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What can be learned about carbon cycle climate feedbacks from the CO_2 airborne fraction?

M. Gloor[1], **J. L. Sarmiento**[2], **and N. Gruber**[3]

[1]The School of Geography, University of Leeds, Leeds, LS2 9JT, UK
[2]Atmospheric and Oceanic Sciences Department, Princeton University, 300 Forrestal Road, Sayre Hall, Princeton, NJ 08544, USA
[3]Institute of Biogeochemistry and Pollutant Dynamics, ETH Zürich, Universitätsstr. 16, 8092 Zürich, Switzerland

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Abstract. The ratio of CO_2 accumulating in the atmosphere to the CO_2 flux into the atmosphere due to human activity, the airborne fraction AF, is central to predict changes in earth's surface temperature due to greenhouse gas induced warming. This ratio has remained remarkably constant in the past five decades, but recent studies have reported an apparent increasing trend and interpreted it as an indication for a decrease in the efficiency of the combined sinks by the ocean and terrestrial biosphere. We investigate here whether this interpretation is correct by analyzing the processes that control long-term trends and decadal-scale variations in the AF. To this end, we use simplified linear models for describing the time evolution of an atmospheric CO_2 perturbation. We find firstly that the spin-up time of the system for the AF to converge to a constant value is on the order of 200–300 years and differs depending on whether exponentially increasing fossil fuel emissions only or the sum of fossil fuel and land use emissions are used. We find secondly that the primary control on the decadal time-scale variations of the AF is variations in the relative growth rate of the total anthropogenic CO_2 emissions. Changes in sink efficiencies tend to leave a smaller imprint. Therefore, before interpreting trends in the AF as an indication of weakening carbon sink efficiency, it is necessary to account for trends and variations in AF stemming from anthropogenic emissions and other extrinsic forcing events, such as volcanic eruptions. Using atmospheric CO_2 data and emission estimates for the period 1959 through 2006, and our simple predictive models for the AF, we find that likely omissions in the reported emissions from land use change and extrinsic forcing events are sufficient to explain the observed long-term trend in AF. Therefore, claims for a decreasing long-term trend in the carbon sink efficiency over the last few decades are currently not supported by atmospheric CO_2 data and anthropogenic emissions estimates.

Correspondence to: M. Gloor
(eugloor@googlemail.com)

1 Introduction

Central for predicting future temperatures of the Earth's surface is how much and for how long carbon dioxide from fossil fuel emissions and land use change stays in the atmosphere, and how much gets removed by the carbon sinks on land and in the ocean (e.g. Solomon et al., 2009). A straightforward measure of this redistribution is the ratio between the increase rate in atmospheric CO_2 and the CO_2 emitted to the atmosphere by human activity (fossil fuel burning and land use change). Keeling (1973) termed this quantity the "airborne fraction" (AF) and it was investigated in many subsequent studies (e.g. Bacastow and Keeling, 1979; Oeschger and Heimann, 1983; Enting, 1986). Because of the large uncertainties in land fluxes, these early studies could estimate the value of AF only to within a wide range from 0.38 to 0.78 (Oeschger and Heimann, 1983). Recently several studies have extended the estimation of AF over the last two decades, with a suggestion of a positive trend in AF (Canadell et al., 2007; Raupach et al., 2008; Le Quéré et al., 2009). Moreover, this positive trend has been interpreted as evidence for a decreasing trend in the efficiency of the ocean and land carbon sinks. Given the model-based projection of a substantial reduction in the sink strength of the ocean and land in the future (e.g. by a large-scale dieback of the Amazon old-growth forest, Cox et al., 2000), the notion that the sinks have already begun to deviate from a linear response to the atmospheric CO_2 perturbation is a source of substantial concern. While there remains discussion about whether this trend in the AF is actually statistically significant (Knorr, 2009), we focus our discussion here on whether the inferred conclusion is defensible, i.e. whether an increasing trend in the AF implies a decreasing efficiency of the carbon sinks.

Determinants of the AF are the magnitude and time course of the human induced emissions of CO_2 into the atmosphere and the removal of this anthropogenic carbon by the ocean and land biosphere. It has been known since the early 1970's, possibly earlier, that the AF will eventually asymptote to a constant value if (i) the CO_2 uptake by the oceans and land ecosystems is linear and (ii) if CO_2 emissions to the atmosphere follow exactly an exponential function (Bacastow and Keeling, 1979). Thus, given that fossil fuel emissions have risen approximately exponentially over the last 250 years, and that natural systems tend to respond linearly to small perturbations, it is natural to inquire whether time trends in AF may inform us about changes in the linear behavior of carbon uptake by the oceans and land ecosystems (Canadell et al., 2007; Raupach et al., 2008; Le Quéré et al., 2009; Rafelski et al., 2009). However, closer examination shows that the relative growth rate RGR$\equiv \frac{1}{\mathrm{FF}}\frac{d\mathrm{FF}}{dt}$ of fossil fuel emissions FF has varied by more than a factor of two in the last 100 years (e.g. Raupach et al., 2008). In addition, emissions from land use change exhibited an even more varied time course, so that the total emissions only very approximately followed a single exponential. Furthermore, trends in AF may be an articulation of an incomplete spin-up of the system, so that the AF is still changing along its way toward reaching its asymptotic value. We examine here the impact of these deviations and controls on the AF, and what the consequences are for the interpretation of the AF as an indicator for changes in the efficiency of carbon sinks, and in turn the state of the global carbon cycle. Our study builds on the seminal work of Bacastow and Keeling (1979) who had stated already 30 years ago that "The global average airborne fraction will probably not remain near 56% in the future ... because fossil fuel resources are finite...", i.e. that one important control of the AF is the growth rate of fossil fuel emissions.

Before proceeding, it is important to recognize that definitions of the AF in the literature vary. Studies from the 1970s and 1980s defined airborne fraction from cumulative carbon inventory changes as

$$\mathrm{AF}_{\mathrm{FF}}^{\mathrm{cum}} \equiv \frac{C(t_\mathrm{f}) - C(t_i)}{\int_{t_i}^{t_\mathrm{f}} \mathrm{FF}(t)dt}$$

(Keeling, 1973; Bacastow and Keeling, 1979; Enting, 1986) or alternatively as

$$\mathrm{AF}_{\mathrm{FF+LU}}^{\mathrm{cum}} \equiv \frac{C(t_\mathrm{f}) - C(t_i)}{\int_{t_i}^{t_\mathrm{f}} \mathrm{FF}(t) + \mathrm{LU}(t)dt}$$

(Oeschger and Heimann, 1983) where C is atmospheric carbon dioxide, t_i, t_f are the beginning and the end time of the period considered, FF is fossil fuel emissions and LU is the flux to the atmosphere due to land use change. The more recent studies define airborne fraction from annual or monthly inventory changes as either

$$\mathrm{AF}_{\mathrm{FF}} \equiv \frac{\frac{dC(t)}{dt}}{\mathrm{FF}(t)}, \ or \ \mathrm{AF}_{\mathrm{FF+LU}} \equiv \frac{\frac{dC(t)}{dt}}{\mathrm{FF}(t) + \mathrm{LU}(t)}$$

(Canadell et al., 2007; Raupach et al., 2008; Le Quéré et al., 2009; Knorr, 2009). We analyze here the time-evolution of the AF as defined by these recent studies.

