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Letter to the Editor

Abel transform inversion of radio occultation measurements made with a receiver inside the Earth's atmosphere

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Abstract. Radio occultation measurements made with a receiver inside the Earth's atmosphere can be inverted, assuming local spherical symmetry, with an Abel transform to provide an estimate of the atmospheric refractive index profile. The measurement geometry is closely related to problems encountered when inverting seismic time-travel data and solar occultation measurements, where the Abel solution is well known. The method requires measuring both rays that originate from above and below the local horizon of the receiver. The Abel transform operates on a profile of "partial bending angles" found by subtracting the positive elevation measurement from the negative elevation value with the same impact parameter. In principle, the refractive index profile can be derived from measurements with a single frequency GPS receiver because the ionospheric bending is removed when the partial bending angle is evaluated.

Key words. Atmospheric composition and structure (pressure, density and temperature) – Radio science (remote sensing)

1 Introduction

Radio occultation (RO) measurements of the Earth's atmosphere using the GPS constellation and receivers placed on low Earth orbiting (LEO) satellites can provide useful atmospheric profile information (Kursinski et al., 1996; Rocken et al., 1997; Wickert et al., 2001). The technique, which has been widely used in the study of planetary atmospheres (e.g. Fjeldbo et al., 1971), is based on measuring how radio waves are bent by refractive index gradients in an atmosphere. It can be shown that by assuming spherical symmetry (e.g. Kursinski et al., 1997) this information can be inverted with an Abel transform to give a vertical profile of refractive index and subsequently temperature. The possibility of making RO measurements with a receiver in the atmosphere – probably placed either on a plane or a mountain top – has also been considered by Zuffada et al. (1999). Although fundamentally this is very similar to the LEO measurement, it was originally suggested by Zuffada et al. (1999) that the limits of integration used in the Abel transform meant that it could not be implemented when the receiver is within the Earth's atmosphere and this, in part, led to the development of a more general ray-tracing inversion scheme. (Note that the retrieval scheme outlined by Palmer et al. (2000) could also be applied to this problem.) In fact, an Abel transform can be used in these circumstances. The measurement geometry is similar to that considered by Gaykovich et al. (1983), Beschastnov et al. (1984) and Bruton and Kattawar (1997), who used an Abel transform to invert solar occultation data. Schreiner et al. (1999) also employed the same approach for deriving electron density profiles from ionospheric RO measurements. Furthermore, the problem is analogous to that of inverting seismic travel-time data for velocity structure given earthquake sources within the volume (Haase, 1992). The similarity between RO and methods used to invert seismic data was originally noted by Phinney and Anderson (1968).

The purpose of this letter is firstly to draw attention to the similarity in the problems considered by Zuffada et al. (1999), Haase (1992) and Gaykovich et al. (1983) and then to outline a strategy with an Abel transform for inverting RO measurements when the receiver is in the atmosphere. In Sect. 2 the inversion of RO measurements with a LEO satellite will be briefly reviewed and the strategy for a receiver in the atmosphere is discussed in Sect. 3, where it is shown that the measurement can be made with a single frequency GPS receiver. The discussion and conclusions are presented in Sect. 4.

2 RO measurements with a LEO

The theory of RO with a receiver in space has been described in detail by a number of authors (e.g. Melbourne et al., 1994; Kursinski et al., 1997; Rocken et al., 1997). Briefly, the RO

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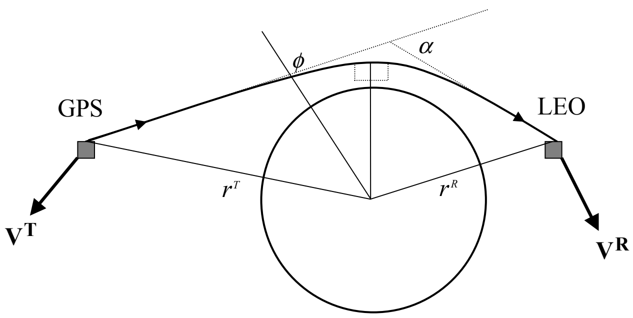

