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Construction of intensity-duration-frequency (IDF) curves for precipitation with annual maxima data in Rwanda, Central Africa

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Abstract. Detailed probabilistic information on the intensity of precipitation in Central Africa is highly needed in order to cope with the risk analysis of natural hazards. In the mountainous areas of Rwanda land slides frequently occur and might cause a heavy toll in human lives. The establishment of Intensity-Duration-Frequency curves for precipitation in Central Africa remains a difficult task as adequate long-term data sets for short aggregation times are usually not available.

In 1962 recording raingauges were installed at several stations in Rwanda. According to the climatological procedures in use at that time in Congo, Rwanda and Burundi, maximum monthly and annual precipitation depths for fixed-time durations of 15, 30, 45, 60 and 120 min were determined from the rainfall charts. The data set is completed by the monthly and annual daily precipitation extremes from the non-recording raingauge at the stations.

The authors used the dataset to establish the IDF-curves for precipitation at 3 stations in Rwanda having more than 20 yr of operation. The fixed-hour intervals of multiple 15 min require the use of a technique converting data from fixed-time intervals into data of arbitrary starting intervals. Therefore, the van Montfort technique was used.

1 Introduction

The purpose of this study is to establish Intensity-Duration-Frequency (IDF) curves for precipitation at 3 climatological stations in Rwanda. These stations are located in zones with a different rainfall regime. The IDF-curves for precipitation provide a mathematical expression linking the intensity, the duration or, better said, the aggregation time of the rainfall and the frequency of a rainfall event (Stedinger et al., 1993). The establishment of IDF-curves for precipitation is a fre-

quently used tool in water resources projects and management, sewer design in urban areas and geomorphologic research. In this case, a Montana-like formula is used as mathematical expression of the IDF-curves relationship. This formula has proven its adaptability to express IDF-curves for precipitation under different rainfall regimes.

Only short-time Annual Maxima (AM) information was available for the sub-daily durations. In Sect. 2, the climate of Rwanda is briefly described and, in particular, the climatological aspects of the 3 stations are given. In Sect. 3, a Gumbel probability distribution is fitted to the AM data. Furthermore, the van Montfort technique is used to assess directly the parameters μ_A and σ_A of the Gumbel probability distributions for A-maxima of 15, 30, 60 min and 1-day.

2 Data and climate

The Republic of Rwanda, known as the *Land of a Thousand Hills*, is a landlocked country located in the Great Lakes region of eastern-central Africa, bordered by Uganda, Burundi, the Democratic Republic of the Congo and Tanzania (Fig. 1). It covers an area of 26 338 km^2. The country is bounded in the west by a mountain zone running south-north with an altitude of between 2000 and 2500 m. To the east of this range, the plateaus are at an altitude of approximately 1500 and 2000 m. To the west of this range is situated Lake Kivu at 1460 m elevation. On the plateau the mean annual rainfall varies between 900 and 1200 mm while in the mountains it varies between 1200 and 1500 mm. In the Kagera plains in the east part of Rwanda, rainfall amounts less 900 mm showing a tendency to aridity (Bultot and Griffiths, 1971).

In 1962 recording raingauges were installed at a number of stations at the Rwanda Agricultural Research Institute/Institut des Sciences Agronomiques du Rwanda (ISAR).

Table 1. Name, geographical co-ordinates, altitude and operating service of the stations Karama-Plateau, Rubona and Rwerere-Colline in Rwanda.

Name	Longitude East	Latitude South	Altitude(m)	Service
Karama-Plateau	30°16′	02°17′	1403	ISAR
Rubona	29°46′	02°29′	1706	ISAR
Rwerere-Colline	29°53′	01°32′	2312	ISAR

Fig. 1. Map of the Republic of Rwanda with the location of the stations, Rwerere-Colline **(a)**, Rubona **(b)** and Karama-Plateau **(c)**.

Monthly and annual precipitation depths, expressed in 0.1 mm, for fixed-time durations of 15, 30, 45, 60 and 120 min were manually determined from the rainfall charts (Crabbé, 1971). The rainfall charts are read in fixed-time calendar 15 min interval. The data set is completed by the monthly and annual daily precipitation extremes from the non-recording raingauge. The sub-daily maxima data from the recording raingauge showed excellent agreement with the daily maxima data from the non-recording raingauge at the same station. The recording raingauge network continued its operation till, more or less, the mid-eighties. Data used in the present research were extracted from the annual publications *Bulletin Climatologique Annuel* mainly edited by ISAR. The AM extremes data set for the 3 stations above-mentioned were available to the authors for the years 1962 till 1985. The 3 stations selected on the basis of the length of their record are respectively: Karama-Plateau, Rubona and Rwerere-Colline (Fig. 1). Table 1 represents their geographical characteristics. The length of the AM series is 20 yr for the stations Karama-Plateau and Rwerere-Colline and 23 yr

for the station of Rubona. These data are the core rainfall intensity data of the present study. Daily data are available for slightly longer to longer reference periods and therefore AM data for the daily maxima are included in the approach.

