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# The auroral $O^+$ non-Maxwellian velocity distribution function revisited

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**Abstract.** New characteristics of $O^+$ ion velocity distribution functions in a background of atomic oxygen neutrals subjected to intense external electromagnetic forces are presented. The one dimensional (1-D) distribution function along the magnetic field displays a core-halo shape which can be accurately fitted by a two Maxwellian model. The Maxwellian shape of the 1-D distribution function around a polar angle of $21 \pm 1°$ from the magnetic field direction is confirmed, taking into account the accuracy of the Monte Carlo simulations. For the first time, the transition of the $O^+$ 1-D distribution function from a core halo shape along the magnetic field direction to the well-known toroidal shape at large polar angles, through the Maxwellian shape at polar angle of $21 \pm 1°$ is properly explained from a generic functional of the velocity moments at order 2 and 4.

## 1 Introduction

In the limiting case of small ion-neutral collision frequency with respect to the ion gyrofrequency as observed in the F region of the auroral ionosphere, the velocity distribution function of the ions in the presence of orthogonal electromagnetic forces is gyrotropic with respect to an axis parallel to the magnetic field direction, and that passes through the ion drift velocity (St-Maurice and Schunk, 1979). Due to the symmetry property of the 3-D velocity distribution, the 1-D distribution defined along a line-of-sight is determined uniquely by the polar angle noted $\varphi$ with respect to the geomagnetic field direction. The shape of the 1-D distribution functions for atomic $O^+$ ions in a background of atomic oxygen neutrals, in the physical conditions of the auroral ionosphere and in the presence of large electric fields has been extensively studied by Kikuchi *et al.* (1989), Barakat and Hubert (1990) and Winkler *et al.* (1992). In their paper, Barakat and Hubert (1990) studied the convergence properties of polynomial expansions, and showed that the generalized polynomial approach proposed by Hubert (1983) accurately fits the Monte Carlo results at third order expansion for electric field intensities as large as 200 mV/m. They also point out a characteristic angle noted $\varphi_m$ where the 1-D line-of-sight distribution for an electric field intensity of 100 mV/m is Maxwellian, and explain the formation of the long tail distribution along the magnetic field direction. Due to the importance of the modelling of the 1-D line-of-sight distributions for the derivation of plasma parameters from incoherent radar spectra (Hubert *et al.*, 1996), we precisely determine the shape of the distribution function along the magnetic field direction; then we analyze the existence of the specific angle $\varphi_m$ for electric field intensities as large as 200 mV/m. We discuss the reasons why the 1-D Raman model (Kikuchi *et al.*, 1989) fits the original distribution well for angles $\varphi$ larger than 21°, but fails for polar angles lower than 21°. Another motivation is to establish definitely the characteristics of the $O^+$ auroral distribution in a background of atomic oxygen neutrals for large electric field intensities. This opens a new field of research on the scattering properties of $O^+$-O charge exchange collisions.

Indeed, in the scenario proposed by Hubert *et al.* (1993) for the measurement of non-equilibrium plasma parameters from incoherent radar spectra, the specific aspect angle of $21 \pm 1°$ for which the 1-D distribution function for the $O^+$ ions should be Maxwellian is of fundamental importance. However, the expansion of the distribution in this scenario is limited to the contributions of velocity moments at order 4; then we need to establish whether this remains true for large electric field intensities when considering a high degree of accuracy of the non-Maxwellian distribution function. For polar angles larger than 30°, Kikuchi *et al.* (1989) have introduced the 1-D Raman model which, from two fitting parameters, provides the line-of-sight temperature; they concluded that this fitting does not work for $\varphi < 30°$. However, they did not give the reasons for these properties: it is still an open question that we want to answer. Another objective is to discuss and propose a fitting model, which characterizes the 1-D non-Maxwellian $O^+$ ion distribution function along the magnetic field direction and for polar angles $\varphi < 21°$. Finally the $O^+$ velocity distributions display three distinct characteristics ordered with respect to the polar angle $\varphi$, and it will be shown how a unique generic function of the velocity moments caused these three distinct features. In addition the scattering properties of the dynamical process of $O^+$-O resonant charge exchange have to be studied in order to predict the microscopic properties in terms of velocity distribution functions.

