Annales Geophysicae (2002) 20: 1553–1558 © European Geosciences Union 2002

Annales Geophysicae

Ducted compressional waves in the magnetosphere in the double-polytropic approximation

I. Ballai[1,2], **R. Erdélyi**[1], **and B. Roberts**[2]

[1]Atmosphere Research Center (SPARC), Dept. of Applied Mathematics, University of Sheffield, Hicks Building, Hounsfield Road, Sheffield, S3 7RH, England, UK

[2]School of Mathematics and Statistics, University of St Andrews, St Andrews, Fife, KY16 9SS, Scotland, UK

Received: 10 October 2001 – Revised: 1 May 2002 – Accepted: 28 May 2002

Abstract. Small-amplitude compressional magnetohydrodynamic-type waves are studied in the magnetosphere. The magnetosphere is treated as a rarefied plasma with anisotropy in the kinetic pressure distribution. The parallel and perpendicular pressures are defined by general polytropic pressure laws. This double-polytropic model can be considered as a natural extension of the magnetohydrodynamic (MHD) model when the plasma is collisionless.

Generalized dispersion relations for surface and body waves are derived and analyzed for an isolated magnetic slab. The waves are confined to the slab. For specific polytropic indices, the results obtained in the (i) Chew-Goldberger-Low (CGL) double-adiabatic and (ii) double-isothermal approximations are recovered.

Key words. Magnetospheric physics (MHD waves and instabilities; plasma sheet; plasma waves and instabilities)

1 Introduction

The study of wave dynamics in inhomogeneous plasmas is of fundamental interest in solar and astrophysical plasmas. Waves are important in their own right since they reflect the stable dynamic behaviour of the objects they occur in. They are also important because they transport momentum and energy. When part of their momentum and energy is dissipated, they can heat and accelerate the plasma. Finally, they can be used as probes for investigating the structure and composition of the plasma in which they are observed.

In a normal gas, collisions between the particles ensure that they have the same temperature, irrespective of type; collisions provide a mechanism to propagate pressure and temperature changes, and dissipation in the form viscosity is a form of collision, which also ensure that the equilibrium distribution of particles speeds is Maxwellian. The plasmas that are found in the extended solar atmosphere and solar wind, in planetary magnetospheres and in interstellar space (excluding cold dense molecular clouds), are very different from an ordinary gas, being collisionless. Having in mind what collisions can introduce in a plasma, we can easily see what the absence of collisions will produce. Different types of particles can have different temperatures. The particle distribution function can be very different from Maxwellian. The important role of magnetic fields in plasmas also means that the distribution function may no longer be isotropic in velocity space.

In the Earth's magnetosphere the mean free path for particles is long compared to other dimensions in the plasma. When the cyclotron frequency is much larger than the collisional frequency, the particles gyrate many times around a line of magnetic force between collisions. The presence of the magnetic field will induce a split in the pressure, introducing parallel and perpendicular components. These components of the pressure are not necessarily equal. However, thermal anisotropy and any strong electron conduction velocity (relative to the ions) generally excite plasma oscillations that scatter the particles, pulling the thermal motions toward isotropy ($p_\parallel \sim p_\perp$). Thus, for instance, in the high-speed tenuous solar wind, the anisotropic expansion maintains a limited but measurable thermal anisotropy. On the other hand, no significant change has been found for wave propagation in the solar corona (e.g. Ballai et al., 2002). In the case of the magnetosphere, for a typical number density ($10^7\,\mathrm{m}^{-3}$) and temperature ($10^6\,\mathrm{K}$), we obtain that the ion collisional time is of the order of 5×10^7 s, so a collisionless plasma means that waves have periods shorter than 5×10^7 s, i.e. waves with a frequency larger than 20 nHz.

The main complication arising from the presence of anisotropy is the fact that changes in anisotropy, or equivalently in pitch-angle scattering, are by definition a kinetic effect, and their proper description within fluid theory remains a fundamental but unresolved problem in plasma physics. Technically speaking, the closure of fluid equations requires the introduction of two equations of state, one for parallel and the other one for perpendicular pressure. In spite of much effort,

Correspondence to: I. Ballai (i.ballai@sheffield.ac.uk)

there is no generally accepted form for the two equations of state.

In such a medium, waves that are generated primarily by the free energy of the particle distribution functions are the means to relax this free energy. Waves, once amplified, can heat particles, permit an exchange of energy between different populations of particles and precipitate magnetospheric particles into the atmosphere.

