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Standing Alfvén waves with $m \gg 1$ in an axisymmetric magnetosphere excited by a non-stationary source

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Abstract. As a continuation of our earlier paper, we consider here the case of the excitation of standing Alfvén waves by a source of the type of sudden impulse. It is shown that, following excitation by such a source, a given magnetic shell will exhibit oscillations with a variable frequency which increases from the shell's poloidal to toroidal frequency. Simultaneously, the oscillations will also switch over from poloidally (radially) to toroidally (azimuthally) polarized. With a reasonably large attenuation, only the start of this process, the stage of poloidal oscillations, will be observed in the ionosphere.

Key words. Ionosphere-magnetosphere interactions · Wave propagation · MHD waves and instabilities

1 Non-stationary oscillations – general formulas

In accordance with the plan outlined in the introduction of Leonovich and Mazur (1998) (henceforth referred to as Paper 1), we now study broad-band standing Alfvén waves excited by a non-stationary correlated source. This implies that the source function $\tilde{\Phi}_N(x^1, \omega)$ which appears in Eq. (4) of Paper 1 will be treated as a specified function of coordinate and frequency rather than a random function out of a certain statistical ensemble. Accordingly, its Fourier transform

$$\Phi_N(x^1, t) = \int_{-\infty}^{\infty} d\omega \tilde{\Phi}_N(x^1, \omega)\, e^{-i\omega t} \qquad (1)$$

is a specified function of coordinate and time. As has already been pointed out, such a treatment is justified for oscillations associated with restructuring processes of the magnetosphere, its response to dramatic effects of an external and internal origin alike. Typical examples are furnished by Pi2 pulsations and the SC phenomenon.

More specifically, the objective of this paper reduces to performing an inverse Fourier transform

$$\Phi(x^1, l, t) = \int_{-\infty}^{\infty} \tilde{\Phi}(x^1, l, \omega)\, e^{-i\omega t}\, d\omega \ , \qquad (2)$$

using the expression for the function $\tilde{\Phi}_N(x^1, l, \omega)$ which we obtained in an earlier publication (Leonovich and Mazur, 1993) and reported in Paper 1 for reference. On substituting Eq. (4) from the cited paper into Eq. (2), we get

$$\Phi(x^1, l, t) = \int_{-\infty}^{\infty} \tilde{\Phi}_N(x^1, \omega) \tilde{Q}_N(x^1, \omega) \times \tilde{Z}_N(x^1, l, \omega)\, e^{-i\omega t}\, d\omega \ . \qquad (3)$$

Using Eq. (1) the last expression may be represented as

$$\Phi(x^1, l, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt' \Phi_N(x^1, t - t') \times \int_{-\infty}^{\infty} d\omega \tilde{Q}_N(x^1, \omega), \tilde{Z}_N(x^1, l, \omega)\, e^{-i\omega t'} \ .$$

Evaluating the integral over frequency in this expression will be based on the fact that the function $\tilde{Q}_N(x^1, \omega)$ varies rapidly with respect to the variable ω, which permits the use of the saddle-point method. The saddle-point depends on x^1 and t' as parameters:

$$\omega = \Omega_N(x^1, t') \ . \qquad (4)$$

It will be shown in the following that the function $\Omega_N(x^1, t)$ has a simple physical meaning. When using the saddle-point method, a relatively slowly varying function $\tilde{Z}_N(x^1, l, \omega)$ can be factored outside the integral sign by taking its value at the saddle-point. On introducing the designations

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$$Z_n(x^1, l, t) = \tilde{Z}_N(x^1, l, \Omega_N(x^1, t)) \ , \tag{5}$$

$$Q_N(x^1, t) = \int_{-\infty}^{\infty} \tilde{Q}_N(x^1, \omega) \ \mathrm{e}^{-i\omega t} \ \mathrm{d}\omega \ , \tag{6}$$

we get

$$\Phi(x^1, l, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_N(x^1, t - t') Q_N(x^1, t') \times Z_N(x^1, l, t') \ \mathrm{d}t' \ . \tag{7}$$