Fig. 1. The geometry of an RO measurement with a receiver on board a LEO satellite.

technique is based on measuring how radio waves emitted by a GPS satellite are bent by refractive index gradients before being received at a low earth orbiting (LEO) satellite. The geometry is shown in Fig. 1. The LEO is typically ~700 km above the surface of the Earth, whereas the GPS satellite is around 22 000 km above the surface. The receiver on the LEO satellite measures the phase and amplitude of the GPS signal. An excess phase delay is introduced because the neutral atmosphere refractive index is greater than unity and the ray path is curved. Both satellites are in motion, so the frequency of the received signal is Doppler shifted. The time derivative of the excess phase gives the additional Doppler shift introduced by the atmosphere, which arises because the ray bending modifies the angle of intersection at the satellites relative to the straight line path.

If the position and velocity vectors of the satellites are known accurately the excess Doppler shift can be inverted, assuming spherical symmetry, using Bouguer's formula (Born and Wolf, 1986), to yield bending angle α and impact parameter $a = nr \sin\phi$ (which is constant along the ray path for a spherically symmetric atmosphere), where n is the refractive index value, r is the radius and ϕ is the angle between the ray vector and the radius vector. Deriving α and a requires an iterative approach and it is usually performed assuming the refractive index is unity at the satellites.

If the atmosphere is spherically symmetric the bending angle, α, can be written as,

$$\alpha(a) = -2a \int_a^\infty \frac{\frac{d\ln n}{dx}}{(x^2 - a^2)^{1/2}} dx \tag{1}$$

where $x = nr$. It is a convention in RO to write the upper limit of this integral as infinity, but in practice if the refractive index at, and above, the receiver is effectively unity this can be replaced with r^R, the radius of receiver. The difference in the satellite radii is not important if there is no ray bending along sections of path where $r > r^R$. Note that the GPS satellite transmits at two frequencies (L1=1575.42 MHz and L2=1227.6 MHz) and the bending due to ionospheric plasma is removed or "corrected", to first order, by taking a linear combination of the L1 and L2 bending angle values (Vorob'ev and Krasil'nikova, 1994). For a spherically symmetric atmosphere, the variation of the corrected bending angle with impact parameter can be inverted with an Abel transform (Phinney and Anderson, 1968; Fjeldbo et al., 1971) to recover the refractive index profile,

$$n(x) = \exp\left(\frac{1}{\pi}\int_x^\infty \frac{\alpha(a)}{(a^2 - x^2)^{1/2}} da\right) \tag{2}$$

which can be evaluated numerically. A useful substitution is $a = x\cosh\theta$. Once again the upper limit of this integral could be replaced with r^R.

The refractive index can be written as $n = 1 + 10^{-6}N$, where N is the refractivity. In the neutral atmosphere the refractivity is related to the temperature and the partial pressures of dry air and water vapour pressure T, P_d and P_w through,

$$N = \frac{k_1 P_d}{T} + \frac{k_2 P_w}{T} + \frac{k_3 P_w}{T^2} \tag{3}$$

where k_1 $(= 77.60 \pm 0.08\mathrm{K/hPa})$, k_2 $(= 70.4 \pm 2.2\mathrm{K/hPa})$ and k_3 $(= (3.739 \pm 0.012) \times 10^5 \mathrm{K^2/hPa})$ have been evaluated experimentally (Bevis et al., 1994, and references therein). For a dry atmosphere ($P_w = 0$), the refractivity is proportional to density and the profile can be used to integrate the hydrostatic equation $dP/dz = -\rho g$ and a vertical temperature profile is derived from the ideal gas law, since $P = \rho RT = NT/k_1$. More generally, the refractivity measurement contains both temperature and water vapour information and this is referred to as the "water vapour ambiguity". Water vapour can be derived from the refractivity using a priori temperature profile information (Kursinski and Hajj, 2001), but it is not advisable to derive a temperature profile with type of approach, because the uncertainty in the a priori water vapour can lead to large errors. Alternative methods for solving the water vapour ambiguity are based on statistically optimal inversion techniques (e.g. Healy and Eyre, 2000; Palmer et al., 2000).