One approach in the problem of physical properties of a rarefied plasma is the CGL theory developed by Chewet al. (1956). The usual MHD equations are derived from the Boltzmann equation, using an expansion in powers of the collisional mean free path. In this case the plasma is collision-dominated and, therefore, the collisional term in the Boltzmann equation is the leading term, with all other terms being treated as perturbations. When the density is so small that the plasma can be considered as a collisionless medium, a different form of equations can be derived from the Boltzmann equation, using an expansion in power of Larmor radius; here, the role of the collisional term is played by the Lorentz force. This approximation can be considered as an adiabatic one, since it depends on the Larmor frequency being large compared to another frequencies.

The aim of the present paper is to consider the effect of pressure anisotropy on wave propagation in a collisionless plasma, such as that found in the magnetosphere. Compressional waves in the magnetosphere were observed both from ground-based telescopes and in situ measurements (Samson et al. 1991; Lin et al., 1992; Ziesolleck and McDiarmid, 1994; Hughes, 1994; Mann et al., 1998, etc.) Parallel to observations, an intensive analytical research has been carried out in order to explain the origin of these waves and their role in the process of, for example, the resonance of the Earth's geomagnetic field lines (Rickard and Wright, 1994, 1995; Wright, 1994, Taroyan and Erdélyi, 2002).

The paper is organized as follows. In Sect. 2 we first introduce the basic equations used in the present paper, emphasizing the differences between the usual MHD and the equations used to describe rarefied plasmas. Section 2 is also devoted to the study of the possible waves that appear in such structures, and we discuss the limitations of the present model due to instabilities. Finally, in Sect. 2, we derive the dispersion relation for waves in an isolated slab, using a double-polytropic pressure law, and we compare our results with those obtained in collisional isotropic plasmas. The possible modes which can arise in the considered magnetic slab are represented in a phase diagram. Finally, we summarize and discuss our results in Sect. 3.

2 Ducted waves in rarefied plasmas

The starting point for the present discussion is the system of modified equations for an anisotropic rarefied plasma. The plasma motion is described by an ideal single-fluid system, where mass conservation and the induction equation are expressed through the usual equations of MHD. The momentum equation has a similar form, but the scalar kinetic pressure is replaced by a pressure tensor denoted as

$$\boldsymbol{P} = p_{\perp}\hat{\boldsymbol{I}} + (p_{\parallel} - p_{\perp})\boldsymbol{bb},$$

where $\hat{\boldsymbol{I}}$ is the unit dyadic, and $\boldsymbol{b}$ is the unit vector parallel to the magnetic field direction. The energy equations in these two directions are (Hau and Lin, 1995; Ballai et al., 2002)

$$\frac{D}{Dt}\left(\frac{p_{\perp}}{\rho B^{\gamma_{\perp}-1}}\right) = 0, \quad \frac{D}{Dt}\left(\frac{p_{\parallel}B^{\gamma_{\parallel}-1}}{\rho^{\gamma_{\parallel}}}\right) = 0, \tag{1}$$

where

$$D/Dt = \partial/\partial t + \boldsymbol{v}\cdot\nabla\,.$$

Here, $\gamma_{\parallel}$ and $\gamma_{\perp}$ are the parallel and perpendicular polytropic indices, respectively. These indices express the increase in temperature upon plasma compression. For $\gamma_{\perp} = 2$ and $\gamma_{\parallel} = 3$, the usual double adiabatic CGL expressions are recovered, whereas for $\gamma_{\parallel} = \gamma_{\perp} = 1$ we obtain the isothermal limit.

Let us perturb the system and write all quantities in the form $f_0 + f$, where f_0 denotes an equilibrium quantity and f is its Eulerian perturbation. The effect of any steady flow is neglected, so we set $\boldsymbol{v}_0 = 0$. The equilibrium magnetic field, $\boldsymbol{B}_0$, is parallel to the z axis and inhomogeneous in the x direction, i.e. $\boldsymbol{B}_0 = B_0(x)\hat{\boldsymbol{z}}$. We are interested only in small disturbances about this equilibrium, so all products or squares of perturbed quantities are neglected. Since the equilibrium quantities depend on x only, the perturbations can be Fourier-analyzed with respect to the y and z coordinates. Perturbations oscillate with a frequency, ω, so they are of the form $\sim \exp[i\omega t + ik_y y + ik_z z]$. The equilibrium quantities satisfy total pressure balance, viz.