The work is organized as follows. After recalling in Sect. 2 the polynomial expansion of the 1-D velocity distribution we concentrate on the modelling of the parallel distribution by two Maxwellian. We discuss in Sect. 3 the foundation of the Maxwellian shape of the 1-D velocity distribution for a specific angle $\varphi_m = 21 \pm 1°$. The modelling of the toroidal 1-D distribution for polar angles larger than 21° is undertaken in Sect. 4. We demonstrate why the 1-D Raman fitting works for polar angles larger than 21°, but fails for lower angles. A short discussion concludes this study.

## 2 The 1-D velocity distribution function polar angles $\varphi < 21°$

The interaction process between atomic $O^+$ ions and atomic oxygen neutrals is considered in Sect. 5 of the paper by Hubert and Barakat (1990) with a neutral temperature of 1000 K. In this Monte Carlo approach similar to the Barakat *et al.* (1983) and Kikuchi *et al.* (1989) models, it was shown that high order velocity moments are provided with an high degree of accuracy. These results are used in a generalized polynomial expansion of the velocity distribution function from departure of a zero order toroidal distribution function (Hubert, 1983) specified to the equality of mass of the $O^+$ ion and the atomic oxygen neutrals

$$f(C_{//}, C_\perp) = \left(\frac{m_i}{2\pi k T_{//}}\right)^{1/2} \frac{m_i}{2\pi k T} \times \exp\left(-\frac{T_n}{T_{//}} C'^2_{//} - \frac{T_n}{T} C'^2_\perp - \alpha D'^2\right) \times Io\left(2\alpha\left(\frac{T_n}{T}\right)^{1/2} D' C'_\perp\right) \times \left(1 + \sum_{p,q\geq 2}^{p+q=8} a_{pq} H_p(C'_{//}) M_q(C'_\perp)\right) \tag{1}$$

In this expression $m_i$ is the ion mass, $k$ the Boltzmann constant, $T_n$ the neutral temperature, $T_{//}$ the temperature parallel to the magnetic field, T an effective perpendicular temperature, Io the modified zero$^{th}$ order Bessel function, and $\alpha D'$ an effective normalized electric field with $D' = D/(2kT_n/m_i)^{1/2}$. The solution in Eq. (1) is limited to the contributions of the velocity moments $\langle c^p_{//} c^q_\perp \rangle$ such that p and q are even with $2 \leq p + q \leq 8$.

The choice of the zero order distribution function generating the Hermite $H_p$ and the generalized polynomial $M_q$, and the construction of the coefficients $a_{pq}$ were presented in the paper of Hubert (1983). From the 3-D gyrotropic distribution given in Eq. (1), 1-D line-of-sight velocity distributions are derived by integration in the velocity plane perpendicular to the direction defined by the polar angle $\varphi$ with respect to the magnetic field direction. Figure 1 displays (solid line) the 1-D distribution for an electric field of 100 mV/m, and four angles $\varphi = 0°, 20°, 54.7°$ and 90°, whilst the respective Maxwellian distributions are represented by dashed lines where temperatures are determined by:

$$T_\varphi = \frac{m_i}{k} \int \mathrm{d}C C^2_\varphi \mathrm{f}_\varphi(C_\varphi)\,. \tag{2}$$

It is clearly seen that the $O^+$ 1-D velocity distributions display different characteristics with respect to the Maxwellian distributions for $\varphi < 20°, \varphi = 20°$ and $\varphi > 20°$; this is the reason why this paper is divided into three sections.