Equation (7) gives a general expression for the Alfvén wave potential in terms of the source function $\Phi_N(x^1, t)$. It is easy to see that the functions $Q_N(x^1, t)$ and $Z_n(x^1, l, t)$ describe the Alfvén wave excited by a source of the type of instantaneous impulse. Assuming

$$\Phi_N(x^1, t) = \Phi_N(x^1)\delta(t) \ ,$$

we have

$$\Phi(x^1, l, t) = \frac{1}{2\pi} \Phi_N(x^1) Q_N(x^1, t) Z_N(x^1, l, t) \ . \tag{8}$$

The function $Z_N(x^1, l, t)$ describes the longitudinal structure of a standing wave, and the main dependence on the coordinate x^1 and time t is concentrated in the slowly varying function $Q_N(x^1, t)$.

The response to a source of the type of instantaneous impulse is in a sense opposite to the case of a monochromatic wave. Investigating the wave structure in these two extreme limits gives also a general idea of all intermediate cases. In this paper we shall therefore restrict our consideration to the response to a source of the type of instantaneous impulse. We are also justified in doing so for the reason that studying a general case Eq. (7) requires specifying a source model $\Phi_N(x^1, t)$, but this does not fall within the scope of our paper.

2 Response to an instantaneous impulse

First we suppose that the saddle-point lies within the interval $(\Omega_{PN}, \Omega_{TN})$ at a reasonable distance from its ends (this condition will be formulated more accurately later). Using Eq. (23) of Paper 1 we then have

$$Q_N(x^1, t) = \int_{-\infty}^{\infty} \mathrm{d}\omega \left[\frac{v^1_{PN}}{v^1_N(x^1, \omega)} \frac{p_0^{-1} k_2^2}{p_0 \tilde{k}^2_{1N}(x^1, \omega) + p_0^{-1} k_2^2} \right]^{1/2} \times \exp[i\Psi_N(x^1, \omega, t) - \tilde{\Gamma}_N(x^1, \omega) + i\pi/4] \ . \tag{9}$$

Here it is designated

$$\Psi_N(x^1, \omega, t) = \tilde{\Psi}_N(x^1, \omega) - \omega t \ .$$

The saddle-point is defined by the equation

$$\frac{\partial \Psi_N(x^1, \omega, t)}{\partial \omega} = 0 \ ,$$

which, in terms of Eq. (18b) of Paper 1, may be represented as

$$\tau(x^1, \omega) = t \ . \tag{10}$$

This equation has a simple physical meaning. It defines the frequency of the monochromatic wave which reaches the shell x^1 during a time t after it has been generated on its poloidal shell. It is this wave that determines the oscillation on the shell x^1 at time t.

For the function $\omega = \Omega_N(x^1, t)$ defined by Eq. (10), it is an easy matter to obtain the limit expressions for small and large values of t. Using Eqs. (18c) and (18d) of Paper 1 we have

$$\Omega_N(x^1, t) = \Omega_{PN}(x^1) + \omega^3_{PN}(x^1) t^2, \quad t \ll m/\Omega \ , \tag{11}$$

$$\Omega_N(x^1, t) = \Omega_{TN}(x^1) - \frac{1}{\omega_{TN}(x^1) t^2}, \quad t \gg m/\Omega \ . \tag{12}$$

Here Ω is the quantity of the order of Ω_{PN} or Ω_{TN}. Hence, as t varies in the interval $(0, \infty)$, the function $\Omega_N(x^1, t)$ varies in the interval $(\Omega_{PN}(x^1), \Omega_{TN}(x^1))$.

We now define the time-dependent quasi-classical wave vector and phase:

$$k_{1N}(x^1, t) = \tilde{k}_{1N}[x^1, \Omega_N(x^1, t)] \ , \tag{13}$$

$$\Psi_N(x^1, t) = \Psi_N[x^1, \Omega_N(x^1, t), t] \equiv \tilde{\Psi}_N[x^1, \Omega_N(x^1, t)] - \Omega_N(x^1, t) t \ . \tag{14}$$