$$\frac{d}{dx}\left(p_{\perp 0} + \frac{B_0^2}{2\mu}\right) = 0. \tag{2}$$

Before we embark on a discussion of wave propagation in a structured plasma, it is convenient to highlight some properties of plasma waves in unbounded media. First of all, slow waves propagate with the speed c_T, modified by the pressure anisotropy and defined through

$$c_T^2 = \frac{c_{S\perp}^2(c_{S\parallel}^2 - c_{S\perp}^2/\gamma_{\perp}^2) + c_{S\parallel}^2 v_A^2}{c_{S\perp}^2 + v_A^2}, \tag{3}$$

where the squares of the Alfvén and sound speeds in the perpendicular and parallel direction are given by

$$v_A^2 = B_0^2/\mu\rho_0, \quad c_{S\perp}^2 = \gamma_{\perp}p_{\perp 0}/\rho_0, \quad c_{S\parallel}^2 = \gamma_{\parallel}p_{\parallel 0}/\rho_0.$$

For the isotropic case (i.e. $c_{S\perp}^2 = c_{S\parallel}^2 = c_S^2$ and $\gamma_{\perp} = 1$), Eq. 3) reduces to the classic cusp speed. However, when $c_{S\parallel}^2 > c_{S\perp}^4/[\gamma_{\perp}^2(v_A^2 + c_{S\perp}^2)]$, c_T^2 can be negative and this condition gives rise to the mirror instability threshold. If this condition is satisfied, the magnetic field develops regions of low field strength separated by regions of enhanced field strength. Where the field is locally strong, particle mirror points shift in such a way that the plasma density decreases. Where the field is locally weak, the plasma density increases. The mirror instability is a purely growing wave with ω purely imaginary. This instability materializes first and grows fastest when the normal to the wave front is oriented nearly perpendicular to the magnetic field lines. If the mirror instability criterion is satisfied, the increase in the destabilizing component of the pressure, $p_{\perp 0}$, that attends an oblique slow mode perturbation exceeds the increase in the restraining tension in the field and in $p_{\parallel 0}$, and the perturbation grows.

The cusp speed exceeds the Alfvén speed only if

$$\beta_\parallel \gamma_\parallel > \beta_\perp \gamma_\perp + 2 - \frac{2\beta_\perp \gamma_\perp}{\beta_\perp \gamma_\perp + 2}, \tag{4}$$

where $\beta_\parallel = 2c_{S\parallel}^2/(\gamma_\parallel v_A^2)$ and $\beta_\perp = 2c_{S\perp}^2/(\gamma_\perp v_A^2)$ are the plasma-beta parameters in parallel and perpendicular directions.

The Alfvén wave propagates with a phase velocity ω/k, where

$$\frac{\omega^2}{k^2} = v_A^{2*} = v_A^2(1-\Gamma)\,. \tag{5}$$

Here, $\Gamma = (p_{\parallel 0} - p_{\perp 0})/c_A^2\rho_0$ is the pressure anisotropy factor. In contrast to the isotropic case, the phase speed of Alfvén waves is determined not only by the magnetic field, but also by the kinetic pressure in the two directions. When $\Gamma = 0$ (i.e. in the isotropic case, $p_{\parallel 0} = p_{\perp 0}$), we recover the usual Alfvén velocity. The effect of pressure anisotropy on the propagation of Alfvén waves is seen through the introduction of the multiplicative factor $(1-\Gamma)$, which may be negative ($p_{\parallel 0} > p_{\perp 0} + B^2/\mu$) and so in this case, the Alfvén mode may exhibit non-propagating, pure exponential growth, whereas in the isotropic case the velocity is positive definite. This instability is called the firehose or ballooning instability. In general, the behaviour of Alfvén waves can be classified according to the sign of $(1-\Gamma)$. Thus, for $(1-\Gamma) > 0$, we have a propagating Alfvén mode (collisional MHD). For $(1-\Gamma) = 0$, we obtain a non-propagating, non-growing perfectly inelastic perturbation. Finally, for $(1-\Gamma) < 0$, we obtain a non-propagating, pure exponential growth, i.e. the firehose instability. Inspecting the expression for the anisotropy factor Γ, we can see that we have to consider the contribution of all three pressures ($p_{\parallel 0}$, $p_{\perp 0}$ and $B_0^2/2\mu$) when we study these waves. In the isotropic case, the frequency is fixed by balancing the inertial force exerted by a volume of plasma that is oscillating transversally to the magnetic field lines, against the magnetic tension. Since an increase in the restoring force increases the frequency, and Γ measures the change in frequency resulting from pressure anisotropy, it is clear that $p_{\perp 0}$ acts to increase the restoring force and $p_{\parallel 0}$ acts to decrease it. When the two pressures are equal (isotropic case), their effects cancel.

