

New electron energy transfer rates for vibrational excitation of N₂

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Abstract. In this paper we present the results of a study of the electron cooling rate, the production rates of vibrationally excited N₂(*v*), and the production frequency of the N₂ vibrational quanta arising from the collisions of electrons with unexcited N₂(0) and vibrationally excited N₂(1) molecules as functions of the electron temperature. The electron energy transfer rates for vibrational excitation of N₂ have been calculated and fit to analytical expressions by use of the revised vibrationally excited N₂ cross sections. These new analytical expressions are available to the researcher for quick reference and accurate computer modeling with a minimum of calculations.

Key words Atmospheric composition and structure · Thermosphere · Ionosphere · Modeling and forecasting

1 Introduction

The theoretical computation of electron temperature distribution in the ionosphere requires the knowledge of various heating, cooling, and energy flow processes. The energy transfer from electrons to neutral gases in the upper atmosphere and ionosphere is one of the dominant electron cooling processes in the ionosphere. The energy transfer rate from electrons to N₂ have been calculated and fitted to analytical expressions by Stubbe and Varnum (1972) for vibrational excitation of N₂. This cooling rate and the analytical expression are based on the N₂ vibrational cross sections of Schulz (1964) and Engelhard *et al.* (1964). However, the measurements of Schulz (1976) and Haddad (1984) indicate much larger values for these cross sections. The cross sections of Schulz (1964) and those shown in Fig. 2.6 of Schulz (1976) differ by about a factor of 2. However, Schulz (1976), on the basis of some unpublished measurements

by Wong *et al.*, has suggested that even these larger cross sections should be doubled.

A normalization factor of 0.7 for Schulz's (1976) doubled nitrogen vibrational cross sections provides a good fit to a large body of experimental swarm data in nitrogen and argon-nitrogen mixture for the energy range above 1.7 eV (Haddad, 1984). As a result, the first cross-section peak for the excitation of the nitrogen molecule from its ground vibrational state *v* = 0 to a vibrational state *v* = 1 has a value of about $4.45 \cdot 10^{-16} \text{ cm}^2$ at 2 eV. The swarm analysis of Haddad results in cross sections consistent with the unpublished measurements of Wong *et al.* of N₂ vibrational cross sections (Itikawa *et al.*, 1986). The value of a normalization factor of 0.7 for Schulz's (1976) nitrogen vibrational cross sections is in agreement with the deduction by Aleksandrov *et al.* (1986) from the analysis of the data on the drift velocity of electrons in a mixture of Ar and N₂.

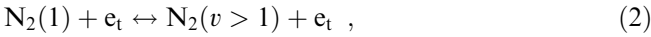
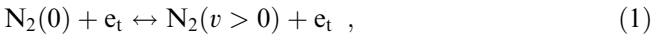
The vibrational cross sections of N₂ can be determined from the results of the differential cross-section (DCS) measurements by extrapolation to 0° and 180° followed by integration over the scattering angle. Comparisons of DCS for N₂(*v* = 0) → N₂(*v* = 1) scattering from many of these experiments have been carried out by Brennan *et al.* (1992) and Sun *et al.* (1995). The discussion given in these papers showed that the positions and shapes of resonant peaks in DCS depend sensitively on both the final vibrational state and the scattering angle. However, the measurements of DCS by Brennan *et al.* (1992) were performed at only a few energies and used a normalization technique which was revised by Sun *et al.* (1995). Robertson *et al.* (1997) used this new technique to calculate vibrational excitation cross sections of N₂ and found that their results are in excellent agreement with the cross sections derived from previous swarm analysis and crossed beam studies at energies above 1.5–1.7 eV, but not compatible with the drift velocity data at energies up to about 1 eV. The first peak of the N₂(0) → N₂(1) cross section has a value of about $4.53 \cdot 10^{-16} \text{ cm}^2$ at 1.95 eV (Robertson *et al.*,

1997) and this value is very close to the value of $4.45 \cdot 10^{-16} \text{ cm}^2$ recommended by Haddad (1984).

The primary object of this study is to use the revised N₂ cross sections (Schulz, 1976; Haddad, 1984; Robertson *et al.*, 1997) to calculate and to fit to a new analytical expression an energy transfer rate from electrons to N₂.