Along the magnetic field, the value of the 1-D ion distribution for small velocities is larger than the Maxwellian distribution defined with the parallel

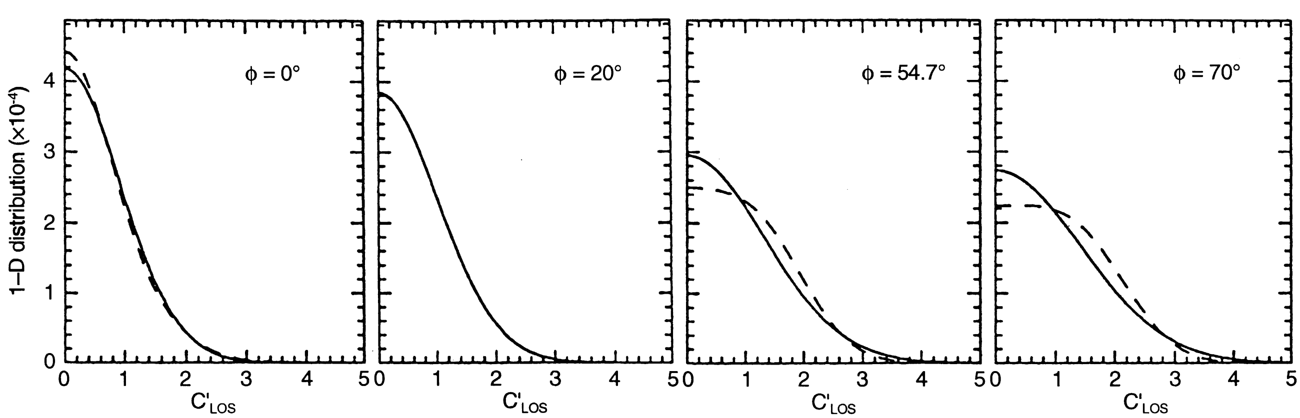


**Fig. 1.** The panels display (*solid line*) the 1-D ion distribution for an electric field of 100 mV/m and four polar angles of 0°, 20°, 54.7°, 70° from the *left* to the *right* respectively. The *dashed lines* represent the corresponding Maxwellian with a temperature $T_\varphi$

temperature $T_{//} = T_0$, and displays a long tail (see the lower left panel in Fig. 2) overpopulated with respect to the Maxwellian for the large velocity $C'_{los} = C_{los}/(2kT_{//}/m_i)^{1/2}$. Barakat and Hubert (1990) explained the long tail formation along the magnetic field as a consequence of the toroidal shape of the 1-D distribution at $\varphi = 90°$, and the coupling between parallel and perpendicular velocities. Indeed, the tail which is above the corresponding Maxwellian for $C'_{//} > 2.6\ C'_{los}$ is a result of the coupling between the parallel and the perpendicular velocities. This reaches a maximum in the velocity range corresponding to the maximum value of the toroidal perpendicular distribution, that is around 2.6 $C'_{los}$; nevertheless Barakat and Hubert (1990) did not explain the formation of the core of the distribution.

We have fitted the 1-D distribution with 2 Maxwellians. For $\varphi = 0°$ we find that the core fits well with a Maxwellian with a temperature of 999 K and 1005 K for the case of 100 mV/m and 200 mV/m respectively. The halo fits with a Maxwellian at 2312 K and 6216 K respectively as shown in the left panels of Fig. 2 for the 100 mV/m and 200 mV/m cases. The ratio of the temperature deduced from the fitting with 2 Maxwellians with respect to the exact parallel temperature is 1 for 100 mV/m and 0.986 for 200 mV/m. Therefore a 2 Maxwellian fitting is well founded, with the core temperature equal to the neutral temperature. The origin of this property is inherent to the $O^+$-O collision process in parent gas, where an ion and a neutral can interchange their identities; then the new $O^+$ ions are created at the neutral temperature with a probability not equal to zero that their trajectory will not be affected by the next collision (Barakat *et al.*, 1983). We also observe this characteristic but less pronounced in the velocity distribution obtained for other small polar angles with $0° < \varphi < 21°$, as shown in the right panels of Fig. 2 for 100 mV/m and 200 mV/m with $\varphi = 10°$.