We return to the discussion of wave propagation in structured plasmas. Let us consider wave propagation in a magnetically isolated double-polytropic plasma confined in a slab of width x_0. The insertion of two interfaces into a homogeneous medium, so that a uniform duct is formed, results in the introduction of a length scale into the model, and the waves are now dispersive. Our treatment is parallel to the theory developed by Roberts (1981) for an MHD plasma, but here we try to emphasize the effects of pressure anisotropy.

The plasmas inside and outside the slab are characterized by the following equilibrium configuration:

$$\begin{cases} B^{(0)},\ p_\perp^{(0)},\ p_\parallel^{(0)},\ \rho^{(0)}, & |x| < x_0, \\ 0,\quad p^{(e)},\ 0,\ \rho^{(e)}, & |x| > x_0, \end{cases} \tag{6}$$

where the superscripts $'0'$ and $'e'$ denote quantities inside and outside the slab, respectively. The velocity perturbation is assumed to be of the form of

$$\mathbf{v} = (v_x(x), 0, v_z(x))e^{i(\omega t - kz)},$$

so $k_y = 0$ and $k_z = k$. We suppose that waves are essentially confined within the slab inhomogeneity ($|x| < x_0$), being evanescent in x outside the slab (so $|v| \to 0$ as $|x| \to \infty$). It is convenient to introduce the so-called *magnetoacoustic parameters* $m^{(0)}$ and $m^{(e)}$, defined by

$$m^{(0)2} = -\frac{(\omega^2 - k^2c_{S\parallel}^{(0)2})[\omega^2 - k^2v_A^2(1-\Gamma)]}{(c_{S\perp}^{(0)2} + v_A^2)(\omega^2 - k^2c_T^{(0)2})},$$

$$m^{(e)2} = k^2 - \frac{\omega^2}{c_S^{(e)2}}. \tag{7}$$

We assume that $m^{(e)2} > 0$, corresponding to waves being trapped within the slab. According to the sign of $m^{(0)2}$, we can have surface ($m^{(0)2} > 0$) or body ($m^{(0)2} < 0$) modes inside the slab when ω^2 and k^2 are considered real.

Waves propagating in a magnetically isolated slab can be classified according to whether $v_x(x)$ is an even or odd function of x. If $v_x(x)$ is an even function, the waves are kink waves, while an odd function of x corresponds to sausage modes.

Requiring that the transverse velocity component, v_x, and the perpendicular component of the total (gas plus magnetic) pressure perturbation are continuous at the boundaries $x = \pm x_0$, we obtain the dispersion relation for surface waves:

$$[k^2v_A^2(1-\Gamma) - \omega^2] = \omega^2\frac{\rho_e m^{(0)}}{\rho_0 m^{(e)}}\begin{Bmatrix}\tanh\\ \coth\end{Bmatrix} m^{(0)}x_0, \tag{8}$$

describing sausage (tanh) and kink (coth) magnetoacoustic oscillations within the slab.

Due to the transcendental form of Eq. (8) we first solve the dispersion relation in the limit of a slender slab. This means that we use the long wavelength approximation, i.e. the wavelengths are much larger that the width of the slab. For a slender slab ($kx_0 \ll 1$), $\tanh m^{(0)}x_0 \approx m^{(0)}x_0$ and the dispersion equation for sausage modes takes the form

$$(\omega^2 - k^2c_T^{(0)2})m^{(e)} = \frac{\rho_e}{\rho_0}\frac{\omega^2(\omega^2 - k^2c_{S\parallel}^{(0)2})}{c_{S\perp}^{(0)2} + v_A^2}x_0. \tag{9}$$

When $c_{S\|}^{(0)} = c_{S\|}^{(0)} = c_S^{(0)}$, the isotropic case, Eq. (9) is similar to the dispersion relation obtained by Roberts (1981). The limit $kx_0 \to 0$ implies the existence of two possible waves, namely $\omega^2 \to k^2 c_T^{(0)2}$ and $\omega^2 \to k^2 c_S^{(e)2}$. It can be shown that there are always modes with phase velocity below the minimum of c_T and $c_S^{(e)}$. Specifically,

$$\omega^2 \approx k^2 c_T^{(0)2}\left[1 - \frac{\rho_e}{\rho_0}\frac{c_S^{(e)}}{(c_S^{(e)2} - c_T^{(0)2})^{1/2}}\psi^2 k x_0\right] \tag{10}$$