2 Electron energy transfer rates for vibrational excitation of N₂

The electron cooling rate, $L(\text{N}_2^*)$, arises from the collisions of thermal electrons, e_t , with unexcited N₂ ($v=0$) and vibrationally excited N₂ ($v>0$) molecules. The number densities $[\text{N}_2(v=0)] \gg [\text{N}_2(v>0)]$ during the undisturbed and geomagnetic storm periods at middle and subauroral latitudes (Pavlov and Namgaladze, 1988; Pavlov 1994a, 1996, 1997; Pavlov and Buonsanto, 1997). As a result, the collisions of thermal electrons with unexcited N₂ yield the main contribution to the electron cooling rate and the production frequency of the N₂ vibrational quanta arising from the collisions of thermal electrons with unexcited and vibrationally excited N₂ molecules (Pavlov, 1997). Therefore, in the present paper we restrict our discussion to the transitions from the ground ($v=0$) and first ($v=1$) vibrational states of N₂:



which produce the electron cooling rate

$$\begin{aligned} L(\text{N}_2^*) = & N_e \sum_{v=1}^{10} Q_{0v} \{ [\text{N}_2(0)] - [\text{N}_2(v)] \exp(E_v T_e^{-1}) \} \\ & + N_e \sum_{v=2}^9 Q_{1v} \{ [\text{N}_2(1)] - [\text{N}_2(v)] \\ & \times \exp((E_v - E_1) T_e^{-1}) \}, \quad (3) \end{aligned}$$

where

$$Q_{0v} = E_v \left\{ 8k T_e (\pi m_e)^{-1} \right\}^{0.5} \int_0^\infty \sigma_{0v}(x) x \exp(-x) dx, \quad (4)$$

$$Q_{1v} = (E_v - E_1) \left\{ 8k T_e (\pi m_e)^{-1} \right\}^{0.5} \int_0^\infty \sigma_{1v}(x) x \exp(-x) dx, \quad (5)$$

$$E_v = vE_1 - v(v-1)\Delta E, \quad (6)$$

$x = E(kT_e)^{-1}$, E is an energy of electrons, E_v is the energy of the v -th vibrational level, $\Delta E = 20.6 \text{ K}$ is anharmonicity of N₂ (Radzig and Smirnov, 1980), $E_1 = 3353 \text{ K}$ (or 0.2889 eV) is the energy of the first vibrational level of N₂ given by Radzig and Smirnov (1980), $\sigma_{0v}(E)$ are the partial cross sections for excitation of N₂(0) by electrons, $\sigma_{1v}(E)$ are the partial cross sections for excitation of N₂($v=1$) by electrons, k is Boltzmann's coefficient, and m_e denotes the mass of electrons.

The results presented in this paper were obtained by use of $\sigma_{01}(E)$ of Engelhard *et al.* (1964) compiled by Itikawa *et al.* (1986) for the region of electron energy from 0.3 to 1.7 eV and $\sigma_{01}(E)$ of Robertson *et al.* (1997) for $E > 1.7 \text{ eV}$. The cross sections for excitation of N₂(0) to N₂(2) by electrons given by Robertson *et al.* (1997) were used in our calculations. We used $\sigma_{0v}(E)$ of the Schulz (1976) for $v=3-10$ with the normalization factor of 0.7 given by Haddad (1984) (when $\sigma_{01}(E) = 4.45 \cdot 10^{-16} \text{ cm}^2$ at $E = 2 \text{ eV}$). The partial cross sections for excitation of N₂($v=1$) to N₂($v>1$) by electrons calculated by Dube and Herzenberg (1979) for $v=2-4$ were adjusted to give the best fit of the calculated $\sigma_{01}(E)$ from Dube and Herzenberg (1979) to the measured $\sigma_{01}(E)$. Chen (1964) calculated $\sigma_{0v}(E)$ and $\sigma_{1v}(E)$ for $v=2-9$ and normalized these cross sections to the Schulz (1964) measurements of $\sigma_{0v}(E)$. In this paper the values of $\sigma_{1v}(E)$ obtained by Chen (1964) for $v=5-9$ were modified by taking into account the new values of $\sigma_{0v}(E)$.

The vibrational cross sections used take into account energetically allowed transitions between rotational levels of different vibrational states of N₂. The theoretical cross sections $\sigma_{0v}(E)$ and $\sigma_{1v}(E)$ equal the sum of rotational-vibrational cross sections for all energetically allowed transitions $0p \rightarrow vq$ for $\sigma_{0v}(E)$ and $1p \rightarrow vq$ for $\sigma_{1v}(E)$, where p is a number of a p -th rotational level and q is a number of a q -th rotational level (Robertson *et al.*, 1997). The energy resolution is not sufficient to resolve rotational structure and the discussed measurements of $\sigma_{0v}(E)$ yield rotationally averaged vibrational excitation cross sections (Sun *et al.*, 1995).