We briefly outline the organization of our paper. We start in Sect. 2 with a characterization of the time course of anthropogenic CO_2 emissions and carbon sinks, thereby highlighting that there have been strong variations in the relative growth rate of fossil fuel emissions over the last century. In Sect. 3 we introduce a simple linear model of the evolution of an atmospheric CO_2 perturbation, thereby also clarifying the meaning of "sink efficiency". In Sect. 4 we explore how the time course of the anthropogenic emissions controls variations in the AF, using a predictive equation implied by our simple model. We demonstrate that (i) for an atmospheric CO_2 perturbation which is not following an exact exponential function, there is an adjustment time for the AF to converge to its constant asymptotic value, which is on the order of centuries, and that (ii) variations in the relative growth rate of the anthropogenic emissions are a major control on variations of the AF. Therefore, in order to unravel trends in the AF caused by trends in carbon sink efficiency or extrinsic non-anthropogenic events, like volcanic eruptions, signatures due to incomplete "spin-up" and fossil fuel growth rate variations, need first to be removed from the observed AF. We can achieve this using our predictive equation for the AF (Sect. 5). We then examine the remaining signal for trends not explained by known extrinsic non-anthropogenic forcings or omissions in anthropogenic fluxes, to conclude whether there is indeed evidence for trends in the carbon sink efficiency trends in the observed AF record (Sect. 6). This terminates our main analysis. Section 7 in addition explores the signal to noise ratio of AF trends caused by sink efficiency trends, and finally we discuss and conclude.

2 Anthropogenic carbon emissions and carbon sinks

The main driver of the rapid increase in atmospheric CO_2 is fossil fuel emissions, which are estimated from national energy statistics with an uncertainty of 6%–10% (90% confidence interval) (Marland, 2006, updated by Boden et al., 2009; Marland, 2008). A logarithmic representation (Fig. 1a) reveals that fossil fuel emissions have increased roughly exponentially, with the time-scale of relative change, the inverse of the relative growth rate, varying roughly between $\sim$20 and 150 years (Fig. 1b).

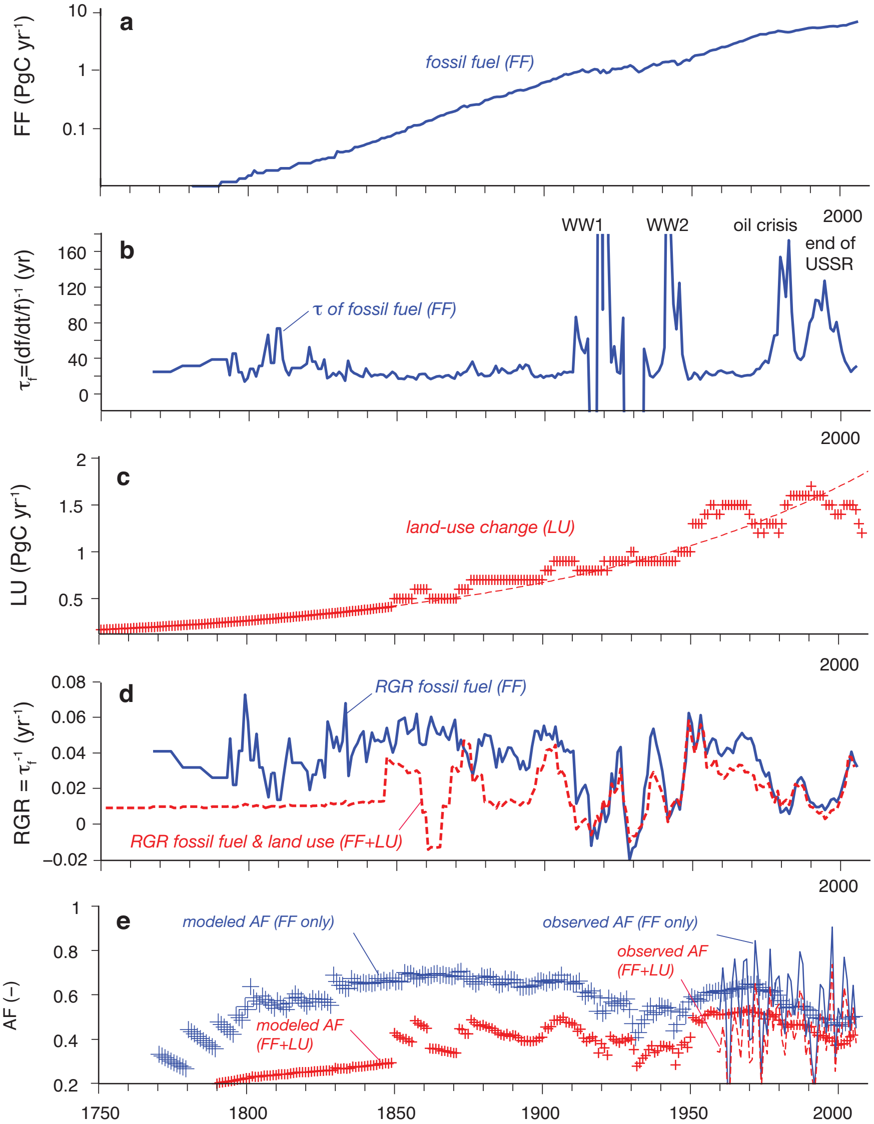

Fig. 1. **(a)** Fossil fuel emissions estimated by Marland (2006), **(b)** time scale τ_f of relative rate of change of FF, **(c)** carbon flux to the atmosphere due to land use change estimated by Houghton et al. (2007), **(d)** relative growth rate of f=FF and f=FF+LU respectively, and **(e)** model predicted and observed AF_{FF} and AF_{FF+LU} .

The time-scale τ_f of relative change of an anthropogenic flux f to the atmosphere is defined as the inverse of its logarithmic derivative:

$$\tau_f \equiv (\frac{1}{f}\frac{df}{dt})^{-1} \simeq (\frac{1}{f}\frac{\Delta f}{\Delta t})^{-1}$$

with $\Delta f \equiv f(t+\Delta t) - f(t)$ and $\Delta t = 1$ year. The time-course of τ_f permits to identify periods with different fossil fuel emissions growth rates particularly well.

Variations in the time-scale of relative change of fossil fuel emissions, τ_f, are mainly due to economic cycles and wars. Thus there was an approximately 80 year period from around 1830 to 1910 (approximately the start date of World War one (WW1)) with a roughly constant $\tau_{FF} \equiv (\frac{1}{FF}\frac{dFF}{dt})^{-1} \simeq (\frac{1}{FF}\frac{\Delta FF}{\Delta t})^{-1}$ of ∼20 years (equivalent to a relative growth rate of 5% per year, Fig. 1d). The WW1, post WW1 and great depression period saw less growth, with both positive and negative time-scales resulting in a substantially longer mean τ_{FF}. After WW2 (starting around 1948) there is again fast growth, paralleling the recovery of industrial countries' economies, until the early 1970s with τ_{FF} of ∼20 years. From the early 1970s until approximately 1999 τ_{FF} increased again to ∼80 years (relative growth rate of ∼1.3% yr^{-1}). Growth returned close to the 1830 to 1910 and post WW2 values starting around 2001.

The second cause of the rise in atmospheric carbon, and at the same time the least well constrained positive component of the atmospheric carbon budget, is carbon fluxes released from land to the atmosphere due to land use change (for example rainforest to pasture conversion in the tropics, or peat burning during 1997/98 in Indonesia due to conversion of swamp forests to rice paddies at a large spatial scale, Page et al., 2002). Estimates of Houghton et al. (2007) indicate that this term has also risen in time, but at a considerably smaller rate than fossil fuel emissions (Fig. 1c). The uncertainty in fluxes associated with land use change is large, on the order of 40–100%, as revealed by the range of published estimates (e.g. Grainger, 2008; Houghton et al., 2007; DeFries et al., 2002; Achard et al., 2002).