The station Karama-Plateau belongs to the Semi Arid Research Centre (SARC) of the Rwanda Agricultural Research Institute (ISAR) and is situated in the Eastern Province, district of Bugesera, at an altitude of 1403 m. The average annual precipitation is approximately 850 mm. The station Rubona-Colline is at an altitude of 1706 m. It received an average annual precipitation of 1240 mm over the reference period 1955–1990. The station Rwerere-Colline is part of the High Altitude Research Center (HARC) of ISAR. The station is located in the Northern Province, Rwerere sector, Burera district, at an altitude of 2312 m with an average annual rainfall of 1200 mm.

3   Methodology

The Montana-type IDF-curve for precipitation was selected for the 3 stations:

$$i_{T,d} = \frac{a(T)}{(d+\theta)^\eta} \tag{1}$$

where d is the length of the precipitation and T is the return period of the event. The nominator $a(T)$ is independent of the aggregation time so that the family of curves in T are parallel. The expression in the nominator is determined by the inverse function of $a(T)$ (Koutsoyiannis et al., 1998).

The denominator $(d+\theta)^\eta$ of the Montana-type of IDF-curve provides the shape of the IDF-curves; the dynamical parameters θ and η describe the dynamics of the extreme rainfall process in function of the duration d and are climate-related.

As only annual maxima are available, the adequate probability distribution functions are the 2-parameter Gumbel distribution G and the 3-parameter General Extreme Value (GEV) distribution. The cumulative distribution function of G is given by:

$$G(x; \mu, \sigma) = \exp\left(-\exp\left(-\frac{x-\mu}{\sigma}\right)\right) \tag{2}$$

where x is the function variable, μ and σ are the parameters of the Gumbel distribution function. They are called the
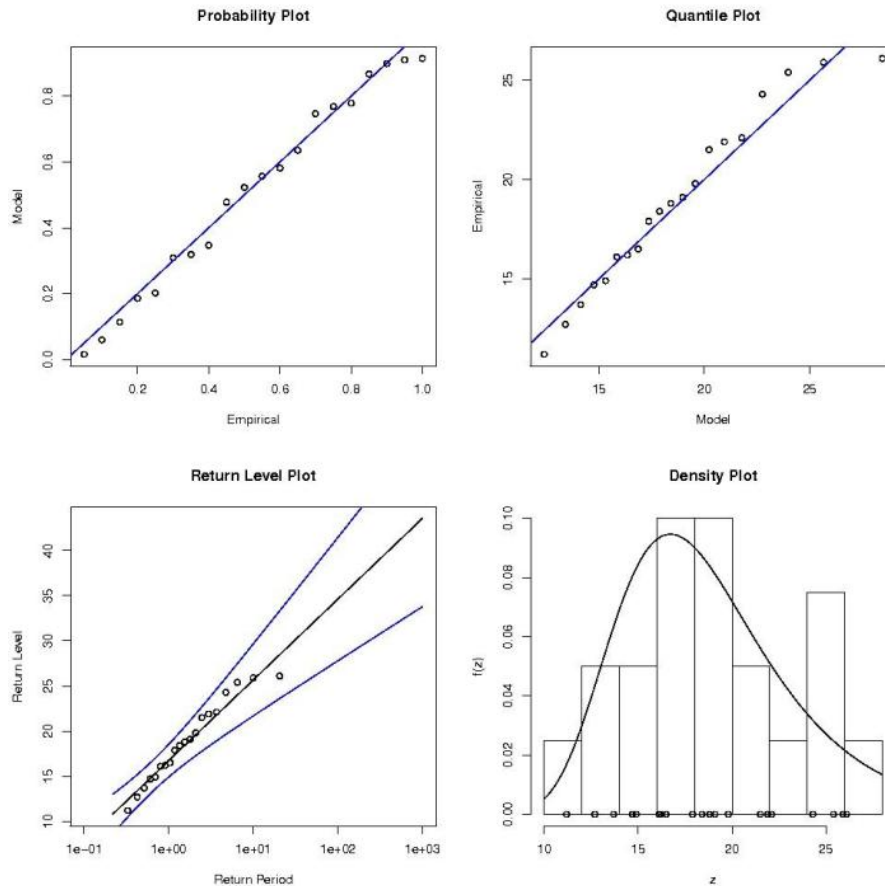
Fig. 2. Diagnostic plots for the Gumbel model fitted to the annual maxima of the 15-min precipitation data at Karama-Plateau (Coles, 2001).