The role of transitions between rotational levels of N₂($v=0$) in the ionosphere was studied by Stubbe (1971) and Stubbe and Varnum (1972). Using the formula of Mentzoni and Row (1963) and rotational excitation cross sections given by Gerjuoy and Stein (1955) the rate of electron cooling through rotational excitation was approximated by analytical formula (Stubbe, 1971; Stubbe and Varnum, 1972) which is used in models of the ionosphere. Taking into account the comparison of theoretical and experimental cross sections given by Robertson *et al.* (1997), this rate of electron cooling must be multiplied by a factor of 1.255 (see Appendix).

Equation (3) includes both excitation and deexcitation, which are related through the principle of detailed balancing. To calculate $[\text{N}_2(v)]$ it is necessary to solve the vibrational quanta continuity equation [a Boltzmann distribution of $[\text{N}_2(v)]$] or the full continuity equations for densities of N₂(v) [a non-Boltzmann distribution of N₂(v)].

The system of full continuity equations is described by Pavlov (1988, 1994) and Pavlov and Namgaladze (1988) using the harmonic and anharmonic oscillator energy level approximations, the vibrational-vibrational and vibrational-translational energy exchange of N₂(v), the diffusion of N₂(v) in the mixture of N₂(0), O₂ and O, and the production rates of N₂(v). The values of the production rates, q_v , of N₂(v) due to the thermal electron excitation of N₂(0) and N₂(1) and the

deexcitation of N₂(*v*) in the reactions given by Eqs. (1) and (2) can be calculated as

$$q_1 = N_e Q_{01} E_1^{-1} \{ [N_2(0)] - [N_2(1)] \exp(E_1 T_e^{-1}) \} , \quad (7)$$

$$q_v (v > 1) = N_e Q_{0v} E_v^{-1} \{ [N_2(0)] - [N_2(v)] \exp(E_v T_e^{-1}) \} \\ + N_e Q_{1v} (E_v - E_1)^{-1} \times \{ [N_2(0)] \\ - [N_2(v)] \exp((E_v - E_1) T_e^{-1}) \} , \quad (8)$$

where Q_{0v} and Q_{1v} are the same as in Eqs. (4) and (5).

In earlier studies, Newton *et al.* (1974) and Pavlov (1989) found that deviation from a Boltzmann distribution of N₂(*v*) was large in the SAR-arc region at the vibrational energy levels $v > 2$, and this deviation significantly affected the loss rate of the O⁺(⁴S) ions at F2-region altitudes. Pavlov and Namgaladze (1988) calculated a vibrational distribution of N₂(*v*) and found that the difference between the Boltzmann and non-Boltzmann distribution of N₂(*v*) does not give any effect on the electron density and temperature at solar minimum. During geomagnetic storms at solar maximum the calculated distribution is highly non-Boltzmann at the vibrational levels $j > 2$ and the Boltzmann distribution assumption results in the increase of 10–30% in the calculated F2 peak electron density (Pavlov and Buonsanto, 1997).

The main contribution to $L(N_2^*)$ arising from the deexcitation of N₂(*v*) is brought by the deexcitation of N₂(1) and N₂(2). As a result, it is possible to use a Boltzmann distribution of N₂(*v*) in calculations of $L(N_2^*)$:

$$[N_2(v)] = [N_2(0)] \left\{ \alpha(1 + \alpha)^{-1} \right\}^v = [N_2(0)] \exp(-v T_{\text{vib}}) , \quad (9)$$

where the vibrational quanta

$$\alpha = \sum_{v=1}^{\infty} v [N_2(v)] / [N_2] , \quad [N_2] = \sum_{v=0}^{\infty} [N_2(v)] ,$$

and the vibrational temperature $T_{\text{vib}} = -E_1 / \ln[\alpha(1 + \alpha)^{-1}]$.

In this case, from Eq. (9) it follows that the sum of the vibrationally unexcited and excited molecular nitrogen concentrations (Pavlov and Namgaladze, 1988) is

$$[N_2] = [N_2(0)](1 + \alpha) . \quad (10)$$

There is no possibility to differentiate the vibrationally unexcited N₂ from the vibrationally excited N₂ during the mass-spectrometer measurements of the neutral species in the atmosphere, and therefore it is necessary to point out that the MSIS-86 model (Hedin, 1987), which is usually used in calculations of $L(N_2^*)$, calculates the value of $[N_2]$. The difference between $[N_2]$ and $[N_2(0)]$ is not negligible for $T_{\text{vib}} > 1100$ –1300 K. For example, $[N_2]/[N_2(0)] = 1.12$ for $T_{\text{vib}} = 1500$ K. The difference between $[N_2]$ and $[N_2(0)]$ is important in subauroral red arc regions and at solar maximum in the mid-latitude atmosphere.

