W. Smith and another referee.

Topical Editor R. Forsyth thanks C. W. Smith and another referee for their help in evaluating this paper.

References

Axford, W. I.: The interaction of the solar wind with the interstellar medium, in Solar Wind, NASA SP-308, 609–658, 1972.

Chalov, S. V. and Fahr, H. J.: Entropy generation at the multifluid solar wind termination shock producing anomalous cosmic ray particles, Planet. Space Sci., 43, 1035, 1995.

Chalov, S. V. and Fahr, H. J.: The three-fluid structure of the particle-modulated solar wind termination shock, Astron. Astrophys., 326, 860, 1997.

Chalov, S. V., Fahr, H. J., and Izmodenov, V.: Spectra of energized pick-up ions upstream of the heliospheric termination shock: The role of Alfvénic turbulences, Astron.Astrophys., 304, 609, 1995.

Chalov, S. V., Fahr, H. J., and Izmodenov, V.: Spectra of energized pick-up ions upstream of the 2-d termination shock: Alfvénic and large scale magnetosonic turbulences, Astron. Astrophys., 320, 659, 1997.

Chashei I. V.: The acceleration of the solar wind by Alfvén waves, Geomagnetizm and Aeron., 26, 486, 1986.

Chashei I. V. and Shishov, V. I.: On the energy source of the solar wind turbulence, Geomagnetizm and Aeron., 22, 529, 1982.

Coleman, P.: Turbulence, viscosity and dissipation in the solar wind plasma, Astrophys. J., 153, 371, 1968.

Decker, R. B., Krimigis, S. M., Armstrong, T. P., Mosley, C. J., Hamilton, D. C., and Gloeckler, G.: Observations of low-energy oxygen at VOYAGERS 1 and 2, Advances in Space Research, COSPAR Houston/Texas, in press, 2002.

Fahr, H. J.: The interplanetary hydrogen cone and its solar cycle variations, Astron. Astrophys., 14, 263, 1971.

Fahr, H. J.: Nonthermal solar wind heating by suprathermal ions, Solar Physics, 30, 193, 1973.