if $c_T^{(0)} < c_S^{(e)}$, where

$$\psi = \frac{c_{S\perp}^{(0)2}}{\gamma_\perp (c_{S\perp}^{(0)2} + v_A^2)}.$$

A second mode arises when $c_{S\|}^{(0)} < c_S^{(e)}$, and then the dispersion Eq. (8) gives

$$\omega^2 \approx \times k^2 c_S^{(e)2}$$

$$\left[1 - \left(\frac{\rho_e}{\rho_0}\frac{c_S^{(e)2}(c_S^{(e)2} - c_{S\perp}^{(0)2})}{(c_S^{(e)2} - c_T^2)(c_{S\perp}^{(0)2} + v_A^2)} k x_0\right)^2\right]. \tag{11}$$

Dispersion causes shorter wavelength harmonics to travel slower than larger wavelength harmonics.

The dispersion relation for kink waves in the long wavelength approximation can be written as

$$\omega^2 = k^2 v_A^2 (1 - \Gamma)\frac{\rho_0}{\rho_e}(k x_0). \tag{12}$$

Let us now look at the extreme of wide slabs ($kx_0 \gg 1$), where we suppose that $m^{(0)} x_0 \to \infty$ for $kx_0 \to \infty$. We approximate $\tanh m^{(0)} x_0$ by unity and obtain a dispersion relation for both sausage and kink modes of the form

$$[k^2 v_A^2 (1 - \Gamma)] m^{(e)} = \left(\frac{\rho_e}{\rho_0}\right)\omega^2 m^{(0)}, \tag{13}$$

provided by $m^{(0)} > 0$ and $m^{(e)} > 0$. This relation coincides with the dispersion relation obtained by Hau and Lin (1995) for surface waves at a magnetic surface. Therefore, we can conclude that the propagation of surface non-leaky waves is equivalent to propagation at a single interface. In the approximation of a wide slab, there is no difference between sausage and kink waves.

For body waves ($m^{(0)2} = -n^{(0)2} < 0$), the dispersion relation for sausage and kink waves is

$$[k^2 v_A^2 (1 - \Gamma) - \omega^2] = \omega^2 \frac{\rho_e n^{(0)}}{\rho_0 m^{(e)}}\left\{\begin{matrix} -\tan \\ \cot \end{matrix}\right\} n^{(0)} x_0, \tag{14}$$

with the tan/cot terms corresponding to the sausage and kink modes, respectively.

We are interested in the solution of Eq. (14) with ω^2 tending to $k^2 c_T^2$, as kx_0 tends to zero. To determine this mode, we suppose that for a small value of kx_0, $\omega^2 \approx k^2 c_T^2 (1 + \nu(k^2 x_0^2))$, where ν is a positive constant to be determined. For sausage modes, in order to have finite values for $\tan n_0 x_0$, we require that the product $n_0 x_0$ tends to the roots of $\tan n_0 x_0$, i.e. $n_0 x_0 \to n\pi$, where $n = 1, 2, \ldots$. Thus, from Eqs. (7) and (14) we obtain

$$n_0^2 x_0^2 = \frac{(c_T^2 - c_{S\|}^{(0)2})(c_T^2 - v_A^{*2})}{(c_{S\perp}^{(0)2} + v_A^2) c_T^2 \nu} = n^2 \pi^2. \tag{15}$$

From this relation we determine the coefficient ν. The behaviour of sausage body waves in a slender slab is given by

$$\omega^2 \approx k^2 c_T^2 \left(1 + \frac{\psi^4 - \frac{c_{S\|}^{(0)2} - v_A^{*2}}{c_{S\perp}^{(0)} + v_A^2}\psi^2}{n^2\pi^2\left(\frac{c_{S\|}^{(0)2}}{c_{S\perp}^{(0)2} + v_A^2} - \psi^2\right)} k^2 x_0^2\right), \tag{16}$$

provided $c_{S\|}^{(0)2} > v_A^{*2}$. The condition imposed here is that the coefficient ν has to be positive, i.e.

$$\frac{\beta_\perp^2}{2 + \beta_\perp \gamma_\perp} < \beta_\| \gamma_\| < 2(1 - \Gamma) + \frac{\beta_\perp^2}{2 + \beta_\perp \gamma_\perp}.$$

Kink modes can be studied in a similar fashion, resulting in a relation close to Eq. (16), but with $(n - 1/2)^2 \pi^2$ in place of $n^2 \pi^2$, which means that kink waves will have a higher propagation speed than sausage modes.