For the Boltzmann distribution given by Eq. (9)

$$L(N_2^*) = N_e [N_2] \{ 1 - \exp(-E_1/T_{\text{vib}}) \} \\ \times \sum_{v=1}^{10} Q_{0v} \{ 1 - \exp[v E_1 (T_e^{-1} - T_{\text{vib}}^{-1})] \} \\ + N_e [N_2] \{ 1 - \exp(-E_1/T_{\text{vib}}) \} \exp(-E_1/T_{\text{vib}}) \\ \times \sum_{v=2}^9 Q_{1v} \{ 1 - \exp[(v-1) E_1 (T_e^{-1} - T_{\text{vib}}^{-1})] \} , \quad (11)$$

To calculate α or T_{vib} it is necessary to solve the vibrational quanta continuity equation (Pavlov and Namgaladze, 1988; Pavlov, 1988, 1994a). The vibrational-translational energy exchange of N₂(*v*), the diffusion of N₂(*v*) in the mixture of N₂(0), O₂, and O, and the production frequencies of α are taken into account in the vibrational quanta continuity equation. The values of the production frequency, W , of the vibrational quanta due to the thermal electron excitation of N₂(0) and N₂(1) and the deexcitation of N₂(*v*) can be calculated as

$$W = L(N_2^*) \{ E_1 [N_2] \}^{-1} . \quad (12)$$

Pavlov and Buonsanto (1996) studied the effects of vibrationally excited N₂ using two approaches: (1) obtaining a full solution of the vibrational quanta continuity equation, and (2) calculating an analytical solution of the steady-state vibrational quanta continuity equation (Pavlov, 1994a) using a simple FORTRAN subroutine. It is shown that during both undisturbed periods and magnetic storms the full solution and the analytical approach give very similar results for the calculated F2 peak electron density and peak height of the F2-layer derived from the time-dependent one-dimensional mathematical model of the Earth ionosphere and plasmasphere, so the analytical approach can be recommended for use in ionospheric models during both quiet and disturbed conditions. A FORTRAN subroutine which carries out this steady-state analytical calculation of the vibrational temperature and takes into account the results of this paper is available from the author by e-mail.

Finally, the most simple approach in calculations of T_{vib} is

$$T_{\text{vib}} = T_n , \quad (13)$$

where T_n is a neutral temperature.

The production frequency of N₂(*v*) arises from the collision of N₂ with thermal electrons and O(¹D) and N with NO. However, at F-region altitudes a marked difference between T_{vib} and T_n is found only for $T_e > 1800$ –2000 K due to the collisions of N₂ with thermal electrons (Pavlov, 1988, 1994a; Pavlov and Namgaladze, 1988). There is absolutely no reason to assert that the condition given by Eq. (13) gives good approximation of T_{vib} during all geomagnetic conditions, but sometimes (during periods with

$T_e < 1800\text{--}2000$ K at the F-region altitudes) the Eq. (13) may be valid with some small errors.

Figures 1 and 2 show Q_{0v} and Q_{1v} as functions of T_e and v . Curves 1–10 of Fig. 1 are the calculated Q_{0v} for $v = 1\text{--}10$ of N₂(v), and curves 2–9 of Fig. 2 are the calculated Q_{1v} for $v = 2\text{--}9$ of N₂(v). The values of $\sum_{v=1}^{10} Q_{0v}$ and $\sum_{v=2}^9 Q_{1v}$ are shown in Figs. 1 and 2 as dashed lines.