Fahr, H. J.: The multifluid character of the Baranov interface, Astrophys. Space Sci., 274, 35, 2000.
Fahr, H. J.: Global energy transfer from pick-up ions to solar wind protons, Solar Physics, 208, 335, 2002a.
Fahr, H. J.: Solar wind heating by an embedded quasi-iosthermal pick-up ion fluid, Ann. Geophys., 20, 1509, 2002b.
Fahr, H. J. and Chashei, I. V.: On the thermodynamics of MHD wave-heated solar wind protons, Astron. Astrophys., 395, 991–1000, 2002.
Fahr, H. J. and Fichtner, H.: The influence of pick-up ion-induced wave pressures on the dynamics of the mass-loaded solar wind, Solar Physics, 158, 353, 1995.
Fahr, H. J. and Rucinski, D., Neutral interstellar gas atoms reducing the solar wind Mach number and fractionally neutralizing the solar wind, Astron. Astrophys., 350, 1071, 1999.
Fahr, H. J. and Rucinski, D.: Modification of properties and dynamics of the distant solar wind due to its interaction with the neutral interstellar gas, Space Sci.Rev., 97, 407, 2001.
Fahr, H. J. and Rucinski, D.: Heliospheric pick-up ions influencing thermodynamics and dynamics of the distant solar wind, Nonlinear Processes in Geophys., 9, 377, 2002.
Fahr, H. J., Ripken, H. W., and Lay, G., Plasma – dust interactions in the solar vicinity and their observational consequences, Astron. Astrophys., 102, 359, 1981.
Fichtner, H., le Roux, J. A., Mall, U., and Rucinski, D.: On the transport of pick-up ions in the heliosphere, Astron. Astrophys., 314, 650, 1996.
Fisk, L. A., Gloeckler, G., Zurbuchen, T. H., and Schwadron, N. A.: "Ubiquitous statistical acceleration in the solar wind", in: ACE Symposium 2000, (Ed) Mewaldt, R. A., American Institue of Physics, pp. 229–233, 2000.
Freund, H. P. and Wu, C. S.: Stability of spherical shell distributions of pick-up ions, J.Geophys.Res., 93, 14 277, 1988.
Gary, S. P. and Feldman, W. C.: A second-order theory field-parallel electromagnetic instabilities, Phys. Fluids, 21, 7, 1978.
Gazis, P. R., Barnes, A., Mihalov, J. D., and Lazarus, A. J.: Solar wind velocity and temperature in the outer heliosphere, J. Geophys. Res., 99, 6561, 1994.
Gloeckler, G.: Ubiquitous suprathermal tails in solar wind and pick-up ion distributions, SOLAR WIND 10, Pisa/Italy, 2003, in press.
Grappin, R., Velli, M., and Mangeney, A.: Nonlinear wave evolution in the expanding solar wind, Phys. Rev. Lett., 70, 2190, 1993.
Gray, P. C., Smith, C. W., Matthaeus, W. H., and Otani, N. F.: Heating of the solar wind by pickup ion driven Alfvén ion cyclotron instability, Geophys. Res. Lett., 23, 113, 1996.
Hartle, R. E. and Wu, C. S.: Effects of electrostatic instabilities on planetary and interstellar ions in the solar wind, J. Geophys. Res., 78, 5802, 1973.
Hollweg, J. V.: Alfvén wave propagation in the solar wind, Rev. Geophys. Space Phys., 13, 263, 1975.
Holzer, T. E.: Interaction of the solar wind with theneutral component of the interstellar gas, J. Geophys. Res., 77, 5407, 1972.
Holzer, T. E. and Leer, E.: Solar wind heating by secondary ions, Astropys. Space Sci., 24, 335, 1973.
Horbury T. S. and Balogh, A. J.: Evolution of magnetic field fluctuations in high-speed solar wind streams: ULYSSES and HELIOS observations, J. Geophys. Res., 106, 15 929, 2001.
Huddleston, D. E. and Johnstone, A. D.: Relationship between wave energy and free energy from pick-up ions in the comet Halley environment, J. Geophys. Res., 97, 12 217, 1992.
Isenberg, P. A.: Interaction of the solar wind with interstellar neutral hydrogen: Three fluid model, J. Geophys. Res., 91, 9965, 1986.
Isenberg, P. A.: Evolution of interstellar pick-up ions in the solar wind, J. Geophys. Res., 92, 1067, 1987.
Izmodenov, V. V.: Physics and gasdynamics of the interface plasma, Astrophys. Space Sci., 274, 55, 2000.
Jokipii, J. R. and Kota, J.: The polar heliosphere magnetic field, Geophys. Res. Lett., 16, 1, 1989.
Kennel, C. F. and Engelmann, F.: Velocity space diffusion from weak plasma turbulence in a magnetic field, Phys.Fluids, 9, 2377, 1996.
Krimigis, S. M. and Decker, T. B.: Observations of pick-up ions in the outer heliosphere by VOYAGER 1 and 2 and implications on pressure balance, in: COSPAR Colloquia Series Vol.11, "The Outer Heliosphere", (Eds. K.Scherer et al., 229, 2001.
Leamon, R. J., Smith, C. W., Ness, N. F., Matthaeus, W. H., and Wong, H. K.: Observational constraints on the dynmics of the interplanetary magnetic field dissipation range, J. Geophys. Res., 104, 22 331, 1998.
Lee, M. A.: "Effects of cosmic rays and interstellar gas on the dynamics of a wind", in: Stellar Winds, (Eds) Jokopii, J. R., Sonett, C. P. and Giampapa, M., Tucson, Arizona, 857–886, 1997.
Lee, M. A. and Ip, W. H.: Hydromagnetic wave excitation by ionized interstellar hydrogen and helium in the solar wind, J. Geophys. Res., 92, 11 041, 1987.
Le Roux, J. A. and Fichtner, H.: A selfconsistent determination of the heliospheric termination shock structure in the presence of pick-up, anomalous and galactic cosmic ray protons, J. Geophys. Res., 102, 17 365, 1997.
Malara, F. and Elaoufir, J.: Oblique propagation of nonlinear hydromagnetic waves: One- and two-dimensional behavior, J. Geophys. Res., 96, 7641, 1991.
Matthaeus, W. H. and Goldstein, M. L.: Low frequency noise in the interplanetary magnetic field, Phys. Rev. Lett., 57, 495, 1986.
Matthaeus, W., Oughton, D., Pontius, D., and Zhou, Y.: Evolution of energy-containing turbulent eddies in the solar wind, J. Geophys. Res., 99, 19 267, 1994.
McComas, D. J., Barraclough, B. L., Funsten, H. O., Gosling, J. T., Santiago-Munoz, E., Skoug, R. M., Goldstein, B. E., Neugebauer, M., Riley, P., and Balogh, A.: Solar wind observations over ULYSSES' first full polar orbit, J. Geophys. Res., 105, 10 419, 2000.
McComas, D. J., Elliott, H. A., Gosling, J. T., Reisenfeld, D. B., Skoug, R. M., Goldstein, B. E., Neugebauer, M., and Balogh, A.: ULYSSES' second fast-latitude scan: Complexity near solar maximum and the reformation of polar coronal holes, Geophys. Res. Lett., 29, 9, 2002.
Parker, E. N.: Dynamical properties of stellar coronae and stellar winds, Astrophys. J., 139, 93, 1964.
Richardson, J. D., Paularena, K. I., Lazarus, A. J., and Belcher, J. W.: Radial evolution of the solar wind from IMP 8 to Voyager 2, Geophys. Res. Lett. 22, 325, 1995.
Smith, C. V., Matthaeus, W. H., Zank, G. P., Ness, N. F., Oughton, S., and Richardson,J.: Heating of the low latitude solar wind by dissipation of turbulent magnetic fluctuations, J. Geophys. Res., 106, 8523, 2001.
Tu, C. Y. and Marsch, E.: MHD-structures, waves and turbulence in the solar wind: Observations and theory, Space Sci. Rev., 73, 1, 1995.
Völk, H. J. and Alpers, W.: The propagation of Alfvén waves and their directional anisotropy in the solar wind, Astrophys. Space Sci., 20, 267, 1973.