To consider body waves in a wide slab we look for the solutions of Eq. (14) for $kx_0 \gg 1$. Investigating the possible modes in similar fashion as for surface waves, we find that when $c_{S\|}^{(0)2} < v_A^{*2}$,

$$\omega^2 = k^2 c_{S\|}^{(0)2}\left(1 + \frac{(c_{S\perp}^{(0)2} + v_A^2)^2 \psi^2 \pi^2 n^2}{c_{S\|}^{(0)2}(c_{S\|}^{(0)2} - v_A^{*2})}\frac{1}{k^2 x_0^2}\right). \tag{17}$$

If modified Alfvén waves propagate slower than parallel sound waves in the slab, the dispersion relation becomes

$$\omega^2 = k^2 v_A^{*2} \times$$

$$\times \left[1 + \frac{(c_{S\perp}^{(0)2} + v_A^2)^2 \left(\frac{v_A^{*2} - c_{S\|}^{(0)2}}{c_{S\perp}^{(0)2} + v_A^2} + \psi^2\right) n^2 \pi^2}{v_A^{*2}(v_A^{*2} - c_{S\|}^{(0)2})}\frac{1}{k^2 x_0^2}\right]. \tag{18}$$

If the modified Alfvén speed and the parallel component of the sound speed are equal, there will be a wave with dispersion relation

$$\omega^2 = k^2 c_{S\|}^{(0)2} \times$$

$$\left\{1 + \left[\left(\frac{c_{S\perp}^{(0)}}{c_{S\|}^{(0)}}\right)^2 + \frac{1}{1 - \Gamma}\right]\psi \arctan(\zeta)\frac{1}{k x_0}\right\}, \tag{19}$$

provided $c_S^{(e)} > c_{S\|}^{(0)}$. In the above equation, ζ is defined as

$$\zeta = \frac{\rho_0}{\rho_e}\frac{(c_S^{(e)2} - c_{S\|}^{(0)2})^{1/2}(c_{S\perp}^{(0)2} + v_A^2)\psi}{c_S^{(e)} c_{S\|}^{(0)2}}.$$

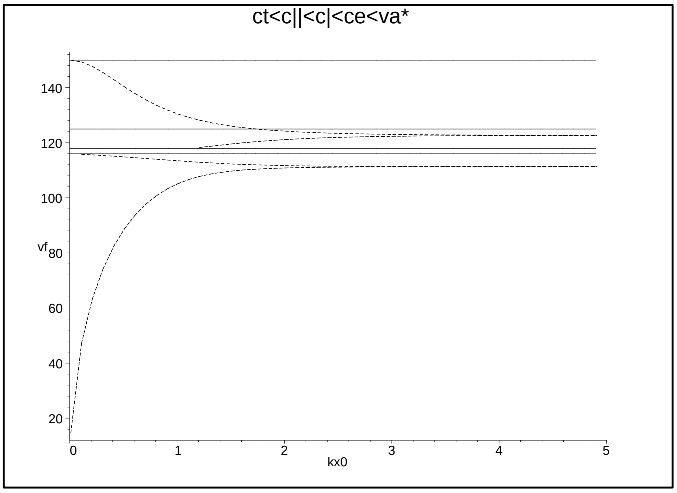

Fig. 1. The phase speed as a function of kx_0 under the circumstances given in the text for surface waves.

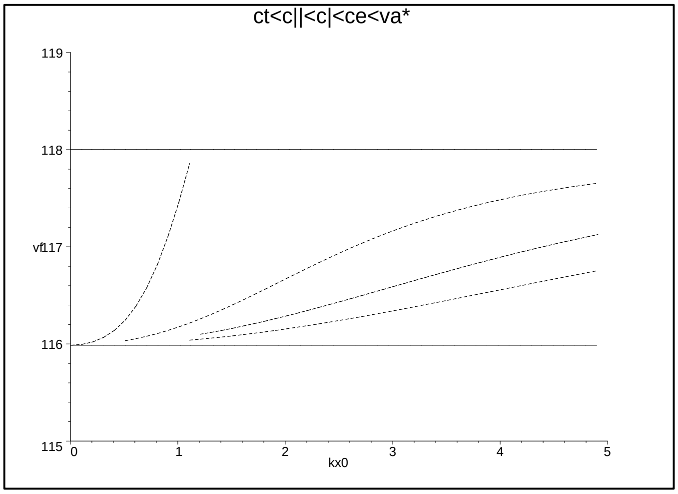

Fig. 2. The phase speed as a function of kx_0 for body waves.