It is easy to see that

$$\frac{\partial \Psi_N(x^1, t)}{\partial x^1} = k_{1N}(x^1, t) \ ;$$

$$\frac{\partial \Psi_N(x^1, t)}{\partial t} = -\Omega_N(x^1, t) \ . \tag{15}$$

Hence it follows that

$$\frac{\partial k_{1N}(x^1, t)}{\partial t} = -\frac{\partial \Omega_N(x^1, t)}{\partial x^1} \ .$$

Using these relationships and the limit expressions in Eqs. (11) and (12) for $\Omega_N(x^1, t)$ it is possible to obtain appropriate expressions for $k_{1N}(x^1, t)$ and $\Psi_N(x^1, t)$:

$$k_{1N}(x^1, t) = \begin{cases} -\Omega'_{PN}(x^1) t, & t \ll m/\Omega, \quad (16a) \\ -\Omega'_{TN}(x^1) t, & t \gg m/\Omega, \quad (16b) \end{cases}$$

$$\Psi_N(x^1, t) = \begin{cases} -\Omega_{PN}(x^1) t - \frac{1}{3} \omega^3_{PN}(x^1) t^3, \quad t \ll m/\Omega, & (17a) \\ \bar{\Psi}_N(x^1) - \Omega_{TN}(x^1) t - \frac{1}{\omega_{TN}(x^1) t}, \quad t \gg m/\Omega \ . & (17b) \end{cases}$$

Here it is designated

$$\bar{\Psi}_N(x^1) = \int_0^{\infty} [\Omega_{TN}(x^1) - \Omega_N(x^1, t)] \ \mathrm{d}t \ . \tag{18}$$

It can be proved that there is a simple relationsip between the function $\bar{\Psi}_N(x^1)$ and the function $\tilde{\Psi}_N(\omega)$ defined by the relationship (15b) of Paper 1:

$$\bar{\Psi}_N(x^1) = \tilde{\Psi}_N[\Omega_{TN}(x^1)] \ . \tag{19}$$

We will also need the time-dependent coefficient of wave attenuation

$$\Gamma_N(x^1, t) = \tilde{\Gamma}_N[(x^1, \Omega_N(x^1, t)] \ . \tag{20}$$

Using Eqs. (20b), (20c), (21) and (22a) of Paper 1 we obtain the limit expressions for this function

$$\Gamma_N(x^1, t) = \begin{cases} \gamma_{PN}(x^1)t, \quad t \ll m/\Omega, & \text{(21a)} \\ \Gamma_N(x^1) + \gamma_{TN}(x^1)t, \ t \gg m/\Omega, & \text{(21b)} \end{cases}$$

where $\Gamma_N(x^1) = \tilde{\Gamma}_N(\Omega_{TN}(x^1))$, and the function $\tilde{\Gamma}_N(\omega)$ is defined by Eq. (22a) of Paper 1.

Returning to evaluating the integral in Eq. (9) by the saddle-point method we get

$$\frac{\partial^2 \Psi_N}{\partial \omega^2} = \frac{\partial \tau N}{\partial \omega} = -\frac{1}{\Omega'_{PN}}\frac{\partial \tilde{k}_{1N}}{\partial \omega} = -\frac{1}{\Omega'_{PN}}\frac{1}{v^1_N} \ .$$

Here Eq. (20a) from Paper 1 is employed. Thus, near the saddle-point

$$\Psi_N(x^1, \omega, t) \approx \Psi_N(x^1, t) - \frac{[\omega - \Omega_N(x^1, t)]^2}{2\Omega'_{PN} v^1_N(x^1, \Omega_N)} \ .$$

The peak width, when integrating over ω by the saddle-point method, on the order of magnitude is

$$\Delta\omega \sim |\Omega'_{PN} v^1_N|^{1/2} \sim \frac{\Delta\Omega_N}{(\alpha_N m)^{1/2}} \ .$$

It will be recalled that $\alpha_N = \Delta\Omega_N/\Omega_{TN}$. Considering that it is supposed that $\alpha_N m \gg 1$, then $\Delta\omega \ll \Delta\Omega_N$ and the use of the saddle-point method is justified if $\Omega_N(x^1, \omega)$ is not too close to the end of the interval $(\Omega_{PN}, \Omega_{TN})$. Based on the foregoing considerations evaluating the integral of Eq. (9) is an easy matter. We have

$$Q_N(x^1, t) = 2\sqrt{\pi}\omega_{PN}\left[\frac{k_2^2/p_0^2}{k_{1N}^2(x^1, t) + k_2^2/p_0^2}\right]^{1/2} \times \exp[i\Psi_N(x^1, t) - \Gamma_N(x^1, t) + i\pi/2] \ . \tag{22}$$

It is taken into consideration here that $\Omega'_{PN} < 0$, and the definition of Eqs. (16) and (17b) from Paper 1 are used.