The atmospheric CO_2 accumulation rate is well constrained by atmospheric concentration records (Keeling, 1960; Etheridge et al., 1996). Estimates of ocean uptake of anthropogenic carbon based on various methods have also converged over recent years to $2.2\pm0.2\,\mathrm{PgC\,yr^{-1}}$ for a nominal period of ∼1995–2000 (Sabine et al., 2004; Sweeney et al., 2007; Sarmiento et al., 2010; Gruber et al., 2009; Khatiwala et al., 2009). The net land sink, the sum of the land sink and the CO_2 flux to the atmosphere due to land use change, can then be calculated as the difference between fossil fuel emissions, the atmospheric CO_2 accumulation rate and ocean uptake. The implied net land sink stayed roughly constant with a mean value of nearly zero from the 1930s to 1990 and then increased to a magnitude of approximately $1\,\mathrm{PgC\,yr^{-1}}$ for the 1990s and early 2000s (e.g., Sarmiento et al., 2010).

3 A simple carbon cycle model

We investigate the processes controlling time-variations of the airborne fraction with simple linear models of the global perturbation of the carbon cycle. This is justified on two grounds. First any claim of a possible non-linear behaviour of the system must be shown to differ from the prediction of such linear models. Secondly, the concept of an efficiency, like the efficiency of a heat engine, is inherently linear. This is because an efficiency is defined as the ratio between the magnitude of an effect and the magnitude of its cause. In our case the cause is the increase in atmospheric CO_2 due to human activity and the effect is the carbon flux from the atmosphere to the ocean and land carbon pools. If we treat the ocean and land each as a single pool of carbon with a constant sink efficiency, then the fluxes from the atmosphere to the oceans and to the land $F_{\mathrm{at}\rightarrow\mathrm{oc}}$ and $F_{\mathrm{at}\rightarrow\mathrm{ld}}$, are given by

$$F_{\mathrm{at}\rightarrow\mathrm{oc}} = \frac{\Delta C}{\tau_{\mathrm{oc}}},\; F_{\mathrm{at}\rightarrow\mathrm{ld}} = \frac{\Delta C}{\tau_{\mathrm{ld}}}\;. \tag{1}$$

Here $\Delta C \equiv C(t) - C(1765)$ is the anthropogenic perturbation of atmospheric carbon dioxide, and τ_{oc} and τ_{ld} are constants. In this context a weakening/strengthening of the sinks means that τ_{oc} and/or, τ_{ld} are increasing/decreasing in time.

With the flux parameterization given by Eq. (1), the time evolution for ΔC implied by the atmospheric CO_2 mass balance is determined by

$$\begin{aligned}\frac{d\Delta C}{dt} &= f(t) - F_{\mathrm{at}\rightarrow\mathrm{oc}} - F_{\mathrm{at}\rightarrow\mathrm{ld}}\\ &= f(t) - \left(\frac{1}{\tau_{\mathrm{oc}}} + \frac{1}{\tau_{\mathrm{ld}}}\right)\Delta C = f(t) - \frac{\Delta C}{\tau_{\mathrm{s}}}.\end{aligned} \tag{2}$$

Here t is time, the subscript s stands for "system",

$$\frac{1}{\tau_{\mathrm{s}}} \equiv \frac{1}{\tau_{\mathrm{oc}}} + \frac{1}{\tau_{\mathrm{ld}}}$$

is the proportionality constant between the atmospheric CO_2 perturbation and the total C flux out of the atmosphere, and $f(t)$ is the anthropogenic CO_2 flux into the atmosphere, which we can view as the forcing of the system. For our problem f is mostly FF+LU although we will also consider the case of f=FF alone.

It is necessary to consider to what extent the assumption of a linear relationship between the flux out of the atmosphere and the anthropogenic atmospheric CO_2 perturbation is justified based on our understanding of the dominant processes. In the case of ocean carbon uptake, this assumption is well underpinned, because the driving force for the uptake is the air-sea CO_2 disequilibrium. In addition, the rate-limiting step of the oceanic uptake, the transport of the anthropogenic CO_2 into the ocean's interior, is a linear process (e.g. Sarmiento et al., 1992; Maier-Reimer and Hasselmann, 1987). The linear scaling of the ocean uptake with the perturbation in atmospheric CO_2 is supported by 3-D ocean model simulations (e.g., Sarmiento and Le Quéré, 1996), although such simulations also show a strong deviation from linearity once atmospheric CO_2 has risen to values where the surface ocean buffer factor begins to change rapidly (Sarmiento and Le Quéré, 1996). Using the model-based scaling used by Gloor et al. (2003) and Mikaloff-Fletcher et al. (2006) and the anthropogenic ocean carbon inventory estimated from oceans surveys for 1995 (Sabine et al., 2004), we obtain an estimate of $\tau_{\mathrm{oc}} \simeq 81.4\,\mathrm{yr}$ (Appendix C).

For uptake by the land vegetation it is less clear whether the linearity concept applies. This is because uptake by land, unlike the oceans, is tied to processes such as productivity and the status of the land vegetation, some of which may be related to the atmospheric CO_2 perturbation (specifically CO_2 uptake during photosynthesis), while others like nutrient and micronutrient availability, plant and soil respiration, vegetation population dynamics, and land use change are not. Even if there were a productivity increase due to CO_2 "fertilization", it would likely be a linear response only during a limited period of time until land vegetation reaches a new steady state balance between growth and mortality. The linear response assumption of the land vegetation thus confounds many processes and time-scales (e.g., Lloyd, 1999). Despite these obvious caveats, we nevertheless described in a first step the land uptake as being linearly related to the atmospheric perturbation with a single time-scale, with the intention to generalize the description if the data were to contain sufficient information.

One could question the realism of our simple model on the grounds that the model treats both the oceans and the land vegetation as just one integral pool, while for both, several pools with different characteristic exchange time scales is more realistic. We tested this for the ocean and it turns out that inclusion of multiple ocean pools does not alter our conclusions for the reason explained in Sect. 5.

4 Airborne fraction for idealized cases

To get a general sense of the implications of our simple model (Eq. 2) for the time-course of AF, we have calculated AF for three idealized forcing functions $f(t)$: (i) exponential forcing $f(t) = f e^{t/\tau_\mathrm{f}}$ with a single characteristic time-scale τ_f (or equivalently relative growth rate $1/\tau_\mathrm{f}$); the subscript f refers to "forcing", (ii) the sum of an exponential function and a constant, and (iii) the sum of several exponential functions with different characteristic time-scales or relative growth rates (Appendix A and Fig. 2). The first case is an idealization of forcing the atmosphere with fossil fuel burning CO_2 alone, while the latter two cases mimic forcing of the atmosphere with the sum of fossil fuel emissions and land use change emissions.

As shown previously (e.g. Bacastow and Keeling, 1979), AF is constant for a purely exponential forcing function (Appendix A and Fig. 2). For the forcing functions differing from an exact exponential function, AF converges to an asymptotic value after some spin up time. The asymptotic value of AF is the same for the three forcing functions and is given by

$$\mathrm{AF}(\infty) = \frac{1}{1+\frac{\tau_\mathrm{f}}{\tau_\mathrm{s}}}\ .$$

It is thus controlled by the ratio between the forcing time-scale and the system response time-scale. If the forcing is not exactly exponential, then the time-scale for convergence is roughly on the order of 200–300 years (Fig. 2), depending on the exact functional form of the forcing. For the case of forcing by the sum of several exponentials it is the τ_f of the fastest growing exponential function that determines the asymptotic value of AF (see last equation in Appendix A).