ULF waves in the magnetosphere can be driven either by external sources (solar wind impulses or buffeting, magnetopause Kelvin-Helmholtz instabilities, upstream waves incident form the bow shock) or internal sources (magnetic reconnection, plasma flows in the magnetotail, resonance with ring current ions). For typical magnetospheric parameters (v_A = 380 km/s, c_{Se} = 150 km/s, $\rho_e/\rho_0 = 6$, $c_{s\perp}^{(0)}$ = 125 km/s, $c_{S\parallel}^{(0)}$ = 118 km/s, $\Gamma = 0.04$, $\gamma_\perp = 1.8$, $\gamma_\parallel = 1.2$, $x_0 = 10^4$ km), we study the possible modes in the case of an anisotropic plasma with a double-polytropic energy law in isolated magnetic slab. In Fig. 1, surface waves are shown. They have two propagation windows, one below the cusp speed (slow waves) and the other (fast waves) situated in the interval ($c_{S\perp}^{(0)}, c_S^{(e)}$). Since we imposed that waves are confined within the slab, there are no modes with phase speed above the external sound speed. For small values of kx_0, we have two modes described by relations (10) and (11). As kx_0 increases, new modes start to appear but there will be no modes between c_T and $c_{S\parallel}^{(0)}$. The possible propagation for body modes is shown in Fig. 2. They are confined within the interval between the cusp speed and the parallel component of the sound speed, and they are slow body modes. For the whole domain, they show an increasing phase velocity with an increasing of the product kx_0.

In general, the effect of anisotropy becomes important for waves propagating in a plasma with not very small plasma beta. That is why, in the vicinity of the Sun, where the plasma beta is very small, the effect of temperature anisotropy can be neglected when studying the wave propagation in homogeneous plasmas, so here the propagation of compressional waves can be studied within the framework of the usual MHD with very high accuracy. However, as we go further from the Sun, the role of anisotropy increases.

Let us make a simple estimation. Compressional waves in the magnetosphere have been observed intensively in the last decades; therefore, there is vast literature covering these phenomena (see, e.g. Strangeway et al., 1988; Takahashi et al., 1987, 1990, 1992a, b; Anderson et al., 1990; Takahashi and Anderson, 1992; Kim and Takahashi, 1999). We try to model the compressional waves observed in the Earth's magnetosphere. They are often seen in the afternoon sector during periods of enhanced geomagnetic activity. Keeping in mind magnetospheric parameters, we can calculate that the frequency of fast surface modes calculated with the aid of Eq. (11) is approximately 20 mHz, which falls in the range of compressional Pc5 waves.

3 Conclusions

The present study was motivated by the observations of compressional waves in the outer atmosphere taken by the Charge Composition Explorer spacecraft as part of the Active Magnetospheric Particle Trace Explorers (AMPTE) program. The dynamics of slow and fast magnetoacoustic waves in the magnetosphere was studied when the anisotropy in the kinetic pressure and a double-polytropic law was considered. Temperature anisotropy becomes significant in plasmas where the kinetic pressure of the plasma is comparable to the magnetic pressure. The dispersion relations for linear waves were obtained in a magnetically isolated plasma slab, modelling the interface between the non-magnetized magnetosheath and homogeneously magnetized magnetosphere. The relative values of the parameters inside and outside the slab determine the types of wave and, in general, it is not possible to give explicit forms. However, by solving the dispersion relations numerically and by considering the special case of slender and wide slabs, it was possible to predict the sort of waves that might be found in rarefied plasmas.

For typical equilibrium quantities, we found that the frequency of slow waves in a slender slab is in the range of the frequency of magnetospheric compressional Pc5 waves.

Acknowledgements. R. Erdélyi acknowledges M. Kéray for patient encouragement. The authors thank Dr A. N. Wright providing the magnetospheric data and M. Homem preparing the figures. RE and IB also acknowledge the financial support obtained from the NSF Hungary (OTKA, ref nr. TO32462). R. Erdélyi also acknowledges the support received from the Nuffield Foundation (NUF-NAL 99).

Topical Editor G. Chanteur thanks D. Gallagher and another referee for their help in evaluating this paper.

References

Anderson, B. J., Engebretson, M. J., Rounds, S. P., Zanetti, L. J., and Potemra, T. A.: A statistical study of Pc 3-5 pulsations observed by the AMPTE/CCE magnetic fields experiment. I – Occurrence distributions, J. Geophys. Res., 95, 10 495, 2002.

Ballai, I., Erdélyi, R., Voytenko, Y., and Goossens, M.: Linear and nonlinear waves in rarefied plasmas, Phys. Plasmas, 9, 2593, 2002.