If the saddle-point $\omega = \Omega_N(x^1, t)$ is close to the poloidal or toroidal frequency, then the above calculation is not justified. Some refinement of the corresponding conditions is in order. For the calculations to be justified near the poloidal frequency, it is necessary that the inequalities

$$\omega - \Omega_{PN} \gg \omega_{PN}, \ \Delta\omega \sim \left(\frac{\Omega_{PN}}{l_N} v^1_N\right)^{1/2} \ll \omega - \Omega_{PN} \tag{23}$$

are satisfied. The former ensures the validity of the WKB approximation for $\tilde{Q}_N(x^1, \omega)$ near the saddle-point, and the latter warrants the validity of the saddle-point method itself. Using Eq. (17a) of Paper 1 we have

$$\Delta\omega \sim \omega_{PN}\left(\frac{\omega - \Omega_{PN}}{\omega_{PN}}\right)^{1/4} ,$$

whence it follows that the latter inequality in Eq. (23) reduces to the former. In a similar manner, near the toroidal frequency it is necessary that the inequalities

$$\Omega_{TN} - \omega \gg \omega_{TN}, \quad \Delta\omega \ll \Omega_{TN} - \omega \tag{24}$$

are satisfied. From Eq. (17a) of Paper 1 it follows that

$$\Delta\omega \sim \omega_{TN}\left(\frac{\Omega_{TN} - \omega}{\omega_{TN}}\right)^{3/4} ,$$

and the latter inequality of Eq. (24a) also reduces to the former.

If Eq. (24) is not satisfied, that is, $\Omega_N - \Omega_{PN} \lesssim \omega_{PN}$, then Eq. (24a) of Paper 1 must be substituted into Eq. (6). Using also Eq. (24b) of Paper 1 and by changing in the resulting iterated integral the order of integration, it is easy to obtain

$$Q_N(x^1, t) = 2i\sqrt{\pi}\omega_{PN}\theta(t) \times \exp\left(-i\Omega_{PN}t\frac{1}{3}i\omega^3_{PN}t^3 - \gamma_{PN}t\right) , \tag{25}$$

where $\theta(t)$ is the Heaviside formula:

$$\theta(t) = \begin{cases} 0, & t < 0, \\ 1, & t > 0. \end{cases}$$

It follows from the inequality $\Omega_N - \Omega_{PN} \lesssim \omega_{PN}$ that

$$t \lesssim \frac{1}{\omega_{PN}} \ll \frac{m}{\Omega} \ . \tag{26}$$

But then

$$k_{1N}(x^1, t) \sim \Omega'_{PN} t \lesssim \frac{1}{\lambda_{PN}} \ll \frac{k_2}{p_0} \ ,$$

and the approximate Eqs. (17a) and (21a) hold for the functions $\Psi_N(x^1, t)$ and $\Gamma_N(x^1, t)$. Based on this, one can see that Eqs. (22) and (25) coincide.

Similarly, when $\Omega_{TN} - \Omega_N \lesssim \omega_{TN}$, Eq. (26a) from Paper 1 should be substituted into Eq. (6):

$$Q_N(x^1, t) = i\frac{k_2\lambda_{PN}}{p_0}\int_{-\infty}^{\infty} \mathrm{d}\omega g\left(\frac{\omega - \Omega_{TN} + i\gamma_{TN}}{\omega_{TN}}\right) \times \exp[-i\omega t + i\tilde{\Psi}_N(\omega) - \tilde{\Gamma}_N(\omega)] \ . \tag{27}$$