An intuitive explanation for the existence of a spin-up period is as follows. The constancy of AF for a purely exponential forcing reflects the balance between two exponential processes, exponential damping of the atmospheric perturbation via carbon sinks and exponential forcing (Appendix B). If the forcing deviates from a pure exponential function, there will be a spin up period until the exponential component of the forcing dominates over other slower growing components of the forcing. An implication of the existence of a spin-up period is that we expect observed $\mathrm{AF}_\mathrm{FF+LU}$ to converge towards AF_FF from lower values. This is because fossil fuel emissions rise approximately exponentially but land use change emissions rise more slowly and thus their sum will not equal an exact exponential function (Sect. 2 and Fig. 1a and c).

5 Predicted and observed airborne fraction

Instead of idealized cases we now predict the time-course of AF using the observed FF and LU emissions. For this purpose we use the differential equation for AF implied by our model Eq. (2)

$$\frac{d\mathrm{AF}}{dt} = \left(\frac{1}{f}\frac{df}{dt} + \frac{1}{\tau_\mathrm{s}}\frac{d\tau_\mathrm{s}}{dt}\right) - \left(\frac{1}{\tau_\mathrm{s}} + \frac{1}{f}\frac{df}{dt} + \frac{1}{\tau_\mathrm{s}}\frac{d\tau_\mathrm{s}}{dt}\right)\mathrm{AF} \qquad (3)$$

derived in Appendix B. In order to interpret this equation it is helpful to notice that it is quite similar to Eq. (2). The analogue of ΔC is AF, the analogue of the "forcing" flux to the atmosphere $f(t)$ is $\frac{1}{f}\frac{df}{dt} + \frac{1}{\tau_\mathrm{s}}\frac{d\tau_\mathrm{s}}{dt}$, and the analogue of the exponential damping term $-\frac{\Delta C}{\tau_\mathrm{s}}$ is $-(\frac{1}{\tau_\mathrm{s}} + \frac{1}{f}\frac{df}{dt} + \frac{1}{\tau_\mathrm{s}}\frac{d\tau_\mathrm{s}}{dt})\mathrm{AF}$. Growth of AF is thus largely dictated by the relative growth rate RGR$= \frac{1}{f}\frac{df}{dt}$ of f (instead of f itself; $\frac{1}{f}\frac{df}{dt} \gg \frac{1}{\tau_s}\frac{d\tau_s}{dt}$ unless there is a very strong feedback), and AF is damped towards zero at a rate $\frac{1}{\tau_\mathrm{s}} + \frac{1}{f}\frac{df}{dt} + \frac{1}{\tau_\mathrm{s}}\frac{d\tau_\mathrm{s}}{dt}$ (instead of $\frac{1}{\tau_\mathrm{s}}$).

To predict the variations in AF according to our simple model, we integrate the equation numerically assuming a constant sink efficiency, i.e. $\tau_\mathrm{s} = \mathrm{const}$. We choose a value for τ_s such that the mean observed and predicted AF are equal over the period 1959–2006 using least squares, which results in τ_s=42 years for AF_FF and τ_s=37.5 years for $\mathrm{AF}_\mathrm{FF+LU}$. Besides using the requirement for agreement of the mean AF over the period from 1959 to 2006 to estimate τ_s, we may also determine τ_s from the mass conservation requirement that predicted and observed increase in atmospheric CO_2 agree. The two estimates agree well. The predicted variations in AF based on the fossil fuel time-series estimated by Marland (2006) alone, as well as the sum of the fossil fuel and land use time-series, used as forcing, are shown in Figs. 1e and 3a.

In order to assess the importance of restricting ourselves to a single ocean and land pool description for our results, we repeated this calculation using a more generalized form of the predictive equation for AF. The generalized form is based on a linear multi-pool representation of the ocean, or equivalently a sum of impulse response functions (Greens functions) with characteristic time-scales of $\tau_0 = \infty, \tau_1 = 433.3\,\mathrm{yr}$, $\tau_2 = 83.9\,\mathrm{yr}$, $\tau_3 = 11.2\,\mathrm{yr}$, and $\tau_4 = 0.8\,\mathrm{yr}$ (see Appendices B and D). The predicted AF is nearly the same as AF predicted by the simple single pool model, confirming that the simple model suffices to analyse the controls on the AF during the 1959–2006 period. The reason is that ocean carbon uptake during this period is primarily governed by one Green's function, the one associated with $\tau_2 = 83.9\,\mathrm{yr}$ (which is close to τ_{oc}).

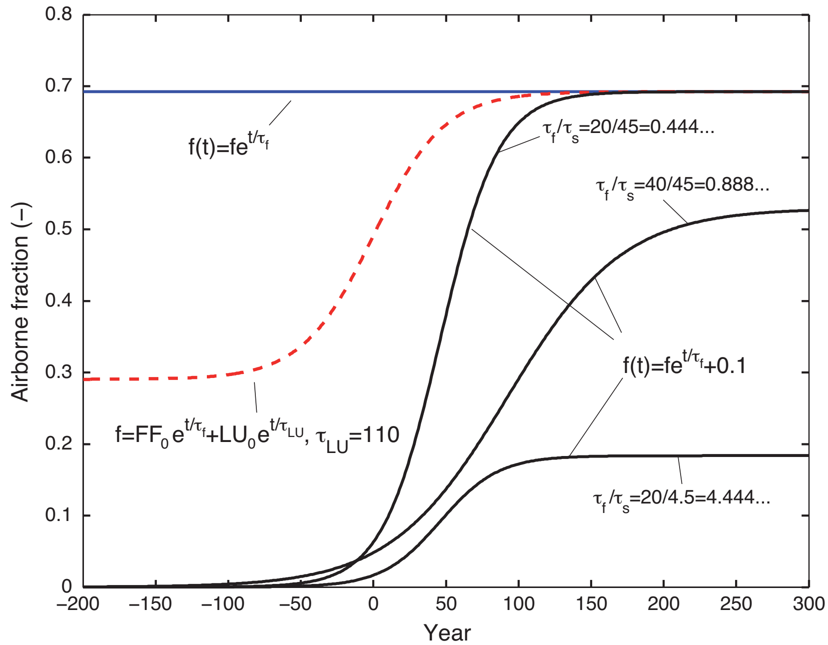

Fig. 2. Predicted air-borne fraction for a range of forcings including purely exponential forcing $f(t) = f e^{t/\tau_f}$ (blue), mixed exponential and constant forcing $f(t) = f e^{t/\tau_f} + f_0$ with $f_0 = 0.1$ (black), and forcing by the sum of two exponential functions, $f(t) = f e^{t/\tau_f} + f_{LU} e^{t/\tau_{LU}}$ (red).

The model computed AF_{FF} and AF_{FF+LU} agree remarkably well with the observed ones, calculated from atmospheric concentration data (from Mauna Loa) and anthropogenic emissions. The numerator of the observed AF_{FF} and AF_{FF+LU} were calculated using the atmospheric rate of change, dC/dt, taken from the monthly mean records from NOAA ESRL (co2_mm_mlo.2009.txt obtained in November 2009 from ftp://ftp.cmdl.noaa.gov/ccg/co2/trends/, Tans, 2009a). From these monthly data we first calculated annual means centered on 31 December/1 January, from which we estimated the time derivative by differencing. We estimated the dC/dt from annual means, because the anthropogenic emissions estimates are annually resolved. Our conclusions are not sensitive to this choice.