Stubbe and Varnum (1972) found the analytical expression for $L(\text{N}_2^*)$ based on the N₂ vibrational cross sections of Schulz (1964) and Engelhard *et al.* (1964) as

$$L_{\text{SV}}(\text{N}_2^*) = [\text{N}_2]N_e\{U(T_e) - H(T_e, T_n)\}, \quad (14)$$

where

$$U(T_e) = 2.99 \cdot 10^{-12} \exp\{f(5 \cdot 10^{-4} - T_e^{-1})\}, \quad (15)$$

$$H(T_e, T_n) = U(T_e) \exp\{-g(T_n^{-1} - T_e^{-1})\}, \quad (16)$$

$$f = 1.06 \cdot 10^4 + 7.51 \cdot 10^3 \tanh[10^{-2}(T_e - 1800)], \quad (17)$$

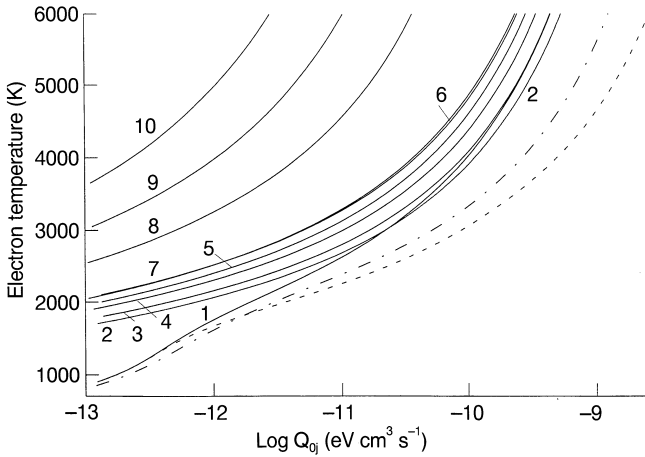


Fig. 1. The calculated Q_{0v} for the N₂ vibrational levels $v = 1\text{--}10$ (solid curves 1–10). The dashed line is $\sum_{v=1}^{10} Q_{0v}$ from the calculation of this paper, and the dashed-dotted line is $U(T_e)$ given by Eq. (12) of Stubbe and Varnum (1972)

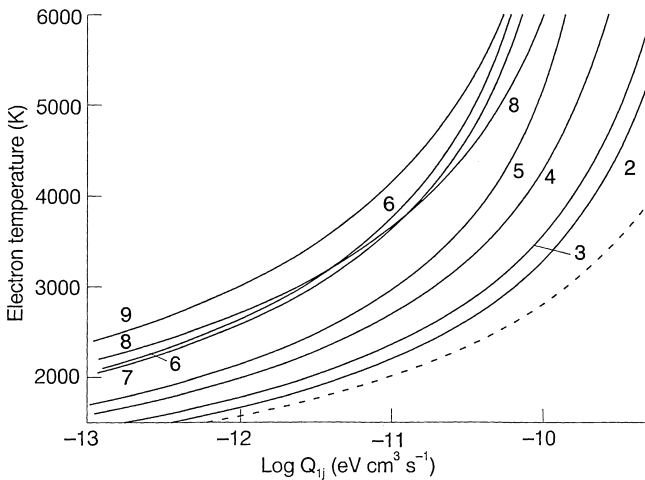


Fig. 2. The calculated Q_{1v} for the N₂ vibrational levels $v = 2\text{--}9$ (curves 2–9). The dashed line is the calculated value of $\sum_{v=2}^9 Q_{1v}$

$$g = 3300 + 1.233(T_e - 1000) - 2.056 \cdot 10^{-4} \times (T_e - 1000)(T_e - 4000), \quad (18)$$

the units of $U(T_e)$ and $H(T_e, T_n)$ are $\text{eV cm}^3 \text{s}^{-1}$.

Our results [Eqs. (3)–(6), (11)] for the cooling rate due to N₂(v) can be compared with those of Stubbe and Varnum (1972). From Eqs. (14)–(18) it follows that the analytical expression for $L(\text{N}_2^*)$ was found by Stubbe and Varnum (1972) only for $T_{\text{vib}} = T_n$ and without consideration of the difference between $[\text{N}_2]$ and $[\text{N}_2(0)]$. The value of $L_{\text{SV}}(\text{N}_2^*)$ includes both excitation ($U(T_e)$) and deexcitation ($H(T_e, T_n)$) of N₂(v) and it is necessary to compare $U(T_e)$ with $\sum_{v=1}^{10} Q_{0v}$.

The calculated values of $U(T_e)$ and $\sum_{v=1}^{10} Q_{0v}$ are shown in Fig. 1 as dashed-dotted and dashed lines. The ratio $R = U(T_e)/\sum_{v=1}^{10} Q_{0v}$ increases with the decrease of T_e . The value of $R = 1.4\text{--}1.0$ within the electron temperature range $300\text{--}1800$ K and $R = 1.0\text{--}0.6$ for $1800 \leq T_e \leq 3000$ K.