The quantity $\exp[-\tilde{\Gamma}_N(\omega)]$ can be factored outside the integral sign at the point $\omega = \Omega_{TN}$ (to give $\exp[-\tilde{\Gamma}_N(x^1)]$), but in the factor $\exp[i\tilde{\Psi}_N(\omega)]$ it is necessary to take into account the dependence on ω because $\tilde{\Psi}_N(\omega)$ is a large phase. To do so, it will suffice to make a linear expansion of this phase near the point $\omega = \Omega_{TN}$:

$$\tilde{\Psi}_N(\omega) = \tilde{\Psi}_N(\Omega_{TN}(x^1)) + \left.\frac{\partial \Psi_N}{\partial \omega}\right|_{\omega=\Omega_{TN}} \cdot (\omega - \Omega_{TN}) = \bar{\Psi}_N(x^1) + \tilde{\tau}_N(\Omega_{TN})(\omega - \Omega_{TN}(x^1)) \ .$$

After that, using the integral representation (26b) from Paper 1 and by changing the order of integration in the resulting iterated integral, we obtain

$$Q_N(x^1,t) = 2i\sqrt{\pi}\frac{k_2\lambda_{PN}}{p_0}\frac{1}{t} \times \exp\left(- i\Omega_{TN}t + i\tilde{\Psi}_N(x^1) - \frac{i}{\omega_{TN}t} - \Gamma_N(x^1) - \gamma_{TN}t\right) \ . \quad (28)$$

Here, the relationship in Eq. (26a) of Paper 1 is taken into account

The inequality $\Omega_{TN} - \Omega_N \lesssim \omega_{TN}$ means that

$$t \gtrsim \frac{1}{\omega_{TN}} \gg \frac{m}{\Omega} \ ,$$

and hence Eqs. (16b), (17b) and (21b) are applicable and inequality $k_{1N}(x^1,t) \gg k_2/p_0$ is executed. But it is easy to see that in such case Eq. (22) goes into Eq. (28).

Thus, Eq. (22) originally received in the interval $(1/\omega_{PN}) \ll t \ll (1/\omega_{TN})$ appears to be applicable at any values of the variable t, and Eqs. (25) and (28) are its limit expressions for $t \ll m/\Omega$ and $t \gg m/\Omega$, respectively.

With the expression for the transverse potential $\Phi(x^1,l,t)$, it is easy to obtain disturbed electric and magnetic fields. We restrict ourselves to calculating the disturbed magnetic field. By analogy with the definition of Eq. (6), we introduce time-dependent longitudinal functions for the magnetic field

$$Y_N^{(i)}(x^1,l,t) = \tilde{Y}_N^{(i)}[x^1,l,\Omega_N(x^1,t)] \ ,$$

time-depending from the amplitude of magnetic field

$$B_N(x^1,t) = \tilde{B}_N(x^1,\Omega_N(x^1,t)) \ ,$$

where the functions $\tilde{B}_N(x^1,\omega)$ and $\tilde{Y}_N^{(i)}(x^1,l,\omega)$ are defined by Eqs. (28b), (28c) and (28d) of Paper 1. Using these definitions from Eqs. (26a) and (3) of Paper 1 we obtain the following expression for physical components of the disturbed magnetic field

$$\hat{B}_i(x^1,l,t) = B_N(x^1,t)Q_N^{(i)}(x^1,t)Y_N^{(i)}(x^1,l,t) \ .$$

Here it is designated [cf. Eq. (30) of Paper 1]:

$$Q_N^{(1)}(x^1,t) = Q_N(x^1,t);$$

$$Q_N^{(2)}(x^1,t) = -\frac{k_{1N}(x^1,t)}{k_2/p_0}Q_N(x^1,t);$$

$$Q_N^{(3)}(x^1,t) = -2i\frac{k_{1N}(x^1,t)\cdot(k_2/p_0)}{k_{1N}^2(x^1,t)+(k_2/p_0)^2}Q_N(x^1,t) \ .$$

3 Discussion of results of the theory

The results obtained may be given a simple and illustrative physical treatment. A source of the type of instantaneous impulse, having a very wide (formally infinite) spectrum, excites Alfvén waves instantaneously on all magnetic shells. And on a given magnetic shell x^1 a monochromatic wave is excited, for which this shell is a poloidal one. Correspondingly, the wave's frequency ω equals the poloidal frequency of this shell $\Omega_{PN}(x^1)$, and the wave's polarization has a poloidal character (the magnetic field oscillates in a radial direction). After that, each of the monochromatic waves travels in a radial direction in a manner described at the beginning of Paper 1, that is, toward its toroidal surface, and as this propagation proceeds, the wave's polarization transforms from poloidal to toroidal.