Observed AF_{FF} and AF_{FF+LU} records (Figs. 1e, 3a) exhibit large inter-annual variability, which is missing in the AF_{FF} and AF_{FF+LU} predicted by the linear model. This is because our model is forced solely by carbon fluxes from fossil fuel and land use change, thus variations due to non-anthropogenic forcings, like volcanic eruptions or climate oscillations, are not captured. The large inter-annual variations in observed air-borne fraction are largely due to inter-annual variability in the rate of change of atmospheric CO_2, $\frac{dC}{dt}$, an observation known since the 1970s to be associated with El Niño/La Niña and post volcanic periods (Agung, El Chichon, Pinatubo; Bacastow, 1976).

The forcing during the period 1959–2006 has three distinctly different phases: 1958–1973 fast growth, small $\tau_f \sim 20$ yr; 1973–1999: slow growth, large $\tau_f \sim 30$–150 years; 2000–2006: fast growth, small $\tau_f \sim 25$ years, (Fig. 1b). We thus expect the predicted AF to decrease during the 1973–1999 period and then to increase again with some lag. This is indeed what we find (Figs. 1e, 3a). The same signature seems to be present in the observations as well, although it tends to be masked by the larger variability. Furthermore for this model there is indeed a tight relation between AF and the relative growth rate RGR of anthropogenic emissions (compare Fig. 1d and e).

As mentioned earlier on, because the forcing used to calculate AF_{FF+LU} is approximately the sum of an exponential function, FF, and a less strongly increasing function, LU, we expect AF_{FF+LU} (red dashed line) to be lower than AF_{FF} (blue solid line) and to slope more upwards than AF_{FF}, eventually converging towards AF_{FF}. This is indeed what is observed and predicted (Fig. 1e).

6 Causes for trends in observed airborne fraction

Given the variation in AF_{FF} and AF_{FF+LU} due to variations in forcing (Figs. 1e, 3a), particularly in fossil fuel emissions, and the considerable time it takes for AF to converge to its asymptotic value, is there nonetheless a possibility to test whether there are trends in sink efficiency from the time course in AF? If our differential equation for AF based on the

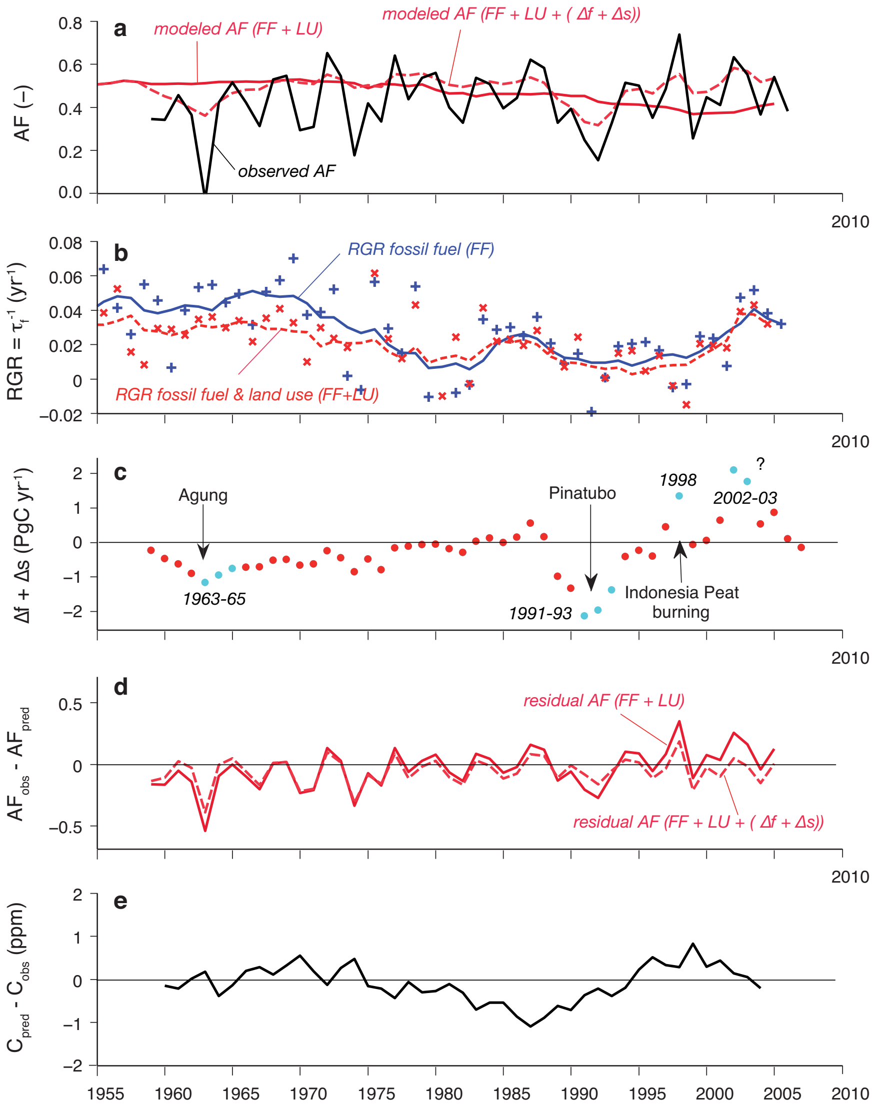

Fig. 3. Time-series from 1955 through 2010: **(a)** model predicted and observed AF_{FF+LU}. Also shown is the model-predicted AF after the addition of the "corrections" shown in **(c)**, **(b)** relative growth rate of f=FF and f=FF+LU, respectively. Shown are the annual values (symbols) and after filtering it with a low-pass filter (11 year running mean); (c) correction to land use and fossil fuel emissions $\Delta f + \Delta s$ calculated by minimizing the least square difference between predicted and observed AF_{FF+LU}. Periods with known extrinsic forcings are indicated with arrows. **(d)** Difference between observed and model predicted AF_{FF+LU} and $AF_{FF+LU+\Delta(f+s)}$, and **(e)** difference between predicted and observed atmospheric CO_2.

assumption of a linear response were to fit the data well, then we would not need to invoke a trend in sink efficiency (i.e. a trend in τ_s). The difference (residuals) between observed and predicted AF can thus give us an indication of potential non-linearities or possibly incompleteness of the linear model to describe the evolution of the anthropogenic atmospheric carbon perturbation.

The trend of the residuals (the difference between observed and predicted AF_{FF+LU}) is positive (Fig. 3d), indicating that something is indeed at odds. There are three possible causes for the trend in the residuals: (i) incomplete forcing, due to the omission of forcings caused by land use change, or associated with indirect, non-anthropogenic mechanisms such as volcanic eruptions, (ii) the response time scale (or equivalently sink efficiency) is changing over time indicating a non-linear behaviour, or (iii) the model is all too simplistic.

We investigate the first explanation for the trend in the residuals by inquiring what corrections $\Delta(f+s)$ to FF+LU would be needed to obtain a better fit between observed and predicted AF. If we can attribute these flux corrections $\Delta(f+s)$ to sources or sinks Δs caused by extrinsic non-anthropogenic forcings, or omissions in land use and fossil fuel fluxes Δf, then there is no need to invoke trends in sink efficiency (i.e. nonlinearities) and vice versa. To estimate the flux corrections $\Delta(f+s)$, we minimize the cost function

$$J(\Delta(f+s)(1959),...,\Delta(f+s)(2006)) = \sum_{yr=1959}^{2006} (AF^{obs}_{FF+LU}(yr) - AF^{pred}_{FF+LU}(yr))^2 + +((C^{pred}_{2006} - C^{pred}_{1959}) - (C^{obs}_{2006} - C^{obs}_{1959}))^2$$

with respect to $\Delta(f+s)$ (1959), ..., $\Delta(f+s)$ (2006) using simulated annealing. The second term of the right hand side ensures that mass is conserved. Because the weighting of the data is uniform, there should not be a significant trend in the residuals after including the flux corrections. To be sure, we used the standard t-test (e.g. Robinson, 1981, Appendix E) and found indeed no significant slope.