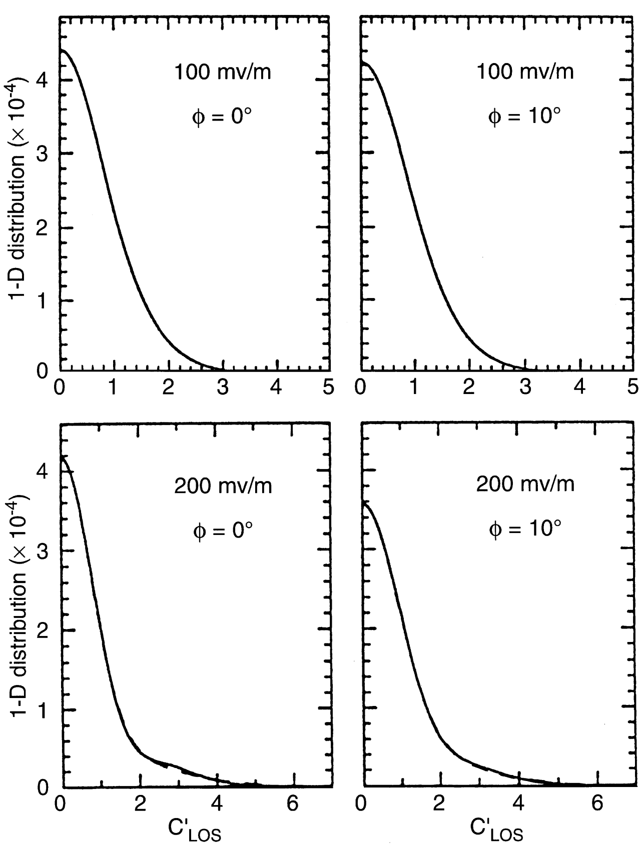


**Fig. 2.** Two-Maxwellian fitting in *dashed line* of the original 1-D ion distributions for $\varphi = 0°$ in the *left panels*, and $\varphi = 10°$ in the *right panels*. The *upper panels* represent the 100 mV/m case, and the *lower panels* the 200 mV/m case

## 3 The Maxwellian shape of the 1-D ion distribution function

The second panel from the left in Fig. 1 shows the 1-D ion velocity distribution function as a solid line compared to the Maxwellian distribution with the temperature at $T_{20°}$ (dashed line) for an electric field of 100 mV/m. The two curves are indistinguishable. The specific angle $\varphi_m$ has been defined such that the line-of-sight velocity moment at fourth order is equal to the fourth order velocity moment of a Maxwellian that is

$$\langle C^4_{\varphi_m} \rangle = 3(\langle C^2_{\varphi_m} \rangle)^2 . \tag{3}$$

The case of a 100 mV/m electric field has been discussed by Barakat and Hubert (1990). Here we concentrate on the case of a 200 mV/m electric field. We wonder whether there is an angle $\varphi_m$ around 21° where the 1-D ion distribution is a Maxwellian. For an electric field intensity of 200 mV/m the specific angle such that Eq. (3) verified is 21.6°. It is better to consider a Maxwellian expansion as was done by Barakat and Hubert (1990) for the case of 100 mV/m: indeed if the line-of-sight distribution is a Maxwellian defined with the temperature $T_{21.6}$, all the polynomial coefficients of the Maxwellian expansion should be identically equal to zero. From the left panel of Fig. 3 it is clear that the

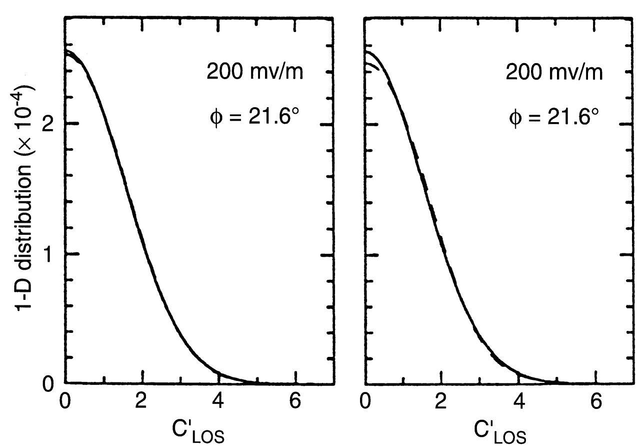


**Fig. 3.** Maxwellian expansion of the 1-D distribution for an electric field of 200 mV/m, and an angle-of-sight of 21.6°. The corresponding Maxwellian is marked by a *solid line*, the third order expansion is a *dotted line* in the *left panel*, and the fourth order expansion is a *dashed line* in the *right panel*