There is no difficulty in understanding what this picture will look like on a fixed magnetic shell x^1. Initially, on oscillation of the poloidal type with the frequency equal to the poloidal frequency of this magnetic shell $\Omega_{PN}(x^1)$ will be excited. The oscillation will at once leave this magnetic shell and move towards its toroidal surface. Its place will be occupied by oscillations arriving from increasingly distant magnetic shells that orignated as poloidal ones there; but as they travel towards the shell x^1, they transform progressively to toroidal ones. Eventually an oscillation reaches this magnetic shell x^1, for which this shell is toroidal, and hence the frequency of this oscillation $\Omega_{TN}(x^1)$.

Thus, this shell will exhibit the oscillation that changes slowly from poloidal to toroidal: its frequency varies from $\Omega_{PN}(x^1)$ to $\Omega_{TN}(x^1)$, and its polarization goes from radial to azimuthal. A typical time of such a variation $\sim m/\omega$, that is, when $t \ll m/\omega$ the oscillation has a poloidal character, and when $t \gg m/\omega$ it has a toroidal character.

The foregoing picture occurs in the case of a reasonably small attenuation when during a typical time m/ω the attenuation coefficient of the wave $\Gamma \sim \gamma m/\omega$ is small

$$m\frac{\gamma}{\omega} \ll 1 \ .$$

Otherwise

$$m\frac{\gamma}{\omega} \gg 1$$

the wave has no time to transform into a toroidal one and attenuates in the stage when it still has a poloidal character.

It has already been pointed out that a source of the type of instantaneous impulse and a monochromatic source are, in a sense, opposite limiting cases: the former is represented by the δ-function of time t, and the latter is represented by the δ-function of frequency ω. It is quite clear that the source can also be considered to be instantaneous for some finite but reasonably short duration of the impulse Δt, and monochromatic for a small spectrum width $\Delta\omega$. We now formulate the relevant requirements explicitly.

When integrating over frequency in Eq. (3) using the saddle-point method, it was evident that near the saddle-point the function $\tilde{Q}_N(x^1,\omega)$ is a sharp peak with respect to the variable ω with a typical width

$$\Delta\Omega \sim \frac{\Delta\Omega_N}{(\alpha_N m)^{1/2}} \sim \left(\frac{\alpha_N}{m}\right)^{1/2}\Omega_N \ . \quad (29)$$

Equation (22) is written when treating the instantaneous impulse when the source function $\tilde{\Phi}_N(x^1,\omega) = const.$

Evaluating the integral in Eq. (9) remains also unchanged when $\tilde{\Phi}_N(x^1, \omega)$ depends on ω, but the typical scale of its variation $\Delta\omega$ with respect to this variable is much larger than the value of (29):

$$\Delta\omega \gg \Delta\Omega \ . \tag{30}$$

In any event this condition is satisfied if the source represents an impulse of a duration

$$\Delta t \ll \frac{1}{\Delta\Omega} \sim \left(\frac{m}{\alpha_N}\right)^{1/2} \frac{1}{\Omega_N} \ . \tag{31}$$

With the typical values of $m \approx 40, \alpha \approx 0.2$ the quantity $(m/\alpha_N)^{1/2} \approx 15$ and (31) imply that the impulse duration must be shorter than 15 oscillation periods. In the case of the SSC phenomenon and substorm breakup, the condition is more likely to be satisfied than not. If the inequality the reverse of the inequality (31) holds

$$\Delta\omega \ll \Delta\Omega \sim \left(\frac{\alpha_N}{m}\right)^{1/2} \Omega_N$$

the source can be considered to be monochromatic.