location- and scale-parameter, respectively. To be more precise, the location-parameter μ specifies the center of the distribution; and the scale-parameter σ determines the size of deviations about the location parameter. It was shown that the 2-parameter Gumbel distribution G fitted the limited amount of data for the 3 stations by the model diagnostics plots developed in Coles (2001). As illustration, the diagnostic plots for the Gumbel model fitted to the annual maxima of the 15-min precipitation data at Karama-Plateau are shown in Fig. 2 (Coles, 2001; ismev, 2013). The inverse function of the Gumbel distribution $G(x; \mu, \sigma)$ is given by the following formula:

$$a(T) = \mu - \sigma \ln\left[-\ln(1 - \frac{1}{T}) \right] \tag{3}$$

Precipitation maxima of specific length with fixed starting points are often derived from digitizing rainfall charts. In this case the specific lengths are determined by the subdivision of the length of the hour on the graph. However, in quantitative hydrology, true maxima, this means maxima in time periods of specific length and with arbitrary starting point are required. In the following, the notation "A-maxima" is used for maxima precipitation amounts with an arbitrary sliding starting point where "F-maxima" relate to fixed starting points. Van Montfort (1990, 1997) proposed a technique to convert maximum precipitation amounts in specific lengths with fixed starting points into maxima of the same specific length and sliding starting points.

The likelihood function L is defined as:

$$L = \prod_{i=1}^{n} g(x_{A,i}; \mu_A, \sigma_A) \tag{4}$$

with $x_i \leq x_{A,i} \leq x_{2i}$, where the indices A are indicative of A-maxima, x_{1i} and x_{2i} are the i-th data points with $i = 1, ..., n$ of the a given length and of its double length, g is the probability density function of the Gumbel distribution G and which is approximated by the expression

$$g(x_{A,i}; \mu_A, \sigma_A) \approx \frac{G(x_{2i}; \mu_A, \sigma_A) - G(x_i; \mu_A, \sigma_A)}{x_{2i} - x_i} \tag{5}$$

Practically it is more convenient to work with the log-likelihood function $\ln L$. Optimizing $\ln L$ for μ_A and σ_A is equivalent to optimizing the equation below for μ_A and σ_A.

$$\ln L = \sum_{i=1}^{n} \ln \left[G(x_{2i}; \mu_A, \sigma_A) - G(x_{1i}; \mu_A, \sigma_A) \right] \tag{6}$$

Table 2. Optimized values of the parameters of the IDF-curves $i_{T,d}$ for the stations Karama-Plateau, Rubona and Rwerere-Colline in Rwanda.

station	μ (mm)	σ (mm)	θ (h)	η
Karama-Plateau	40.48	7.86	0.25	0.94
Rubona	44.71	11.00	0.25	0.95
Rwerere-Colline	40.41	5.73	0.24	0.95

Table 3. Rainfall depths (mm) for return periods T (year), and aggregation times d (h) based on the adjusted IDF-curves of A-data at the station Rwerere-Colline (Rwanda).

T (year)	2	5	10	20	50
$d = 0.25$ h	20.8	24.0	26.1	28.1	30.7
$d = 0.5$ h	28.2	32.5	35.3	38.1	41.6
$d = 0.75$ h	32.1	37.0	40.3	43.4	47.4
$d = 1$ h	34.6	39.9	43.4	46.8	51.1
$d = 24$ h	50.1	57.8	62.8	67.7	74.0

Table 4. Rainfall depths (mm) for return periods T (year), and aggregation times d (h) based on the adjusted IDF-curves of A-data at the station Karama-Plateau (Rwanda).

T (year)	2	5	10	20	50
$d = 0.25$ h	20.9	25.2	28.1	30.8	34.3
$d = 0.5$ h	28.5	34.4	38.3	42.0	46.8
$d = 0.75$ h	32.6	39.3	43.8	48.0	53.6
$d = 1$ h	35.3	42.5	47.3	51.9	57.9
$d = 24$ h	52.2	63.0	70.1	76.9	85.7

Table 5. Rainfall depths (mm) for return periods T (year), and aggregation times d (h) based on the adjusted IDF-curves of A-data at the station Rubona (Rwanda).