The estimation procedure identifies four events (Fig. 3c): increased sinks for atmospheric carbon in the aftermaths of the 1963 Agung and 1991 Pinatubo eruptions and carbon flux pulses to the atmosphere in 1997/98 and similarly in 2002/03. A dip in the increase rate of atmospheric carbon is well known to occur after major volcanic eruptions, especially those that inject material into the stratosphere (e.g., Rödenbeck et al., 2003). The decrease in atmospheric CO_2 is generally attributed to a land sink in the aftermath of the eruption. The mechanism may possibly be an increase in the ratio between diffuse and direct radiation, enhancing photosynthesis (Roderick et al., 2001) and/or reduced soil respiration due to temporary cooling of the earth surface (Jones and Cox, 2001). The onset of increased land uptake in the early 1990's is actually before the Pinatubo eruption as noticed by Keeling et al. (1995). To our knowledge the mechanism for this early onset remains unclear. The 1997/98 carbon flux pulse to the atmosphere is also well studied, and largely attributed to peat burning in Indonesia in 1997/1998 (Page et al., 2002). This carbon flux to the atmosphere seems to be missing from the Houghton et al. (2007) land use change flux estimate, although it is the result of land use change (Page et al., 2002). Finally there are indications from several studies of what the causes of the 2002 and 2003 flux pulses to the atmosphere could be (Yurganov et al., 2005; Balzter et al., 2005; Jones and Cox, 2005). Specifically Yurganov et al. (2005) documented air-column CO anomalies on the order of 50% at northern hemisphere mid-to high latitude stations, with anomalies occurring during the second half of the year 2002 and 2003. They associated these signatures with boreal forest fires in Siberia, consistent with results from remote sensing fire spot data, and results based on more refined remote sensing methods (Balzter et al., 2005). Besides boreal forest fires, the 2002/2003 events may also be related to the drought in Europe in summer 2003, which reduced net primary production of the land vegetation (Ciais et al., 2005), although decreases in primary production are likely to be paralleled by compensating anomalies in respiration. Thus overall, with the possible exception of the 2002/2003 event, the four events in the residuals can be attributed to extrinsic forcings and omissions in land use change fluxes.

We may finally test whether there is a declining trend in sink efficiency by investigating the slope of $\Delta(f+s)$ but with the post-Agung, post-Pinatubo, Indonesian peat pulse and 2002/2003 events excluded, as indicated by the red dashed line in Fig. 3c. The result of a t-test indicates that the chance for this slope to be significantly differently from zero is very small ($p < 0.01$). Thus, after removing the four events there is no evidence for a sink efficiency trend in the AF.

Our analysis is ambiguous regarding the possible sudden "positive feedback event" in 2002/2003. If the event is indeed due to Siberian forest fires then it may not necessarily be a sign of an nonlinear, irreversible event but rather part of a natural cycle of boreal forest population dynamics (birth, aging, death, caused e.g. by fire) and thus carbon uptake and release (Wirth et al., 2002; Mollicone et al., 2002). Therefore measurements of future net carbon fluxes in this region are necessary to determine to what extent these fluxes could indeed reflect a positive feedback.

7 Detecting trends in the efficiency of sinks

Although our analysis suggests that the decadal-scale observed variations and trends in the AF primarily reflect changes in the relative growth rate of the total anthropogenic CO_2 and incomplete spin-up, it is still interesting to analyze the relation between trends in sink efficiency and trends in AF within the framework of our simple model. For this purpose we investigate the hypothetical case where we impose a 50% decrease in the sink efficiency by the year 2008 compared to 1959. We achieve this strong feedback by setting $\tau_s(t) = 42$ yr for $t < 1959$ and $\tau_s(t) = 42\,\text{yr} + \epsilon * (t - 1959)$ with $\epsilon = 0.5$ for $t \geq 1959$. We then integrate Eq. (3) forward in time, starting from 1765 and compare the result with the record for the AF calculated for a constant τ_s. Such a weakening trend since 1959 would induce a difference in the trend of AF of

$$\delta \frac{\Delta AF}{\Delta t} = \frac{\Delta AF_{\epsilon=0.5}}{\Delta t} - \frac{\Delta AF_{\epsilon=0.0}}{\Delta t} \sim 0.1 (50\ \text{yr})^{-1}$$

where

$$\frac{\Delta AF}{\Delta t} \equiv \frac{AF(t=2008) - AF(1959)}{2008-1959}.$$

This shows firstly that a fairly strong positive feedback, operating over a period of 50 years, causes a trend that is roughly of similar magnitude as variations caused by relative growth rate variations in fossil fuel emissions over the 1959–2009 period. Secondly, the signal would be difficult to detect. Using a standard t-test, such a 50% sink efficiency decrease over a period of 50 years is detectable only at the 90% significance level, but not at the 95% significance level. This is because the'natural' variation in AF of the order of 0.15 (Fig. 3a) will tend to mask any trend. In conclusion, variations in emissions and "noise" due to extrinsic non-anthropogenic forcings make the AF not a very suitable diagnostic for detecting trends in carbon sink efficiency.

8 Discussion

A key motivation to undertake this study has been the recent claims by Canadell et al. (2007) and Raupach et al. (2008) that they have detected a long-term increasing trend in the AF and that this trend is due to positive feedbacks in the coupled carbon-climate system. Knorr (2009) already challenged these authors with regard to the detection of the trends, arguing that given the noise in the data, the trend is not detectable. Here we challenged the second claim of Canadell et al. (2007) and Raupach et al. (2008) that a positive trend is indicative of a positive feedback between climate and the global carbon cycle. Our analyses suggest that this assumption cannot be made because trends in AF are not only caused by trends in sink efficiency but also by (i) variations in the relative growth rate of the emissions, (ii) incomplete spin-up, and (iii) omissions in the anthropogenic emissions.

An alternative method to investigate carbon-climate feedbacks on the basis of the atmospheric CO_2 record was recently proposed by Rafelski et al. (2009). They analysed a quantity that they termed the "constant airborne fraction anomaly". This quantity is defined as the difference between the atmospheric CO_2 record and a fixed fraction (57%) of the cumulated (time-integrated) fossil fuel emissions. While related, this quantity differs fundamentally from our AF in two ways. First it uses the time-integrated emissions, whereas we use the instantaneous emissions. Secondly, it is expressed in terms of an absolute anomaly, i.e. the amount of anomalous CO_2 in the atmosphere, whereas our AF is expressed relative to the magnitude of the emissions. One may argue that an analysis in terms of absolute anomalies is preferable, as the magnitude of the anomaly does not depend on the magnitude of the emissions. This dependency is actually a disadvantage of the AF analysis, since the same flux anomaly leads to a larger deviation early in time when the emissions are small, and to a much smaller deviation in the latter part of the record, when emissions are large. A potential downside of the analysis of the "constant airborne fraction anomaly" is that because of its cumulative nature, it tends to suppress shorter-term variations. We focused our analysis here on the AF itself, primarily because our primary target was to investigate the robustness of the conclusions of the analyses by Canadell et al. (2007) and Raupach et al. (2008).