The convenient measure of the importance of the reactions in Eqs. (1) and (2) for the electron cooling rate and the production frequency W of the vibrational quanta is the ratio $R = \sum_{v=2}^9 Q_{1v}/\sum_{v=1}^{10} Q_{0v}$. This ratio is 0.14 for $T_e = 1200$ K and 1.101 for $T_e = 1500$ K. The value of R increases with the increase of T_e up to $R = 2.59$ for $T_e = 2000$ K, and after that this ratio decreases with the increase of T_e , reaching $R = 1.0$ for $T_e = 4500$ K. The part of cooling rate due to the reaction in Eq. (2) is proportional to $\exp(-E_1/T_{\text{vib}})$. Therefore, the processes of Eq. (2) can be important for the calculation of $L(\text{N}_2^*)$ and W [in comparison with Eq. (1)] only for very high T_{vib} , which may be in the subauroral red arcs (Newton *et al.*, 1974; Pavlov, 1989, 1997).

If the number of vibrational level increases then Q_{0v} decreases for most values of T_e . The relative contribution to $L(\text{N}_2^*)$ and W of the vibrational levels $v = 8\text{--}10$ can be evaluated from the ratio $R = \sum_{v=8}^{10} Q_{0v}/\sum_{v=1}^{10} Q_{0v}$. We have $R < 0.01$ within the electron temperature range $300\text{--}3600$ K and $R < 0.02$ for $T_e \leq 6000$ K. Therefore, the vibrational levels $v = 1\text{--}7$ are enough to calculate $L(\text{N}_2^*)$ and W .

3 Analytical expressions

The electron cooling rate, the production rates of N₂(v), and the production frequency of the N₂ vibrational quanta arising from the collisions of electrons with unexcited N₂(0) and vibrationally excited N₂($v > 0$) molecules depend on Q_{0v} and Q_{1v} which are functions of the electron temperature. We now give analytical expressions for Q_{0v} and Q_{1v} available to the researcher for quick reference and accurate computer modeling with a minimum of calculations:

$$\log Q_{0v}(\text{an}) = A_{0v} + B_{0v}T_e + C_{0v}T_e^2 + D_{0v}T_e^3 + F_{0v}T_e^4 - 16, \quad (19)$$

$$\log Q_{1v}(\text{an}) = A_{1v} + B_{1v}T_e + C_{1v}T_e^2 + D_{1v}T_e^3 + F_{1v}T_e^4 - 16, \quad (20)$$

Table 1. Coefficients for calculations of Q_{0v} for $1500 \leq T_e \leq 6000$ K

v	A_{0v}	B_{0v}, K^{-1}	C_{0v}, K^{-2}	D_{0v}, K^{-3}	F_{0v}, K^{-4}	δ_{0v}
1	2.025	$8.782 \cdot 10^{-4}$	$2.954 \cdot 10^{-7}$	$-9.562 \cdot 10^{-11}$	$7.252 \cdot 10^{-15}$	0.06
2	-7.066	$1.001 \cdot 10^{-2}$	$-3.066 \cdot 10^{-6}$	$4.436 \cdot 10^{-10}$	$-2.449 \cdot 10^{-14}$	0.08
3	-8.211	$1.092 \cdot 10^{-2}$	$-3.369 \cdot 10^{-6}$	$4.891 \cdot 10^{-10}$	$-2.706 \cdot 10^{-14}$	0.10
4	-9.713	$1.204 \cdot 10^{-2}$	$-3.732 \cdot 10^{-6}$	$5.431 \cdot 10^{-10}$	$-3.008 \cdot 10^{-14}$	0.10
5	-10.353	$1.243 \cdot 10^{-2}$	$-3.850 \cdot 10^{-6}$	$5.600 \cdot 10^{-10}$	$-3.100 \cdot 10^{-14}$	0.13
6	-10.819	$1.244 \cdot 10^{-2}$	$-3.771 \cdot 10^{-6}$	$5.385 \cdot 10^{-10}$	$-2.936 \cdot 10^{-14}$	0.15
7	-10.183	$1.185 \cdot 10^{-2}$	$-3.570 \cdot 10^{-6}$	$5.086 \cdot 10^{-10}$	$-2.769 \cdot 10^{-14}$	0.15
8	-12.698	$1.309 \cdot 10^{-2}$	$-3.952 \cdot 10^{-6}$	$5.636 \cdot 10^{-10}$	$-3.071 \cdot 10^{-14}$	0.15
9	-14.710	$1.409 \cdot 10^{-2}$	$-4.249 \cdot 10^{-6}$	$6.058 \cdot 10^{-10}$	$-3.300 \cdot 10^{-14}$	0.15
10	-17.538	$1.600 \cdot 10^{-2}$	$-4.916 \cdot 10^{-6}$	$7.128 \cdot 10^{-10}$	$-3.941 \cdot 10^{-14}$	0.15