T (year)	2	5	10	20	50
$d = 0.25$ h	23.5	29.6	33.5	37.4	42.3
$d = 0.5$ h	32.1	40.3	45.7	50.9	57.6
$d = 0.75$ h	36.6	46.0	52.2	58.1	65.8
$d = 1$ h	39.5	49.6	56.3	62.7	71.1
$d = 24$ h	57.4	72.1	81.8	91.1	103.2

Then the parameters of the Gumbel probability distribution function G for arbitrary starting points are obtained by optimizing the log-likelihood function $\ln L$.

The above-mentioned technique (van Montfort, 1990) is applied to the following couples of lengths (15–30) minutes, (30–60) minutes, (60–120) minutes, and the (1-day, 2-days) precipitation data providing estimates of the parameters μ_A and σ_A of the Gumbel probability distributions for A-maxima of 15, 30, 60 min and 1-day.
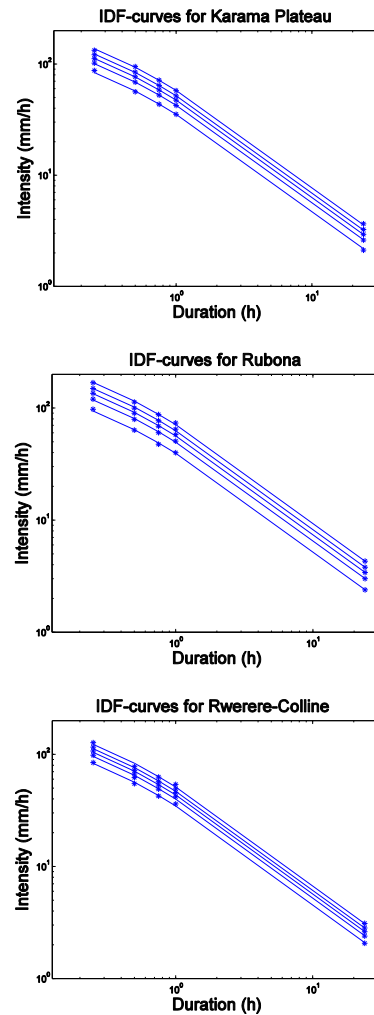

Fig. 3. Intensity-Duration-Frequency (IDF) curves for precipitation at the stations (from top to bottom) Karama-Plateau, Rubona and Rwerere-Colline, Rwanda, using A-type statistical parameters values produced by the van Montfort technique.

Once the parameters of the Gumbel distribution functions are determined the quantiles of the Gumbel distributions are computed corresponding to return periods of 2, 5, 10, 20 and 50 yr. The parameters μ, σ, θ and η of the Montana-type formula $i_{T,d}$ are obtained by global optimization techniques (Table 2).

4   Results

Figure 3a, b and c show the Intensity-Duration-Frequency (IDF) Montana-type curves for precipitation at the stations Karama-Plateau, Rubona and Rwerere-Colline using the A-type statistical parameters determined by the van Montfort technique (1990). The quantiles for aggregation times d of 15, 30, 45, 60 and 1440 min (0.25, 0.5, 0.75, 1 and 24 h) and

for return periods T of 2, 5, 10, 20 and 50 yr are computed and are given in Tables 3, 4 and 5.

The 3 sets of IDF-curves for precipitation have approximately the same dynamical parameters θ and η but showing differences in the magnitude of intensity; the order of magnitude of the intensity in the stations is (from low to high): Rwerere-Colline (Table 3), Karama-Plateau (Table 4) and Rubona (Table 5).

5   Discussion

Intensity-Duration-Frequency (IDF) curves for precipitation were obtained for 3 rainfall stations in different climate regions of Rwanda. Although that information is highly needed as natural hazards such as landslides are frequently observed in the country no such information was available until now for Rwanda. The numerical results obtained for the 3 sites in Rwanda are of similar magnitude to the results for Lubumbashi, Congo (Van de Vyver and Demarée, 2010).

The information is based upon A-maxima of aggregation times with arbitrary starting points. That information was obtained by a van Montfort technique using F-data of aggregation times with fixed starting points. The novelty in the paper is the use of a set of sub-daily precipitation F-data obtained from digitizing precipitation charts in combination with the data set of daily precipitation amounts.

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