4 Inferences of theory and observations

Based on statistical observations from geostationary satellite AMPTE/CCE, Anderson *et al.* (1990) and Takahashi and Anderson (1992) showed that about 50% of observed oscillations correspond to unstructured stochastic oscillations. About 30% of the other 50% are harmoic toroidal oscillations, 10% correspond to oscillations with a larger share of the compressible component ($B_3 \sim B_1, B_2$), and less than 5% refer to transverse oscillations of the poloidal type. In other publications (Takahashi and McPerron, 1984; Engebretson *et al.*, 1988; Takahashi *et al.*, 1990) it was shown that transverse poloidal MHD oscillations ($B_1 > B_2 \gg B_3$) were observed largely in magnetically quiet conditions. In a disturbed magnetosphere, oscillations of the poloidal type with a substantial share of the compressible component are mostly recorded (Walker *et al.*, 1982; Takahashi, 1988; Odera *et al.*, 1991). Our calculations apply for oscillations of the former type: without a substantial share of the compressible component. Oscillations of the latter type are described in terms of a theory assuming finite plasma pressure $\beta \sim 1$ (Southwood and Saunders, 1985; Walker, 1987; Taylor and Walker, 1987; Klimushkin, 1997).

It may be suggested that the propagation of poloidal Alfvén oscillations across magnetic shells caused by their transverse dispersion has a universal character irrespectively of the way in which they are excited. In this connection, the preceding observations may be given the following interpretation. Poloidal Alfvén waves can be recorded only during the time-interval $t \sim m/\Omega$ that has elapsed from the time of their excitement. After that, in the process of their propagation across magnetic shells, they transfer to toroidal oscillations at the expense of their transverse dispersion. At the same time, toroidal oscillations, such as field line resonance, do nothing but enhance their toroidal character in the process of propagation across magnetic shells (see Leonovich and Mazur, 1989). Hence the probability of observation of poloidal oscillations is significantly lower compared to toroidal oscillations.

Leonovich and Mazur (1993) compared the behaviour of hodographs of monochromatic oscillations ensuring from theory and hodographs constructed by Walker *et al.*, (1982) using STAR radar observations. In that experiment at a reasonably dense grid of points of ionospheric observations, hodographs of poloidal oscillations of ionospheric plasma were constructed. If it is assumed that these oscillations are caused by the poloidal Alfvén wave in the magnetosphere, then the behaviour of these hodographs must be described by the theory presented in this paper. It was found that the orientation of observed hodographs coincides with that of monochromatic Alfvén oscillations with $m \gg 1$ in the region between resonance surfaces, poloidal and toroidal. At the same time, according to Walker *et al.* (1982), the oscillation frequency varied during the observations. This may be explained in the following manner by invoking the results of this paper. Hodographs of oscillations constructed for the period of time $t \ll m/\Omega$

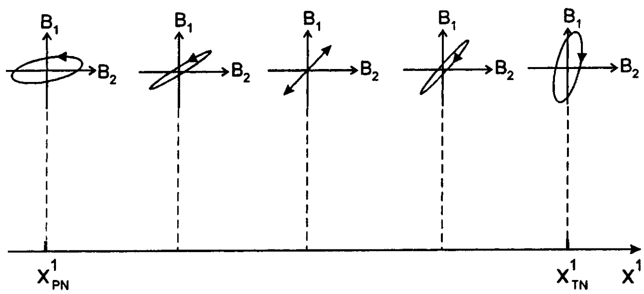

Fig. 1. Hodographs of monochromatic Alfvén oscillations with $m \gg 1$ at different points inside the transparency region: between the resonance surfaces $x^1_{PN} < x^1 < x^1_{TN}$