third order Maxwellian expansion at temperature $T_{21.6}$ contains some added contributions from the velocity moments $\langle C^6_{21.6}\rangle$ : the Maxwellian shown by the solid line is different from the expansion in the dashed line. On the right panel of Fig. 3 the expansions using the $\langle C^6_{21.6}\rangle$ and $\langle C^8_{21.6}\rangle$ contributions are also different from the Maxwellian distribution function. These results indicate that the velocity moments $\langle C^6_{21.6}\rangle$ and $\langle C^8_{21.6}\rangle$ are not the velocity moments of the Maxwellian at the 6th and 8th order, that is:

$$\langle C^6_{21.6}\rangle \neq 15\langle C^2_{21.6}\rangle^3$$

and

$$\langle C^8_{21.6}\rangle \neq 105\langle C^2_{21.6}\rangle^4 . \quad (4)$$

This can be seen in the right panels of Fig. 4 where the Maxwellian velocity moments are shown by dashed lines, and the Monte Carlo velocity moments by solid lines versus the polar angle $\varphi$. If the Monte Carlo velocity moments $\langle C^6_{21.6}\rangle$ and $\langle C^8_{21.6}\rangle$ are decreased by 5% and 8% respectively, we find exactly the velocity moments at order 6 and 8 of the Maxwellian at temperature $T_{21.6}$, and then no correcting term rises in the Maxwellian expansion.

These 5% and 8% quantities are in the range of the percentage error of the computed Monte Carlo values with respect to the exact values as discussed by Hubert and Barakat (1990). Indeed it was shown that the error increases as the moment order increases, as well as the error in the moments has a random nature; moreover the error increases with an increasing electric field intensity. Another aspect is that the specific angle $\varphi_m$ increases to 22° when the velocity moment $\langle C^4_\varphi\rangle$ is increased by 1%, then decreasing by 2.5% and 4% the velocity moments $\langle C^6_{22}\rangle$ and $\langle C^8_{22}\rangle$ respectively we find also exactly the velocity moments at order 6 and 8 of the Maxwellian at temperature $T_{22}$. For completeness the 100 mV/m case on the left of Fig. 4 shows that $\langle C^6_{20}\rangle$ and $\langle C^8_{20}\rangle$ are nearly exactly the velocity moments of order 6 and 8 of the respective Maxwellian at temperature $T_{20}$. In conclusion, the Maxwellian property for the 1-D ion distribution for an angle-of-sight around 21°, and electric field intensities as large as 200 mV/m is established when the inherent uncertainty of the velocity moments derived from the Monte Carlo simulations is taken into account.