Whang, Y. C.: Solar wind in the distant heliosphere, J. Geophys. Res., 103, 17 419, 1998.

Whang, Y. C., LU, J. Y., and Burlaga, L. F.: The termination shock: 1979–1995, J. Geophys. Res., 104, 28 255, 1999.

Williams, L. L. and Zank, G. P.: The effect of magnetic field geometry on the wave signature of the pick-up of interstellar neutrals, J. Geophys. Res., 99, 19 229, 1994.

Williams, L. L., Zank, G. P., and Matthaeus, W. H.: Dissipation of pick-up ion induced waves : A solar wind temperature increase in the outer heliosphere, J. Geophys. Res., 100, 17 059, 1995.

Zank, G. P. and Matthaeus, W. H.: Waves and turbulences in the solar wind, J. Geophys. Res., 97, 17 189, 1992.

Zank, G. P. and Pauls, H. L.: Shock propagation in the outer heliosphere. I: Pick-up ions and gas dynamics, J. Geophys. Res., 102, 7037, 1997.

Zank, G. P., Matthaeus, W. H., and Smith, C. W.: Evolution of turbulent magnetic fluctuation power with heliospheric distance, J. Geophys.Res., 101, 17 093, 1996.

Zank, G. P., Matthaeus, W. H., Smith, C. W., and Oughton, S.: Heating of solar wind beyond 1 AU by turbulent dissipation, in: Solar Wind Nine, (Eds) Habbal, S., Esser, R., Hollweg, J. V., and Isenberg, P. A., AIP-CP 471, 523–527, 1998.

Zhou, Y. and Matthaeus, W. H.; Transport and turbulence modeling of solar wind fluctuations, J. Geophys. Res., 95, 14 881, 1990.