Despite the fundamental differences in the approach, it is nevertheless of interest to compare our conclusions with those of Rafelski et al. (2009). Two main conclusions of their study are (i) that they expected a decrease in the airborne fraction anomaly after the early 1970s due to the decrease in the fossil fuel growth rates, and (ii) that the absence of this decrease in the observed anomaly is caused by enhanced land emissions due to a warming trend that began around the same time. Regarding their first conclusion, it seems as if variations in the growth of the fossil fuel emissions matter irrespective of whether the AF is expressed instantaneously or cumulatively. This is likely a consequence of the fact that the integral of an exponential function is an exponential with the same exponent, i.e. time-scale. Thus changes in the time-scale τ_f affect both definitions of the AF. In their second conclusion, their statement is equivalent to invoking the detection of a positive feedback between the carbon cycle and climate, i.e. they essentially support the conclusions of Canadell et al. (2007) and Raupach et al. (2008). This second conclusion of Rafelski et al. (2009) is based on a slightly better fit of their model predicted time-evolution of the airborne fraction anomaly if they included a temperature dependent model of the land (and ocean). However, the fit of this temperature-dependent model was only marginally better, and inclusion in the forcing of the additional processes we have identified in land-use change and variability could have equally led to an improvement over the temperature-independent model. Given our previous finding that relatively small omissions in the emissions from fossil fuel burning and land-use change can alter the fit (and trend) substantially, it may well be the case that once these omissions are added, the temperature-independent model may produce an equally good fit. We thus conclude that the evidence for the detection of a carbon-cycle climate feedback is weak and not robust. A critical element to advance is the availability of much more accurate emission data, as this would permit to distinguish between alternative explanations.

9 Conclusions

We have investigated the question of what controls trends and decadal scale variations in CO_2 airborne fraction (AF) using simple linear models describing the evolution of an atmospheric perturbation in CO_2. Our analysis suggests firstly that variations of the relative growth rate of anthropogenic CO_2 emissions are a major control of variations in AF. Secondly, it suggests that there is a long spin-up time for AF to converge to its asymptotic value if the forcing is not exactly exponential. If the forcing is not exactly an exponential function, as it is the case for the sum of fossil fuel burning and land use change emissions, this time-scale is of the order of 200–300 years. A first consequence is that there is no one-to-one association between positive trends in AF_{FF+LU} and negative trends in sink efficiency. A second consequence is that in order to detect trends in sink efficiencies from the time course of AF_{FF+LU}, it is necessary to disentangle the spin-up time and fossil fuel growth rate variation signatures in the AF from signatures due to other causes. Our differential equation for AF permits us to do so by predicting the time course of AF due solely to these two factors. The remaining trends and variations in the residuals can then be explained by variations in extrinsic forcings like volcanic eruptions and climate variations, omissions in the anthropogenic fluxes to the atmosphere, trends in sink efficiencies, or inadequacies in our model. We do indeed find a positive trend in the residuals, but argue that this trend is not statistically significant after correcting for known events such as the temporal distribution of the extrinsic forcings and likely omissions in the emissions (particularly from land-use change). We thus do not need to invoke a trend in carbon sink efficiencies to explain the trend in the AF. Our analysis also suggests that trends in AF are not a very good diagnostic to detect changes in carbon sink efficiency because variations in the signal are complex and the signal-to-noise ratio is small.

Although the surprisingly linear behaviour of the global carbon cycle for the past 50 years may suggest otherwise, it would be a mistake to assume that it will continue to operate in such a linear fashion into the future. For one thing, the continuous acidification of the ocean will inevitably lead to a decrease in the oceanic uptake capacity for anthropogenic carbon (Sarmiento and Le Quéré, 1996). Our analysis does not dispute a future reduction in sinks of anthropogenic carbon compared to a linear system response. Rather, we argue that atmospheric concentration data if analysed adequately do not yet reveal a statistically significantly signal.

Appendix A

Solutions of the differential equation for ΔC for idealized cases

In order to calculate the AF for idealized cases we integrate the differential equation

$$\frac{d\Delta C}{dt} = -\frac{\Delta C}{\tau_s} + f(t)$$

with initial condition $\Delta C(-\infty) = 0$ (since ΔC is the perturbation of atmospheric carbon). For purely exponential forcing $f(t) = f e^{\frac{t}{\tau_f}}$, τ_f constant, we find by the method of "variation of constant"

$$\Delta C(t) = \frac{f}{\frac{1}{\tau_s} + \frac{1}{\tau_f}} e^{\frac{t}{\tau_f}}$$

and thus

$$\mathrm{AF} \equiv \frac{\frac{dC}{dt}}{f e^{\frac{t}{\tau_f}}} = \frac{\frac{d\Delta C}{dt}}{f e^{\frac{t}{\tau_f}}} = \frac{1}{1 + \frac{\tau_f}{\tau_s}} = \text{constant}.$$

The second identity holds because $\frac{dC}{dt} = \frac{d(C(t) - C(-\infty))}{dt} = \frac{d\Delta C}{dt}$.

For a forcing of the form $f e^{t/\tau_f} + f_0$ where f_0 is constant, we may integrate the equation similarly to obtain

$$\mathrm{AF} = \frac{1}{1 + \frac{\tau_f}{\tau_s}} \times \frac{e^{\frac{t}{\tau_f}}}{e^{\frac{t}{\tau_f}} + (f_0/f)}.$$

Finally for the case of a sum of exponential forcings with different time-scales, i.e. $\sum_{i=1}^{n} f_i e^{t/\tau_i}$, we find in a similar way

$$\mathrm{AF} = \frac{\sum_{i=1}^{n} \frac{1}{1 + \frac{\tau_i}{\tau_s}} f_i e^{t/\tau_i}}{\sum_{i=1}^{n} f_i e^{t/\tau_i}}.$$

Appendix B

Derivation of the differential equation for the time evolution of AF

The basis for the derivation of the differential equation for the AF is the general solution of the Eq. (2) for an arbitrary forcing function $f(t)$, which is again obtained with the method of "variation of constant":

$$\Delta C = \int_{-\infty}^{t} G(t,t') f(t') dt' \text{ with } G(t,t') = e^{-\int_{t'}^{t} \frac{dt''}{\tau_s(t'')}}. \quad \text{(B1)}$$

$G(t,t')$ is called the Greens function of the problem. The interpretation of this expression is as follows. The atmospheric perturbation at time t is given by the sum of "flux pulses" to the atmosphere, each of them damped exponentially in time by $G(t,t')$ from the moment they have been emitted into the atmosphere. From the definition of AF we then find

$$\mathrm{AF} = \frac{\frac{dC}{dt}}{f} = \frac{\frac{d\Delta C}{dt}}{f} = 1 - \frac{1}{\tau_s}\frac{\Delta C}{f} = 1 - \frac{1}{\tau_s}\frac{\int_{-\infty}^{t} G(t,t') f(t') dt'}{f}$$

or equivalently

$$(1 - \mathrm{AF}) = \frac{1}{\tau_s}\frac{\int_{-\infty}^{t} G(t,t') f(t') dt'}{f}.$$

The time-derivative of AF is thus

$$\frac{d\mathrm{AF}}{dt} = \frac{1}{f}\frac{df}{dt}\frac{1}{\tau_s}\frac{\int_{-\infty}^{t} G(t,t') f(t') dt'}{f}$$
$$-\frac{d\frac{1}{\tau_s}}{dt}\frac{\int_{-\infty}^{t} G(t,t') f(t') dt'}{f} - \frac{1}{\tau_s}\frac{\frac{d}{dt}\int_{-\infty}^{t} G(t,t') f(t') dt'}{f}.$$