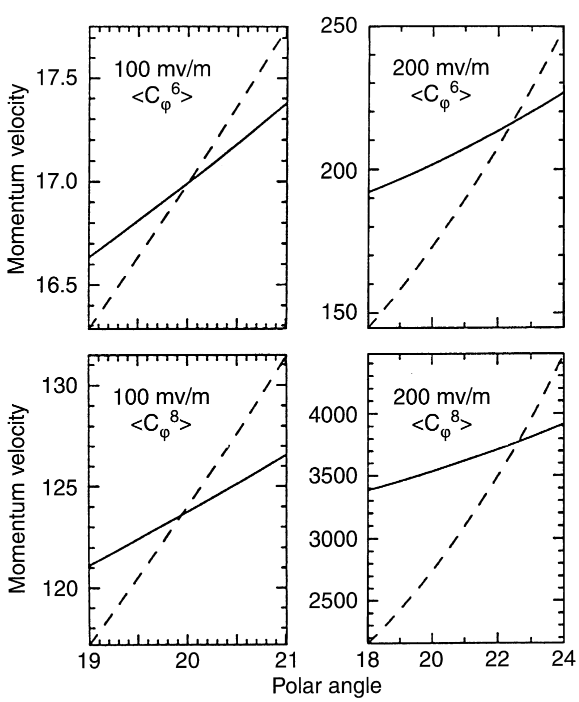


**Fig. 4.** Comparison of the velocity moments of order 6 and 8 of a Maxwellian at temperature $T_\varphi$ (*dashed lines*) to the Monte Carlo velocity moments at similar order (*solid lines*). Examples of an electric field of 100 mV/m is on the *left panels*, and of 200 mV/m on the *right panels*

## 4 The $O^+$ ion distribution for large polar angles

Figure 5 shows the 1-D line-of-sight distributions (solid line) for angles $\varphi$ equal to $0°, 30°, 70°, 90°$ for an electric field of 100 mV/m, in the top panels, and an electric field of 200 mV/m in the lower panels. The dashed line is the 1-D Raman model fitting defined by Kikuchi *et al.* (1989)

$$g_\varphi(u') = \frac{2}{\pi}\int \mathrm{d}r' \exp\left(-D^{*2}_\varphi - (r'^2 + u'^2)\right) Io \times \left(2D^*_\varphi (r'^2 + u'^2)^{1/2}\right) \quad (5)$$

where $u' = u/(2kT^*_\varphi/m_i)^{1/2}$. The parameters $T^*_\varphi$ and $D^*_\varphi$ are fitting parameters which have been determined for 50 mV/m and 100 mV/m electric fields by Kikuchi *et al.* (1989) for polar angles larger than 30°. Indeed for smaller angles the 1-D Raman fitting does not provide a unique fit in terms of $D^*_\varphi$ and $T^*_\varphi$ while for polar angles $\varphi \geq 30°$ it provides a unique fit with accurate line-of-sight temperature when compared to the exact values. The line-of-sight temperature derived from the fitting model as given in Eq. (5) is

$$T_\varphi = T^*_\varphi(1 + D^{*2}_\varphi) . \quad (6)$$

The great interest of the 1-D Raman model is that it can fit any toroidal distribution function with only two fitting parameters: $T^*_\varphi$ plays the role of an effective temperature, $D^*_\varphi$ of an effective electric field. This model has been successfully used for the interpretation of $O^+$ and $NO^+$ non-Maxwellian microscopic state on space-craft data by St-Maurice *et al.* (1976), and in theoretical discussion of experimental results by Hubert (1983). More recently Lockwood and Winser (1988) and Kikuchi *et al.* (1989) pointed out the usefulness of this model for the derivation of non-equilibrium plasma parameters from incoherent radar spectra. Nevertheless from the paper of Kikuchi *et al.* (1989), it is not clear why for polar angles $\varphi$ lower than some 30°, the 1-D Raman model does not fit accurately the 1-D distribution function; and why the fitting works for $\varphi \geq 30°$. Indeed a toroidal distribution as defined in Eq. (5)