3 Inverting measurements with a receiver in the atmosphere

As with the space-based measurement the time derivative of the excess phase gives the Doppler shift, which is then inverted using Bouguer's formula to provide $\alpha(a)$. Note that it is now necessary to measure the refractive index at the receiver, n^R, to perform this step since the impact parameter value at the receiver is now $a = n^R r^R \sin\phi^R$, where r^R is the radius of the receiver. Unlike the LEO case, there may be significant ray bending along sections of path where $r > r^R$, so the bending will not be symmetric either side of the tangent point. However, Zuffada et al. (1999) noted that when the receiver is within the atmosphere it is possible to measure both positive and negative elevation rays. These refer to rays that intersect the receiver from above and below the local horizon, as shown in Fig. 2. They went on to point out that, assuming spherical symmetry, for every negative elevation ray, with bending angle α_N, there is a corresponding positive elevation value α_P with the same impact parameter value a. (Note that in Fig. 2 the positive elevation ray has been extrapolated beyond the receiver to emphasise that the ray paths have the same impact parameter, a.) These bending angles can be written as,

$$\alpha_P(a) = -a \int_{n^R r^R}^{n^T r^T} \frac{\frac{d\ln n}{dx}}{(x^2 - a^2)^{1/2}} dx \tag{4}$$

and,

$$\alpha_N(a) = \alpha_P(a) - 2a \int_{a}^{n^R r^R} \frac{\frac{d\ln n}{dx}}{(x^2 - a^2)^{1/2}} dx \tag{5}$$

Adding the positive and negative elevation values,

$$\alpha(a) = \alpha_N(a) + \alpha_P(a) \tag{6}$$

gives the total or "space-based equivalent" bending angle. This is the bending angle that would be observed if the receiver was placed outside the atmosphere, as in the LEO measurement. Although it may be useful to derive the total bending angles in this manner, it remains difficult to use the Abel transform given by Eq. (2) because we can only derive $\alpha(a)$ with tangent heights below the receiver, but we require values up to infinity. An approach might be to simulate bending angles above the receiver from, for example, a numerical forecast model, but this is theoretically questionable because the derived refractive index profiles will then contain errors caused by both measurement error and uncertainty in the numerical forecast state and they may be difficult to characterise for assimilation purposes. However, using the same arguments as Zuffada et al. (1999), and assuming spherical symmetry, subtracting the positive elevation bending angle from the negative gives,

$$\alpha'(a) = \alpha_N(a) - \alpha_P(a) \tag{7}$$

where $\alpha'(a)$ is the "partial bending angle" which is ray bending that occurs along the section of path below the receiver (e.g. see Bruton and Kattawa, 1997). By definition, as $a \to n^R r^R$ then $\alpha'(a) \to 0$. Note that subtracting the positive elevation bending angle from the negative also removes the ionospheric signal (again, assuming spherical symmetry) so in principle it is not necessary to have a dual frequency receiver.

The partial bending angle, along the section of path below the receiver, can be written as,

$$\alpha'(a) = -2a \int_{a}^{n^R r^R} \frac{\frac{d\ln n}{dx}}{(x^2 - a^2)^{1/2}} dx \tag{8}$$

which can be inverted with (e.g. Gaykovich et al., 1983),

$$n(x) = n^R \exp\left(\frac{1}{\pi} \int_{x}^{n^R r^R} \frac{\alpha'(a)}{(a^2 - x^2)^{1/2}} da\right) \tag{9}$$

The Abel transform essentially evaluates a weighted sum of the partial bending angle values. As a result, the fractional errors in refractivity tend to be smaller than fractional bending angle errors. Temperature, humidity and surface pressure information can be derived simultaneously from the refractive index profile using the statistically optimal retrieval routine outlined by Healy and Eyre (2000). Preliminary information content studies suggest that the measurements with a receiver at 10–11 km will mainly provide water vapour profile information because numerical weather prediction temperature errors are typically of order ∼1 K, whereas the errors in specific humidity are ∼ 20%.