Table 2. Coefficients for calculations of Q_{0v} for $300 \leq T_e \leq 1500$ K

v	A_{0v}	B_{0v}, K^{-1}	C_{0v}, K^{-2}	D_{0v}, K^{-3}	F_{0v}, K^{-4}	δ_{0v}
1	-6.462	$3.151 \cdot 10^{-2}$	$-4.075 \cdot 10^{-5}$	$2.439 \cdot 10^{-8}$	$-5.479 \cdot 10^{-12}$	0.14

Table 3. Coefficients for calculations of Q_{1v} for $1500 \leq T_e \leq 6000$ K

v	A_{1v}	B_{1v}, K^{-1}	C_{1v}, K^{-2}	D_{1v}, K^{-3}	F_{1v}, K^{-4}	δ_{1v}
2	-3.413	$7.326 \cdot 10^{-3}$	$-2.200 \cdot 10^{-6}$	$3.128 \cdot 10^{-10}$	$-1.702 \cdot 10^{-14}$	0.11
3	-4.160	$7.803 \cdot 10^{-3}$	$-2.352 \cdot 10^{-6}$	$3.352 \cdot 10^{-10}$	$-1.828 \cdot 10^{-14}$	0.11
4	-5.193	$8.360 \cdot 10^{-3}$	$-2.526 \cdot 10^{-6}$	$3.606 \cdot 10^{-10}$	$-1.968 \cdot 10^{-14}$	0.12
5	-5.939	$8.807 \cdot 10^{-3}$	$-2.669 \cdot 10^{-6}$	$3.806 \cdot 10^{-10}$	$-2.073 \cdot 10^{-14}$	0.08
6	-8.261	$1.010 \cdot 10^{-2}$	$-3.039 \cdot 10^{-6}$	$4.318 \cdot 10^{-10}$	$-2.347 \cdot 10^{-14}$	0.10
7	-8.185	$1.010 \cdot 10^{-2}$	$-3.039 \cdot 10^{-6}$	$4.318 \cdot 10^{-10}$	$-2.347 \cdot 10^{-14}$	0.12
8	-10.823	$1.199 \cdot 10^{-2}$	$-3.620 \cdot 10^{-6}$	$5.159 \cdot 10^{-10}$	$-2.810 \cdot 10^{-14}$	0.09
9	-11.273	$1.283 \cdot 10^{-2}$	$-3.879 \cdot 10^{-6}$	$5.534 \cdot 10^{-10}$	$-3.016 \cdot 10^{-14}$	0.09

where the constants $A_{0v}, B_{0v}, C_{0v}, D_{0v}, F_{1v}, A_{1v}, B_{1v}, C_{1v}, D_{1v}$, and F_{1v} are given in Tables 1–3, the units of $Q_{0v}(\text{an})$ and $Q_{1v}(\text{an})$ are $\text{eV cm}^3 \text{s}^{-1}$, the units of T_e are K.

To establish how good or bad the analytical Eqs. (11) and (12) can be, it is necessary to define quantitative measure of maximum error for given range of T_e as

$$\delta_{0v} = \max |1 - Q_{0v}(\text{an})/Q_{0v}|, \quad (21)$$

$$\delta_{1v} = \max |1 - Q_{1v}(\text{an})/Q_{1v}|. \quad (22)$$

The calculated values of δ_{0v} and δ_{1v} are presented in Tables 1–3.

The ratio $Q_{01}/\sum_{v=1}^{10} Q_{0v} > 0.94$ for $T_e < 1500$ K. The vibrational temperature of N₂ is close to the neutral temperature for $T_e < 1500$ K (Pavlov, 1988, 1994a; Pavlov and Namgaladze, 1988). Therefore the processes of Eqs. (1) and (2) with $v > 1$ are unimportant for the calculation of $L(\text{N}_2^*)$ and W for $T_e < 1500$ K.