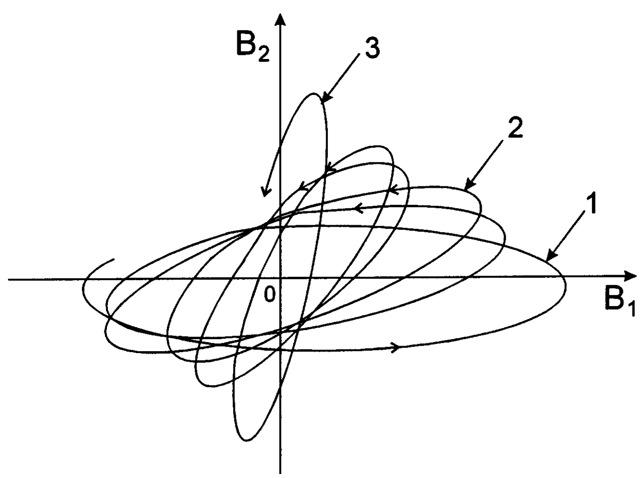

Fig. 2. Hodographs of non-stationary Alfvén oscillations with $m \gg 1$ excited by a source of the type of "instantaneous impulse" at different instants of time: 1 – oscillations of the poloidal type ($B_1 \gg B_2$), 2 – oscillations of the intermediate type ($B_1 \sim B_2$), 3 – oscillations of the toroidal type ($B_1 \ll B_2$)

reflect at the observational point just the behaviour of the monochromatic wave which travels slowly across magnetic shells. Therefore, if the oscillation source persists during a reasonably long time-interval and changes little, then at a different point of space we must observe the oscillation that changes gradually from poloidal to toroidal. Note that such a behaviour is possible only with a sufficiently weak dissipation of the wave in the ionosphere. In the case of a strong dissipation, the oscillation at all observation points will be poloidal.

We are quite aware that observational data available to date are inadequate unambiguously to reveal the oscillations described in this paper. Therefore, we wish to point out those characteristic properties of the oscillations which may serve as an indicator of these oscillations in observations.

One peculiarity refers to the behaviour of hodographs of monochromatic oscillations. As follows from Leonovich and Mazur (1993), in the region between the resonance surfaces where, essentially, the oscillation is just concentrated, the locus behaviour depends critically on the observation point. For the same oscillation near the poloidal surface (where $B_1 \gg B_2$) and near the toroidal surface (where $B_2 \gg B_1$), the locus rotates in different directions. The sense of its rotation depends on the sign of k_2 (or, equivalently of the sign of m). In the region between the resonance surfaces at a reasonable distance from each of them, the oscillations are linearly polarized (see Fig. 1).

The other peculiarity applies to the spectrum of broadband stochastic oscillations. A distinguishing characteristic of such a spectrum, in the case of a moderate attenuation of the oscillations, is the presence of peaks near eigenfrequencies of each of the harmonics of standing Alfvén waves in the magnetosphere. In this case each such peak must be split into two closely spaced peaks, corresponding, respectively, to the poloidal eigenfrequency Ω_{PN} and to the toroidal eigenfrequency Ω_{TN}. The spectral splitting $\Delta\Omega_N = \Omega_{TN} - \Omega_{PN}$ is small compared with each of the eigenfrequencies $\Omega_{PN} : \Omega_{TN}$. This splitting is largest for the fundamental harmonic $N = 1 : \Delta\Omega_1/\Omega_1 \sim 0.25$. Thus, this spectrum splitting is most readily observed near the frequency of the fundamental harmonic of the oscillations $N = 1$. With an increase in N, the ratio $\Delta\Omega_N/\Omega_{PN}$ decreased abruptly (see Figs. 3 and 4 in Paper 1).

The third peculiarity is associated with the observation of Alfvén waves excited by a source of the type of instantaneous impulse. As follows from the results of this paper, oscillations with a variable frequency must be observed at a fixed point of space when recording oscillations from such a source. Also, in the observation process (if the observing time $t > m/\Omega$) the frequency of the recorded oscillations for each harmonic of standing Alfvén waves must increase from poloidal eigenfrequency Ω_{PN} to toroidal frequency Ω_{TN}. As in the case of stochastic oscillations, the greatest effect must be observed for the fundamental harmonic $N = 1$. Figure 2 plots the time-variation of the typical form of locus of such oscillations. Note that this picture is in complete agreement with a numerical study of the evolution of the initial poloidal disturbance made by Mann and Wright (1995) and Mann *et al.* (1997). At the same time our investigation does not confirm the finding obtained numerically by Ding *et al.* (1995), who obtained such an evolution of the initial poloidal disturbance which led to a periodic change of the polarization of oscillations from the poloidal to toroidal type, and vice versa.

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