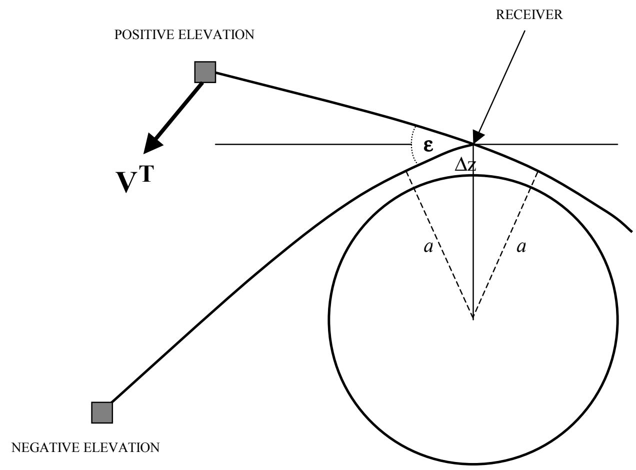

Fig. 2. Positive and negative elevation rays with the same impact parameter, measured by a receiver Δz above the Earth's surface. ϵ is the angle between the ray paths at the receiver. Note that the relative scales of a and Δz are exaggerated for illustration purposes.

4 Discussion and conclusions

It has been shown that RO measurements made with a receiver inside the atmosphere can be inverted with an Abel transform. The method requires measurements at both positive and negative elevation angles and relies on the assumption of local spherical symmetry. This assumption may be poorer than in the LEO case. Recent work (Lesne et al., 2001) indicates that tangent point will drift by around 200 to 500 km for an occultation with a receiver on board an aircraft, compared with 100 km for the LEO. This suggests that we will be assuming local spherical symmetry over a larger horizontal scale. The errors arising from this assumption could be investigated by simulating measurements within the domain of a high resolution mesoscale model (Healy, 2001).

It is interesting that in deriving the partial bending angle $\alpha'(a)$ by subtracting the positive elevation value from the negative, we effectively perform an ionospheric correction. The accuracy of this step will also depend on the assumption of spherical symmetry, since the positive and negative elevation rays will have different paths through the ionosphere. For a zero elevation ray, when the tangent point is at the receiver, the paths are identical by definition. As the tangent point moves lower into the atmosphere the paths begin to diverge, the largest separation being when the negative elevation ray is close to the surface. Analytical calculations, assuming straight line paths and a receiver at a height of 11 km, indicate that ϵ, the angle between positive and negative rays at the receiver, is around 7 degrees for paths that graze the surface. This translates to a horizontal separation between the paths of $\sim$800 km, when at an altitude of 400 km above the Earth's surface. In addition, typical ray bending for a path near the surface of around 1.2° will increase the separation from 800 km to around 950 km. This appears quite large but it needs to be put in some context. Firstly, Hajj and Romans (1998) obtained reasonable ionospheric profile results with an Abel transform of L1 bending angles assuming spherical symmetry. The geometry of their measurements implies that spherical symmetry is effectively being assumed over much greater angles. In addition, it is important to recognise the significance of ionospheric bending compared to neutral bending for a path near the surface. The total bending angle will be of order 1 degree, but the ionospheric contribution will only be of order $\sim$0.01° given daytime, solar maximum conditions (Fig. 3, Kursinski et al., 1997). It seems reasonable to expect that the error will be smaller than the ionospheric signal, so we would anticipate errors of less than 1%.

To conclude, we have outlined an inversion method based on an Abel transform that can be applied to RO measurements made with a receiver inside the atmosphere. Although further work is required to assess the importance of the assumption of spherical symmetry in both neutral atmosphere and ionosphere, we nevertheless believe this may be an interesting area for future experimental work.

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