4 Conclusions

The electron energy transfer rates for vibrational excitation of N₂ have been calculated and fitted to analytical expressions by the use of the revised vibrationally excited N₂ cross sections (Schulz, 1976; Haddad, 1984). Our results for the cooling rate due to N₂(v) are smaller than those of Stubbe and Varnum (1972) in the E region of the ionosphere and at F-region altitudes

for $T_e < 1800$ K. The new cooling rate is larger than the widely used Stubbe and Varnum (1972) cooling rate for $T_e > 1800$ K. We found that the processes given by Eq. (1) give the main contribution to the values of the electron cooling rate and the production frequency of the N₂ vibrational quanta arising from the collisions of electrons with unexcited N₂(0) and vibrationally excited N₂(1) molecules. The processes of Eq. (2) can be important for the calculation of $L(\text{N}_2^*)$ and W [in comparison with Eq. (1)] only for very high T_{vib} , which may be in the subauroral red arcs. We have also shown that the relative contribution of the vibrational levels $v = 8$ –10 to the electron cooling rate and the production frequency of the N₂ vibrational quanta is less 1% for $T_e \leq 3600$ K and 2% for $T_e \leq 6000$ K. Therefore, the vibrational levels $v = 1$ –7 are enough to calculate $L(\text{N}_2^*)$ and W .

The electron cooling rate due to rotational excitation of N₂($v = 0$) was found by Stubbe (1971) and Stubbe and Varnum (1972). It is shown that this rate of electron energy loss associated with rotational transitions in N₂ must be multiplied by a factor or 1.255.

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Appendix

The electron cooling rate by means of rotational excitation of N₂

The general expression for the rate of electron energy loss associated with rotational transitions in N₂($v = 0$) is given by Eq. (1) of Stubbe and Varnum (1972). Using the quadrupole Born approximation formula for rotational excitation cross sections given by Gerjuoy and Stein (1955), and making use of the fact that the rotational states are so close together that the summation can be changed into an integration (Mentzoni and Row, 1963), the electron cooling rate by means of rotational excitation of N₂($v = 0$) can be calculated as

$$L_{\text{rot}}(\text{N}_2(v=0)) = C[\text{N}_2(v=0)]N_e(T_e - T_n)T_e^{-0.5}, \quad (\text{A1})$$

where $C = (64\pi B a_0^2 Q^2 2^{0.5}) / (15 \pi^{0.5} m_e^{0.5})$, B is the rotational constant for the ground vibrational state of N₂, Q is the quadrupole moment averaged over the ground vibrational state N₂, a_0 is the Bohr radius.

Mentzoni and Row (1963) used the values of $B = 2.49 \cdot 10^{-4}$ eV and $Q = -0.98 e a_0^2$, where e is the electronic charge. As a result, Stubbe (1971) and Stubbe and Varnum (1972) found $C = 2.8 \cdot 10^{-14}$ eV cm³ s⁻¹ K^{-0.5} where the units of [N₂] and N_e are cm⁻³, the units of $L_{\text{rot}}(\text{N}_2(v=0))$ are eV cm⁻³ s⁻¹, and the units of T_e are K.

Robertson *et al.* (1997) treated the quadrupole moment Q as an adjustable parameter and obtained the best fit of the quadrupole Born approximation for rotational excitation cross sections to the experimental cross sections with a value of $Q = -1.1 e a_0^2$. This value of Q and more accurate value of $B = 2.4805 \cdot 10^{-4}$ eV (Robertson *et al.*, 1997) give $C = 3.5 \cdot 10^{-14}$ eV cm³ s⁻¹ K^{-0.5}.

The total rate of electron energy loss associated with rotational transitions within each vibrational level of N₂ is $L_{\text{rot}}(\text{N}_2) = \sum_{v=0}^{\infty} L_{\text{rot}}(\text{N}_2(v))$. To simplify the problem, Stubbe (1971) and Stubbe and Varnum (1972) used

$$L_{\text{rot}}(\text{N}_2) = C[\text{N}_2]N_e(T_e - T_n)T_e^{-0.5}, \quad (\text{A2})$$

meaning that they did not distinguish between rotational excitation cross sections of N₂($v = 0$) and N₂($v > 0$) (as far as the author knows, rotational excitation cross sections of N₂($v > 0$) are not measured). The number densities give $[\text{N}_2(v=0)] \gg [\text{N}_2(v > 0)]$ during the undisturbed and geomagnetic storm periods at middle and subauroral latitudes (Pavlov and Namgaladze, 1988; Pavlov, 1994a, 1996, 1997; Pavlov and Buonsanto, 1997). As a result, the collisions of thermal electrons with vibrationally unexcited N₂ yield the main contribution for the rate of electron energy loss associated with rotational transitions within N₂($v = 0$) and N₂($v > 0$), and the possible error or Eq. (A2) is expected to be negligible.

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