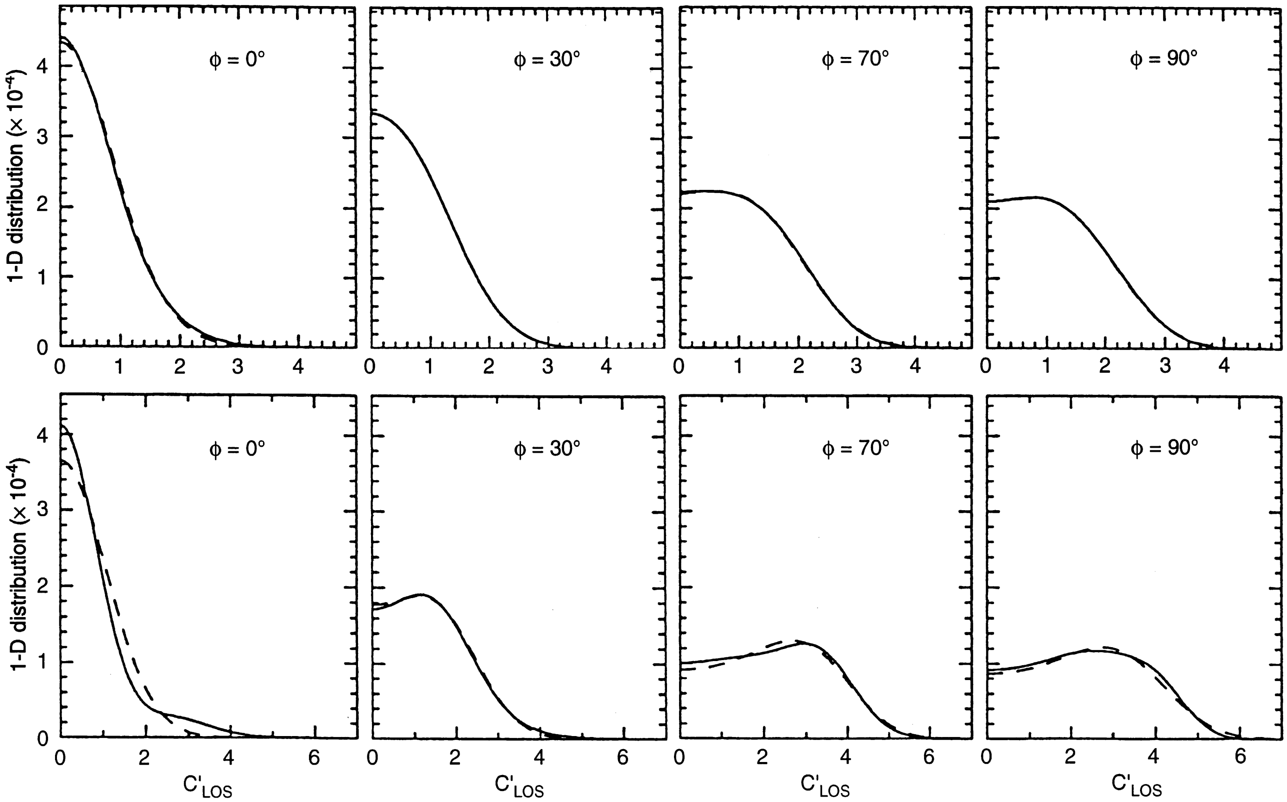


**Fig. 5.** The *top panels* show (*solid line*) the 1-D distribution for an electric field intensity of 100 mV/m and four angles-of-sight of 0°, 30°, 70°, 90° from *left* to *right*. The *dashed curves* represent the fitted 1-D Raman distribution. The *four lower panels* show similar results for an electric field intensity of 200 mV/m

implies a relationship between the parameters $T^*_\varphi$ and $D^*_\varphi$ via the velocity moments of order 2 and 4 as shown by Hubert (1983) for the case where $\varphi = 90°$. The required conditions for polar angles $\varphi < 90°$ are easily generalized from Eqs. (10) and (11) from the paper of Hubert (1983) into:

$$(\alpha D'_\varphi)^4\left(2\langle C'^4_\varphi\rangle - 3(\langle C'^2_\varphi\rangle)^2\right) + 2(\alpha D'_\varphi)^2 \times \left(2\langle C'^4_\varphi\rangle - 6(\langle C'^2_\varphi\rangle)^2\right) + 2\langle C'^4_\varphi\rangle - 6(\langle C'^2_\varphi\rangle)^2 = 0\,, \tag{7}$$

where $\alpha D'_\varphi$ is a generalized effective electric field, $\langle C'^p_\varphi\rangle$ is the velocity moment of order $p$ along the direction defined by the polar angle $\varphi$. The theoretical model similar to the fitting model given by Eq. (5) is then completely defined by:

$$T^+_\varphi = \frac{m_i}{k}\,\frac{\langle c^2_\varphi\rangle}{1 + \alpha D'^2_\varphi}\,. \tag{8}$$