Chew, G. F., Goldberger, M. L., and Low, F. E.: The Boltzmann equation and trhe one-fluid hydromagnetic equations in the absence of particle collisions, Proc. Roy. Soc. London, Ser A, 236, 112, 1956.

Hau, L.-N. and Lin, C. A.: Wave propagation in an inhomogeneous anisotropic plasma, Phys. Plasmas, 2, 294, 1995.

Hughes, W. J.: in: Solar Wind Sources of Magnetospheric Ultra-Low-Frequency Waves, (Eds) Engebretson et al., Geophys. Monogr. Ser, AGU, Washington D. C., 81 p. 1–11, 1994.

Kim, K. H. and Takahashi, K.: Statistical analysis of compressional Pc3-4 pulsations observed by AMPTE CCE at $L = 2 - 3$ in the dayside magnetosphere, J. Geophys. Res., 104, 4539, 1999.

Lin, N., Engebretson, M. J., Reinleitner, L. A., Olson, J. V., Gallagher, D. L., Cahill, L. J., Slavin, J. A., and Persoon, A. M.: Field and thermal plasma observations of ULF pulsations during a magnetically disturbed interval, J. Geophys. Res., 97, 14 859, 1992.

Mann, I. R., Crisham, G., Bale, S. D., and Stuart, D.: Multisatellite and ground-based observations of a tailward propagating Pc5 magnetospheric waveguide mode, J. Geophys. Res., 103, 4657, 1998.

Rickard, G. J., and Wright, A. N.: Alfvén resonance excitation and fast wave propagation in magnetospheric waveguides, J. Geophys. Res., 99, 13 455, 1994.

Rickard, G. J. and Wright, A. N.: ULF pulsations in a magnetospheric waveguide: Comparison of real and simulated satellite data, J. Geophys. Res., 100, 3531, 1995.

Roberts, B.: Wave Propagation in a Magnetically Structured Atmosphere – Part Two – Waves in a Magnetic Slab, Solar Phys., 69, 39, 1981.

Samson, J. C., Greenwald, R. A., Rouhoniemi, J. M., Hughes, T. J., and Wallis, D. D: Magnetometer and radar observations of magnetohydrodynamic cavity modes in the Earth's magnetosphere, Can. J. Phys. 69, 929, 1991.

Strangeway, R. J., Zanetti, L. J., Klumpar, D. M., and Scarf, F. L.: AMPTE CCE plasma wave measurements during magnetospheric compressions, J. Geophys. Res., 93, 14 357, 1988.

Takahashi, K., McEntire, R. W., Zanetti, L. J, Lopez, R. E., and Kistler, L. M.: An eastward propagating compressional Pc-5 wave observed by AMPTE/CCE in the postmidnight sector, J. Geophys. Res., 92, 13 472, 1987.

Takahashi, K., McEntire, R. W., Cheng, C. Z., and Kistler, L. M.: Observation and theory of Pc-5 waves with harmonically related transverse and compressional components, J. Geophys. Res., 95, 977, 1990.

Takahashi, K. and Anderson, B. J.: Distribution of ULF energy (f is less than 80 mHz) in the inner magnetosphere – A statistical analysis of AMPTE CCE magnetic field data, J. Geophys. Res., 97, 10 751, 1992.

Takahashi, K., Sato, N., Warwecke, J., Luehr, H., Spence, H. E., and Tonegawa, Y.: On the standing wave mode of giant pulsations, J. Geophys. Res., 97, 10 717, 1992a.

Takahashi, K., Ohtani, S. I., and Yumoto, K.: AMPTE CCE observations of Pi 2 pulsations in the inner magnetosphere, Geophys. Res. Lett., 19, 1447, 1992b.

Taroyan, Y. and Erdélyi, R.: Resonant and Kelvin-Helmholtz instabilities on the magnetopause Phys. Plasmas, 9, 3121, 2002.

Walker, A. D. M., Rouhoniemi, J. M., Baker, K. B., and Greenwald, R. A.: Spatial and temporal behavior of ULF pulsations observed by the Goose Bay HF radar, J. Geophys. Res., 97, 12 187, 1992.

Wright, A. N.: Dispersion and wave coupling in inhomogeneous MHD waveguides J. Geophys. Res., 99, 159, 1994.

Ziesolleck, C. W. S. and McDiarmid, D. R.: Auroral latitude Pc-5 field line resonances: Quantized frequencies, spatial characteristics, and diurnal variation, J. Geophys. Res., 99, 5817, 1994.