Applying Leibniz's rule

$$\frac{d}{dt}\int_{g(t)}^{h(t)} m(t,s) ds$$
$$= m(t,h(t))\frac{dh}{dt} - m(t,g(t))\frac{dg}{dt} + \int_{g(t)}^{h(t)} \frac{dm(t,s)}{dt} ds$$

to the third term on the right gives

$$\frac{d}{dt}\int_0^t G(t,t')f(t')dt' = G(t,t)f(t)\frac{dt}{dt}$$
$$+\int_{-\infty}^t \frac{dG(t,t')}{dt}f(t')dt' = f(t) - \frac{1}{\tau_s}\int_{-\infty}^t G(t,t')f(t')dt'.$$

Therefore

$$\frac{d\mathrm{AF}}{dt} = (\frac{1}{f}\frac{df}{dt} + \frac{1}{\tau_s}\frac{d\tau_s}{dt})(1-AF) - \frac{1}{\tau_s}\mathrm{AF}$$
$$= (\frac{1}{f}\frac{df}{dt} + \frac{1}{\tau_s}\frac{d\tau_s}{dt}) - (\frac{1}{\tau_s} + \frac{1}{f}\frac{df}{dt} + \frac{1}{\tau_s}\frac{d\tau_s}{dt})\mathrm{AF}.$$

Appendix C

Estimation of atmosphere ocean and atmosphere land exchange time constants τ_{oc} and τ_{ld}

Coupled carbon cycle ocean general circulation models show that there is an approximately linear relationship between the atmospheric perturbation of CO_2 and ocean carbon uptake (e.g. Sarmiento and Le Quéré, 1996). Furthermore we know ocean anthropogenic carbon inventories from ocean surveys (Sabine et al., 2004; Gruber et al., 2009). Based on the approximate linearity

$$F_{\mathrm{at}\to\mathrm{oc}}(t) = F_{\mathrm{at}\to\mathrm{oc}}(t_{\mathrm{ref}})\frac{p\mathrm{CO}_2^{\mathrm{at}}(t) - p\mathrm{CO}_2^{\mathrm{at}}(1765)}{p\mathrm{CO}_2^{\mathrm{at}}(\mathrm{t}_{\mathrm{ref}}) - p\mathrm{CO}_2^{\mathrm{at}}(1765)}$$

and thus

$$\tau_{\mathrm{oc}} = \frac{p\mathrm{CO}_2^{\mathrm{at}}(\mathrm{t}_{\mathrm{ref}}) - p\mathrm{CO}_2^{\mathrm{at}}(1765)}{F(t_{\mathrm{ref}})} = 81.4\ \mathrm{yr}.$$

Here $t_{\mathrm{ref}} = 1995$ and $F(1995) = 2.2\ \mathrm{PgC\,yr^{-1}}$ is from Gruber et al. (2009), $pCO_2(1765)$=276.7 ppm (Etheridge et al., 1996), $pCO_2(1995)$=360.9 ppm, and 1 ppm CO_2=2.1276 Pg C for the earth's atmosphere (e.g., Sarmiento et al., 2010). Given $\tau_s = 37.5$ years (from the main text) and using the relation

$$\frac{1}{\tau_s} = \frac{1}{\tau_{\mathrm{ld}}} + \frac{1}{\tau_{\mathrm{oc}}}$$

from Eq. (2) from the main text we furthermore find $\tau_{\mathrm{ld}} \simeq$ 69.5 yr.

Appendix D

Derivation of a predictive equation for AF for multiple ocean pools

Instead of one differential equation for the evolution of atmospheric ΔC we consider a system of ordinary differential equations describing carbon exchange between different volumes of the ocean. In this case the sink efficiencies are given by the inverse of the exchange time constants between the different ocean volumes. The solution of a system of ordinary differential equations is similar to the solution ΔC given in Eq. (B1), Appendix B, for Eq. (2) but with Greens function $G(t,t') = G_{\mathrm{ld}}(t,t')G_{\mathrm{oc}}(t,t')$ and $G_{\mathrm{oc}}(t,t') = A_0 + \sum_{j=1}^N A_j e^{-\frac{t-t'}{\tau_j}}, \sum_{j=0}^N A_j = 1$. The Greens function $G_{\mathrm{oc}}(t,t')$ for the oceans is available from Sarmiento et al. (1992) and Maier-Reimer and Hasselmann (1987), calculated using coupled ocean circulation carbon cycle models. The perturbation of atmospheric carbon ΔC due to anthropogenic emissions is then given by

$$\Delta C = \int_{-\infty}^t e^{-\frac{t-t'}{\tau_{\mathrm{ld}}}}(A_0 + \sum_{j=1}^4 A_j e^{-\frac{t-t'}{\tau_j}})f(t')dt'.$$

Thus

$$\frac{d\Delta C}{dt} = \sum_{j=0}^4 A_j f(t) +$$
$$-\int_{-\infty}^t e^{-\frac{t-t'}{\tau_{\mathrm{ld}}}}(\frac{A_0}{\tau_{\mathrm{ld}}} + \sum_{j=1}^4 A_j(\frac{1}{\tau_{\mathrm{ld}}} + \frac{1}{\tau_j})e^{-\frac{t-t'}{\tau_j}})f(t')dt'$$
$$= f(t) - I(t)$$

using $\sum_{j=0}^4 A_j = 1$ and with

$$I(t) \equiv \int_{-\infty}^t e^{-\frac{t-t'}{\tau_{\mathrm{ld}}}}(\frac{A_0}{\tau_{\mathrm{ld}}} + \sum_{j=1}^4 A_j(\frac{1}{\tau_{\mathrm{ld}}} + \frac{1}{\tau_j})e^{-\frac{t-t'}{\tau_j}})f(t')dt'.$$

Therefore

$$\frac{d\mathrm{AF}}{dt} = \frac{d}{dt}(\frac{\frac{d\Delta C}{dt}}{f}) = \frac{1}{f}\frac{df}{dt}\frac{I(t)}{f(t)} - \frac{1}{f}\frac{dI}{dt} = \frac{1}{f}\frac{df}{dt}(1-\mathrm{AF})$$
$$-\frac{1}{f}\frac{dI}{dt}$$

with

$$\frac{dI}{dt} = (\frac{A_0}{\tau_{\mathrm{ld}}} + \sum_{j=1}^4 A_j(\frac{1}{\tau_{\mathrm{ld}}} + \frac{1}{\tau_j}))f(t) +$$
$$-\int_{-\infty}^t e^{-\frac{t-t'}{\tau_{\mathrm{ld}}}}(\frac{A_0}{\tau_{\mathrm{ld}}^2} + \sum_{j=1}^4 A_j(\frac{1}{\tau_{\mathrm{ld}}} + \frac{1}{\tau_j})^2 e^{-\frac{t-t'}{\tau_j}})f(t')dt'.$$

Appendix E

t-test for significance of a trend

For completeness we give here the test statistic for the significance of the slope b of a regression line $y = bx + a$ to data (x_i, y_i), $i = 1, ..., N$:

$$t = \frac{b-\beta}{\sqrt{(\frac{s^2}{Ns_x^2})}}$$

with

$$s^2 = \frac{1}{N-2}\sum_{i=1}^{N}(y_i - a - bx_i)^2$$

and

$$s_x^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2 .$$

The t statistic is distributed as a t-distribution with N-2 degrees of freedom. Because we want to test whether b differs significantly from 0, we use the statistic for $\beta = 0$.

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