The condition required for an efficient fitting is that $\alpha D'_\varphi$ be a real quantity which implies from the quadratic Eq. (7) that:

$$\langle C'^4_\varphi\rangle < 3(\langle C'^2_\varphi\rangle)^2\,,$$
$$\langle C'^4_\varphi\rangle > \frac{3}{2}(\langle C'^2_\varphi\rangle)^2\,. \tag{9}$$

These conditions are realized for $\varphi > 20°$ and $\varphi > 21.6°$ for electric field intensities of 100 mV/m and 200 mV/m respectively, but fail for $\varphi < 21 \pm 1°$. When $\langle C'^4_\varphi\rangle = 3(\langle C'^2_\varphi\rangle)^2$, the value of $\alpha D'_\varphi$ derived from Eq. (7) is equal to zero, and the 1-D Raman theoretical model reduces to a Maxwellian distribution, with $T^+_\varphi = T_\varphi$. This occurs for $\varphi = 20°$ and $\varphi = 21.6°$ for electric fields of 100 mV/m and 200 mV/m respectively as discussed in Sect. 3.

## 5 Conclusions

New features of the $O^+$ ion velocity distribution function in a background of neutral atomic oxygen submitted to an external crossed electro-magnetic field have been established. The 1-D ion velocity distribution is derived from polynomial expansion whose velocity moments have been provided by Monte Carlo simulations of the first generation (Barakat *et al*., 1983) Kikuchi *et al*., 1989). The 1-D distribution function parallel to the magnetic field displays a core-halo structure with a core temperature equal to the neutral temperature. The 1-D distribution function is confirmed to be a Maxwellian for a typical polar angle around $21 \pm 1°$ for electric fields intensity of 100 mV/m, and for a very high electric field intensity of 200 mV/m. The Maxwellian property is proved when taking into account the precision on the velocity moments at order

6 and 8. For the first time the reasons why the 1-D Raman model fits well the $O^+$ non-Maxwellian distribution function for polar angles larger than some $21 \pm 1°$ have been explained; as well as why the fitting does not work for $\varphi < 21 \pm 1°$. A good fitting is obtained when the fourth order velocity moment of a distribution is lower than the corresponding velocity moment of the respective Maxwellian velocity distribution function, but larger than half of this fourth order Maxwellian velocity moments.

The results obtained are important in the perspective of the non-equilibrium plasma parameter measurements from incoherent radar data. The property that the $O^+$ velocity distribution is a Maxwellian for a specific angle is an important ingredient for a measurement scenario at large aspect angles: indeed the electron temperature which can only be derived from a measurement at the specific angle, is required for the measurement of the ion parameters from the other aspect angles (Hubert *et al.*, 1993; 1996). The fitting with 2 Maxwellians of the 1-D distribution, for angles smaller than 21°, completes the empirical modelling of the 1-D distribution by a toroidal distribution for $\varphi > 21°$. Similar properties could be found in $N_2^+$ distributions which could be abundant in the auroral ionosphere when they are produced by soft electron impacts (Winkler *et al.*, 1992). Finally the characteristics of the $O^+$ velocity distributions with respect to the polar angle $\varphi$, imply intriguing properties of $O^+$-O charge exchange collisions. These properties from a Monte Carlo code of the second generation are being studied (Winkler *et al.*, 1992; Gaimard, 1996).

The transition of the $O^+$ 1-D velocity distribution function from a core-halo shape to the well-known toroidal shape at large polar angle through the Maxwellian shape at $21 \pm 1°$ discussed from a generic functional of the velocity moments at order 2 and 4 represents an interesting new approach. As is well known, the characteristics of a polynomial expansion of any velocity distribution function is very dependent on the shape of the zero$^{th}$ order approximation, but the generic function Eq. (7) provides directly from the velocity moments the core-halo, Maxwellian or toroidal features of a symmetric distribution function. This could have applications in other fields of plasma physics, as for example, the microscopic description of the solar wind, or the modelling of new-born ions in cometary physics.

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